Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/126-4.5.7-d-trig-^m-a+b-c-sec-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [470]. This is test number [126].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.57 (468)	0.43 (2)
Maple	91.70 (431)	8.30 (39)
Mathematica	90.21 (424)	9.79 (46)
Fricas	85.53 (402)	14.47 (68)
Maxima	60.85 (286)	39.15 (184)
Giac	58.51 (275)	41.49 (195)
Mupad	51.70 (243)	48.30 (227)
Sympy	5.32 (25)	94.68 (445)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

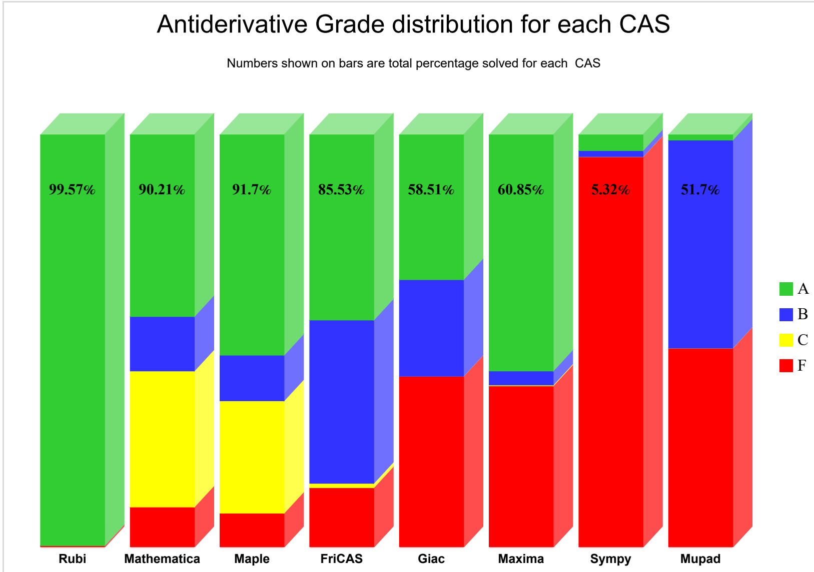
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

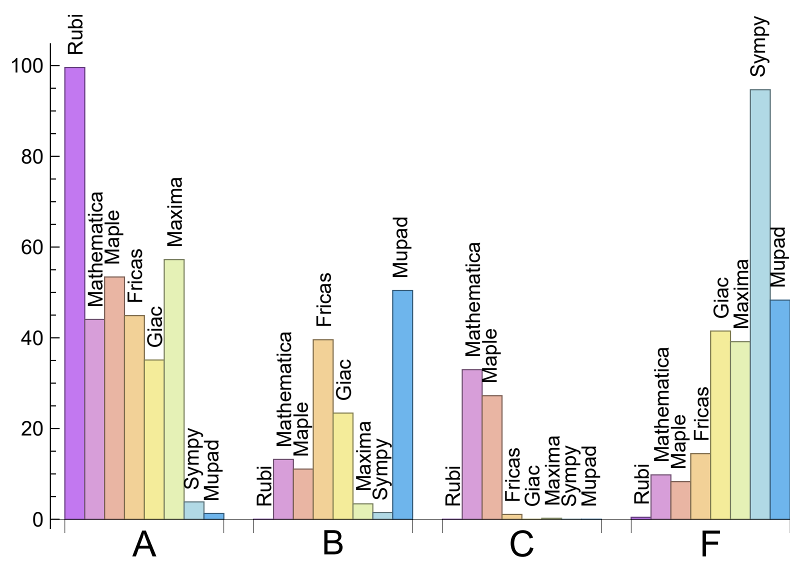
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.57	0.00	0.00	0.43
Maxima	57.23	3.40	0.21	39.15
Maple	53.40	11.06	27.23	8.30
Fricas	44.89	39.57	1.06	14.47
Mathematica	44.04	13.19	32.98	9.79
Giac	35.11	23.40	0.00	41.49
Sympy	3.83	1.49	0.00	94.68
Mupad	N/A	50.43	0.00	48.30

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	46	100.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Fricas	68	79.41 %	0.00 %	20.59 %
Giac	195	90.77 %	2.05 %	7.18 %
Maxima	184	86.41 %	10.33 %	3.26 %
Sympy	445	72.81 %	19.78 %	7.42 %
Mupad	227	96.04 %	3.96 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

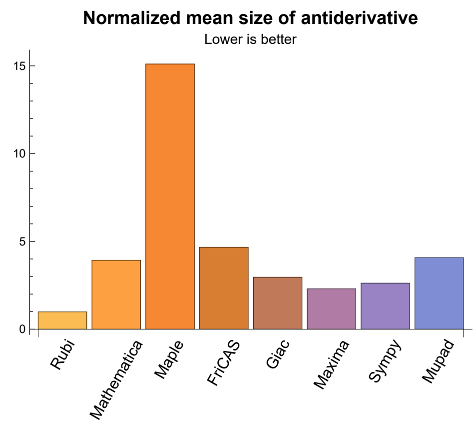
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

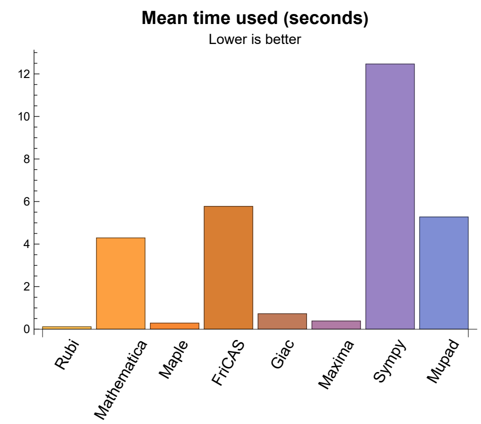
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	123.47	0.99	103.50	1.00
Mathematica	4.29	460.28	3.93	172.00	1.63
Maple	0.29	2720.42	15.11	158.00	1.41
Maxima	0.39	196.04	2.30	103.50	1.22
Fricas	5.78	624.18	4.67	411.00	3.79
Sympy	12.47	163.84	2.62	66.00	1.34
Giac	0.73	304.12	2.96	172.00	1.43
Mupad	5.28	522.42	4.07	105.00	1.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{462, 466, 467, 468, 469, 470\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {28, 29, 30, 34, 35, 36, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 78, 79, 88, 89, 125, 126, 127, 128, 132, 133, 136, 137, 138, 139, 140, 180, 181, 193, 196, 199, 202, 205, 208, 209, 211, 213, 214, 215, 217, 218, 219, 240, 250, 251, 254, 264, 268, 269, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 392, 393, 394, 397, 398, 399, 416, 428, 429, 430, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

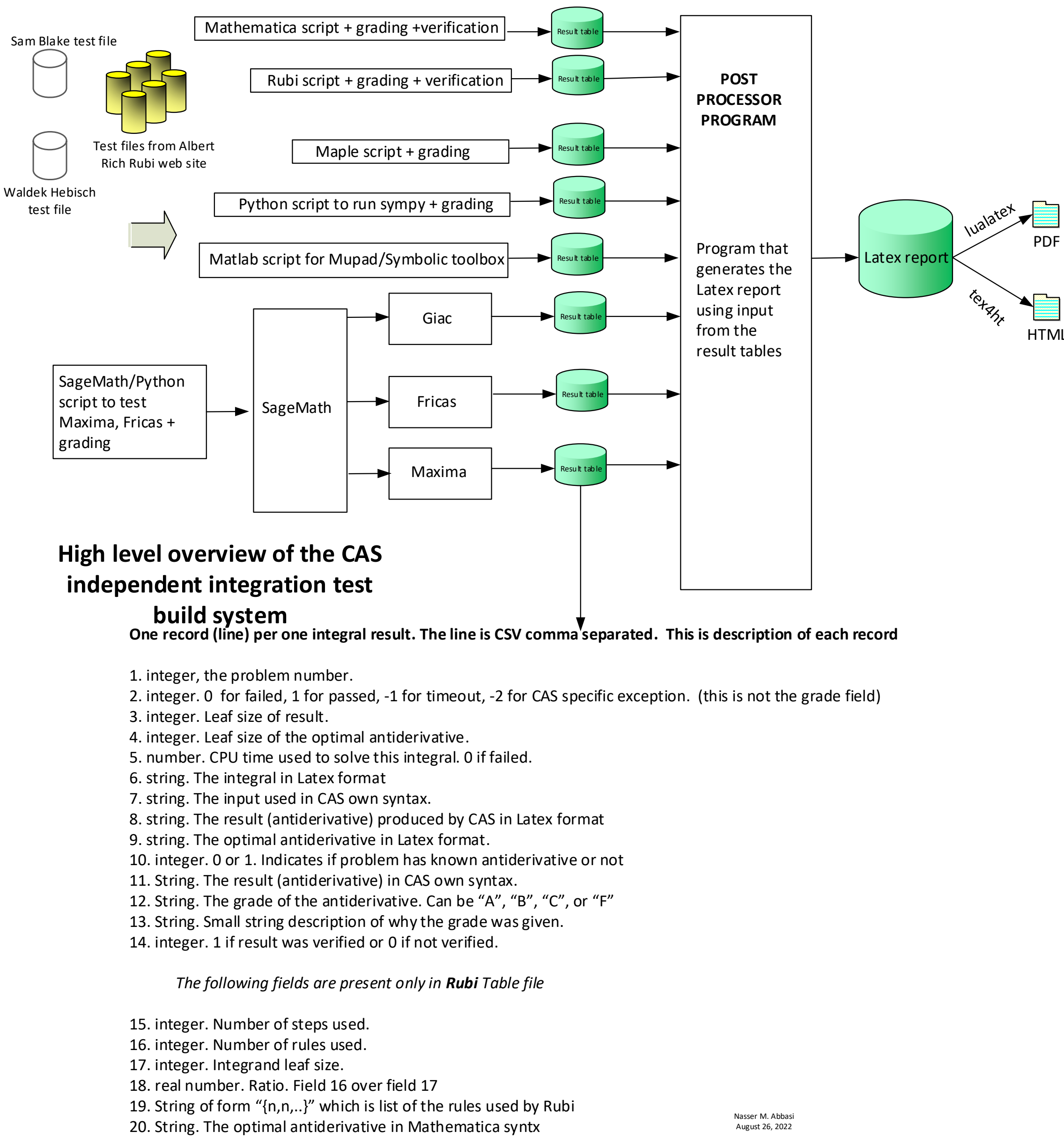
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { }

C grade: { }

F grade: { 132, 298 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 175, 176, 177, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 197, 198, 200, 203, 204, 206, 207, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 382, 383, 386, 387, 388, 395, 396, 408, 409, 412, 413, 414, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 18, 19, 21, 24, 25, 27, 102, 106, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 170, 174, 178, 179, 236, 256, 265, 266, 279, 291, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 333, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 181, 186, 187, 189, 193, 196, 199, 201, 202, 205, 208, 209, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 391, 392, 393, 394, 397, 398, 399, 400, 401, 416, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F grade: { 73, 74, 76, 86, 228, 229, 230, 233, 237, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 376, 377, 379, 381, 385, 389, 390, 402, 403, 404, 405, 406, 407, 410, 415, 418, 419, 420, 431, 432, 433 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 93, 94, 95, 103, 104, 105, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 466, 467, 468, 469, 470 }

B grade: { 67, 68, 70, 71, 72, 80, 81, 83, 84, 85, 96, 97, 98, 106, 107, 109, 110, 111, 122, 123, 124, 222, 223, 224, 225, 226, 227, 376, 377, 379, 380, 381, 389, 390, 392, 393, 394, 402, 403, 405, 406, 407, 415, 416, 418, 419, 420, 428, 429, 431, 432, 433 }

C grade: { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 112, 113, 114, 115, 125, 126, 127, 128, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 426, 427, 434, 435, 436, 437, 438, 439, 440 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 247, 248, 249, 263, 264, 265, 276, 277, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 466, 467, 468, 469, 470 }

B grade: { 59, 65, 102, 115, 219, 266, 279, 289, 297, 355, 367, 368, 411, 416, 423, 424 }

C grade: { 379 }

F grade: { 70, 71, 72, 73, 74, 75, 76, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 109, 110, 111, 112, 113, 114, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 71, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 252, 253, 263, 268, 269, 276, 281, 282, 290, 295, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459, 462, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 11, 18, 20, 33, 38, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 72, 75, 76, 77, 78, 79, 88, 89, 101, 102, 109, 110, 111, 114, 115, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 254, 255, 256, 264, 265, 266, 267, 277, 278, 279, 280, 289, 291, 292, 293, 294, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade: { 259, 457, 458, 460, 461 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.6 Sympy

A grade: { 162, 312, 313, 317, 318, 319, 325, 326, 417, 430, 452, 453, 454, 462, 466, 468, 469, 470 }

B grade: { 311, 324, 339, 363, 364, 365, 459 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 457, 458, 460, 461, 463, 464, 465, 467 }

2.1.7 Giac

A grade: { 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 42, 43, 47, 48, 49, 50, 51, 52, 53, 55, 56, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 108, 121, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 317, 318, 319, 320, 330, 331, 332, 333, 334, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 455, 462, 466, 467, 468, 469, 470 }

B grade: { 1, 2, 5, 6, 7, 15, 18, 19, 20, 28, 32, 33, 41, 44, 45, 46, 54, 57, 58, 59, 67, 68, 70, 71, 72, 80, 81, 93, 94, 96, 98, 106, 107, 110, 111, 119, 120, 123, 124, 148, 149, 150, 151, 222, 223, 278, 290, 291, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 335, 336, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 378, 379, 380, 381, 389, 390, 391, 392, 402, 403, 404, 405, 406, 407, 415, 416, 417, 428, 429, 430, 452, 453, 454, 456, 459 }

C grade: { }

F grade: { 73, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 99, 100, 101, 102, 103, 104, 105, 109, 112, 113, 114, 115, 116, 117, 118, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 382, 383, 384, 385, 386, 387, 388, 393, 394, 395, 396, 397, 398, 399, 400, 401, 408, 409, 410, 411, 412, 413, 414, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 460, 461, 463, 464, 465 }

2.1.8 Mupad

A grade: { 462, 466, 467, 468, 469, 470 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 103, 104, 105, 108, 116, 117, 121, 129, 135, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

C grade: { }

F grade: { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 107, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	120	102	78	80	0	266	70
N.S.	1	1.00	1.45	1.23	0.94	0.96	0.00	3.20	0.84
time (sec)	N/A	0.044	0.058	0.167	0.267	2.848	0.000	0.497	0.096

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	88	82	62	64	0	197	55
N.S.	1	1.00	1.33	1.24	0.94	0.97	0.00	2.98	0.83
time (sec)	N/A	0.039	0.036	0.099	0.271	3.447	0.000	0.494	4.212

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	62	43	45	0	57	39
N.S.	1	1.00	1.20	1.41	0.98	1.02	0.00	1.30	0.89
time (sec)	N/A	0.029	0.028	0.073	0.271	4.031	0.000	0.479	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	25	27	29	0	26	25
N.S.	1	1.00	1.46	1.04	1.12	1.21	0.00	1.08	1.04
time (sec)	N/A	0.015	0.015	0.030	0.268	4.633	0.000	0.465	0.044

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	84	51	47	65	0	60	29
N.S.	1	1.00	3.11	1.89	1.74	2.41	0.00	2.22	1.07
time (sec)	N/A	0.025	0.033	0.072	0.271	7.313	0.000	0.429	0.094

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	236	90	81	133	0	193	62
N.S.	1	1.00	4.45	1.70	1.53	2.51	0.00	3.64	1.17
time (sec)	N/A	0.040	0.278	0.111	0.273	5.183	0.000	0.470	4.203

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	198	120	108	191	0	262	86
N.S.	1	1.00	2.44	1.48	1.33	2.36	0.00	3.23	1.06
time (sec)	N/A	0.057	1.302	0.102	0.277	5.585	0.000	0.476	4.287

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	112	119	92	0	104	105
N.S.	1	1.00	0.80	1.14	1.21	0.94	0.00	1.06	1.07
time (sec)	N/A	0.076	0.222	0.103	0.479	2.859	0.000	0.496	4.887

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	92	88	73	0	82	79
N.S.	1	1.00	0.77	1.31	1.26	1.04	0.00	1.17	1.13
time (sec)	N/A	0.050	0.226	0.082	0.488	1.323	0.000	0.465	4.400

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	54	46	51	54	0	47	35
N.S.	1	1.00	1.29	1.10	1.21	1.29	0.00	1.12	0.83
time (sec)	N/A	0.034	0.075	0.053	0.479	1.939	0.000	0.439	4.244

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	34	0	15	17
N.S.	1	1.00	1.00	1.07	1.07	2.27	0.00	1.00	1.13
time (sec)	N/A	0.010	0.004	0.040	0.269	3.839	0.000	0.484	4.305

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	36	43	28	42	0	26	28
N.S.	1	1.00	1.38	1.65	1.08	1.62	0.00	1.00	1.08
time (sec)	N/A	0.025	0.050	0.057	0.268	1.994	0.000	0.463	4.208

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	73	46	71	0	50	46
N.S.	1	1.00	1.83	1.59	1.00	1.54	0.00	1.09	1.00
time (sec)	N/A	0.034	0.038	0.072	0.266	1.202	0.000	0.472	4.307

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	128	101	68	98	0	76	60
N.S.	1	1.00	1.88	1.49	1.00	1.44	0.00	1.12	0.88
time (sec)	N/A	0.042	0.038	0.085	0.271	1.200	0.000	0.477	4.523

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	155	94	95	0	414	87
N.S.	1	1.00	1.22	1.60	0.97	0.98	0.00	4.27	0.90
time (sec)	N/A	0.063	0.428	0.115	0.281	4.640	0.000	0.581	4.302

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	125	71	71	0	91	66
N.S.	1	1.00	1.15	1.74	0.99	0.99	0.00	1.26	0.92
time (sec)	N/A	0.051	0.315	0.084	0.283	1.820	0.000	0.517	4.137

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	45	47	0	44	45
N.S.	1	1.00	1.63	0.91	0.98	1.02	0.00	0.96	0.98
time (sec)	N/A	0.025	0.080	0.056	0.288	1.191	0.000	0.513	0.062

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	108	94	86	107	0	172	53
N.S.	1	1.00	2.08	1.81	1.65	2.06	0.00	3.31	1.02
time (sec)	N/A	0.047	0.369	0.077	0.293	2.014	0.000	0.522	0.118

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	1021	163	132	203	0	341	96
N.S.	1	1.00	9.82	1.57	1.27	1.95	0.00	3.28	0.92
time (sec)	N/A	0.081	6.415	0.102	0.284	1.220	0.000	0.518	4.291

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	218	211	173	300	0	493	135
N.S.	1	1.00	1.55	1.50	1.23	2.13	0.00	3.50	0.96
time (sec)	N/A	0.104	1.333	0.123	0.298	1.288	0.000	0.554	4.419

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	499	199	173	138	0	183	163
N.S.	1	1.00	3.37	1.34	1.17	0.93	0.00	1.24	1.10
time (sec)	N/A	0.129	1.020	0.062	0.494	1.436	0.000	0.592	4.791

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	153	123	127	113	0	123	116
N.S.	1	1.00	1.34	1.08	1.11	0.99	0.00	1.08	1.02
time (sec)	N/A	0.091	1.151	0.099	0.488	1.249	0.000	0.572	4.325

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	126	71	72	86	0	67	94
N.S.	1	1.00	1.73	0.97	0.99	1.18	0.00	0.92	1.29
time (sec)	N/A	0.070	0.677	0.078	0.493	7.326	0.000	0.488	4.460

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	47	62	0	49	42
N.S.	1	1.00	2.65	1.20	1.18	1.55	0.00	1.22	1.05
time (sec)	N/A	0.021	0.259	0.059	0.290	1.647	0.000	0.446	4.333

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	109	96	57	75	0	60	56
N.S.	1	1.00	2.18	1.92	1.14	1.50	0.00	1.20	1.12
time (sec)	N/A	0.040	0.784	0.073	0.269	0.843	0.000	0.509	4.402

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	151	144	84	107	0	98	85
N.S.	1	1.00	1.99	1.89	1.11	1.41	0.00	1.29	1.12
time (sec)	N/A	0.056	0.885	0.083	0.270	1.945	0.000	0.522	4.449

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	353	190	112	147	0	141	108
N.S.	1	1.00	3.43	1.84	1.09	1.43	0.00	1.37	1.05
time (sec)	N/A	0.068	1.175	0.066	0.276	3.469	0.000	0.554	4.803

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	425	115	106	239	0	349	123
N.S.	1	1.00	4.34	1.17	1.08	2.44	0.00	3.56	1.26
time (sec)	N/A	0.076	2.301	0.237	0.466	3.298	0.000	0.477	4.307

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	376	67	66	162	0	85	76
N.S.	1	1.00	5.30	0.94	0.93	2.28	0.00	1.20	1.07
time (sec)	N/A	0.057	0.944	0.145	0.472	5.371	0.000	0.462	0.121

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	329	44	42	124	0	42	39
N.S.	1	1.00	7.00	0.94	0.89	2.64	0.00	0.89	0.83
time (sec)	N/A	0.027	0.607	0.068	0.482	5.718	0.000	0.428	0.067

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	239	71	67	164	0	85	123
N.S.	1	1.00	4.35	1.29	1.22	2.98	0.00	1.55	2.24
time (sec)	N/A	0.049	1.148	0.139	0.472	7.731	0.000	0.467	0.206

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	371	115	133	345	0	209	392
N.S.	1	1.00	4.31	1.34	1.55	4.01	0.00	2.43	4.56
time (sec)	N/A	0.070	1.712	0.208	0.482	2.275	0.000	0.459	4.912

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	549	181	238	721	0	389	870
N.S.	1	1.00	4.26	1.40	1.84	5.59	0.00	3.02	6.74
time (sec)	N/A	0.106	6.981	0.242	0.478	5.065	0.000	0.493	7.895

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	357	170	217	446	0	237	1448
N.S.	1	1.00	2.15	1.02	1.31	2.69	0.00	1.43	8.72
time (sec)	N/A	0.220	4.544	0.210	0.476	2.267	0.000	0.444	5.706

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	303	119	143	348	0	152	494
N.S.	1	1.00	2.59	1.02	1.22	2.97	0.00	1.30	4.22
time (sec)	N/A	0.117	2.128	0.167	0.475	2.570	0.000	0.478	4.583

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	245	78	81	271	0	92	111
N.S.	1	1.00	3.22	1.03	1.07	3.57	0.00	1.21	1.46
time (sec)	N/A	0.069	0.921	0.119	0.470	3.279	0.000	0.467	4.448

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	46	241	0	65	460
N.S.	1	1.00	4.04	1.02	1.02	5.36	0.00	1.44	10.22
time (sec)	N/A	0.032	0.369	0.092	0.477	2.370	0.000	0.420	4.562

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	189	52	52	287	0	71	46
N.S.	1	1.00	3.50	0.96	0.96	5.31	0.00	1.31	0.85
time (sec)	N/A	0.052	0.741	0.131	0.467	3.295	0.000	0.479	4.277

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	226	69	85	419	0	103	80
N.S.	1	1.00	2.97	0.91	1.12	5.51	0.00	1.36	1.05
time (sec)	N/A	0.069	2.313	0.158	0.485	3.601	0.000	0.494	4.311

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	318	93	141	615	0	173	112
N.S.	1	1.00	3.03	0.89	1.34	5.86	0.00	1.65	1.07
time (sec)	N/A	0.101	1.970	0.191	0.480	4.297	0.000	0.509	5.051

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	454	159	154	421	0	509	195
N.S.	1	1.00	2.82	0.99	0.96	2.61	0.00	3.16	1.21
time (sec)	N/A	0.126	7.249	0.296	0.476	3.467	0.000	0.520	0.161

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	403	102	109	311	0	136	130
N.S.	1	1.00	3.54	0.89	0.96	2.73	0.00	1.19	1.14
time (sec)	N/A	0.080	4.471	0.207	0.474	5.023	0.000	0.500	0.145

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	393	69	74	213	0	72	72
N.S.	1	1.00	4.68	0.82	0.88	2.54	0.00	0.86	0.86
time (sec)	N/A	0.036	4.093	0.099	0.476	3.020	0.000	0.475	4.561

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	384	103	143	408	0	272	2188
N.S.	1	1.00	3.88	1.04	1.44	4.12	0.00	2.75	22.10
time (sec)	N/A	0.075	1.534	0.223	0.479	5.660	0.000	0.557	5.945

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	468	148	238	726	0	550	1845
N.S.	1	1.00	3.18	1.01	1.62	4.94	0.00	3.74	12.55
time (sec)	N/A	0.121	2.285	0.280	0.485	4.282	0.000	0.573	5.646

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	450	213	378	1240	0	669	2500
N.S.	1	1.00	2.28	1.08	1.92	6.29	0.00	3.40	12.69
time (sec)	N/A	0.172	2.755	0.349	0.486	3.743	0.000	0.589	9.326

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	2738	204	312	700	0	294	1461
N.S.	1	1.00	10.25	0.76	1.17	2.62	0.00	1.10	5.47
time (sec)	N/A	0.289	24.709	0.172	0.481	4.311	0.000	0.506	6.658

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	1105	147	213	546	0	193	435
N.S.	1	1.00	5.79	0.77	1.12	2.86	0.00	1.01	2.28
time (sec)	N/A	0.181	14.500	0.217	0.488	2.474	0.000	0.528	5.767

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	825	109	132	463	0	149	816
N.S.	1	1.00	6.35	0.84	1.02	3.56	0.00	1.15	6.28
time (sec)	N/A	0.117	11.969	0.178	0.483	3.076	0.000	0.533	5.209

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	110	455	0	114	2056
N.S.	1	1.00	2.61	0.98	1.20	4.95	0.00	1.24	22.35
time (sec)	N/A	0.065	2.171	0.153	0.485	3.144	0.000	0.467	6.350

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	242	78	121	429	0	127	91
N.S.	1	1.00	2.66	0.86	1.33	4.71	0.00	1.40	1.00
time (sec)	N/A	0.061	2.549	0.171	0.484	2.964	0.000	0.578	4.403

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	637	106	198	691	0	185	141
N.S.	1	1.00	5.18	0.86	1.61	5.62	0.00	1.50	1.15
time (sec)	N/A	0.130	6.871	0.236	0.487	3.091	0.000	0.559	5.494

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	777	124	274	1021	0	254	198
N.S.	1	1.00	4.13	0.66	1.46	5.43	0.00	1.35	1.05
time (sec)	N/A	0.194	3.787	0.224	0.490	2.950	0.000	0.603	6.305

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	1641	190	212	601	0	781	255
N.S.	1	1.00	7.67	0.89	0.99	2.81	0.00	3.65	1.19
time (sec)	N/A	0.183	12.900	0.187	0.478	3.503	0.000	0.637	4.509

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1392	125	156	459	0	174	172
N.S.	1	1.00	9.04	0.81	1.01	2.98	0.00	1.13	1.12
time (sec)	N/A	0.135	12.178	0.355	0.483	3.126	0.000	0.574	0.168

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	656	83	109	317	0	92	105
N.S.	1	1.00	5.66	0.72	0.94	2.73	0.00	0.79	0.91
time (sec)	N/A	0.048	7.668	0.173	0.480	3.364	0.000	0.556	0.147

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	447	156	268	807	0	592	2500
N.S.	1	1.00	2.90	1.01	1.74	5.24	0.00	3.84	16.23
time (sec)	N/A	0.145	2.672	0.342	0.486	3.231	0.000	0.612	8.601

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	532	198	408	1370	0	750	2728
N.S.	1	1.00	2.50	0.93	1.92	6.43	0.00	3.52	12.81
time (sec)	N/A	0.208	4.105	0.414	0.483	2.431	0.000	0.695	6.921

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	549	259	539	1881	0	1316	2500
N.S.	1	1.00	2.14	1.01	2.10	7.32	0.00	5.12	9.73
time (sec)	N/A	0.256	6.045	0.379	0.488	3.211	0.000	0.615	9.604

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	1639	247	430	964	0	351	2117
N.S.	1	1.00	5.22	0.79	1.37	3.07	0.00	1.12	6.74
time (sec)	N/A	0.344	20.331	0.194	0.501	3.357	0.000	0.640	7.717

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	2838	193	309	835	0	306	1317
N.S.	1	1.00	11.92	0.81	1.30	3.51	0.00	1.29	5.53
time (sec)	N/A	0.240	26.740	0.184	0.490	3.392	0.000	0.627	7.193

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	1915	155	280	845	0	208	2628
N.S.	1	1.00	10.41	0.84	1.52	4.59	0.00	1.13	14.28
time (sec)	N/A	0.199	17.911	0.302	0.486	3.052	0.000	0.585	7.834

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	237	847	0	196	2500
N.S.	1	1.00	2.31	1.02	1.65	5.88	0.00	1.36	17.36
time (sec)	N/A	0.135	6.403	0.234	0.475	3.243	0.000	0.463	8.606

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	987	97	225	643	0	177	146
N.S.	1	1.00	7.96	0.78	1.81	5.19	0.00	1.43	1.18
time (sec)	N/A	0.080	6.914	0.253	0.478	2.011	0.000	0.627	5.107

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	994	140	330	1043	0	264	207
N.S.	1	1.00	6.06	0.85	2.01	6.36	0.00	1.61	1.26
time (sec)	N/A	0.176	4.694	0.240	0.498	2.058	0.000	0.588	6.885

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	479	180	442	1463	0	367	267
N.S.	1	1.00	1.98	0.74	1.83	6.05	0.00	1.52	1.10
time (sec)	N/A	0.269	6.665	0.296	0.506	2.197	0.000	0.641	7.490

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	152	1840	183	313	0	1281	-1
N.S.	1	1.00	1.09	13.24	1.32	2.25	0.00	9.22	-0.01
time (sec)	N/A	0.101	0.950	1.081	0.474	2.902	0.000	0.849	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	1525	124	248	0	805	-1
N.S.	1	1.00	1.20	15.25	1.24	2.48	0.00	8.05	-0.01
time (sec)	N/A	0.066	0.399	0.185	0.480	2.512	0.000	0.629	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	98	94	94	196	0	55	87
N.S.	1	1.00	1.48	1.42	1.42	2.97	0.00	0.83	1.32
time (sec)	N/A	0.036	0.157	0.033	0.489	2.128	0.000	0.441	6.768

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	119	688	0	528	0	405	-1
N.S.	1	1.00	1.45	8.39	0.00	6.44	0.00	4.94	-0.01
time (sec)	N/A	0.059	0.148	0.187	0.000	1.893	0.000	0.868	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	163	3015	0	923	0	578	-1
N.S.	1	1.00	1.31	24.31	0.00	7.44	0.00	4.66	-0.01
time (sec)	N/A	0.096	0.470	0.135	0.000	2.306	0.000	0.996	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	198	9758	0	1548	0	867	-1
N.S.	1	1.00	1.08	53.32	0.00	8.46	0.00	4.74	-0.01
time (sec)	N/A	0.152	1.493	0.157	0.000	2.633	0.000	1.352	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	2665	0	1805	0	0	-1
N.S.	1	1.00	0.00	11.10	0.00	7.52	0.00	0.00	-0.00
time (sec)	N/A	0.253	9.661	0.467	0.000	6.787	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	1939	0	1651	0	0	-1
N.S.	1	1.00	0.00	10.71	0.00	9.12	0.00	0.00	-0.01
time (sec)	N/A	0.152	5.988	0.207	0.000	3.282	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	432	1290	0	1499	0	0	-1
N.S.	1	1.00	3.51	10.49	0.00	12.19	0.00	0.00	-0.01
time (sec)	N/A	0.089	6.322	0.138	0.000	3.370	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	1293	0	0	-1
N.S.	1	1.00	0.00	7.44	0.00	16.37	0.00	0.00	-0.01
time (sec)	N/A	0.034	1.358	0.257	0.000	2.452	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	1004	53	330	0	0	-1
N.S.	1	1.00	0.90	14.76	0.78	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.243	0.191	0.274	1.459	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	285	3847	87	466	0	0	-1
N.S.	1	1.00	2.71	36.64	0.83	4.44	0.00	0.00	-0.01
time (sec)	N/A	0.070	3.809	0.161	0.279	3.059	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	941	8587	152	692	0	0	-1
N.S.	1	1.00	6.32	57.63	1.02	4.64	0.00	0.00	-0.01
time (sec)	N/A	0.098	9.726	0.198	0.276	4.466	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	188	2537	297	373	0	1952	-1
N.S.	1	1.00	0.96	12.94	1.52	1.90	0.00	9.96	-0.01
time (sec)	N/A	0.121	1.454	0.345	0.484	3.254	0.000	2.095	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	164	1913	268	298	0	1388	-1
N.S.	1	1.00	1.01	11.81	1.65	1.84	0.00	8.57	-0.01
time (sec)	N/A	0.095	0.796	0.162	0.481	3.259	0.000	1.416	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	120	152	249	0	87	61
N.S.	1	1.00	0.73	1.20	1.52	2.49	0.00	0.87	0.61
time (sec)	N/A	0.049	0.675	0.033	0.480	3.063	0.000	0.474	6.111

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	171	2563	0	767	0	0	-1
N.S.	1	1.00	1.40	21.01	0.00	6.29	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.637	0.141	0.000	5.057	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	202	5178	0	1052	0	0	-1
N.S.	1	1.00	1.25	32.16	0.00	6.53	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.585	0.163	0.000	3.703	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	262	10199	0	1595	0	0	-1
N.S.	1	1.00	1.20	46.78	0.00	7.32	0.00	0.00	-0.00
time (sec)	N/A	0.228	3.702	0.188	0.000	3.127	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	0	3067	0	1957	0	0	-1
N.S.	1	1.00	0.00	10.29	0.00	6.57	0.00	0.00	-0.00
time (sec)	N/A	0.307	11.225	0.500	0.000	34.119	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	211	2309	0	1765	0	0	-1
N.S.	1	1.00	0.97	10.64	0.00	8.13	0.00	0.00	-0.00
time (sec)	N/A	0.231	5.924	0.286	0.000	17.807	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	493	1583	0	1629	0	0	-1
N.S.	1	1.00	3.06	9.83	0.00	10.12	0.00	0.00	-0.01
time (sec)	N/A	0.133	6.478	0.161	0.000	13.541	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1547	0	0	-1
N.S.	1	1.00	4.47	13.19	0.00	13.11	0.00	0.00	-0.01
time (sec)	N/A	0.066	5.903	0.128	0.000	7.411	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	2032	104	398	0	0	-1
N.S.	1	1.00	0.61	19.35	0.99	3.79	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.242	0.155	0.273	3.992	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	369	3925	257	506	0	0	-1
N.S.	1	1.00	2.15	22.82	1.49	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.109	8.873	0.184	0.282	6.960	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	512	8726	289	722	0	0	-1
N.S.	1	1.00	2.45	41.75	1.38	3.45	0.00	0.00	-0.00
time (sec)	N/A	0.142	10.775	0.269	0.275	17.019	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	93	105	174	92	0	928	-1
N.S.	1	1.00	0.76	0.85	1.41	0.75	0.00	7.54	-0.01
time (sec)	N/A	0.090	1.033	0.213	0.277	1.128	0.000	0.835	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	69	89	61	0	559	-1
N.S.	1	1.00	0.86	0.93	1.20	0.82	0.00	7.55	-0.01
time (sec)	N/A	0.060	0.289	0.137	0.282	1.516	0.000	0.745	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	30	40	0	31	46
N.S.	1	1.00	1.60	1.03	1.00	1.33	0.00	1.03	1.53
time (sec)	N/A	0.028	0.137	0.085	0.286	1.693	0.000	0.486	5.384

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	86	280	0	148	0	305	-1
N.S.	1	1.00	2.00	6.51	0.00	3.44	0.00	7.09	-0.02
time (sec)	N/A	0.046	0.124	0.149	0.000	1.695	0.000	0.786	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	140	2199	0	323	0	0	-1
N.S.	1	1.00	1.61	25.28	0.00	3.71	0.00	0.00	-0.01
time (sec)	N/A	0.075	1.088	0.195	0.000	2.158	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	78	4983	0	515	0	785	-1
N.S.	1	1.00	0.57	36.11	0.00	3.73	0.00	5.69	-0.01
time (sec)	N/A	0.109	0.169	0.156	0.000	3.152	0.000	1.005	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	163	2425	0	670	0	0	-1
N.S.	1	1.00	0.84	12.56	0.00	3.47	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.534	0.336	0.000	3.063	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	145	1701	0	594	0	0	-1
N.S.	1	1.00	1.07	12.60	0.00	4.40	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.588	0.196	0.000	2.152	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	125	1055	0	524	0	0	-1
N.S.	1	1.00	1.47	12.41	0.00	6.16	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.287	0.145	0.000	3.718	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	1046	427	0	0	-1
N.S.	1	1.00	2.23	9.74	26.82	10.95	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.072	0.162	0.589	3.464	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	35	51	0	0	74
N.S.	1	1.00	1.67	1.45	1.06	1.55	0.00	0.00	2.24
time (sec)	N/A	0.050	0.123	0.125	0.270	3.423	0.000	0.000	4.646

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	66	102	102	0	0	123
N.S.	1	1.00	0.95	0.85	1.31	1.31	0.00	0.00	1.58
time (sec)	N/A	0.071	0.223	0.139	0.269	5.380	0.000	0.000	9.922

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	100	101	203	179	0	0	723
N.S.	1	1.00	0.76	0.77	1.54	1.36	0.00	0.00	5.48
time (sec)	N/A	0.100	0.369	0.172	0.278	6.615	0.000	0.000	15.145

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	432	35190	268	143	0	1439	-1
N.S.	1	1.00	2.53	205.79	1.57	0.84	0.00	8.42	-0.01
time (sec)	N/A	0.124	7.637	1.993	0.275	3.579	0.000	1.250	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	93	12782	151	104	0	852	-1
N.S.	1	1.00	0.82	112.12	1.32	0.91	0.00	7.47	-0.01
time (sec)	N/A	0.079	3.705	0.415	0.280	1.720	0.000	1.052	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	59	61	72	0	48	155
N.S.	1	1.00	1.03	0.95	0.98	1.16	0.00	0.77	2.50
time (sec)	N/A	0.039	1.415	0.029	0.266	3.363	0.000	0.513	10.869

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	113	1094	0	362	0	0	-1
N.S.	1	1.00	1.41	13.68	0.00	4.52	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.842	0.170	0.000	2.293	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	97	3289	0	571	0	903	-1
N.S.	1	1.00	0.77	26.10	0.00	4.53	0.00	7.17	-0.01
time (sec)	N/A	0.110	0.361	0.145	0.000	4.944	0.000	1.315	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	100	8268	0	903	0	1625	-1
N.S.	1	1.00	0.56	46.71	0.00	5.10	0.00	9.18	-0.01
time (sec)	N/A	0.150	0.519	0.175	0.000	2.526	0.000	1.776	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	256	2437	0	850	0	0	-1
N.S.	1	1.00	1.06	10.07	0.00	3.51	0.00	0.00	-0.00
time (sec)	N/A	0.255	8.812	0.440	0.000	20.237	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	229	1714	0	738	0	0	-1
N.S.	1	1.00	1.31	9.79	0.00	4.22	0.00	0.00	-0.01
time (sec)	N/A	0.153	4.001	0.202	0.000	10.823	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	190	1069	0	640	0	0	-1
N.S.	1	1.00	1.57	8.83	0.00	5.29	0.00	0.00	-0.01
time (sec)	N/A	0.102	1.348	0.139	0.000	5.125	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2169	632	0	0	-1
N.S.	1	1.00	2.18	13.08	28.17	8.21	0.00	0.00	-0.01
time (sec)	N/A	0.035	1.459	0.154	0.705	3.021	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	89	68	108	0	0	2151
N.S.	1	1.00	1.12	1.31	1.00	1.59	0.00	0.00	31.63
time (sec)	N/A	0.063	1.790	0.138	0.268	2.734	0.000	0.000	12.122

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	102	137	166	197	0	0	2500
N.S.	1	1.00	0.83	1.11	1.35	1.60	0.00	0.00	20.33
time (sec)	N/A	0.092	0.722	0.161	0.270	4.839	0.000	0.000	27.528

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	204	300	324	0	0	-1
N.S.	1	1.00	0.69	1.11	1.64	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.015	0.237	0.277	8.833	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	182	229	358	198	0	1638	-1
N.S.	1	1.00	0.83	1.05	1.63	0.90	0.00	7.48	-0.00
time (sec)	N/A	0.143	3.441	1.217	0.287	4.338	0.000	2.033	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	129	159	209	146	0	1017	-1
N.S.	1	1.00	0.88	1.09	1.43	1.00	0.00	6.97	-0.01
time (sec)	N/A	0.090	2.522	0.518	0.275	5.984	0.000	1.455	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	92	108	0	70	2500
N.S.	1	1.00	0.91	0.93	0.95	1.11	0.00	0.72	25.77
time (sec)	N/A	0.045	1.421	0.032	0.270	4.201	0.000	0.612	17.230

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	108	5056	0	616	0	0	-1
N.S.	1	1.00	0.85	39.81	0.00	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.098	5.353	0.332	0.000	3.335	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	151	11110	0	971	0	1980	-1
N.S.	1	1.00	0.88	64.97	0.00	5.68	0.00	11.58	-0.01
time (sec)	N/A	0.140	1.519	0.534	0.000	3.551	0.000	1.785	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	129	15551	0	1343	0	3235	-1
N.S.	1	1.00	0.55	66.46	0.00	5.74	0.00	13.82	-0.00
time (sec)	N/A	0.228	1.950	0.918	0.000	5.064	0.000	2.307	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	1705	4477	0	1046	0	0	-1
N.S.	1	1.00	5.92	15.55	0.00	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.303	19.775	0.993	0.000	89.903	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	1315	3223	0	914	0	0	-1
N.S.	1	1.00	5.79	14.20	0.00	4.03	0.00	0.00	-0.00
time (sec)	N/A	0.219	13.012	0.454	0.000	26.513	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	983	3158	0	918	0	0	-1
N.S.	1	1.00	5.89	18.91	0.00	5.50	0.00	0.00	-0.01
time (sec)	N/A	0.145	7.693	0.348	0.000	9.322	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	918	0	0	-1
N.S.	1	1.00	15.42	24.19	0.00	7.34	0.00	0.00	-0.01
time (sec)	N/A	0.070	17.458	0.226	0.000	5.672	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	133	100	200	0	0	336
N.S.	1	1.00	1.02	1.25	0.94	1.89	0.00	0.00	3.17
time (sec)	N/A	0.074	2.134	0.233	0.282	5.256	0.000	0.000	16.300

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	138	212	230	330	0	0	-1
N.S.	1	1.00	0.87	1.34	1.46	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.111	4.964	0.175	0.284	11.841	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	173	324	397	472	0	0	-1
N.S.	1	1.00	0.77	1.43	1.76	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.165	7.608	0.193	0.291	29.622	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0	-1
N.S.	1	0.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	4.296	0.122	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	253	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	8.566	0.422	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	178	0	0	0	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	4.282	0.418	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	79
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.16
time (sec)	N/A	0.033	1.900	0.039	0.000	0.000	0.000	0.000	4.919

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	1532	0	0	0	0	0	-1
N.S.	1	1.00	19.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	17.461	0.088	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	266	0	0	0	0	0	-1
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	4.978	0.086	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	5878	0	0	0	0	0	-1
N.S.	1	1.00	66.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	25.965	0.420	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	3781	0	0	0	0	0	-1
N.S.	1	1.00	42.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	21.600	0.282	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	15.387	0.015	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	1.216	0.082	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	132	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	2.539	0.089	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	149	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	2.305	0.092	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	125	129	82	0	66	61
N.S.	1	1.00	0.97	1.69	1.74	1.11	0.00	0.89	0.82
time (sec)	N/A	0.036	0.044	0.102	0.274	2.710	0.000	0.453	4.619

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	81	81	69	0	53	49
N.S.	1	1.00	1.04	1.45	1.45	1.23	0.00	0.95	0.88
time (sec)	N/A	0.030	0.046	0.053	0.282	2.529	0.000	0.459	4.468

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	49	45	56	0	39	33
N.S.	1	1.00	1.11	1.29	1.18	1.47	0.00	1.03	0.87
time (sec)	N/A	0.023	0.025	0.039	0.280	2.978	0.000	0.428	4.396

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	16	32	0	16	16
N.S.	1	1.00	1.62	1.06	1.00	2.00	0.00	1.00	1.00
time (sec)	N/A	0.010	0.009	0.026	0.265	2.444	0.000	0.400	4.442

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	31	24	26	31	0	45	19
N.S.	1	1.00	1.63	1.26	1.37	1.63	0.00	2.37	1.00
time (sec)	N/A	0.017	0.028	0.049	0.478	2.442	0.000	0.427	4.704

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	34	40	75	0	80	31
N.S.	1	1.00	0.97	0.92	1.08	2.03	0.00	2.16	0.84
time (sec)	N/A	0.024	0.032	0.052	0.486	3.522	0.000	0.439	4.410

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	44	50	109	0	111	41
N.S.	1	1.00	0.65	0.80	0.91	1.98	0.00	2.02	0.75
time (sec)	N/A	0.031	0.060	0.060	0.481	3.466	0.000	0.431	4.538

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	36	54	60	147	0	139	51
N.S.	1	1.00	0.49	0.74	0.82	2.01	0.00	1.90	0.70
time (sec)	N/A	0.037	0.022	0.071	0.488	3.575	0.000	0.468	4.926

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	108	134	122	0	121	102
N.S.	1	1.00	0.77	1.10	1.37	1.24	0.00	1.23	1.04
time (sec)	N/A	0.046	0.375	0.128	0.276	3.725	0.000	0.450	4.655

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	85	103	102	0	98	78
N.S.	1	1.00	0.77	1.21	1.47	1.46	0.00	1.40	1.11
time (sec)	N/A	0.037	0.154	0.081	0.280	2.612	0.000	0.458	0.143

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	62	78	0	60	41
N.S.	1	1.00	1.20	1.38	1.55	1.95	0.00	1.50	1.02
time (sec)	N/A	0.018	0.030	0.043	0.278	2.264	0.000	0.437	4.410

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	41	43	0	40	22
N.S.	1	1.00	1.46	1.25	1.71	1.79	0.00	1.67	0.92
time (sec)	N/A	0.021	0.024	0.056	0.283	3.052	0.000	0.439	0.060

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	33	29	30	0	34	28
N.S.	1	1.00	1.67	1.10	0.97	1.00	0.00	1.13	0.93
time (sec)	N/A	0.032	0.033	0.083	0.266	2.710	0.000	0.426	0.047

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	54	46	48	0	57	43
N.S.	1	1.00	1.48	1.08	0.92	0.96	0.00	1.14	0.86
time (sec)	N/A	0.048	0.032	0.108	0.273	3.063	0.000	0.415	4.318

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	64	79	0	79	56
N.S.	1	1.00	0.93	0.90	0.74	0.91	0.00	0.91	0.64
time (sec)	N/A	0.036	0.360	0.079	0.278	2.740	0.000	0.435	4.308

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	46	60	0	57	42
N.S.	1	1.00	0.94	0.89	0.71	0.92	0.00	0.88	0.65
time (sec)	N/A	0.033	0.237	0.059	0.271	3.459	0.000	0.424	4.503

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	37	40	0	34	28
N.S.	1	1.00	0.84	0.81	0.86	0.93	0.00	0.79	0.65
time (sec)	N/A	0.028	0.103	0.040	0.271	3.264	0.000	0.432	4.459

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	34	0	15	17
N.S.	1	1.00	1.00	1.07	1.07	2.27	0.00	1.00	1.13
time (sec)	N/A	0.010	0.005	0.000	0.287	2.792	0.000	0.433	4.416

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	40	30	51	37	25
N.S.	1	1.00	1.06	1.19	1.29	0.97	1.65	1.19	0.81
time (sec)	N/A	0.021	0.038	0.056	0.479	1.951	3.447	0.422	4.313

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	78	52	0	73	67
N.S.	1	1.00	0.74	1.07	1.28	0.85	0.00	1.20	1.10
time (sec)	N/A	0.031	0.109	0.098	0.488	2.458	0.000	0.413	4.478

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	86	110	72	0	96	91
N.S.	1	1.00	0.76	0.97	1.24	0.81	0.00	1.08	1.02
time (sec)	N/A	0.041	0.129	0.115	0.479	2.868	0.000	0.434	4.937

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	119	180	210	177	0	221	170
N.S.	1	1.00	0.72	1.09	1.27	1.07	0.00	1.34	1.03
time (sec)	N/A	0.102	0.596	0.155	0.277	2.333	0.000	0.483	4.684

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	94	147	174	151	0	183	134
N.S.	1	1.00	0.73	1.14	1.35	1.17	0.00	1.42	1.04
time (sec)	N/A	0.094	0.446	0.102	0.275	2.894	0.000	0.472	4.555

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	63	107	125	123	0	120	86
N.S.	1	1.00	0.69	1.18	1.37	1.35	0.00	1.32	0.95
time (sec)	N/A	0.052	0.165	0.072	0.280	3.324	0.000	0.465	4.465

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	69	94	101	0	79	55
N.S.	1	1.00	1.43	1.23	1.68	1.80	0.00	1.41	0.98
time (sec)	N/A	0.049	0.046	0.082	0.271	3.220	0.000	0.445	0.109

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	55	67	70	0	70	48
N.S.	1	1.00	1.47	1.12	1.37	1.43	0.00	1.43	0.98
time (sec)	N/A	0.044	0.036	0.089	0.270	3.617	0.000	0.440	4.529

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	109	67	58	62	0	76	44
N.S.	1	1.00	2.06	1.26	1.09	1.17	0.00	1.43	0.83
time (sec)	N/A	0.048	0.039	0.118	0.273	2.446	0.000	0.451	4.440

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	134	108	126	0	152	94
N.S.	1	1.00	0.91	1.26	1.02	1.19	0.00	1.43	0.89
time (sec)	N/A	0.063	0.479	0.075	0.275	2.407	0.000	0.462	4.544

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	104	85	99	0	114	70
N.S.	1	1.00	0.94	1.30	1.06	1.24	0.00	1.42	0.88
time (sec)	N/A	0.052	0.429	0.089	0.266	2.637	0.000	0.473	4.673

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	71	77	73	0	76	44
N.S.	1	1.00	0.91	1.34	1.45	1.38	0.00	1.43	0.83
time (sec)	N/A	0.043	0.307	0.066	0.275	3.281	0.000	0.445	4.543

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	47	62	0	49	42
N.S.	1	1.00	2.65	1.20	1.18	1.55	0.00	1.22	1.05
time (sec)	N/A	0.022	0.445	0.000	0.270	2.673	0.000	0.425	4.550

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	57	60	0	53	66
N.S.	1	1.00	1.11	1.09	1.21	1.28	0.00	1.13	1.40
time (sec)	N/A	0.051	0.177	0.077	0.490	3.592	0.000	0.459	4.521

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	78	92	65	0	87	76
N.S.	1	1.00	0.72	0.96	1.14	0.80	0.00	1.07	0.94
time (sec)	N/A	0.064	0.142	0.092	0.481	2.704	0.000	0.458	4.566

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	116	142	93	0	150	123
N.S.	1	1.00	0.83	0.97	1.19	0.78	0.00	1.26	1.03
time (sec)	N/A	0.099	0.236	0.063	0.476	3.507	0.000	0.450	5.323

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	268	84	83	90	0	91	73
N.S.	1	1.00	3.67	1.15	1.14	1.23	0.00	1.25	1.00
time (sec)	N/A	0.034	1.131	0.069	0.266	3.535	0.000	0.456	4.507

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	455	130	134	130	0	148	119
N.S.	1	1.00	4.10	1.17	1.21	1.17	0.00	1.33	1.07
time (sec)	N/A	0.048	1.696	0.099	0.275	3.030	0.000	0.462	4.579

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	2519	106	130	290	0	113	591
N.S.	1	1.00	29.29	1.23	1.51	3.37	0.00	1.31	6.87
time (sec)	N/A	0.083	6.949	0.211	0.483	5.287	0.000	0.441	4.949

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	1022	63	87	165	0	74	456
N.S.	1	1.00	18.58	1.15	1.58	3.00	0.00	1.35	8.29
time (sec)	N/A	0.049	2.178	0.125	0.486	3.509	0.000	0.441	4.583

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	52	121	0	38	28
N.S.	1	1.00	1.00	0.78	1.44	3.36	0.00	1.06	0.78
time (sec)	N/A	0.031	0.085	0.087	0.488	2.353	0.000	0.464	0.111

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	70	170	0	53	44
N.S.	1	1.00	1.00	0.87	1.35	3.27	0.00	1.02	0.85
time (sec)	N/A	0.044	0.117	0.131	0.477	2.151	0.000	0.445	4.403

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	105	70	92	238	0	85	72
N.S.	1	1.00	1.38	0.92	1.21	3.13	0.00	1.12	0.95
time (sec)	N/A	0.065	0.316	0.162	0.486	3.241	0.000	0.433	4.452

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	136	110	122	315	0	129	111
N.S.	1	1.00	1.26	1.02	1.13	2.92	0.00	1.19	1.03
time (sec)	N/A	0.074	0.731	0.210	0.471	3.278	0.000	0.434	0.144

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	224	70	68	372	0	96	72
N.S.	1	1.00	2.91	0.91	0.88	4.83	0.00	1.25	0.94
time (sec)	N/A	0.066	2.508	0.208	0.478	3.269	0.000	0.435	4.368

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	192	45	47	302	0	66	44
N.S.	1	1.00	3.69	0.87	0.90	5.81	0.00	1.27	0.85
time (sec)	N/A	0.048	0.848	0.134	0.477	3.421	0.000	0.439	4.416

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	28	219	0	48	31
N.S.	1	1.00	1.00	0.78	0.78	6.08	0.00	1.33	0.86
time (sec)	N/A	0.039	0.073	0.083	0.481	2.707	0.000	0.456	4.508

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	46	241	0	65	460
N.S.	1	1.00	4.04	1.02	1.02	5.36	0.00	1.44	10.22
time (sec)	N/A	0.032	0.373	0.002	0.482	2.794	0.000	0.442	4.708

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	76	76	286	0	94	373
N.S.	1	1.00	0.89	1.01	1.01	3.81	0.00	1.25	4.97
time (sec)	N/A	0.070	0.276	0.134	0.483	3.073	0.000	0.430	5.238

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	116	132	359	0	141	1114
N.S.	1	1.00	0.81	0.99	1.13	3.07	0.00	1.21	9.52
time (sec)	N/A	0.111	0.461	0.168	0.481	2.665	0.000	0.434	5.260

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	165	197	442	0	216	1979
N.S.	1	1.00	0.82	1.01	1.21	2.71	0.00	1.33	12.14
time (sec)	N/A	0.166	0.987	0.217	0.483	3.116	0.000	0.436	6.153

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	980	110	152	410	0	126	2039
N.S.	1	1.00	9.61	1.08	1.49	4.02	0.00	1.24	19.99
time (sec)	N/A	0.104	4.556	0.324	0.491	2.814	0.000	0.473	5.869

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	88	68	102	272	0	76	62
N.S.	1	1.00	1.19	0.92	1.38	3.68	0.00	1.03	0.84
time (sec)	N/A	0.052	0.334	0.170	0.485	3.073	0.000	0.491	0.135

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	80	115	311	0	91	71
N.S.	1	1.00	0.99	0.96	1.39	3.75	0.00	1.10	0.86
time (sec)	N/A	0.048	0.443	0.185	0.481	3.792	0.000	0.446	4.434

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	945	92	138	403	0	113	94
N.S.	1	1.00	9.36	0.91	1.37	3.99	0.00	1.12	0.93
time (sec)	N/A	0.093	3.762	0.204	0.476	3.782	0.000	0.439	0.182

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	139	120	160	504	0	146	124
N.S.	1	1.00	1.10	0.95	1.27	4.00	0.00	1.16	0.98
time (sec)	N/A	0.114	1.296	0.263	0.486	3.896	0.000	0.446	4.623

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	171	158	190	599	0	188	173
N.S.	1	1.00	1.09	1.01	1.21	3.82	0.00	1.20	1.10
time (sec)	N/A	0.121	2.289	0.349	0.472	3.910	0.000	0.445	0.183

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	248	89	114	538	0	120	113
N.S.	1	1.00	2.48	0.89	1.14	5.38	0.00	1.20	1.13
time (sec)	N/A	0.094	2.720	0.155	0.473	3.777	0.000	0.442	4.788

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	76	91	424	0	89	70
N.S.	1	1.00	1.02	0.93	1.11	5.17	0.00	1.09	0.85
time (sec)	N/A	0.058	0.319	0.240	0.476	3.246	0.000	0.442	4.409

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	211	64	74	386	0	83	69
N.S.	1	1.00	2.89	0.88	1.01	5.29	0.00	1.14	0.95
time (sec)	N/A	0.049	1.052	0.152	0.480	3.647	0.000	0.449	4.471

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	110	455	0	114	2056
N.S.	1	1.00	2.61	0.98	1.20	4.95	0.00	1.24	22.35
time (sec)	N/A	0.061	2.246	0.066	0.493	2.729	0.000	0.453	6.668

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	103	120	181	566	0	192	2401
N.S.	1	1.00	0.73	0.85	1.27	3.99	0.00	1.35	16.91
time (sec)	N/A	0.138	1.476	0.208	0.487	3.133	0.000	0.441	7.692

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	138	160	276	680	0	195	2880
N.S.	1	1.00	0.68	0.79	1.36	3.35	0.00	0.96	14.19
time (sec)	N/A	0.207	1.788	0.280	0.492	3.229	0.000	0.446	11.907

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	499	210	379	815	0	270	2500
N.S.	1	1.00	1.79	0.76	1.36	2.93	0.00	0.97	8.99
time (sec)	N/A	0.247	4.883	0.191	0.490	4.510	0.000	0.465	8.558

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	128	108	185	488	0	117	113
N.S.	1	1.00	1.19	1.00	1.71	4.52	0.00	1.08	1.05
time (sec)	N/A	0.071	0.584	0.156	0.486	3.356	0.000	0.472	0.223

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	218	560	0	155	129
N.S.	1	1.00	1.30	0.99	1.74	4.48	0.00	1.24	1.03
time (sec)	N/A	0.074	0.685	0.397	0.487	3.679	0.000	0.498	4.681

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	2256	142	239	629	0	178	149
N.S.	1	1.00	15.67	0.99	1.66	4.37	0.00	1.24	1.03
time (sec)	N/A	0.098	7.492	0.316	0.487	3.887	0.000	0.489	0.254

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	2382	149	260	745	0	197	175
N.S.	1	1.00	15.27	0.96	1.67	4.78	0.00	1.26	1.12
time (sec)	N/A	0.137	7.704	0.338	0.483	3.286	0.000	0.480	4.742

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	194	177	280	876	0	229	256
N.S.	1	1.00	1.07	0.98	1.55	4.84	0.00	1.27	1.41
time (sec)	N/A	0.176	4.854	0.439	0.493	2.687	0.000	0.506	0.390

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	2670	214	312	1017	0	271	257
N.S.	1	1.00	12.48	1.00	1.46	4.75	0.00	1.27	1.20
time (sec)	N/A	0.192	7.763	0.275	0.492	2.565	0.000	0.502	0.327

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	125	137	215	746	0	185	149
N.S.	1	1.00	0.88	0.96	1.51	5.25	0.00	1.30	1.05
time (sec)	N/A	0.112	1.111	0.205	0.491	3.479	0.000	0.464	5.211

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	283	118	192	678	0	163	125
N.S.	1	1.00	2.30	0.96	1.56	5.51	0.00	1.33	1.02
time (sec)	N/A	0.070	4.020	0.165	0.482	2.728	0.000	0.482	5.102

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	265	100	161	604	0	124	112
N.S.	1	1.00	2.50	0.94	1.52	5.70	0.00	1.17	1.06
time (sec)	N/A	0.059	2.888	0.284	0.488	2.438	0.000	0.500	5.001

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	237	847	0	196	2500
N.S.	1	1.00	2.31	1.02	1.65	5.88	0.00	1.36	17.36
time (sec)	N/A	0.129	6.323	0.000	0.492	3.313	0.000	0.437	9.331

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	177	346	1000	0	228	2500
N.S.	1	1.00	0.78	0.88	1.72	4.98	0.00	1.13	12.44
time (sec)	N/A	0.237	4.166	0.359	0.499	3.777	0.000	0.492	9.973

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	1430	215	474	1161	0	464	2500
N.S.	1	1.00	5.32	0.80	1.76	4.32	0.00	1.72	9.29
time (sec)	N/A	0.271	6.621	0.226	0.493	4.298	0.000	0.484	9.586

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	1770	267	623	1330	0	350	2500
N.S.	1	1.00	5.03	0.76	1.77	3.78	0.00	0.99	7.10
time (sec)	N/A	0.338	6.695	0.239	0.498	3.114	0.000	0.503	10.107

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1411	229	401	1323	0	324	2500
N.S.	1	1.00	6.92	1.12	1.97	6.49	0.00	1.59	12.25
time (sec)	N/A	0.244	6.926	0.398	0.507	2.483	0.000	0.449	9.462

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	70	167	81	100	0	217	-1
N.S.	1	1.00	0.52	1.25	0.60	0.75	0.00	1.62	-0.01
time (sec)	N/A	0.048	2.358	0.712	0.486	3.251	0.000	0.704	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	60	157	62	87	0	182	-1
N.S.	1	1.00	0.59	1.55	0.61	0.86	0.00	1.80	-0.01
time (sec)	N/A	0.035	0.605	0.182	0.479	3.209	0.000	0.583	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	147	40	68	0	137	-1
N.S.	1	1.00	0.75	2.30	0.62	1.06	0.00	2.14	-0.02
time (sec)	N/A	0.029	0.124	0.138	0.484	3.681	0.000	0.529	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	109	21	53	0	141	-1
N.S.	1	1.00	1.00	3.30	0.64	1.61	0.00	4.27	-0.03
time (sec)	N/A	0.023	0.043	0.145	0.491	3.220	0.000	0.486	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	75	37	56	0	0	-1
N.S.	1	1.00	1.25	2.34	1.16	1.75	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.059	0.149	0.489	2.063	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	139	60	94	0	108	-1
N.S.	1	1.00	0.85	2.07	0.90	1.40	0.00	1.61	-0.01
time (sec)	N/A	0.032	0.174	0.132	0.492	2.418	0.000	0.577	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	203	79	125	0	147	-1
N.S.	1	1.00	0.69	2.03	0.79	1.25	0.00	1.47	-0.01
time (sec)	N/A	0.038	0.286	0.141	0.497	3.008	0.000	0.713	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	79	265	94	162	0	171	-1
N.S.	1	1.00	0.59	1.99	0.71	1.22	0.00	1.29	-0.01
time (sec)	N/A	0.044	0.376	0.172	0.495	3.343	0.000	0.676	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	6560	0	0	0	0	-1
N.S.	1	1.00	0.00	17.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	29.017	1.307	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	0	4739	0	0	0	0	-1
N.S.	1	1.00	0.00	16.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	12.150	0.217	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	0	3454	0	0	0	0	-1
N.S.	1	1.00	0.00	15.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	12.657	0.140	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	3408	0	0	0	0	-1
N.S.	1	1.00	0.86	42.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.297	0.198	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	539	4623	0	0	0	0	-1
N.S.	1	1.00	2.19	18.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	12.705	0.236	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	0	6392	0	0	0	0	-1
N.S.	1	1.00	0.00	18.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	12.506	0.394	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	407	2518	335	496	0	0	-1
N.S.	1	1.00	2.19	13.54	1.80	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.131	9.343	0.324	0.275	5.010	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	322	1769	183	416	0	0	-1
N.S.	1	1.00	2.64	14.50	1.50	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.076	8.030	0.190	0.271	3.513	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	210	1096	73	344	0	0	-1
N.S.	1	1.00	2.76	14.42	0.96	4.53	0.00	0.00	-0.01
time (sec)	N/A	0.057	1.831	0.123	0.263	3.498	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	1293	0	0	-1
N.S.	1	1.00	0.00	7.44	0.00	16.37	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.106	0.000	0.000	4.959	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	136	1066	0	526	0	0	-1
N.S.	1	1.00	1.66	13.00	0.00	6.41	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.746	0.144	0.000	4.096	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	152	1713	0	596	0	0	-1
N.S.	1	1.00	1.09	12.24	0.00	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.352	0.204	0.000	4.540	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	1902	2436	0	672	0	0	-1
N.S.	1	1.00	9.70	12.43	0.00	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.147	16.836	0.302	0.000	5.426	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	0	7996	0	0	0	0	-1
N.S.	1	1.00	0.00	17.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	10.462	0.809	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	6562	0	0	0	0	-1
N.S.	1	1.00	0.00	17.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	17.673	0.372	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	0	5185	0	0	0	0	-1
N.S.	1	1.00	0.00	17.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	12.429	0.221	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	0	3632	0	0	0	0	-1
N.S.	1	1.00	0.00	16.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	13.563	0.180	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	179	5069	0	0	0	0	-1
N.S.	1	1.00	0.74	21.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	2.085	0.232	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	858	6396	0	0	0	0	-1
N.S.	1	1.00	2.69	20.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	16.984	0.386	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	512	3343	439	596	0	0	-1
N.S.	1	1.00	2.11	13.76	1.81	2.45	0.00	0.00	-0.00
time (sec)	N/A	0.153	11.604	0.516	0.282	18.894	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	400	2519	257	498	0	0	-1
N.S.	1	1.00	2.42	15.27	1.56	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.097	9.771	0.283	0.276	5.292	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	1768	110	416	0	0	-1
N.S.	1	1.00	0.76	15.93	0.99	3.75	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.357	0.173	0.274	4.084	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1547	0	0	-1
N.S.	1	1.00	4.47	13.19	0.00	13.11	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.208	0.122	0.000	5.130	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	466	1511	0	1485	0	0	-1
N.S.	1	1.00	3.76	12.19	0.00	11.98	0.00	0.00	-0.01
time (sec)	N/A	0.098	7.652	0.151	0.000	5.348	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	191	1713	0	592	0	0	-1
N.S.	1	1.00	1.53	13.70	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.079	0.204	0.000	5.414	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	165	2439	0	678	0	0	-1
N.S.	1	1.00	0.85	12.64	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.951	0.296	0.000	6.571	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	706	2231	0	1611	0	0	-1
N.S.	1	1.00	4.25	13.44	0.00	9.70	0.00	0.00	-0.01
time (sec)	N/A	0.122	10.338	0.622	0.000	6.512	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	109	429	0	160	0	0	-1
N.S.	1	1.00	2.60	10.21	0.00	3.81	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.193	0.300	0.000	2.656	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	57	190	0	131	0	0	-1
N.S.	1	1.00	2.38	7.92	0.00	5.46	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.056	0.146	0.000	2.687	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	0	4753	0	0	0	0	-1
N.S.	1	1.00	0.00	14.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	12.247	0.284	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	0	3033	0	0	0	0	-1
N.S.	1	1.00	0.00	17.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	11.595	0.156	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	269	0	305	0	0	-1
N.S.	1	1.00	0.86	3.36	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.216	0.217	0.000	0.844	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	279	2985	0	0	0	0	-1
N.S.	1	1.00	2.66	28.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	8.974	0.230	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	0	4637	0	0	0	0	-1
N.S.	1	1.00	0.00	18.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	7.928	0.266	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	0	6382	0	0	0	0	-1
N.S.	1	1.00	0.00	18.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	13.832	0.407	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	326	1756	169	422	0	0	-1
N.S.	1	1.00	2.38	12.82	1.23	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.091	8.214	0.221	0.270	5.147	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	326	1086	78	348	0	0	-1
N.S.	1	1.00	4.02	13.41	0.96	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.062	10.286	0.151	0.273	3.728	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	379	24	229	0	0	-1
N.S.	1	1.00	2.23	9.72	0.62	5.87	0.00	0.00	-0.03
time (sec)	N/A	0.050	0.139	0.145	0.266	2.998	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	1046	427	0	0	-1
N.S.	1	1.00	2.23	9.74	26.82	10.95	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.090	0.000	0.605	3.302	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	126	1056	0	529	0	0	-1
N.S.	1	1.00	1.45	12.14	0.00	6.08	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.268	0.155	0.000	3.950	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	1840	1701	0	596	0	0	-1
N.S.	1	1.00	12.87	11.90	0.00	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.097	16.536	0.204	0.000	3.735	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1739	2425	0	674	0	0	-1
N.S.	1	1.00	8.52	11.89	0.00	3.30	0.00	0.00	-0.00
time (sec)	N/A	0.144	16.811	0.301	0.000	6.441	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	0	12510	0	0	0	0	-1
N.S.	1	1.00	0.00	43.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	21.107	0.339	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	113	6601	0	0	0	0	-1
N.S.	1	1.00	0.75	44.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	2.508	0.201	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	822	6593	0	0	0	0	-1
N.S.	1	1.00	3.59	28.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	15.343	0.210	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	8684	0	0	0	0	-1
N.S.	1	1.00	0.00	36.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	14.939	0.205	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	0	11939	0	0	0	0	-1
N.S.	1	1.00	0.00	35.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	17.155	0.305	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	0	15199	0	0	0	0	-1
N.S.	1	1.00	0.00	34.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	16.324	0.523	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	375	4338	171	554	0	0	-1
N.S.	1	1.00	2.72	31.43	1.24	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.104	10.324	0.208	0.264	4.075	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	405	3068	83	436	0	0	-1
N.S.	1	1.00	5.26	39.84	1.08	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.068	7.176	0.183	0.266	3.819	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	59	32	70	0	103	199
N.S.	1	1.00	1.78	1.84	1.00	2.19	0.00	3.22	6.22
time (sec)	N/A	0.051	0.708	0.105	0.266	3.015	0.000	0.774	6.291

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2169	632	0	0	-1
N.S.	1	1.00	2.18	13.08	28.17	8.21	0.00	0.00	-0.01
time (sec)	N/A	0.034	1.524	0.000	0.725	4.497	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	2059	1645	0	732	0	0	-1
N.S.	1	1.00	15.72	12.56	0.00	5.59	0.00	0.00	-0.01
time (sec)	N/A	0.110	15.632	0.162	0.000	4.083	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	2046	2372	0	846	0	0	-1
N.S.	1	1.00	10.55	12.23	0.00	4.36	0.00	0.00	-0.01
time (sec)	N/A	0.154	17.162	0.238	0.000	9.268	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	2068	3171	0	978	0	0	-1
N.S.	1	1.00	7.63	11.70	0.00	3.61	0.00	0.00	-0.00
time (sec)	N/A	0.219	20.496	0.375	0.000	17.126	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	167	14353	0	0	0	0	-1
N.S.	1	1.00	0.52	44.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	2.892	0.367	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	1156	10271	0	0	0	0	-1
N.S.	1	1.00	3.62	32.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	17.309	0.314	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	0	14353	0	0	0	0	-1
N.S.	1	1.00	0.00	43.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	13.371	0.353	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	0	17494	0	0	0	0	-1
N.S.	1	1.00	0.00	50.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	17.122	0.523	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	0	20922	0	0	0	0	-1
N.S.	1	1.00	0.00	47.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	13.282	0.997	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	0	26983	0	0	0	0	-1
N.S.	1	1.00	0.00	48.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	23.833	1.635	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	607	3018	293	720	0	0	-1
N.S.	1	1.00	4.56	22.69	2.20	5.41	0.00	0.00	-0.01
time (sec)	N/A	0.096	10.333	0.259	0.282	4.414	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	76	125	141	0	336	153
N.S.	1	1.00	0.94	0.96	1.58	1.78	0.00	4.25	1.94
time (sec)	N/A	0.063	4.636	0.145	0.268	3.551	0.000	1.063	12.666

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	215	85	65	141	0	337	172
N.S.	1	1.00	3.03	1.20	0.92	1.99	0.00	4.75	2.42
time (sec)	N/A	0.057	6.144	0.139	0.266	3.715	0.000	1.059	13.902

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	918	0	0	-1
N.S.	1	1.00	15.42	24.19	0.00	7.34	0.00	0.00	-0.01
time (sec)	N/A	0.074	6.578	0.000	0.000	5.033	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1775	4270	0	1062	0	0	-1
N.S.	1	1.00	9.49	22.83	0.00	5.68	0.00	0.00	-0.01
time (sec)	N/A	0.163	17.800	0.394	0.000	10.056	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	1777	5600	0	1228	0	0	-1
N.S.	1	1.00	6.81	21.46	0.00	4.70	0.00	0.00	-0.00
time (sec)	N/A	0.228	21.010	0.586	0.000	22.889	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	1776	6934	0	1380	0	0	-1
N.S.	1	1.00	5.35	20.89	0.00	4.16	0.00	0.00	-0.00
time (sec)	N/A	0.287	23.877	1.032	0.000	82.969	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	1777	6116	0	1241	0	0	-1
N.S.	1	1.00	9.93	34.17	0.00	6.93	0.00	0.00	-0.01
time (sec)	N/A	0.135	18.754	1.108	0.000	13.094	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	37	142	388	53	0	0	-1
N.S.	1	1.00	2.64	10.14	27.71	3.79	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.030	0.128	0.524	3.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	2195	0	0	0	0	0	-1
N.S.	1	0.00	19.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	18.693	0.134	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	1989	0	0	0	0	0	-1
N.S.	1	1.00	19.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	17.360	0.075	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	1995	0	0	0	0	0	-1
N.S.	1	1.00	19.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	17.106	0.102	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	1983	0	0	0	0	0	-1
N.S.	1	1.00	19.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	16.770	0.114	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	1987	0	0	0	0	0	-1
N.S.	1	1.00	19.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	17.450	0.256	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	1997	0	0	0	0	0	-1
N.S.	1	1.00	19.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	17.577	0.202	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	2.319	0.082	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	126	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	2.406	0.077	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	1.102	0.072	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	6.251	0.001	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	-1
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	16.147	0.164	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1912	0	0	0	0	0	-1
N.S.	1	1.00	23.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	16.720	0.187	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	-1
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	17.524	0.224	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	57	101	74	116	280	52
N.S.	1	1.00	0.76	0.79	1.40	1.03	1.61	3.89	0.72
time (sec)	N/A	0.044	0.182	0.069	0.259	5.007	0.760	1.838	4.699

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	45	68	54	80	236	46
N.S.	1	1.00	0.88	0.92	1.39	1.10	1.63	4.82	0.94
time (sec)	N/A	0.037	0.094	0.052	0.258	3.782	0.312	0.836	4.785

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	35	40	42	190	32
N.S.	1	1.00	1.00	0.87	1.17	1.33	1.40	6.33	1.07
time (sec)	N/A	0.018	0.022	0.032	0.259	3.550	0.136	0.479	4.927

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	24	35	37	0	98	32
N.S.	1	1.00	1.57	0.86	1.25	1.32	0.00	3.50	1.14
time (sec)	N/A	0.034	0.042	0.063	0.261	3.164	0.000	0.448	4.868

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	39	31	53	0	146	51
N.S.	1	1.00	1.62	1.22	0.97	1.66	0.00	4.56	1.59
time (sec)	N/A	0.038	0.190	0.056	0.255	3.864	0.000	0.454	6.143

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	64	55	52	89	0	238	61
N.S.	1	1.00	1.25	1.08	1.02	1.75	0.00	4.67	1.20
time (sec)	N/A	0.051	0.204	0.059	0.254	4.300	0.000	0.488	6.099

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	73	61	61	95	66	56	51
N.S.	1	1.00	1.14	0.95	0.95	1.48	1.03	0.88	0.80
time (sec)	N/A	0.042	0.037	0.060	0.464	5.152	1.845	2.334	4.986

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	50	49	77	54	45	40
N.S.	1	1.00	1.19	1.04	1.02	1.60	1.12	0.94	0.83
time (sec)	N/A	0.039	0.027	0.050	0.458	3.859	1.140	1.103	4.681

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	41	36	57	42	33	29
N.S.	1	1.00	1.28	1.28	1.12	1.78	1.31	1.03	0.91
time (sec)	N/A	0.035	0.021	0.059	0.462	3.591	0.847	0.646	4.520

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	34	0	15	17
N.S.	1	1.00	1.00	1.07	1.07	2.27	0.00	1.00	1.13
time (sec)	N/A	0.010	0.005	0.001	0.259	2.973	0.000	0.416	4.515

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	33	27	37	0	53	19
N.S.	1	1.00	2.26	1.74	1.42	1.95	0.00	2.79	1.00
time (sec)	N/A	0.038	0.041	0.056	0.463	3.541	0.000	0.456	4.498

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	51	48	44	82	0	111	35
N.S.	1	1.00	1.55	1.45	1.33	2.48	0.00	3.36	1.06
time (sec)	N/A	0.042	0.028	0.049	0.460	2.929	0.000	0.486	4.671

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	63	56	119	0	170	46
N.S.	1	1.00	1.00	1.24	1.10	2.33	0.00	3.33	0.90
time (sec)	N/A	0.045	0.036	0.054	0.460	3.397	0.000	0.484	4.886

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	126	101	155	105	190	431	124
N.S.	1	1.00	1.26	1.01	1.55	1.05	1.90	4.31	1.24
time (sec)	N/A	0.073	0.467	0.094	0.263	2.791	1.610	2.106	4.509

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	107	89	120	84	128	385	92
N.S.	1	1.00	1.39	1.16	1.56	1.09	1.66	5.00	1.19
time (sec)	N/A	0.059	0.273	0.072	0.261	2.207	0.785	1.038	4.544

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	41	71	57	61	334	61
N.S.	1	1.00	1.71	0.85	1.48	1.19	1.27	6.96	1.27
time (sec)	N/A	0.030	0.127	0.059	0.261	3.074	0.299	0.577	4.501

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	50	67	84	0	247	58
N.S.	1	1.00	1.58	0.94	1.26	1.58	0.00	4.66	1.09
time (sec)	N/A	0.052	0.267	0.092	0.258	2.860	0.000	0.487	4.483

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	81	64	63	105	0	233	68
N.S.	1	1.00	1.42	1.12	1.11	1.84	0.00	4.09	1.19
time (sec)	N/A	0.057	0.197	0.068	0.261	2.883	0.000	0.525	4.582

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	77	71	64	103	0	332	83
N.S.	1	1.00	1.51	1.39	1.25	2.02	0.00	6.51	1.63
time (sec)	N/A	0.062	0.273	0.066	0.259	3.725	0.000	0.527	4.613

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	275	105	90	144	0	91	126
N.S.	1	1.00	2.89	1.11	0.95	1.52	0.00	0.96	1.33
time (sec)	N/A	0.074	2.155	0.080	0.460	3.507	0.000	2.646	4.541

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	395	94	76	119	0	78	97
N.S.	1	1.00	5.13	1.22	0.99	1.55	0.00	1.01	1.26
time (sec)	N/A	0.067	1.197	0.069	0.465	3.235	0.000	1.377	4.575

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	281	85	62	91	0	65	69
N.S.	1	1.00	4.76	1.44	1.05	1.54	0.00	1.10	1.17
time (sec)	N/A	0.064	0.875	0.057	0.463	3.081	0.000	0.814	4.658

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	47	62	0	49	42
N.S.	1	1.00	2.65	1.20	1.18	1.55	0.00	1.22	1.05
time (sec)	N/A	0.022	0.408	0.000	0.263	2.307	0.000	0.418	4.581

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	66	49	72	0	46	44
N.S.	1	1.00	2.28	1.83	1.36	2.00	0.00	1.28	1.22
time (sec)	N/A	0.057	0.788	0.047	0.464	2.431	0.000	0.480	4.642

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	160	73	62	104	0	176	53
N.S.	1	1.00	3.56	1.62	1.38	2.31	0.00	3.91	1.18
time (sec)	N/A	0.060	0.909	0.054	0.464	3.156	0.000	0.473	4.608

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	256	107	76	145	0	273	68
N.S.	1	1.00	3.94	1.65	1.17	2.23	0.00	4.20	1.05
time (sec)	N/A	0.066	1.101	0.064	0.462	3.488	0.000	0.519	4.837

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	99	67	84	89	0	373	103
N.S.	1	1.00	1.43	0.97	1.22	1.29	0.00	5.41	1.49
time (sec)	N/A	0.076	0.306	0.075	0.259	3.262	0.000	1.745	4.612

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	42	52	43	0	222	64
N.S.	1	1.00	0.91	0.93	1.16	0.96	0.00	4.93	1.42
time (sec)	N/A	0.055	0.115	0.062	0.257	3.101	0.000	0.769	4.486

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	35	27	22	117	129	63
N.S.	1	1.00	1.13	1.52	1.17	0.96	5.09	5.61	2.74
time (sec)	N/A	0.022	0.206	0.042	0.259	3.068	7.236	0.697	4.558

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	68	52	44	0	166	65
N.S.	1	1.00	0.93	1.48	1.13	0.96	0.00	3.61	1.41
time (sec)	N/A	0.058	0.110	0.097	0.258	3.910	0.000	0.491	4.695

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	100	119	90	131	0	294	98
N.S.	1	1.00	1.35	1.61	1.22	1.77	0.00	3.97	1.32
time (sec)	N/A	0.077	0.273	0.093	0.258	3.701	0.000	0.512	4.935

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	138	180	149	274	0	513	160
N.S.	1	1.00	1.28	1.67	1.38	2.54	0.00	4.75	1.48
time (sec)	N/A	0.108	0.664	0.112	0.260	4.210	0.000	0.524	4.905

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	229	101	99	393	0	126	1109
N.S.	1	1.00	2.76	1.22	1.19	4.73	0.00	1.52	13.36
time (sec)	N/A	0.185	3.051	0.128	0.464	3.162	0.000	2.204	4.923

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	206	72	69	315	0	87	410
N.S.	1	1.00	3.49	1.22	1.17	5.34	0.00	1.47	6.95
time (sec)	N/A	0.114	1.242	0.099	0.465	3.015	0.000	0.999	4.684

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	184	48	47	236	0	66	126
N.S.	1	1.00	4.00	1.04	1.02	5.13	0.00	1.43	2.74
time (sec)	N/A	0.087	0.357	0.104	0.460	2.664	0.000	0.575	4.717

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	46	241	0	65	460
N.S.	1	1.00	4.04	1.02	1.02	5.36	0.00	1.44	10.22
time (sec)	N/A	0.033	0.271	0.000	0.458	3.453	0.000	0.411	4.856

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	204	68	69	328	0	87	637
N.S.	1	1.00	3.29	1.10	1.11	5.29	0.00	1.40	10.27
time (sec)	N/A	0.118	1.506	0.151	0.462	2.840	0.000	0.472	6.252

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	390	90	110	559	0	134	2644
N.S.	1	1.00	4.53	1.05	1.28	6.50	0.00	1.56	30.74
time (sec)	N/A	0.172	3.795	0.140	0.463	3.531	0.000	0.497	8.973

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	671	118	166	867	0	212	2500
N.S.	1	1.00	5.59	0.98	1.38	7.22	0.00	1.77	20.83
time (sec)	N/A	0.243	3.052	0.171	0.473	3.203	0.000	0.524	10.170

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	72	101	123	0	532	170
N.S.	1	1.00	1.42	0.94	1.31	1.60	0.00	6.91	2.21
time (sec)	N/A	0.074	0.489	0.086	0.258	3.411	0.000	1.806	4.655

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	50	61	56	0	314	97
N.S.	1	1.00	1.59	0.98	1.20	1.10	0.00	6.16	1.90
time (sec)	N/A	0.057	0.771	0.073	0.257	4.036	0.000	0.845	4.591

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	79	61	59	55	0	384	90
N.S.	1	1.00	1.61	1.24	1.20	1.12	0.00	7.84	1.84
time (sec)	N/A	0.038	0.523	0.074	0.259	4.195	0.000	0.571	4.481

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	112	91	120	143	0	396	106
N.S.	1	1.00	1.35	1.10	1.45	1.72	0.00	4.77	1.28
time (sec)	N/A	0.082	0.385	0.144	0.260	2.943	0.000	0.500	4.851

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	130	141	197	321	0	811	160
N.S.	1	1.00	1.17	1.27	1.77	2.89	0.00	7.31	1.44
time (sec)	N/A	0.111	1.471	0.128	0.260	4.391	0.000	0.592	5.051

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	162	201	285	570	0	913	206
N.S.	1	1.00	1.16	1.44	2.04	4.07	0.00	6.52	1.47
time (sec)	N/A	0.143	2.027	0.147	0.267	4.843	0.000	0.624	6.094

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	286	125	132	540	0	155	765
N.S.	1	1.00	2.40	1.05	1.11	4.54	0.00	1.30	6.43
time (sec)	N/A	0.179	5.092	0.135	0.464	3.485	0.000	2.364	4.989

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	249	85	100	413	0	120	285
N.S.	1	1.00	2.77	0.94	1.11	4.59	0.00	1.33	3.17
time (sec)	N/A	0.120	2.785	0.118	0.466	3.410	0.000	1.117	4.780

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	346	77	79	478	0	94	711
N.S.	1	1.00	4.07	0.91	0.93	5.62	0.00	1.11	8.36
time (sec)	N/A	0.106	8.475	0.127	0.464	3.823	0.000	0.690	4.999

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	240	90	110	455	0	114	2056
N.S.	1	1.00	2.61	0.98	1.20	4.95	0.00	1.24	22.35
time (sec)	N/A	0.064	2.110	0.000	0.463	3.654	0.000	0.418	6.393

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	288	102	168	630	0	175	2500
N.S.	1	1.00	2.38	0.84	1.39	5.21	0.00	1.45	20.66
time (sec)	N/A	0.184	4.270	0.169	0.464	4.841	0.000	0.518	9.296

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	1896	124	241	1013	0	212	2500
N.S.	1	1.00	11.85	0.78	1.51	6.33	0.00	1.32	15.62
time (sec)	N/A	0.247	7.074	0.207	0.469	3.573	0.000	0.569	10.462

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	3028	152	326	1547	0	298	2500
N.S.	1	1.00	14.63	0.73	1.57	7.47	0.00	1.44	12.08
time (sec)	N/A	0.303	7.490	0.247	0.479	3.396	0.000	0.634	10.865

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	136	80	116	122	921	531	166
N.S.	1	1.00	1.74	1.03	1.49	1.56	11.81	6.81	2.13
time (sec)	N/A	0.078	2.373	0.093	0.258	3.352	83.207	2.053	4.602

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	131	73	117	117	933	656	153
N.S.	1	1.00	1.62	0.90	1.44	1.44	11.52	8.10	1.89
time (sec)	N/A	0.077	1.157	0.089	0.257	3.183	83.003	1.039	4.449

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	129	83	106	108	782	782	142
N.S.	1	1.00	1.74	1.12	1.43	1.46	10.57	10.57	1.92
time (sec)	N/A	0.053	1.571	0.049	0.264	3.877	82.863	0.708	4.404

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	158	144	248	316	0	767	190
N.S.	1	1.00	1.22	1.11	1.91	2.43	0.00	5.90	1.46
time (sec)	N/A	0.118	1.125	0.129	0.261	6.504	0.000	0.583	4.863

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	176	193	351	597	0	1069	272
N.S.	1	1.00	1.14	1.25	2.28	3.88	0.00	6.94	1.77
time (sec)	N/A	0.148	1.908	0.158	0.271	6.924	0.000	0.704	6.033

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	208	254	462	876	0	1912	327
N.S.	1	1.00	1.08	1.32	2.41	4.56	0.00	9.96	1.70
time (sec)	N/A	0.194	5.454	0.178	0.271	9.357	0.000	0.669	6.420

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	760	134	199	692	0	200	615
N.S.	1	1.00	5.17	0.91	1.35	4.71	0.00	1.36	4.18
time (sec)	N/A	0.206	6.567	0.152	0.465	4.285	0.000	2.623	4.899

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1473	124	169	791	0	159	1117
N.S.	1	1.00	10.75	0.91	1.23	5.77	0.00	1.16	8.15
time (sec)	N/A	0.176	15.865	0.165	0.466	3.790	0.000	1.334	5.403

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	1473	125	197	888	0	172	2405
N.S.	1	1.00	10.67	0.91	1.43	6.43	0.00	1.25	17.43
time (sec)	N/A	0.164	13.669	0.175	0.468	5.953	0.000	0.864	7.442

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	332	147	237	847	0	196	2500
N.S.	1	1.00	2.31	1.02	1.65	5.88	0.00	1.36	17.36
time (sec)	N/A	0.139	6.206	0.001	0.463	4.481	0.000	0.429	8.427

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	2089	149	318	1094	0	247	2500
N.S.	1	1.00	11.54	0.82	1.76	6.04	0.00	1.36	13.81
time (sec)	N/A	0.265	7.181	0.230	0.479	5.291	0.000	0.591	10.706

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	3340	171	417	1691	0	302	2500
N.S.	1	1.00	14.52	0.74	1.81	7.35	0.00	1.31	10.87
time (sec)	N/A	0.328	7.835	0.271	0.485	4.074	0.000	0.596	11.983

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	976	199	529	2279	0	388	2500
N.S.	1	1.00	3.42	0.70	1.86	8.00	0.00	1.36	8.77
time (sec)	N/A	0.422	8.533	0.300	0.493	4.707	0.000	0.617	15.210

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	924	0	484	0	1280	-1
N.S.	1	1.00	0.00	8.32	0.00	4.36	0.00	11.53	-0.01
time (sec)	N/A	0.109	2.403	0.753	0.000	11.855	0.000	1.987	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	648	0	412	0	800	-1
N.S.	1	1.00	0.00	8.10	0.00	5.15	0.00	10.00	-0.01
time (sec)	N/A	0.074	1.085	0.142	0.000	5.008	0.000	0.920	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	119	58	0	332	0	377	46
N.S.	1	1.00	2.20	1.07	0.00	6.15	0.00	6.98	0.85
time (sec)	N/A	0.045	0.499	0.029	0.000	3.943	0.000	0.579	5.487

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	C	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	570	3495	1019	0	402	-1
N.S.	1	1.00	0.00	8.14	49.93	14.56	0.00	5.74	-0.01
time (sec)	N/A	0.077	1.309	0.171	0.726	5.173	0.000	0.821	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	527	2528	0	1422	0	574	-1
N.S.	1	1.00	4.83	23.19	0.00	13.05	0.00	5.27	-0.01
time (sec)	N/A	0.103	6.356	0.134	0.000	5.286	0.000	0.956	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	0	7346	0	2049	0	866	-1
N.S.	1	1.00	0.00	45.63	0.00	12.73	0.00	5.38	-0.01
time (sec)	N/A	0.154	5.532	0.154	0.000	12.381	0.000	1.377	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	263	2756	0	1873	0	0	-1
N.S.	1	1.00	1.20	12.58	0.00	8.55	0.00	0.00	-0.00
time (sec)	N/A	0.292	3.933	0.329	0.000	18.447	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	208	2005	0	1715	0	0	-1
N.S.	1	1.00	1.26	12.15	0.00	10.39	0.00	0.00	-0.01
time (sec)	N/A	0.203	2.926	0.211	0.000	11.649	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	526	1331	0	1561	0	0	-1
N.S.	1	1.00	4.46	11.28	0.00	13.23	0.00	0.00	-0.01
time (sec)	N/A	0.143	4.620	0.147	0.000	5.234	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-2)	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	1293	0	0	-1
N.S.	1	1.00	0.00	7.44	0.00	16.37	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.105	0.000	0.000	6.883	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	130	1004	0	528	0	0	-1
N.S.	1	1.00	1.88	14.55	0.00	7.65	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.659	0.197	0.000	5.533	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	176	3855	0	664	0	0	-1
N.S.	1	1.00	1.54	33.82	0.00	5.82	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.782	0.155	0.000	7.448	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	178	8605	0	890	0	0	-1
N.S.	1	1.00	1.07	51.53	0.00	5.33	0.00	0.00	-0.01
time (sec)	N/A	0.217	1.765	0.191	0.000	15.226	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	2606	0	557	0	2026	-1
N.S.	1	1.00	0.00	19.30	0.00	4.13	0.00	15.01	-0.01
time (sec)	N/A	0.111	3.087	0.230	0.000	28.507	0.000	3.318	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	0	2150	0	471	0	1431	-1
N.S.	1	1.00	0.00	20.67	0.00	4.53	0.00	13.76	-0.01
time (sec)	N/A	0.086	2.091	0.168	0.000	11.443	0.000	1.760	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	84	77	0	399	0	867	66
N.S.	1	1.00	1.08	0.99	0.00	5.12	0.00	11.12	0.85
time (sec)	N/A	0.058	0.304	0.024	0.000	6.257	0.000	1.120	7.158

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	506	2424	0	1139	0	718	-1
N.S.	1	1.00	5.56	26.64	0.00	12.52	0.00	7.89	-0.01
time (sec)	N/A	0.094	5.729	0.120	0.000	6.299	0.000	4.086	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	622	1609	0	1380	0	0	-1
N.S.	1	1.00	5.46	14.11	0.00	12.11	0.00	0.00	-0.01
time (sec)	N/A	0.114	6.104	0.168	0.000	6.358	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	684	4955	0	1897	0	0	-1
N.S.	1	1.00	4.30	31.16	0.00	11.93	0.00	0.00	-0.01
time (sec)	N/A	0.168	6.141	0.131	0.000	9.265	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	353	3583	0	2075	0	0	-1
N.S.	1	1.00	1.22	12.36	0.00	7.16	0.00	0.00	-0.00
time (sec)	N/A	0.375	6.903	0.532	0.000	57.191	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	258	2757	0	1875	0	0	-1
N.S.	1	1.00	1.21	12.88	0.00	8.76	0.00	0.00	-0.00
time (sec)	N/A	0.308	4.530	0.319	0.000	18.016	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	703	2002	0	1721	0	0	-1
N.S.	1	1.00	4.23	12.06	0.00	10.37	0.00	0.00	-0.01
time (sec)	N/A	0.230	6.936	0.193	0.000	9.542	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	1547	0	0	-1
N.S.	1	1.00	4.47	13.19	0.00	13.11	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.158	0.119	0.000	4.543	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	410	1952	0	1536	0	0	-1
N.S.	1	1.00	3.69	17.59	0.00	13.84	0.00	0.00	-0.01
time (sec)	N/A	0.153	6.629	0.198	0.000	5.686	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	2014	0	632	0	0	-1
N.S.	1	1.00	0.89	17.98	0.00	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.364	0.154	0.000	5.892	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	139	5850	0	808	0	0	-1
N.S.	1	1.00	0.84	35.45	0.00	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.234	1.447	0.192	0.000	15.885	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	358	0	436	0	765	-1
N.S.	1	1.00	0.00	4.02	0.00	4.90	0.00	8.60	-0.01
time (sec)	N/A	0.087	2.062	0.205	0.000	6.400	0.000	2.019	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	303	0	348	0	373	-1
N.S.	1	1.00	0.00	5.41	0.00	6.21	0.00	6.66	-0.02
time (sec)	N/A	0.063	1.628	0.160	0.000	4.828	0.000	1.025	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	B	F	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	0	42	0	277	0	111	27
N.S.	1	1.00	0.00	1.27	0.00	8.39	0.00	3.36	0.82
time (sec)	N/A	0.038	0.127	0.044	0.000	4.499	0.000	0.677	5.090

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	376	0	1071	0	403	-1
N.S.	1	1.00	0.00	5.37	0.00	15.30	0.00	5.76	-0.01
time (sec)	N/A	0.073	2.439	0.162	0.000	4.014	0.000	1.020	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	4245	0	1630	0	595	-1
N.S.	1	1.00	0.00	36.59	0.00	14.05	0.00	5.13	-0.01
time (sec)	N/A	0.113	5.203	0.143	0.000	5.544	0.000	1.077	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	0	10441	0	2353	0	932	-1
N.S.	1	1.00	0.00	62.90	0.00	14.17	0.00	5.61	-0.01
time (sec)	N/A	0.169	7.919	0.158	0.000	7.965	0.000	1.378	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	230	1993	0	1767	0	0	-1
N.S.	1	1.00	1.33	11.52	0.00	10.21	0.00	0.00	-0.01
time (sec)	N/A	0.220	4.671	0.229	0.000	7.941	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	196	1322	0	1597	0	0	-1
N.S.	1	1.00	1.63	11.02	0.00	13.31	0.00	0.00	-0.01
time (sec)	N/A	0.157	2.935	0.168	0.000	4.917	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	404	0	1325	0	0	-1
N.S.	1	1.00	0.00	5.05	0.00	16.56	0.00	0.00	-0.01
time (sec)	N/A	0.124	2.933	0.165	0.000	3.889	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	1046	427	0	0	-1
N.S.	1	1.00	2.23	9.74	26.82	10.95	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.141	0.000	0.583	3.214	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	127	1865	0	554	0	0	-1
N.S.	1	1.00	1.72	25.20	0.00	7.49	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.225	0.193	0.000	3.865	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	168	5619	0	758	0	0	-1
N.S.	1	1.00	1.41	47.22	0.00	6.37	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.949	0.168	0.000	4.397	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	199	11267	0	1028	0	0	-1
N.S.	1	1.00	1.16	65.51	0.00	5.98	0.00	0.00	-0.01
time (sec)	N/A	0.226	4.488	0.204	0.000	9.919	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	6593	0	484	0	560	-1
N.S.	1	1.00	0.00	74.92	0.00	5.50	0.00	6.36	-0.01
time (sec)	N/A	0.102	5.180	0.260	0.000	4.343	0.000	2.444	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	187	2371	2674	443	0	251	-1
N.S.	1	1.00	2.97	37.63	42.44	7.03	0.00	3.98	-0.02
time (sec)	N/A	0.082	4.168	0.158	0.738	3.434	0.000	1.328	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	382	62	0	418	53	216	49
N.S.	1	1.00	6.70	1.09	0.00	7.33	0.93	3.79	0.86
time (sec)	N/A	0.052	6.963	0.025	0.000	3.588	7.332	0.985	5.558

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	9693	0	1649	0	0	-1
N.S.	1	1.00	0.00	96.93	0.00	16.49	0.00	0.00	-0.01
time (sec)	N/A	0.107	6.460	0.198	0.000	3.941	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	0	19968	0	2443	0	0	-1
N.S.	1	1.00	0.00	130.51	0.00	15.97	0.00	0.00	-0.01
time (sec)	N/A	0.175	10.639	0.668	0.000	6.876	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	0	32565	0	3613	0	0	-1
N.S.	1	1.00	0.00	152.89	0.00	16.96	0.00	0.00	-0.00
time (sec)	N/A	0.232	14.191	1.014	0.000	16.216	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	247	3860	0	2001	0	0	-1
N.S.	1	1.00	1.44	22.44	0.00	11.63	0.00	0.00	-0.01
time (sec)	N/A	0.229	10.096	0.200	0.000	8.487	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	201	2662	0	1749	0	0	-1
N.S.	1	1.00	1.73	22.95	0.00	15.08	0.00	0.00	-0.01
time (sec)	N/A	0.162	4.543	0.197	0.000	4.535	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	169	569	2119	579	0	0	-1
N.S.	1	1.00	2.38	8.01	29.85	8.15	0.00	0.00	-0.01
time (sec)	N/A	0.141	2.680	0.161	0.669	3.020	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	2169	632	0	0	-1
N.S.	1	1.00	2.18	13.08	28.17	8.21	0.00	0.00	-0.01
time (sec)	N/A	0.034	1.475	0.000	0.699	3.484	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	182	2836	0	776	0	0	-1
N.S.	1	1.00	1.53	23.83	0.00	6.52	0.00	0.00	-0.01
time (sec)	N/A	0.186	4.622	0.161	0.000	5.248	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	224	7541	0	1102	0	0	-1
N.S.	1	1.00	1.29	43.34	0.00	6.33	0.00	0.00	-0.01
time (sec)	N/A	0.245	5.834	0.181	0.000	9.486	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	237	14137	0	1564	0	0	-1
N.S.	1	1.00	0.98	58.66	0.00	6.49	0.00	0.00	-0.00
time (sec)	N/A	0.319	11.307	0.237	0.000	24.310	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	187	10947	0	596	0	466	-1
N.S.	1	1.00	1.93	112.86	0.00	6.14	0.00	4.80	-0.01
time (sec)	N/A	0.112	8.871	1.269	0.000	5.805	0.000	3.046	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	613	10839	0	554	0	418	-1
N.S.	1	1.00	6.89	121.79	0.00	6.22	0.00	4.70	-0.01
time (sec)	N/A	0.090	10.683	1.321	0.000	5.313	0.000	1.871	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	613	86	0	526	78	370	68
N.S.	1	1.00	7.39	1.04	0.00	6.34	0.94	4.46	0.82
time (sec)	N/A	0.063	7.197	0.025	0.000	4.419	13.507	1.446	8.168

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	68989	0	2375	0	0	-1
N.S.	1	1.00	0.00	503.57	0.00	17.34	0.00	0.00	-0.01
time (sec)	N/A	0.141	9.550	1.894	0.000	7.055	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	0	105237	0	3619	0	0	-1
N.S.	1	1.00	0.00	526.18	0.00	18.10	0.00	0.00	-0.00
time (sec)	N/A	0.228	21.036	2.404	0.000	16.288	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	145925	0	4879	0	0	-1
N.S.	1	1.00	0.00	544.50	0.00	18.21	0.00	0.00	-0.00
time (sec)	N/A	0.320	31.986	23.221	0.000	54.066	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	316	2256	0	2145	0	0	-1
N.S.	1	1.00	2.01	14.37	0.00	13.66	0.00	0.00	-0.01
time (sec)	N/A	0.233	11.841	1.068	0.000	9.720	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	409	1142	0	698	0	0	-1
N.S.	1	1.00	3.41	9.52	0.00	5.82	0.00	0.00	-0.01
time (sec)	N/A	0.168	6.609	1.040	0.000	6.352	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	410	2112	0	810	0	0	-1
N.S.	1	1.00	3.45	17.75	0.00	6.81	0.00	0.00	-0.01
time (sec)	N/A	0.171	4.822	1.076	0.000	5.233	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	918	0	0	-1
N.S.	1	1.00	15.42	24.19	0.00	7.34	0.00	0.00	-0.01
time (sec)	N/A	0.070	6.504	1.163	0.000	5.484	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	247	7586	0	1138	0	0	-1
N.S.	1	1.00	1.42	43.60	0.00	6.54	0.00	0.00	-0.01
time (sec)	N/A	0.248	7.914	1.267	0.000	10.530	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	234	15128	0	1626	0	0	-1
N.S.	1	1.00	0.99	64.10	0.00	6.89	0.00	0.00	-0.00
time (sec)	N/A	0.315	14.859	1.406	0.000	24.500	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	272	22712	0	2112	0	0	-1
N.S.	1	1.00	0.86	72.10	0.00	6.70	0.00	0.00	-0.00
time (sec)	N/A	0.396	24.856	1.490	0.000	83.142	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	259	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	3.865	0.155	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	106	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	4.301	0.105	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	2.760	0.086	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.097	0.107	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	115	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	2.320	0.093	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	139	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	3.847	0.092	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2777	0	0	0	0	0	-1
N.S.	1	1.00	31.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	18.616	0.106	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2465	0	0	0	0	0	-1
N.S.	1	1.00	28.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	16.883	0.082	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	-1
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	6.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	2469	0	0	0	0	0	-1
N.S.	1	1.00	29.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	17.411	0.089	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	3033	0	0	0	0	0	-1
N.S.	1	1.00	34.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	18.881	0.092	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	141	79	88	119	340	227
N.S.	1	1.00	0.95	1.53	0.86	0.96	1.29	3.70	2.47
time (sec)	N/A	0.048	0.300	0.116	0.281	2.557	1.138	1.829	8.788

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	101	55	64	82	271	167
N.S.	1	1.00	0.97	1.66	0.90	1.05	1.34	4.44	2.74
time (sec)	N/A	0.038	0.160	0.091	0.285	3.040	0.494	0.868	8.392

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	26	30	40	42	181	83
N.S.	1	1.00	1.00	0.87	1.00	1.33	1.40	6.03	2.77
time (sec)	N/A	0.016	0.021	0.046	0.283	2.815	0.198	0.530	5.257

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	65	42	48	66	0	87	72
N.S.	1	1.00	1.20	0.78	0.89	1.22	0.00	1.61	1.33
time (sec)	N/A	0.048	0.076	0.089	0.287	2.776	0.000	0.463	4.638

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	114	63	66	105	0	172	86
N.S.	1	1.00	1.58	0.88	0.92	1.46	0.00	2.39	1.19
time (sec)	N/A	0.044	1.112	0.101	0.274	3.400	0.000	0.489	4.653

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	251	267	223	4438	0	0	2500
N.S.	1	1.00	1.15	1.22	1.02	20.26	0.00	0.00	11.42
time (sec)	N/A	0.218	0.391	0.352	0.499	7.238	0.000	0.000	7.258

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	133	163	2294	0	0	1620
N.S.	1	1.00	1.46	0.80	0.98	13.82	0.00	0.00	9.76
time (sec)	N/A	0.102	0.276	0.211	0.504	54.377	0.000	0.000	8.540

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	22	22	139	177	114
N.S.	1	1.00	1.00	1.52	0.96	0.96	6.04	7.70	4.96
time (sec)	N/A	0.022	0.021	0.062	0.283	3.543	21.469	0.556	5.189

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	290	303	312	6487	0	0	2500
N.S.	1	1.00	0.98	1.03	1.06	21.99	0.00	0.00	8.47
time (sec)	N/A	0.358	0.437	0.362	0.503	6.299	0.000	0.000	7.898

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	336	374	496	10762	0	0	2500
N.S.	1	1.00	0.85	0.95	1.26	27.38	0.00	0.00	6.36
time (sec)	N/A	0.448	2.136	0.753	0.501	10.565	0.000	0.000	19.524

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.039	3.438	0.137	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	245	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	10.396	0.082	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	162	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	4.809	0.079	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.096	0.042	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	4.165	0.073	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	38.356	0.092	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	2.719	0.053	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	1.389	0.059	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	2.070	0.077	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [67] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	3	2	1.00	21	0.095
3	A	3	2	1.00	21	0.095
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	4	4	1.00	21	0.190
7	A	5	4	1.00	21	0.190
8	A	6	6	1.00	21	0.286
9	A	5	5	1.00	21	0.238
10	A	4	4	1.00	21	0.190
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	21	0.095
13	A	3	2	1.00	21	0.095
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	23	0.087
16	A	3	2	1.00	23	0.087
17	A	3	2	1.00	21	0.095
18	A	4	3	1.00	21	0.143
19	A	5	5	1.00	23	0.217
20	A	6	5	1.00	23	0.217
21	A	7	6	1.00	23	0.261
22	A	6	5	1.00	23	0.217
23	A	5	5	1.00	23	0.217
24	A	4	3	1.00	14	0.214
25	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	23	0.087
27	A	3	2	1.00	23	0.087
28	A	4	3	1.00	23	0.130
29	A	4	4	1.00	23	0.174
30	A	3	3	1.00	21	0.143
31	A	4	4	1.00	21	0.190
32	A	5	5	1.00	23	0.217
33	A	6	6	1.00	23	0.261
34	A	7	7	1.00	23	0.304
35	A	6	6	1.00	23	0.261
36	A	5	5	1.00	23	0.217
37	A	3	3	1.00	14	0.214
38	A	3	3	1.00	23	0.130
39	A	4	4	1.00	23	0.174
40	A	4	3	1.00	23	0.130
41	A	6	5	1.00	23	0.217
42	A	5	4	1.00	23	0.174
43	A	4	4	1.00	21	0.190
44	A	5	5	1.00	21	0.238
45	A	6	6	1.00	23	0.261
46	A	7	6	1.00	23	0.261
47	A	8	7	1.00	23	0.304
48	A	7	6	1.00	23	0.261
49	A	6	6	1.00	23	0.261
50	A	5	5	1.00	14	0.357
51	A	4	4	1.00	23	0.174
52	A	5	4	1.00	23	0.174
53	A	6	5	1.00	23	0.217
54	A	6	5	1.00	23	0.217
55	A	6	5	1.00	23	0.217
56	A	5	4	1.00	21	0.190
57	A	6	6	1.00	21	0.286
58	A	7	7	1.00	23	0.304
59	A	8	7	1.00	23	0.304
60	A	9	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	6	1.00	23	0.261
62	A	7	6	1.00	23	0.261
63	A	6	6	1.00	14	0.429
64	A	5	4	1.00	23	0.174
65	A	6	5	1.00	23	0.217
66	A	7	6	1.00	23	0.261
67	A	6	6	1.00	25	0.240
68	A	5	5	1.00	25	0.200
69	A	4	4	1.00	23	0.174
70	A	6	6	1.00	23	0.261
71	A	7	7	1.00	25	0.280
72	A	8	8	1.00	25	0.320
73	A	9	8	1.00	25	0.320
74	A	8	8	1.00	25	0.320
75	A	7	7	1.00	25	0.280
76	A	6	6	1.00	16	0.375
77	A	4	4	1.00	25	0.160
78	A	5	5	1.00	25	0.200
79	A	6	6	1.00	25	0.240
80	A	7	7	1.00	25	0.280
81	A	6	6	1.00	25	0.240
82	A	5	5	1.00	23	0.217
83	A	7	7	1.00	23	0.304
84	A	8	8	1.00	25	0.320
85	A	9	9	1.00	25	0.360
86	A	10	10	1.00	25	0.400
87	A	9	9	1.00	25	0.360
88	A	8	8	1.00	25	0.320
89	A	7	7	1.00	16	0.438
90	A	5	5	1.00	25	0.200
91	A	6	6	1.00	25	0.240
92	A	7	7	1.00	25	0.280
93	A	4	4	1.00	25	0.160
94	A	3	3	1.00	25	0.120
95	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	23	0.130
97	A	5	5	1.00	25	0.200
98	A	6	6	1.00	25	0.240
99	A	7	7	1.00	25	0.280
100	A	6	6	1.00	25	0.240
101	A	5	5	1.00	25	0.200
102	A	3	3	1.00	16	0.188
103	A	2	2	1.00	25	0.080
104	A	3	3	1.00	25	0.120
105	A	4	4	1.00	25	0.160
106	A	5	5	1.00	25	0.200
107	A	4	4	1.00	25	0.160
108	A	3	3	1.00	23	0.130
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	25	0.240
111	A	7	6	1.00	25	0.240
112	A	8	7	1.00	25	0.280
113	A	7	6	1.00	25	0.240
114	A	6	6	1.00	25	0.240
115	A	4	4	1.00	16	0.250
116	A	3	3	1.00	25	0.120
117	A	4	4	1.00	25	0.160
118	A	5	5	1.00	25	0.200
119	A	6	6	1.00	25	0.240
120	A	5	5	1.00	25	0.200
121	A	4	4	1.00	23	0.174
122	A	6	6	1.00	23	0.261
123	A	7	6	1.00	25	0.240
124	A	8	6	1.00	25	0.240
125	A	9	7	1.00	25	0.280
126	A	8	6	1.00	25	0.240
127	A	7	6	1.00	25	0.240
128	A	6	6	1.00	16	0.375
129	A	4	4	1.00	25	0.160
130	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	6	1.00	25	0.240
132	F	0	0	N/A	0.	N/A
133	A	5	5	1.00	23	0.217
134	A	4	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	3	3	1.00	21	0.143
137	A	3	3	1.00	23	0.130
138	A	3	3	1.00	23	0.130
139	A	3	3	1.00	23	0.130
140	A	3	3	1.00	14	0.214
141	A	3	3	1.00	23	0.130
142	A	4	4	1.00	23	0.174
143	A	5	5	1.00	23	0.217
144	A	6	3	1.00	15	0.200
145	A	5	3	1.00	15	0.200
146	A	4	3	1.00	15	0.200
147	A	3	2	1.00	13	0.154
148	A	3	3	1.00	15	0.200
149	A	4	3	1.00	15	0.200
150	A	5	3	1.00	15	0.200
151	A	6	3	1.00	15	0.200
152	A	4	3	1.00	21	0.143
153	A	3	3	1.00	21	0.143
154	A	2	2	1.00	19	0.105
155	A	2	2	1.00	19	0.105
156	A	3	2	1.00	21	0.095
157	A	4	3	1.00	21	0.143
158	A	3	2	1.00	21	0.095
159	A	3	2	1.00	21	0.095
160	A	3	3	1.00	21	0.143
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	21	0.095
163	A	3	3	1.00	21	0.143
164	A	4	3	1.00	21	0.143
165	A	6	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	5	5	1.00	23	0.217
167	A	4	4	1.00	21	0.190
168	A	5	4	1.00	21	0.190
169	A	4	3	1.00	23	0.130
170	A	3	2	1.00	23	0.087
171	A	3	2	1.00	23	0.087
172	A	3	2	1.00	23	0.087
173	A	3	2	1.00	23	0.087
174	A	4	3	1.00	14	0.214
175	A	5	4	1.00	23	0.174
176	A	4	4	1.00	23	0.174
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	14	0.214
179	A	4	3	1.00	14	0.214
180	A	5	5	1.00	23	0.217
181	A	4	4	1.00	23	0.174
182	A	2	2	1.00	21	0.095
183	A	3	3	1.00	21	0.143
184	A	4	3	1.00	23	0.130
185	A	4	3	1.00	23	0.130
186	A	4	3	1.00	23	0.130
187	A	3	3	1.00	23	0.130
188	A	2	2	1.00	23	0.087
189	A	3	3	1.00	14	0.214
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	7	6	1.00	23	0.261
193	A	5	5	1.00	23	0.217
194	A	3	3	1.00	23	0.130
195	A	3	3	1.00	21	0.143
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	23	0.174
198	A	5	4	1.00	23	0.174
199	A	5	4	1.00	23	0.174
200	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.00	23	0.130
202	A	5	5	1.00	14	0.357
203	A	6	6	1.00	23	0.261
204	A	7	6	1.00	23	0.261
205	A	8	6	1.00	23	0.261
206	A	4	3	1.00	23	0.130
207	A	4	4	1.00	23	0.174
208	A	4	4	1.00	21	0.190
209	A	6	5	1.00	21	0.238
210	A	6	5	1.00	23	0.217
211	A	6	5	1.00	23	0.217
212	A	4	4	1.00	23	0.174
213	A	4	4	1.00	23	0.174
214	A	4	3	1.00	23	0.130
215	A	6	6	1.00	14	0.429
216	A	7	6	1.00	23	0.261
217	A	8	6	1.00	23	0.261
218	A	9	6	1.00	23	0.261
219	A	7	6	1.00	14	0.429
220	A	6	4	1.00	17	0.235
221	A	5	4	1.00	17	0.235
222	A	4	4	1.00	17	0.235
223	A	3	3	1.00	17	0.176
224	A	3	3	1.00	17	0.176
225	A	4	4	1.00	17	0.235
226	A	5	4	1.00	17	0.235
227	A	6	4	1.00	17	0.235
228	A	11	10	1.00	25	0.400
229	A	10	10	1.00	25	0.400
230	A	10	10	1.00	23	0.435
231	A	5	5	1.00	23	0.217
232	A	9	9	1.00	25	0.360
233	A	10	10	1.00	25	0.400
234	A	6	6	1.00	25	0.240
235	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	25	0.160
237	A	6	6	1.00	16	0.375
238	A	4	4	1.00	25	0.160
239	A	5	5	1.00	25	0.200
240	A	7	6	1.00	25	0.240
241	A	12	10	1.00	25	0.400
242	A	11	10	1.00	25	0.400
243	A	10	10	1.00	23	0.435
244	A	9	9	1.00	23	0.391
245	A	9	9	1.00	25	0.360
246	A	10	10	1.00	25	0.400
247	A	7	6	1.00	25	0.240
248	A	6	5	1.00	25	0.200
249	A	5	4	1.00	25	0.160
250	A	7	7	1.00	16	0.438
251	A	7	7	1.00	25	0.280
252	A	5	4	1.00	25	0.160
253	A	6	5	1.00	25	0.200
254	A	8	8	1.00	16	0.500
255	A	6	6	1.00	10	0.600
256	A	5	5	1.00	10	0.500
257	A	10	10	1.00	25	0.400
258	A	7	7	1.00	25	0.280
259	A	5	5	1.00	23	0.217
260	A	5	5	1.00	23	0.217
261	A	9	9	1.00	25	0.360
262	A	10	10	1.00	25	0.400
263	A	5	5	1.00	25	0.200
264	A	4	4	1.00	25	0.160
265	A	3	3	1.00	25	0.120
266	A	3	3	1.00	16	0.188
267	A	4	4	1.00	25	0.160
268	A	6	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240
270	A	10	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	7	7	1.00	25	0.280
272	A	9	9	1.00	23	0.391
273	A	9	9	1.00	23	0.391
274	A	10	10	1.00	25	0.400
275	A	11	10	1.00	25	0.400
276	A	5	5	1.00	25	0.200
277	A	4	4	1.00	25	0.160
278	A	2	2	1.00	25	0.080
279	A	4	4	1.00	16	0.250
280	A	6	6	1.00	25	0.240
281	A	7	6	1.00	25	0.240
282	A	8	6	1.00	25	0.240
283	A	10	10	1.00	25	0.400
284	A	10	10	1.00	25	0.400
285	A	10	10	1.00	23	0.435
286	A	10	10	1.00	23	0.435
287	A	11	11	1.00	25	0.440
288	A	12	11	1.00	25	0.440
289	A	5	5	1.00	25	0.200
290	A	3	3	1.00	25	0.120
291	A	3	3	1.00	25	0.120
292	A	6	6	1.00	16	0.375
293	A	7	6	1.00	25	0.240
294	A	8	6	1.00	25	0.240
295	A	9	6	1.00	25	0.240
296	A	7	6	1.00	16	0.375
297	A	3	3	1.00	10	0.300
298	F	0	0	N/A	0.	N/A
299	A	5	5	1.00	23	0.217
300	A	5	5	1.00	21	0.238
301	A	5	5	1.00	21	0.238
302	A	5	5	1.00	23	0.217
303	A	5	5	1.00	23	0.217
304	A	5	5	1.00	23	0.217
305	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	3	1.00	23	0.130
307	A	3	3	1.00	14	0.214
308	A	3	3	1.00	23	0.130
309	A	3	3	1.00	23	0.130
310	A	3	3	1.00	23	0.130
311	A	4	3	1.00	21	0.143
312	A	4	3	1.00	21	0.143
313	A	3	2	1.00	19	0.105
314	A	4	3	1.00	19	0.158
315	A	4	3	1.00	21	0.143
316	A	4	3	1.00	21	0.143
317	A	4	3	1.00	21	0.143
318	A	4	3	1.00	21	0.143
319	A	4	3	1.00	21	0.143
320	A	3	2	1.00	12	0.167
321	A	4	3	1.00	21	0.143
322	A	4	3	1.00	21	0.143
323	A	4	3	1.00	21	0.143
324	A	4	3	1.00	23	0.130
325	A	4	3	1.00	23	0.130
326	A	4	3	1.00	21	0.143
327	A	4	3	1.00	21	0.143
328	A	4	3	1.00	23	0.130
329	A	4	3	1.00	23	0.130
330	A	4	3	1.00	23	0.130
331	A	4	3	1.00	23	0.130
332	A	4	3	1.00	23	0.130
333	A	4	3	1.00	14	0.214
334	A	4	3	1.00	23	0.130
335	A	4	3	1.00	23	0.130
336	A	4	3	1.00	23	0.130
337	A	4	3	1.00	23	0.130
338	A	4	3	1.00	23	0.130
339	A	2	2	1.00	21	0.095
340	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	3	1.00	23	0.130
342	A	4	3	1.00	23	0.130
343	A	7	7	1.00	23	0.304
344	A	6	6	1.00	23	0.261
345	A	5	5	1.00	23	0.217
346	A	3	3	1.00	14	0.214
347	A	6	6	1.00	23	0.261
348	A	7	7	1.00	23	0.304
349	A	8	7	1.00	23	0.304
350	A	4	3	1.00	23	0.130
351	A	4	3	1.00	23	0.130
352	A	4	3	1.00	21	0.143
353	A	4	3	1.00	21	0.143
354	A	4	3	1.00	23	0.130
355	A	4	3	1.00	23	0.130
356	A	7	7	1.00	23	0.304
357	A	6	6	1.00	23	0.261
358	A	6	6	1.00	23	0.261
359	A	5	5	1.00	14	0.357
360	A	7	7	1.00	23	0.304
361	A	8	7	1.00	23	0.304
362	A	9	7	1.00	23	0.304
363	A	4	3	1.00	23	0.130
364	A	4	3	1.00	23	0.130
365	A	4	3	1.00	21	0.143
366	A	4	3	1.00	21	0.143
367	A	4	3	1.00	23	0.130
368	A	4	3	1.00	23	0.130
369	A	7	7	1.00	23	0.304
370	A	7	7	1.00	23	0.304
371	A	7	7	1.00	23	0.304
372	A	6	6	1.00	14	0.429
373	A	8	8	1.00	23	0.348
374	A	9	8	1.00	23	0.348
375	A	10	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	6	1.00	25	0.240
377	A	6	6	1.00	25	0.240
378	A	5	5	1.00	23	0.217
379	A	7	5	1.00	23	0.217
380	A	8	6	1.00	25	0.240
381	A	9	7	1.00	25	0.280
382	A	10	9	1.00	25	0.360
383	A	9	9	1.00	25	0.360
384	A	8	8	1.00	25	0.320
385	A	6	6	1.00	16	0.375
386	A	6	6	1.00	25	0.240
387	A	7	7	1.00	25	0.280
388	A	8	7	1.00	25	0.280
389	A	8	6	1.00	25	0.240
390	A	7	6	1.00	25	0.240
391	A	6	5	1.00	23	0.217
392	A	8	6	1.00	23	0.261
393	A	8	6	1.00	25	0.240
394	A	9	7	1.00	25	0.280
395	A	11	9	1.00	25	0.360
396	A	10	9	1.00	25	0.360
397	A	9	9	1.00	25	0.360
398	A	7	7	1.00	16	0.438
399	A	8	8	1.00	25	0.320
400	A	7	7	1.00	25	0.280
401	A	8	7	1.00	25	0.280
402	A	6	5	1.00	25	0.200
403	A	5	5	1.00	25	0.200
404	A	4	4	1.00	23	0.174
405	A	7	5	1.00	23	0.217
406	A	8	6	1.00	25	0.240
407	A	9	7	1.00	25	0.280
408	A	9	9	1.00	25	0.360
409	A	8	8	1.00	25	0.320
410	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	3	3	1.00	16	0.188
412	A	6	6	1.00	25	0.240
413	A	7	7	1.00	25	0.280
414	A	8	7	1.00	25	0.280
415	A	6	5	1.00	25	0.200
416	A	5	5	1.00	25	0.200
417	A	5	5	1.00	23	0.217
418	A	8	6	1.00	23	0.261
419	A	9	7	1.00	25	0.280
420	A	10	8	1.00	25	0.320
421	A	9	9	1.00	25	0.360
422	A	8	8	1.00	25	0.320
423	A	5	5	1.00	25	0.200
424	A	4	4	1.00	16	0.250
425	A	7	7	1.00	25	0.280
426	A	8	7	1.00	25	0.280
427	A	9	7	1.00	25	0.280
428	A	6	5	1.00	25	0.200
429	A	6	6	1.00	25	0.240
430	A	6	5	1.00	23	0.217
431	A	9	7	1.00	23	0.304
432	A	10	7	1.00	25	0.280
433	A	11	8	1.00	25	0.320
434	A	9	9	1.00	25	0.360
435	A	7	7	1.00	25	0.280
436	A	7	7	1.00	25	0.280
437	A	6	6	1.00	16	0.375
438	A	8	8	1.00	25	0.320
439	A	9	8	1.00	25	0.320
440	A	10	8	1.00	25	0.320
441	A	4	4	1.00	25	0.160
442	A	5	4	1.00	23	0.174
443	A	4	4	1.00	23	0.174
444	A	3	3	1.00	21	0.143
445	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	6	6	1.00	23	0.261
447	A	4	4	1.00	23	0.174
448	A	4	4	1.00	23	0.174
449	A	3	3	1.00	14	0.214
450	A	4	4	1.00	23	0.174
451	A	4	4	1.00	23	0.174
452	A	3	2	1.00	21	0.095
453	A	3	2	1.00	21	0.095
454	A	3	2	1.00	19	0.105
455	A	3	2	1.00	19	0.105
456	A	5	4	1.00	21	0.190
457	A	11	10	1.00	23	0.435
458	A	9	9	1.00	23	0.391
459	A	2	2	1.00	21	0.095
460	A	11	10	1.00	21	0.476
461	A	11	10	1.00	23	0.435
462	A	0	0	0.00	0	0.000
463	A	15	8	1.00	25	0.320
464	A	11	8	1.00	25	0.320
465	A	5	5	1.00	23	0.217
466	A	0	0	0.00	0	0.000
467	A	0	0	0.00	0	0.000
468	A	0	0	0.00	0	0.000
469	A	0	0	0.00	0	0.000
470	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx$

Optimal. Leaf size=83

$$-\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-3*b)*cos(f*x+e)/f+(a-b)*cos(f*x+e)^3/f-1/5*(3*a-b)*cos(f*x+e)^5/f+1/7*a
*cos(f*x+e)^7/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4218, 459}

$$-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/f - ((3*a - b)*Cos
[e + f*x]^5)/(5*f) + (a*Cos[e + f*x]^7)/(7*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3(b+ax^2)}{x^2}\,dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{3b}{a}\right)+\frac{b}{x^2}-3(a-b)x^2+(3a-b)x^4-ax^6\right)\,dx,x\right.}{f}$$
$$= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(3a-b)\cos^5(e+}{5f}$$

Mathematica [A]
time = 0.06, size = 120, normalized size = 1.45

$$-\frac{35a\cos(e+fx)}{64f}+\frac{19b\cos(e+fx)}{8f}+\frac{7a\cos(3(e+fx))}{64f}-\frac{3b\cos(3(e+fx))}{16f}-\frac{7a\cos(5(e+fx))}{320f}+\frac{b\cos(5(e+fx))}{80f}+\frac{a\cos(7(e+fx))}{448f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] (-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e + f
*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(320*
f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec[e
+ f*x])/f
```

Maple [A]
time = 0.17, size = 102, normalized size = 1.23

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin^8(fx+e)}{\cos(fx+e)}+\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2\right.}{}\right.\right.}{f}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin^8(fx+e)}{\cos(fx+e)}+\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2\right.}{}\right.\right.}{f}$
norman	$\frac{\frac{32a-224b}{35f}-\frac{32(a+b)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{6(32a-224b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35f}+\frac{2(32a-224b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f}+\frac{2(32a-224b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^7}$
risch	$\frac{19\,e^{i(fx+e)}b}{16f}-\frac{35\,e^{i(fx+e)}a}{128f}+\frac{19\,e^{-i(fx+e)}b}{16f}-\frac{35\,e^{-i(fx+e)}a}{128f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a\cos(7fx+7e)}{448f}-\frac{7\cos(5fx}{320f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/7*a*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e
)+b*(sin(f*x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*
x+e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.27, size = 78, normalized size = 0.94

$$\frac{5\,a\cos\left(fx+e\right)^7-7\,(3\,a-b)\cos\left(fx+e\right)^5+35\,(a-b)\cos\left(fx+e\right)^3-35\,(a-3\,b)\cos\left(fx+e\right)+\frac{35\,b}{\cos(fx+e)}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
+ e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

Fricas [A]
time = 2.85, size = 80, normalized size = 0.96

$$\frac{5\,a\cos\left(fx+e\right)^8-7\,(3\,a-b)\cos\left(fx+e\right)^6+35\,(a-b)\cos\left(fx+e\right)^4-35\,(a-3\,b)\cos\left(fx+e\right)^2+35\,b}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
+ e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(79) = 158.
time = 0.50, size = 266, normalized size = 3.20

$$\frac{2\left(\frac{35\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{16\,a-77\,b-\frac{112\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{504\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{336\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{1337\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{560\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{1680\,b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{1015\,b(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{280\,b(\cos(fx+e)-1)^5}{(\cos(fx+e)+1)^5}-\frac{35\,b(\cos(fx+e)-1)^6}{(\cos(fx+e)+1)^6}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^7}\right)}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/35*(35*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (16*a - 77*b - 112
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 504*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 336*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1337*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 560*a*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 1680*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1015*b*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 280*b*(cos(f*x + e) - 1)^5/(cos(f
*x + e) + 1)^5 - 35*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 1)^7)/f
```

Mupad [B]
time = 0.10, size = 70, normalized size = 0.84

$$\frac{\frac{a\cos(e+f\,x)^7}{7}-\cos\left(e+f\,x\right)\,\left(a-3\,b\right)-\cos\left(e+f\,x\right)^5\,\left(\frac{3\,a}{5}-\frac{b}{5}\right)+\frac{b}{\cos(e+f\,x)}+\cos\left(e+f\,x\right)^3\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*cos(e + f*x)^7)/7 - cos(e + f*x)*(a - 3*b) - cos(e + f*x)^5*((3*a)/5 -
b/5) + b/cos(e + f*x) + cos(e + f*x)^3*(a - b))/f
```

3.2 $\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx$

Optimal. Leaf size=66

$$-\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-2*b)*cos(f*x+e)/f+1/3*(2*a-b)*cos(f*x+e)^3/f-1/5*a*cos(f*x+e)^5/f+b*sec
(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4218, 459}

$$\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((2*a - b)*Cos[e + f*x]^3)/(3*f) - (a*Cos[e
 + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{2b}{a}\right)+\frac{b}{x^2}-(2a-b)x^2+ax^4\right)\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.33

$$-\frac{5a\cos(e+fx)}{8f}+\frac{7b\cos(e+fx)}{4f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{b\cos(3(e+fx))}{12f}-\frac{a\cos(5(e+fx))}{80f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x)
])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (b*
Sec[e + f*x])/f
```

Maple [A]
time = 0.10, size = 82, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)\right)}{f}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{16a-80b}{15f}-\frac{32(a+b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{4(16a-80b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15f}+\frac{(16a-80b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5}$
risch	$-\frac{5\,e^{i(fx+e)}a}{16f}+\frac{7\,e^{i(fx+e)}b}{8f}-\frac{5\,e^{-i(fx+e)}a}{16f}+\frac{7\,e^{-i(fx+e)}b}{8f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{5\cos(3fx+3}{48f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^6/
cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.94

$$-\frac{3\,a\cos\left(fx+e\right)^{5}-5\,(2\,a-b)\cos\left(fx+e\right)^{3}+15\,(a-2\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos(fx+e)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(f
*x + e) - 15*b/cos(f*x + e))/f
```

Fricas [A]
time = 3.45, size = 64, normalized size = 0.97

$$-\frac{3\,a\cos\left(fx+e\right)^{6}-5\,(2\,a-b)\cos\left(fx+e\right)^{4}+15\,(a-2\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(f
*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sin^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(62) = 124.
time = 0.49, size = 197, normalized size = 2.98

$$\frac{2\left(\frac{15\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{8\,a-25\,b-\frac{40\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{110\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{160\,b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{90\,b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{15\,b(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^{5}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")
```

[Out] 2/15*(15*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (8*a - 25*b - 40*a *(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 110*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 160*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5)/f

Mupad [B]
time = 4.21, size = 55, normalized size = 0.83

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{2\,a}{3}-\frac{b}{3}\right)-\cos\left(e+f\,x\right)\,\left(a-2\,b\right)-\frac{a\cos\left(e+f\,x\right)^5}{5}+\frac{b}{\cos\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2),x)

[Out] (cos(e + f*x)^3*((2*a)/3 - b/3) - cos(e + f*x)*(a - 2*b) - (a*cos(e + f*x)^5)/5 + b/cos(e + f*x))/f

3.3 $\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-b)*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 459}

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
[Out] -(((a - b)*Cos[e + f*x])/f) + (a*Cos[e + f*x]^3)/(3*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.20

$$-\frac{3a\cos(e+fx)}{4f}+\frac{b\cos(e+fx)}{f}+\frac{a\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
[Out] (-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*f
) + (b*Sec[e + f*x])/f
```

Maple [A]
time = 0.07, size = 62, normalized size = 1.41

method	result	size
derivativedivides	$\frac{-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+b\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+\left(2+\sin^2(fx+e)\right)\cos(fx+e)\right)}{f}$	62
default	$\frac{-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+b\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+\left(2+\sin^2(fx+e)\right)\cos(fx+e)\right)}{f}$	62
norman	$\frac{\frac{4a-12b}{3f}-\frac{4(a+b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2(4a-12b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$	87
risch	$-\frac{3\,e^{i(fx+e)}a}{8f}+\frac{e^{i(fx+e)}b}{2f}-\frac{3\,e^{-i(fx+e)}a}{8f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f
*x+e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.98

$$\frac{a\cos\left(fx+e\right)^3-3\left(a-b\right)\cos\left(fx+e\right)+\frac{3\,b}{\cos(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
[Out] 1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

Fricas [A]
time = 4.03, size = 45, normalized size = 1.02

$$\frac{a\cos\left(fx+e\right)^4-3\left(a-b\right)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\sin^{3}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**3, x)
```

Giac [A]
time = 0.48, size = 57, normalized size = 1.30

$$\frac{b}{f\cos\left(fx+e\right)} + \frac{af^{5}\cos\left(fx+e\right)^{3} - 3\,af^{5}\cos\left(fx+e\right) + 3\,bf^{5}\cos\left(fx+e\right)}{3\,f^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - 3*a*f^5*cos(f*x + e) + 3*b
*f^5*cos(f*x + e))/f^6
```

Mupad [B]
time = 0.07, size = 39, normalized size = 0.89

$$\frac{\frac{a\cos(e+f\,x)^{3}}{3} - \cos\left(e+f\,x\right)\,\left(a-b\right) + \frac{b}{\cos(e+f\,x)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*cos(e + f*x)^3)/3 - cos(e + f*x)*(a - b) + b/cos(e + f*x))/f
```

3.4 $\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx$

Optimal. Leaf size=24

$$-\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -a*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4218, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.46

$$-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\sec(e+fx)}{f}+\frac{a\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

Maple [A]
time = 0.03, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
default	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
risch	$-\frac{e^{i(fx+e)}a}{2f}-\frac{e^{-i(fx+e)}a}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}$	60
norman	$\frac{\frac{2a-2b}{f}-\frac{2(a+b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(sec(f*x+e)*b-a/sec(f*x+e))
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.12

$$-\frac{a\cos\left(fx+e\right)-\frac{b}{\cos(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
[Out] -(a*cos(f*x + e) - b/cos(f*x + e))/f
```

Fricas [A]
time = 4.63, size = 29, normalized size = 1.21

$$-\frac{a\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
[Out] -(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\sin\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A]
time = 0.46, size = 26, normalized size = 1.08

$$-\frac{a\cos\left(fx + e\right)}{f} + \frac{b}{f\cos\left(fx + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
[Out] -a*cos(f*x + e)/f + b/(f*cos(f*x + e))
```

Mupad [B]
time = 0.04, size = 25, normalized size = 1.04

$$-\frac{a\,\cos\left(e+f\,x\right)-\frac{b}{\cos\left(e+f\,x\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] -(a*cos(e + f*x) - b/cos(e + f*x))/f
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=27

$$-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a+b)*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {4218, 464, 212}

$$\frac{b\sec(e+fx)}{f}-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= \frac{b\sec(e+fx)}{f} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f} + \frac{b\sec(e+fx)}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.
time = 0.03, size = 84, normalized size = 3.11

$$-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (a*Log[Sin
[e/2 + (f*x)/2]])/f + (b*Log[Sin[(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

Maple [A]
time = 0.07, size = 51, normalized size = 1.89

method	result	size
norman	$-\frac{2b}{f\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)} + \frac{(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$	40
derivativedivides	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
default	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
risch	$\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)} + \frac{\ln\left(e^{i(fx+e)}-1\right)a}{f} + \frac{\ln\left(e^{i(fx+e)}-1\right)b}{f} - \frac{\ln\left(e^{i(fx+e)}+1\right)a}{f} - \frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*ln(csc(f*x+e)-cot(f*x+e))+b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))
)
```

Maxima [A]
time = 0.27, size = 47, normalized size = 1.74

$$-\frac{(a+b)\log(\cos(fx+e)+1)-(a+b)\log(\cos(fx+e)-1)-\frac{2b}{\cos(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/c
os(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 7.31, size = 65, normalized size = 2.41

$$-\frac{(a+b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-2b}{2f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)\csc(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.43, size = 60, normalized size = 2.22

$$\frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{4b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 4*b/((cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B]
time = 0.09, size = 29, normalized size = 1.07

$$\frac{b}{f\cos(e+fx)}-\frac{\mathrm{atanh}(\cos(e+fx))\,(a+b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x),x)
[Out] b/(f*cos(e + f*x)) - (atanh(cos(e + f*x))*(a + b))/f
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -1/2*(a+3*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)*cot(f*x+e)*csc(f*x+e)/f+b*sec(
f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 467, 464, 212}

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 3*b)*ArcTanh[Cos[e + f*x]])/f - ((a + b)*Cot[e + f*x]*Csc[e + f*
x])/(2*f) + (b*Sec[e + f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{\text{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}\,dx,x,\cos\right.}{2f}\\ &= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}-\frac{(a+3b)\text{Subs}}{}\\ &= -\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(53) = 106.
time = 0.28, size = 236, normalized size = 4.45

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{3b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{3b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(e + f*x)/2]^2)/f - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(
e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e + f
*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)
/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2]))
```

Maple [A]
time = 0.11, size = 90, normalized size = 1.70

method	result
derivativedivides	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right.}{f}$
default	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right.}{f}$
norman	$\frac{\frac{a+b}{8f}+\frac{(a+b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{(a+9b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$
risch	$\frac{a\,e^{5i(fx+e)}+3b\,e^{5i(fx+e)}+2a\,e^{3i(fx+e)}-2b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}+3b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f}-\frac{3\ln\left(e^{i(fx+e)}+\right.}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+b*(-1/2/s
in(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.53

$$-\frac{(a+3\,b)\log\left(\cos\left(fx+e\right)+1\right)-(a+3\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+3\,b)\cos(fx+e)^2-2\,b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) - 2
*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(53) = 106$.
time = 5.18, size = 133, normalized size = 2.51

$$\frac{2\,(a+3\,b)\cos\left(fx+e\right)^2-\left((a+3\,b)\cos\left(fx+e\right)^3-(a+3\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a+3\,b)\cos\left(fx+e\right)^3-(a+3\,b)\cos\left(fx+e\right)\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-4\,b}{4\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a + 3
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(49) = 98.
time = 0.47, size = 193, normalized size = 3.64

$$\frac{2\left(a+3b\right)\log\left(\frac{\left|-\cos(fx+e)+1\right|}{\left|\cos(fx+e)+1\right|}\right)-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a+b+\frac{14\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+\frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - a*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + (a + b + 14*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/
((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2))/f
```

Mupad [B]
time = 4.20, size = 62, normalized size = 1.17

$$\frac{b-\cos\left(e+f\,x\right)^2\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)-\cos\left(e+f\,x\right)^3\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^3,x)
```

```
[Out] (b - cos(e + f*x)^2*(a/2 + (3*b)/2))/(f*(cos(e + f*x) - cos(e + f*x)^3)) -
(atanh(cos(e + f*x))*(a/2 + (3*b)/2))/f
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=81

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}+\frac{b\,\mathrm{s}}{}$$

```
[Out] -3/8*(a+5*b)*arctanh(cos(f*x+e))/f-1/8*(3*a+7*b)*cot(f*x+e)*csc(f*x+e)/f-1/
4*(a+b)*cot(f*x+e)*csc(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 467, 464, 212}

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-3*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f) + (b*Sec[e +
f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{-4b-3(a+b)x^2}{x^2(1-x^2)^2}\,dx, x, \text{c}\right.}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+}{4f}$$

$$= -\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+}{8f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(81) = 162.
time = 1.30, size = 198, normalized size = 2.44

$$\frac{-2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right)-(a+b)\csc^4\left(\frac{1}{2}(e+fx)\right)+\frac{2\left(-3(a+13b)+4\cos(e+fx)\left(8b+3(a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-3(a+5b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)-(a+b)\sec^4\left(\frac{1}{2}(e+fx)\right)+(4(a+2b)+(3a+7b)\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)}{-1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(a
 + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a +
 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)/2]
^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e + f
*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

Maple [A]
time = 0.10, size = 120, normalized size = 1.48

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos\right.}{f}$
default	$\frac{a\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos\right.}{f}$
norman	$\frac{\frac{a+b}{64f}+\frac{(a+b)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}+\frac{(7a+15b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}+\frac{(7a+15b)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}-\frac{(a+10b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3(a+5b)\ln}{}$
risch	$\frac{3a\,e^{9i(fx+e)}+15b\,e^{9i(fx+e)}-8a\,e^{7i(fx+e)}-40b\,e^{7i(fx+e)}-22a\,e^{5i(fx+e)}+18b\,e^{5i(fx+e)}-8a\,e^{3i(fx+e)}-40b\,e^{3i(fx+e)}+3a}{4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-cot
(f*x+e)))+b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)+15/8/
cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.33

$$-\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos(fx+e)^4-5\,(a+5\,b)\cos(fx+e)^2+8\,b\right)}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx+e)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(81) = 162$.
time = 5.59, size = 191, normalized size = 2.36

$$\frac{6\,(a+5\,b)\cos\left(fx+e\right)^4-10\,(a+5\,b)\cos\left(fx+e\right)^2-3\,\left((a+5\,b)\cos\left(fx+e\right)^5-2\,(a+5\,b)\cos\left(fx+e\right)^3+(a+5\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+3\,\left((a+5\,b)\cos\left(fx+e\right)^5-2\,(a+5\,b)\cos\left(fx+e\right)^3+(a+5\,b)\cos\left(fx+e\right)\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+16\,b}{16\,\left(f\cos\left(fx+e\right)^5-2\,f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5*
b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f
```

```
*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)\csc^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(75) = 150.
time = 0.48, size = 262, normalized size = 3.23

$$\frac{12\left(a+5b\right)\log\left(\frac{\left|-\cos\left(fx+e\right)+1\right|}{\left|\cos\left(fx+e\right)+1\right|}\right)-\frac{\left(a+b-\frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{128b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(12*(a + 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - (a +
 b - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e)
 - 1)^2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 1))/f
```

Mupad [B]
time = 4.29, size = 86, normalized size = 1.06

$$\frac{\left(\frac{3a}{8}+\frac{15b}{8}\right)\cos\left(e+fx\right)^4+\left(-\frac{5a}{8}-\frac{25b}{8}\right)\cos\left(e+fx\right)^2+b}{f\left(\cos\left(e+fx\right)^5-2\cos\left(e+fx\right)^3+\cos\left(e+fx\right)\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+fx\right)\right)\left(\frac{3a}{8}+\frac{15b}{8}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^5,x)
```

```
[Out] (b + cos(e + f*x)^4*((3*a)/8 + (15*b)/8) - cos(e + f*x)^2*((5*a)/8 + (25*b)
/8))/(f*(cos(e + f*x) - 2*cos(e + f*x)^3 + cos(e + f*x)^5)) - (atanh(cos(e
+ f*x))*((3*a)/8 + (15*b)/8))/f
```

3.8 $\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx$

Optimal. Leaf size=98

$$\frac{5}{16}(a-6b)x-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$

```
[Out] 5/16*(a-6*b)*x-1/16*(11*a-18*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-6*b)*cos
(f*x+e)^3*sin(f*x+e)/f-1/6*a*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4217, 466, 1828, 1171, 396, 209}

$$\frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a - 6*b)*x)/16 - ((11*a - 18*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a*Cos[e + f*x]^5*Sin[e +
f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)}{(1+x^2)^4}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{a\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+6ax^2-6ax^4-6bx^6}{(1+x^2)^3}\,dx,x,\text{ta}\right.}{6f}$$

$$= \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\,\text{si}}{24f}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\,\text{si}}{24f}$$

$$= \frac{5}{16}(a-6b)x - \frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\,\text{c}}{}$$

Mathematica [A]
time = 0.22, size = 78, normalized size = 0.80

$$\frac{60ae-360be+60afx-360bfx+(-45a+96b)\sin(2(e+fx))+(9a-6b)\sin(4(e+fx))-a\sin(6(e+fx))+192b\tan(e+fx)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
+ (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])/(
192*f)
```

Maple [A]
time = 0.10, size = 112, normalized size = 1.14

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\ldots\right.\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\ldots\right.\right.}{f}$
risch	$\frac{5ax}{16}-\frac{15xb}{8}+\frac{15i e^{2i(fx+e)}a}{128f}-\frac{i e^{2i(fx+e)}b}{4f}-\frac{15i e^{-2i(fx+e)}a}{128f}+\frac{i e^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}-\frac{a\sin(6f\ldots}{192\ldots}$
norman	$\left(-\frac{5a}{16}+\frac{15b}{8}\right)x+\left(-\frac{45a}{16}+\frac{135b}{8}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/
8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 1.21

$$\frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-18\,b)\tan(fx+e)^5+8\,(5\,a-12\,b)\tan(fx+e)^3+3\,(5\,a-14\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*x
 + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.86, size = 92, normalized size = 0.94

$$\frac{15\,(a-6\,b)fx\cos\left(fx+e\right)-\left(8\,a\cos\left(fx+e\right)^{6}-2\,(13\,a-6\,b)\cos\left(fx+e\right)^{4}+3\,(11\,a-18\,b)\cos\left(fx+e\right)^{2}-48\,b\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)*
cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*co
s(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 104, normalized size = 1.06

$$\frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{33\,a\tan(fx+e)^{5}-54\,b\tan(fx+e)^{5}+40\,a\tan(fx+e)^{3}-96\,b\tan(fx+e)^{3}+15\,a\tan(fx+e)-42\,b\tan(fx+e)}{\left(\tan(fx+e)^{2}+1\right)^{3}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 - 5
4*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan(f
*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B]
time = 4.89, size = 105, normalized size = 1.07

$$x\left(\frac{5\,a}{16}-\frac{15\,b}{8}\right)-\frac{\left(\frac{11\,a}{16}-\frac{9\,b}{8}\right)\,\tan(e+f\,x)^{5}+\left(\frac{5\,a}{6}-2\,b\right)\,\tan(e+f\,x)^{3}+\left(\frac{5\,a}{16}-\frac{7\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^{6}+3\tan\left(e+f\,x\right)^{4}+3\tan\left(e+f\,x\right)^{2}+1\right)}+\frac{b\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/6 - 2*b) + tan(e + f*x)^5*
((11*a)/16 - (9*b)/8) + tan(e + f*x)*((5*a)/16 - (7*b)/8))/(f*(3*tan(e + f*
x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))/f
```

3.9 $\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx$

Optimal. Leaf size=70

$$\frac{3}{8}(a-4b)x-\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*sin(
f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]

time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4217, 466, 1171, 396, 209}

$$-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-4ax^2-4bx^4}{(1+x^2)^2}\,dx, x, \tan(e+f$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\text{s}}{4f}$$

Mathematica [A]
time = 0.23, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(e+fx) - 8(a-b)\sin(2(e+fx)) + a\sin(4(e+fx)) + 32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)] +
 32*b*Tan[e + f*x])/(32*f)
```

Maple [A]
time = 0.08, size = 92, normalized size = 1.31

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{3}(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^{5}(fx+e)}{\cos(fx+e)}+\left(\sin^{3}(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}}{f}$
default	$\frac{a\left(-\frac{\left(\sin^{3}(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^{5}(fx+e)}{\cos(fx+e)}+\left(\sin^{3}(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}}{f}$
risch	$\frac{3ax}{8}-\frac{3xb}{2}+\frac{i e^{2i(fx+e)}a}{8f}-\frac{i e^{2i(fx+e)}b}{8f}-\frac{i e^{-2i(fx+e)}a}{8f}+\frac{i e^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e)a}{32f}$
norman	$\left(-\frac{3a}{8}+\frac{3b}{2}\right)x+\left(-\frac{9a}{8}+\frac{9b}{2}\right)x\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{3a}{4}+3b\right)x\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{3a}{4}-3b\right)x\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{3a}{8}-\frac{3b}{2}\right)x($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e
))
```

Maxima [A]
time = 0.49, size = 88, normalized size = 1.26

$$\frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-4\,b)\tan(fx+e)^{3}+(3\,a-4\,b)\tan(fx+e)}{\tan(fx+e)^{4}+2\,\tan(fx+e)^{2}+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^3
 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 1.32, size = 73, normalized size = 1.04

$$\frac{3\,(a-4\,b)fx\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^{4}-(5\,a-4\,b)\cos\left(fx+e\right)^{2}+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(f
*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\sin^{4}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [A]
time = 0.46, size = 82, normalized size = 1.17

$$\frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{5\,a\tan(fx+e)^{3}-4\,b\tan(fx+e)^{3}+3\,a\tan(fx+e)-4\,b\tan(fx+e)}{\left(\tan(fx+e)^{2}+1\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*t
an(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2
)/f
```

Mupad [B]
time = 4.40, size = 79, normalized size = 1.13

$$x\left(\frac{3\,a}{8}-\frac{3\,b}{2}\right)-\frac{\left(\frac{5\,a}{8}-\frac{b}{2}\right)\,\tan(e+f\,x)^{3}+\left(\frac{3\,a}{8}-\frac{b}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^{4}+2\tan\left(e+f\,x\right)^{2}+1\right)}+\frac{b\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 - (3*b)/2) - (tan(e + f*x)^3*((5*a)/8 - b/2) + tan(e + f*x)*((3*
a)/8 - b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(e + f*x))
/f
```

3.10 $\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx$

Optimal. Leaf size=42

$$\frac{1}{2}(a-2b)x-\frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 1/2*(a-2*b)*x-1/2*a*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4217, 466, 396, 209}

$$\frac{1}{2}x(a-2b)-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] ((a - 2*b)*x)/2 - (a*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$
$$= \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 1.29

$$\frac{a(e+fx)}{2f} - \frac{b\text{ArcTan}(\tan(e+fx))}{f} - \frac{a\sin(2(e+fx))}{4f} + \frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
[Out] (a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4*
f) + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.05, size = 46, normalized size = 1.10

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
risch	$\frac{ax}{2} - xb + \frac{i e^{2i(fx+e)}a}{8f} - \frac{i e^{-2i(fx+e)}a}{8f} + \frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$

norman	$\frac{\left(-\frac{a}{2}+b\right)x+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(a-2b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\left(-\frac{a}{2}+b\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{a}{2}-b\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{a}{2}-b\right)x}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.48, size = 51, normalized size = 1.21

$$\frac{(fx+e)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

Fricas [A]
time = 1.94, size = 54, normalized size = 1.29

$$\frac{(a-2\,b)fx\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
[Out] 1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(f
*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.12

$$\frac{(fx+e)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

Mupad [B]
time = 4.24, size = 35, normalized size = 0.83

$$\frac{b\tan(e+f\,x)-\frac{a\,\sin(2\,e+2\,f\,x)}{4}+f\,x\,\left(\frac{a}{2}-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x) - (a*sin(2*e + 2*f*x))/4 + f*x*(a/2 - b))/f
```

3.11 $\int (a+b\sec^2(e+fx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3852, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.04, size = 16, normalized size = 1.07

method	result	size
default	$ax+\frac{b\tan\left(fx+e\right)}{f}$	16
derivativedivides	$\frac{\left(fx+e\right)a+b\tan\left(fx+e\right)}{f}$	21
risch	$ax+\frac{2ib}{f\left(\mathrm{e}^{2i\left(fx+e\right)}+1\right)}$	25
norman	$\frac{ax\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.07

$$ax+\frac{b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 3.84, size = 34, normalized size = 2.27

$$\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [**A**]
time = 0.48, size = 15, normalized size = 1.00

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [**B**]
time = 4.30, size = 17, normalized size = 1.13

$$\frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+b)*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 1.38

$$-\frac{a\cot(e+fx)}{f}-\frac{b\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]

[Out] -((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f

Maple [A]
time = 0.06, size = 43, normalized size = 1.65

method	result	size
derivativedivides	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
default	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
risch	$-\frac{2i\left(a\,\mathrm{e}^{2i(fx+e)}+a+2b\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}-1\right)}$	49
norman	$\frac{\frac{a+b}{2f}+\frac{(a+b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{(a+3b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-cot(f*x+e)*a+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))

Maxima [A]
time = 0.27, size = 28, normalized size = 1.08

$$\frac{b\tan(fx+e)-\frac{a+b}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")

[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f

Fricas [A]
time = 1.99, size = 42, normalized size = 1.62

$$-\frac{(a+2\,b)\cos(fx+e)^2-b}{f\cos(fx+e)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\csc^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [A]
time = 0.46, size = 26, normalized size = 1.00

$$\frac{b\tan\left(fx + e\right) - \frac{a+b}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Mupad [B]
time = 4.21, size = 28, normalized size = 1.08

$$\frac{b\tan(e + f\,x)}{f} - \frac{a + b}{f\tan\left(e + f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x))/f - (a + b)/(f*tan(e + f*x))
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=46

$$-\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+2*b)*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 459}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((a + b)*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + 
 f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{a+2b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 84, normalized size = 1.83

$$-\frac{2a\cot(e+fx)}{3f}-\frac{5b\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc[
e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*x]
)/f
```

Maple [A]
time = 0.07, size = 73, normalized size = 1.59

method	result	size
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$	73
default	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$	73
risch	$\frac{4i\left(3a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+8b\,e^{2i(fx+e)}-a-4b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)}$	76
norman	$\frac{\frac{a+b}{24f}+\frac{(a+b)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{3(a+5b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{(2a+5b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}+\frac{(2a+5b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4
/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 1.00

$$\frac{3\,b\tan(fx+e)-\frac{3\,(a+2\,b)\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^3
)/f
```

Fricas [A]
time = 1.20, size = 71, normalized size = 1.54

$$-\frac{2(a+4b)\cos(fx+e)^4-3(a+4b)\cos(fx+e)^2+3b}{3\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)\csc^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [A]
time = 0.47, size = 50, normalized size = 1.09

$$\frac{3b\tan(fx+e)-\frac{3a\tan(fx+e)^2+6b\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/t
an(f*x + e)^3)/f
```

Mupad [B]
time = 4.31, size = 46, normalized size = 1.00

$$\frac{b\tan(e+fx)}{f}-\frac{(a+2b)\tan(e+fx)^2+\frac{a}{3}+\frac{b}{3}}{f\tan(e+fx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^4,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/3 + b/3 + tan(e + f*x)^2*(a + 2*b))/(f*tan(e + f*x)
^3)
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=68

$$-\frac{(a+3b)\cot(e+fx)}{f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+3*b)*cot(f*x+e)/f-1/3*(2*a+3*b)*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4217, 459}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 3*b)*Cot[e + f*x])/f) - ((2*a + 3*b)*Cot[e + f*x]^3)/(3*f) - ((a +
b)*Cot[e + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^6}+\frac{2a+3b}{x^4}+\frac{a+3b}{x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+3b)\cot(e+fx)}{f} - \frac{(2a+3b)\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e}{5f}$$

Mathematica [A]
time = 0.04, size = 128, normalized size = 1.88

$$-\frac{8a\cot(e+fx)}{15f}-\frac{11b\cot(e+fx)}{5f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}-\frac{3b\cot(e+fx)\csc^2(e+fx)}{5f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}-\frac{b\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]*
Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot[e
 + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (b*
Tan[e + f*x])/f
```

Maple [A]
time = 0.08, size = 101, normalized size = 1.49

method	result
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-24b\,e^{2i(fx+e)}+a+6b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}+1\right)}$
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos}\right.}{f}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos}\right.}{f}$
norman	$\frac{\frac{a+b}{160f}+\frac{(a+b)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{160f}-\frac{5(a+7b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}+\frac{5(5a+21b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96f}+\frac{5(5a+21b)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96f}+\frac{(11a+21b)(}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/5/sin(f*
x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+e)-16/
5*cot(f*x+e)))
```

Maxima [A]
time = 0.27, size = 68, normalized size = 1.00

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+3\,b)\tan(fx+e)^4+5\,(2\,a+3\,b)\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan(
f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Fricas [A]
time = 1.20, size = 98, normalized size = 1.44

$$-\frac{8\,(a+6\,b)\cos\left(fx+e\right)^6-20\,(a+6\,b)\cos\left(fx+e\right)^4+15\,(a+6\,b)\cos\left(fx+e\right)^2-15\,b}{15\,\left(f\cos\left(fx+e\right)^5-2\,f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a + 6
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 76, normalized size = 1.12

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan(fx+e)^4+45\,b\tan(fx+e)^4+10\,a\tan(fx+e)^2+15\,b\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Mupad [B]
time = 4.52, size = 60, normalized size = 0.88

$$\frac{b\tan(e+f\,x)}{f}-\frac{(a+3\,b)\,\tan(e+f\,x)^4+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^2+\frac{a}{5}+\frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/sin(e + f*x)^6,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/5 + b/5 + tan(e + f*x)^2*((2*a)/3 + b) + tan(e + f*
x)^4*(a + 3*b))/(f*tan(e + f*x)^5)
```

3.15 $\int (a+b\sec^2(e+fx))^2 \sin^5(e+fx)\,dx$

Optimal. Leaf size=97

$$-\frac{(a^2-4ab+b^2)\cos(e+fx)}{f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a^2-4*a*b+b^2)*cos(f*x+e)/f+2/3*a*(a-b)*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e)
^5/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4218, 459}

$$-\frac{(a^2-4ab+b^2)\cos(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -(((a^2 - 4*a*b + b^2)*Cos[e + f*x])/f) + (2*a*(a - b)*Cos[e + f*x]^3)/(3*f
) - (a^2*Cos[e + f*x]^5)/(5*f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e
+ f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^2\sin^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(-4a+b)}{a^2}\right)+\frac{b^2}{x^4}+\frac{2(a-b)b}{x^2}-2a(a-b)x^2+a^2 \ldots\right.}{f}$$
$$= -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}-\frac{a^2\cos \ldots}{}$$

Mathematica [A]
time = 0.43, size = 118, normalized size = 1.22

$$-\frac{(425a^2-4400ab+2000b^2+24(22a^2-215ab+120b^2)\cos(2(e+fx))+12(7a^2-60ab+20b^2)\cos(4(e+fx))-16a^2\cos(6(e+fx))+40ab\cos(6(e+fx))+3a^2\cos(8(e+fx)))\sec^3(e+fx)}{1920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -1/1920*((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*C
os[2*(e + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*Co
s[6*(e + f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e +
f*x]^3)/f
```

Maple [A]
time = 0.12, size = 155, normalized size = 1.60

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)+b^2 \ldots}{f}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)+b^2 \ldots}{f}$
norman	$\frac{\frac{16a^2-160ab+80b^2}{15f}+\frac{16\left(5a^2-2ab-7b^2\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{2\left(16a^2-160ab+80b^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15f}-\frac{2\left(16a^2-160ab+80b^2\right)\left(\tan^4\left(\ldots\right)\right)}{15f}\ldots}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\ldots\right)}$
risch	$-\frac{a^2 e^{5i(fx+e)}}{160f}+\frac{5\,e^{3i(fx+e)}a^2}{96f}-\frac{e^{3i(fx+e)}ab}{12f}-\frac{5\,e^{i(fx+e)}a^2}{16f}+\frac{7\,e^{i(fx+e)}ab}{4f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{5\,e^{-i(fx+e)}a^2}{16f}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*s
in(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*sin(
f*x+e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.28, size = 94, normalized size = 0.97

$$-\frac{3\,a^2\cos\left(fx+e\right)^5-10\,\left(a^2-ab\right)\cos\left(fx+e\right)^3+15\,\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)-\frac{5\,\left(6\,\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Fricas [A]
time = 4.64, size = 95, normalized size = 0.98

$$-\frac{3\,a^2\cos\left(fx+e\right)^8-10\,\left(a^2-ab\right)\cos\left(fx+e\right)^6+15\,\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-30\,\left(ab-b^2\right)\cos\left(fx+e\right)^2-5\,b^2}{15\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(f*
x + e)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(91) = 182.
time = 0.58, size = 414, normalized size = 4.27

$$\frac{2\left(\frac{5\left(6\,ab-5\,b^2+\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}+\frac{8\,a^2-50\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{220\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{60\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{320\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{180\,ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{60\,b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{30\,ab(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{15\,b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(5*(6*a*b - 5*b^2 + 12*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3 + (8*a^2 - 50*a*b + 15*b^2 - 40*a^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 220*a*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 320*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 30*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*
(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 1)^5)/f
```

Mupad [B]
time = 4.30, size = 87, normalized size = 0.90

$$\frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-2\,b^2\right)}{\cos(e+f\,x)^3}-\cos\left(e+f\,x\right)\,\left(a^2-4\,a\,b+b^2\right)-\frac{a^2\,\cos(e+f\,x)^5}{5}+\frac{2\,a\,\cos(e+f\,x)^3\,\left(a-b\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - 2*b^2))/cos(e + f*x)^3 - cos(e + f*x)*(a^
2 - 4*a*b + b^2) - (a^2*cos(e + f*x)^5)/5 + (2*a*cos(e + f*x)^3*(a - b))/3)
/f
```

3.16 $\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx$

Optimal. Leaf size=72

$$-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a*(a-2*b)*cos(f*x+e)/f+1/3*a^2*cos(f*x+e)^3/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b
^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4218, 459}

$$\frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cos[e + f*x])/f) + (a^2*Cos[e + f*x]^3)/(3*f) + ((2*a - b)*b
*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+f}{f}$$

Mathematica [A]
time = 0.32, size = 83, normalized size = 1.15

$$\frac{(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cos(2(e+fx))-6a(a-4b)\cos(4(e+fx))+a^2\cos(6(e+fx)))\sec^3(e+fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x)
] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/
(96*f)
```

Maple [A]
time = 0.08, size = 125, normalized size = 1.74

method	result
derivativedivides	$\frac{-\frac{a^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2ab\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+(2+\sin^2(fx+e))\cos(fx+e)\right)+b^2\left(\frac{\sin^4(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{3\cos(fx+e)}-\frac{(2+\sin^2}{}\right.}{f}$
default	$\frac{-\frac{a^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2ab\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+(2+\sin^2(fx+e))\cos(fx+e)\right)+b^2\left(\frac{\sin^4(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{3\cos(fx+e)}-\frac{(2+\sin^2}{}\right.}{f}$
norman	$\frac{\frac{4a^2-24ab+4b^2}{3f}+\frac{32(a^2-b^2)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{(4a^2+8ab+4b^2)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{(8a^2-16ab+8b^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{3\,e^{i(fx+e)}a^2}{8f}+\frac{e^{i(fx+e)}ab}{f}-\frac{3\,e^{-i(fx+e)}a^2}{8f}+\frac{e^{-i(fx+e)}ab}{f}+\frac{e^{-3i(fx+e)}a^2}{24f}-\frac{2b(-6a\,e^{5i(fx+e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2
+sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*sin(f*x+e)^4/cos(f*x+e)^3-1/3*sin(f*x+e
)^4/cos(f*x+e)-1/3*(2+sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.99

$$\frac{a^2\cos\left(fx+e\right)^3-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-b^2\right)\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*c
os(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A]
time = 1.82, size = 71, normalized size = 0.99

$$\frac{a^2\cos\left(fx+e\right)^6-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sin^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

Giac [A]
time = 0.52, size = 91, normalized size = 1.26

$$\frac{6\,ab\cos\left(fx+e\right)^2-3\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}+\frac{a^2f^{11}\cos\left(fx+e\right)^3-3\,a^2f^{11}\cos\left(fx+e\right)+6\,abf^{11}\cos\left(fx+e\right)}{3\,f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 6*a*b*f^11*cos(f
*x + e))/f^12
```

Mupad [B]
time = 4.14, size = 66, normalized size = 0.92

$$\frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-b^2\right)}{\cos(e+f\,x)^3}+\frac{a^2\,\cos(e+f\,x)^3}{3}-a\,\cos\left(e+f\,x\right)\,\left(a-2\,b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((b^2/3 + cos(e + f*x)^2*(2*a*b - b^2))/cos(e + f*x)^3 + (a^2*cos(e + f*x)^
3)/3 - a*cos(e + f*x)*(a - 2*b))/f
```

3.17 $\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a^2*cos(f*x+e)/f+2*a*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {4218, 276}

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] -((a^2*Cos[e + f*x])/f) + (2*a*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*
f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec^2(e+fx))^2\sin(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 1.63

$$\frac{4(b+a\cos^2(e+fx))^2\left(b^2+6ab\cos^2(e+fx)-3a^2\cos^4(e+fx)\right)\sec^3(e+fx)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
[Out] (4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*x
]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\frac{b^2(\sec^3(fx+e))}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$	42
default	$\frac{\frac{b^2(\sec^3(fx+e))}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$	42
risch	$-\frac{e^{i(fx+e)}a^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{4b\left(3a\,e^{5i(fx+e)}+6a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	104
norman	$\frac{\frac{6a^2-12ab-2b^2}{3f}+\frac{2(3a^2+2ab-b^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(a^2+2ab+b^2)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(9a^2-6ab+b^2)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*b^2*sec(f*x+e)^3+2*a*b*sec(f*x+e)-a^2/sec(f*x+e))
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.98

$$-\frac{3\,a^2\cos(fx+e)-\frac{6\,ab}{\cos(fx+e)}-\frac{b^2}{\cos(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
[Out] -1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

Fricas [A]
time = 1.19, size = 47, normalized size = 1.02

$$-\frac{3\,a^2\cos(fx+e)^4-6\,ab\cos(fx+e)^2-b^2}{3\,f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{2}\sin\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x), x)
```

Giac [A]
time = 0.51, size = 44, normalized size = 0.96

$$-\frac{a^{2}\cos\left(fx + e\right)}{f} + \frac{6\,ab\cos\left(fx + e\right)^{2} + b^{2}}{3\,f\cos\left(fx + e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -a^2*cos(f*x + e)/f + 1/3*(6*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Mupad [B]
time = 0.06, size = 45, normalized size = 0.98

$$\frac{\frac{b^{2}}{3} + 2\,a\,b\cos\left(e + f\,x\right)^{2}}{f\cos\left(e + f\,x\right)^{3}} - \frac{a^{2}\,\cos\left(e + f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2/3 + 2*a*b*cos(e + f*x)^2)/(f*cos(e + f*x)^3) - (a^2*cos(e + f*x))/f
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*arctanh(cos(f*x+e))/f+b*(2*a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/
f
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4218, 472, 213}

$$\frac{b(2a+b)\sec(e+fx)}{f}-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cos[e + f*x]])/f) + (b*(2*a + b)*Sec[e + f*x])/f + (b^
2*Sec[e + f*x]^3)/(3*f)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x^2}\right.}{f}$$

$$= -\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.
time = 0.37, size = 108, normalized size = 2.08

$$-\frac{4(b+a\cos^2(e+fx))^2\left(-b^2-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec^3(e+fx)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e
+ f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.08, size = 94, normalized size = 1.81

method	result
derivativedivides	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)\right.}{f}$
default	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)\right.}{f}$
norman	$\frac{\frac{(8ab+4b^2)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{12ab+8b^2}{3f}-\frac{(4ab+4b^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}+\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$
risch	$\frac{2b\left(6a\,e^{5i(fx+e)}+3b\,e^{5i(fx+e)}+12a\,e^{3i(fx+e)}+10b\,e^{3i(fx+e)}+6a\,e^{i(fx+e)}+3b\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{\ln\left(e^{i(fx+e)}-1\right)a^2}{f}+\frac{2\ln(e^{i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*ln(csc(f*x+e)-cot(f*x+e))+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*
x+e)))+b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.65

$$-\frac{3\left(a^{2}+2\,ab+b^{2}\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^{2}+2\,ab+b^{2}\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}\right)}{\cos\left(fx+e\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*l
og(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(53) = 106.
time = 2.01, size = 107, normalized size = 2.06

$$-\frac{3\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{3}\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-3\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{3}\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-6\left(2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}-2\,b^{2}}{6\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*
(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b
+ b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(50) = 100.
time = 0.52, size = 172, normalized size = 3.31

$$\frac{3\left(a^{2}+2\,ab+b^{2}\right)\log\left(\frac{\left|-\cos\left(fx+e\right)+1\right|}{\left|\cos\left(fx+e\right)+1\right|}\right)+\frac{8\left(3\,ab+2\,b^{2}+\frac{6\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,b^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,ab\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\,b^{2}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)
) + 8*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B]
time = 0.12, size = 53, normalized size = 1.02

$$\frac{\cos(e+f\,x)^2\,\left(b^2+2\,a\,b\right)+\frac{b^2}{3}}{f\cos(e+f\,x)^3}-\frac{\mathrm{atanh}(\cos(e+f\,x))\,\left(a+b\right)^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x),x)
```

```
[Out] (cos(e + f*x)^2*(2*a*b + b^2) + b^2/3)/(f*cos(e + f*x)^3) - (atanh(cos(e +
f*x))*(a + b)^2)/f
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=104

$$-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}$$

```
[Out] -1/2*(a+b)*(a+5*b)*arctanh(cos(f*x+e))/f-1/6*(3*a^2+6*a*b+5*b^2)*cot(f*x+e)
*csc(f*x+e)/f+1/3*b*(6*a+5*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^2*sec(f*x+e)^
3/f
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 473, 467, 464, 212}

$$-\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)*(a + 5*b)*ArcTanh[Cos[e + f*x]])/f - ((3*a^2 + 6*a*b + 5*b^2)
*Cot[e + f*x]*Csc[e + f*x])/(6*f) + (b*(6*a + 5*b)*Sec[e + f*x])/(3*f) + (b
^2*Csc[e + f*x]^2*Sec[e + f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2(1-x^2)^2}\,dx,x,\cos(e\right.}{3f}$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b^2\csc^2(e+fx)\sec}{3f}$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+}{3f}$$

$$= -\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{(3a^2+6ab+5b^2)\cot(e}{6f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1021 vs. $2(104)=208$.
time = 6.42, size = 1021, normalized size = 9.82

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*
x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2)*
Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*
b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Log[Si
n[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^2)^2)/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e + f*x]^
4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[(f*x)/2])
/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(
b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (2*Cos[e + f*
x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*Sin[(f*x)/2]))/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x
)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^
2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[
e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*
f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2)
- (2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*
Sin[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*
(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

Maple [A]
time = 0.10, size = 163, normalized size = 1.57

method	result
derivativedivides	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+}{2}\right.}{f}$
default	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+}{2}\right.}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{8f}+\frac{(a^2+2ab+b^2)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{(7a^2+46ab+55b^2)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{(9a^2+66ab+65b^2)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{(11a^2}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
risch	$\frac{3a^2e^{9i(fx+e)}+18ab\,e^{9i(fx+e)}+15b^2e^{9i(fx+e)}+12a^2e^{7i(fx+e)}+24ab\,e^{7i(fx+e)}+20b^2e^{7i(fx+e)}+18a^2e^{5i(fx+e)}+12ab\,e^{5i}}{3f\left(e^{2i(fx+e)}+1\right)^3(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*(
-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e)))+
```

```
b^2*(1/3/sin(f*x+e)^2/cos(f*x+e)^3-5/6/sin(f*x+e)^2/cos(f*x+e)+5/2/cos(f*x+
e)+5/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.28, size = 132, normalized size = 1.27

$$-\frac{3\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^{2}+6\,ab+5\,b^{2}\right)\cos(fx+e)^{4}-2\left(6\,ab+5\,b^{2}\right)\cos(fx+e)^{2}-2\,b^{2}\right)}{\cos(fx+e)^{5}-\cos(fx+e)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*b
^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2*(
6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Fricas [A]
time = 1.22, size = 203, normalized size = 1.95

$$\frac{6\left(a^{2}+6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{4}-4\left(6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\,b^{2}-3\left(\left(a^{2}+6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{5}-\left(a^{2}+6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{3}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+3\left(\left(a^{2}+6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{5}-\left(a^{2}+6\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{3}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{12\left(f\cos\left(fx+e\right)^{5}-f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b
^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^2)*
cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e
) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\csc^{3}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(96) = 192.
time = 0.52, size = 341, normalized size = 3.28

$$-\frac{\frac{3\,a^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-6\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{3\left(a^{2}+2\,ab+b^{2}-\frac{2\,a^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{10\,b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{16\left(6\,ab+7\,b^{2}+\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{12\,b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{9\,b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^{3}}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 3*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^
2 + 6*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 3*(a
^2 + 2*a*b + b^2 - 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*a*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 10*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + 7*b^2 + 12*a*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 12*b^2*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^2*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ 1)^3)/f
```

Mupad [B]
time = 4.29, size = 96, normalized size = 0.92

$$\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(\frac{5\,b^2}{3}+2\,a\,b\right)-\cos\left(e+f\,x\right)^4\,\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)}{f\,\left({\cos\left(e+f\,x\right)}^3-{\cos\left(e+f\,x\right)}^5\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(a+b\right)\,\left(a+5\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (5*b^2)/3) - cos(e + f*x)^4*(3*a*b + a^2/2
 + (5*b^2)/2))/(f*(cos(e + f*x)^3 - cos(e + f*x)^5)) - (atanh(cos(e + f*x))
*(a + b)*(a + 5*b))/(2*f)
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=141

$$-\frac{(3a^2+30ab+35b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}-\frac{(3a^2+6ab+7b^2)\cot(e+}{12f}$$

```
[Out] -1/8*(3*a^2+30*a*b+35*b^2)*arctanh(cos(f*x+e))/f-1/24*(3*a+7*b)^2*cot(f*x+e
)*csc(f*x+e)/f-1/12*(3*a^2+6*a*b+7*b^2)*cot(f*x+e)*csc(f*x+e)^3/f+1/3*b*(6*
a+7*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^4*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {4218, 473, 467, 464, 212}

$$-\frac{(3a^2+30ab+35b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b(6a+7b)\sec(e+fx)}{3f}-\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}+\frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e + f*x]])/f - ((3*a + 7*b)^2*C
ot[e + f*x]*Csc[e + f*x])/(24*f) - ((3*a^2 + 6*a*b + 7*b^2)*Cot[e + f*x]*Cs
c[e + f*x]^3)/(12*f) + (b*(6*a + 7*b)*Sec[e + f*x])/(3*f) + (b^2*Csc[e + f*
x]^4*Sec[e + f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
```

```
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+7b)+3a^2x^2}{x^2(1-x^2)^3}\,dx, x, \cos(\right.}{3f}$$

$$= -\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f} + \frac{b^2\csc^4(e+fx)}{3f}$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{(3a^2+6ab+7b^2)\cot(e}{12f}$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{(3a^2+6ab+7b^2)\cot(e}{12f}$$

$$= -\frac{(3a^2+30ab+35b^2)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)^2\cot(e+}{24}$$

Mathematica [A]
time = 1.33, size = 218, normalized size = 1.55

$$-\frac{(b+a\cos^2(e+fx))^2\left((90a^2+132ab-102b^2+(6a^2+60ab+70b^2)\cos(4(e+fx))-3(3a^2+30ab+35b^2)\cos(6(e+fx)))\cot(e+fx)\csc^3(e+fx)+\frac{1}{2}(105a^2+282ab+329b^2)\left(\cos(e+fx)+\cos(3(e+fx))\right)\csc^4(e+fx)+96(3a^2+30ab+35b^2)\cos^4(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec^4(e+fx)}{192f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/192*((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 60
*a*b + 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e + f*
x)])*Cot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e +
f*x] + Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^2)*C
os[e + f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]
^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.12, size = 211, normalized size = 1.50

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2 c}\right.}{}$
default	$\frac{a^2\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2 c}\right.}{}$
norman	$\frac{\frac{a^2+2ab+b^2}{64f}+\frac{\left(a^2+2ab+b^2\right)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}+\frac{\left(5a^2+26ab+21b^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}+\frac{\left(5a^2+26ab+21b^2\right)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}-\frac{\left(3a^2+2\right.}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$
risch	$\frac{9a^2 e^{13i(fx+e)}+90ab\, e^{13i(fx+e)}+105b^2 e^{13i(fx+e)}-6a^2 e^{11i(fx+e)}-60ab\, e^{11i(fx+e)}-70b^2 e^{11i(fx+e)}-105a^2 e^{9i(fx+e)}-28}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-c
ot(f*x+e)))+2*a*b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)
+15/8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e)))+b^2*(-1/4/sin(f*x+e)^4/cos
(f*x+e)^3+7/12/sin(f*x+e)^2/cos(f*x+e)^3-35/24/sin(f*x+e)^2/cos(f*x+e)+35/8
/cos(f*x+e)+35/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.30, size = 173, normalized size = 1.23

$$-\frac{3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^2+30ab+35b^2\right)\cos(fx+e)^6-5\left(3a^2+30ab+35b^2\right)\cos(fx+e)^4+8\left(6ab+7b^2\right)\cos(fx+e)^2+8b^2\right)}{\cos(fx+e)^7-2\cos(fx+e)^5+\cos(fx+e)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a*
b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x
```

```
+ e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*cos
(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/
f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(139) = 278$.
time = 1.29, size = 300, normalized size = 2.13

$$\frac{6\,(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^6-10\,(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^4+16\,(6\,ab+7\,b^2)\cos(fx+e)^2+16\,b^2-3\left((3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^7-2\,(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^5+(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^3\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+3\left((3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^7-2\,(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^5+(3\,a^2+30\,ab+35\,b^2)\cos(fx+e)^3\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{48\,(f\cos(fx+e)^7-2\,f\cos(fx+e)^5+f\cos(fx+e)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35*
b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^
2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x +
 e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/
2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b
^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1/2*cos
(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(131) = 262$.
time = 0.55, size = 493, normalized size = 3.50

$$-\frac{\frac{24\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{96\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{72\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-12\,(3\,a^2+30\,ab+35\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{3\left(a^2+2\,ab+b^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{32\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{180\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{210\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\frac{256\left(3\,ab+5\,b^2+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{9\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{6\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 96*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 72*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3
*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 6*a*b*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1
2*(3*a^2 + 30*a*b + 35*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1
```

```
)) + 3*(a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3
2*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*
b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 210*b^2*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 - 256*(3*a*
b + 5*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 9*b^2*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 1)^3)/f
```

Mupad [B]
time = 4.42, size = 135, normalized size = 0.96

$$\frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(\frac{7\,b^2}{3}+2\,a\,b\right)+\cos\left(e+f\,x\right)^6\,\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)-\cos\left(e+f\,x\right)^4\,\left(\frac{5\,a^2}{8}+\frac{25\,a\,b}{4}+\frac{175\,b^2}{24}\right)}{f\,\left(\cos\left(e+f\,x\right)^7-2\,\cos\left(e+f\,x\right)^5+\cos\left(e+f\,x\right)^3\right)}-\frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

```
[Out] (b^2/3 + cos(e + f*x)^2*(2*a*b + (7*b^2)/3) + cos(e + f*x)^6*((15*a*b)/4 +
(3*a^2)/8 + (35*b^2)/8) - cos(e + f*x)^4*((25*a*b)/4 + (5*a^2)/8 + (175*b^2
)/24))/(f*(cos(e + f*x)^3 - 2*cos(e + f*x)^5 + cos(e + f*x)^7)) - (atanh(co
s(e + f*x))*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/8))/f
```

3.21 $\int (a+b\sec^2(e+fx))^2 \sin^6(e+fx)\,dx$

Optimal. Leaf size=148

$$\frac{5}{16}\left(a^2-12ab+8b^2\right)x-\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f}+\frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f}-$$

```
[Out] 5/16*(a^2-12*a*b+8*b^2)*x-1/16*(3*a^2-36*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)/f
+1/24*a*(a-12*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*(a^2-12*a*b+12*b^2)*tan(f*x+
e)/f+1/6*a^2*sin(f*x+e)^6*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 474, 466, 1828, 1167, 209}

$$-\frac{(a^2-12ab+12b^2)\tan(e+fx)}{6f}-\frac{(3a^2-36ab+8b^2)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a^2-12ab+8b^2)+\frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f}+\frac{a(a-12b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] (5*(a^2 - 12*a*b + 8*b^2)*x)/16 - ((3*a^2 - 36*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(a - 12*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((
a^2 - 12*a*b + 12*b^2)*Tan[e + f*x])/(6*f) + (a^2*Sin[e + f*x]^6*Tan[e + f*
x])/(6*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
```

```
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^2}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{x^6\left(7a^2-6(a+b)^2-6b^2x^2\right)}{(1+x^2)^3}\,dx, x \ldots\right.}{6f}$$

$$= \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+f \ldots}{6f}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(\ldots}{\ldots}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(\ldots}{\ldots}$$

$$= -\frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(\ldots}{\ldots}$$

$$= \frac{5}{16}\left(a^2-12ab+8b^2\right)x - \frac{(3a^2-36ab+8b^2)\cos(e+fx)\sin(e+ \ldots}{16f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 499 vs. $2(148) = 296$.
time = 1.02, size = 499, normalized size = 3.37

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2)
*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f*x
*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e + 3
*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] + 960*
b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*b^2*S
in[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*Sin[2*
e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 1936*b^2*
Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f*x] - 14
4*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e + 5*f*x]
 - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Sin[6*e + 5*
f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b*Sin[6*e +
7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2*Sin[8*e + 9
*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.06, size = 199, normalized size = 1.34

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}\right.\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}\right.\right.}{}$
risch	$\frac{5a^2x}{16}-\frac{15xab}{4}+\frac{5x\,b^2}{2}-\frac{3ie^{4i(fx+e)}a^2}{128f}+\frac{ie^{4i(fx+e)}ab}{32f}+\frac{15ie^{2i(fx+e)}a^2}{128f}-\frac{ie^{2i(fx+e)}ab}{2f}+\frac{ie^{2i(fx+e)}b^2}{8f}-1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)+2*a*b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)
^3+15/8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^7/cos(f
*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e))
```

Maxima [A]
time = 0.49, size = 173, normalized size = 1.17

$$\frac{16\,b^2\tan(fx+e)^3+15\,(a^2-12\,ab+8\,b^2)(fx+e)+96\,(ab-b^2)\tan(fx+e)-\frac{3\,(11\,a^2-36\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2-24\,ab+6\,b^2)\tan(fx+e)^3+3\,(5\,a^2-28\,ab+8\,b^2)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*b
 - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a
^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x +
e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 1.44, size = 138, normalized size = 0.93

$$\frac{15\,(a^2-12\,ab+8\,b^2)fx\cos(fx+e)^3-\left(8\,a^2\cos(fx+e)^8-2\,(13\,a^2-12\,ab)\cos(fx+e)^6+3\,(11\,a^2-36\,ab+8\,b^2)\cos(fx+e)^4-16\,(6\,ab-7\,b^2)\cos(fx+e)^2-16\,b^2\right)\sin(fx+e)}{48\,f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
- 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*x
+ e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*cos(f
*x + e)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.59, size = 183, normalized size = 1.24

$$\frac{16\,b^2\tan\left(fx+e\right)^3+96\,ab\tan\left(fx+e\right)-96\,b^2\tan\left(fx+e\right)+15\left(a^2-12\,ab+8\,b^2\right)(fx+e)-\frac{33\,a^2\tan(fx+e)^5-108\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3-192\,ab\tan(fx+e)^3+48\,b^2\tan(fx+e)^3+15\,a^2\tan(fx+e)-84\,ab\tan(fx+e)+24\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) + 1
5*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan(f
*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(f*x
 + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x + e)
 + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B]
time = 4.79, size = 163, normalized size = 1.10

$$x\left(\frac{5\,a^2}{16}-\frac{15\,a\,b}{4}+\frac{5\,b^2}{2}\right)-\frac{\left(\frac{11\,a^2}{16}-\frac{9\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}-4\,a\,b+b^2\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a^2}{16}-\frac{7\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}+\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\,\left(4\,b^2-2\,b\,(a+b)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((5*a^2)/16 - (15*a*b)/4 + (5*b^2)/2) - (tan(e + f*x)*((5*a^2)/16 - (7*a*
b)/4 + b^2/2) + tan(e + f*x)^3*((5*a^2)/6 - 4*a*b + b^2) + tan(e + f*x)^5*(
(11*a^2)/16 - (9*a*b)/4 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 +
 tan(e + f*x)^6 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(4*b^2 -
 2*b*(a + b)))/f
```

3.22 $\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx$

Optimal. Leaf size=114

$$\frac{1}{8}\left(3a^2-24ab+8b^2\right)x-\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f}+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}$$

```
[Out] 1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(a^2-8
*a*b+4*b^2)*tan(f*x+e)/f+1/4*a^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b^2*tan(f*x+
e)^3/f
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 474, 466, 1167, 209}

$$-\frac{(a^2-8ab+4b^2)\tan(e+fx)}{4f}+\frac{1}{8}x(3a^2-24ab+8b^2)+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{a(a-8b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8
*f) - ((a^2 - 8*a*b + 4*b^2)*Tan[e + f*x])/(4*f) + (a^2*Sin[e + f*x]^4*Tan[
e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{\text{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2-4b^2x^2\right)}{\left(1+x^2\right)^2}\,dx,x$$

$$=-\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2\sin^4(e+fx)\tan(e+f$$

$$=-\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2\sin^4(e+fx)\tan(e+f$$

$$=-\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{\left(a^2-8ab+4b^2\right)\tan(e+$$

$$=\frac{1}{8}\left(3a^2-24ab+8b^2\right)x-\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{(a$$

Mathematica [A]
time = 1.15, size = 153, normalized size = 1.34

$$\frac{\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(32b^2\sec(e)\sin(fx)+64(3a-2b)b\cos^2(e+fx)\sec(e)\sin(fx)+3\cos^3(e+fx)\left(4\left(3a^2-24ab+8b^2\right)fx-8a(a-2b)\sin(2(e+fx))+a^2\sin(4(e+fx))\right)+32b^2\cos(e+fx)\tan(e)\right)}{24f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
- 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24*a
*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)]) +
32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.10, size = 123, normalized size = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\cdots}{f}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\cdots}{f}$
risch	$\frac{3a^2x}{8}-3xab+x\,b^2-\frac{i\mathrm{e}^{4i(fx+e)}a^2}{64f}+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{2i(fx+e)}ab}{4f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^2}{8f}+\frac{i\mathrm{e}^{-2i(fx+e)}ab}{4f}+\cdots$
norman	$\frac{\left(-\frac{3}{8}a^2+3ab-b^2\right)x+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{3}{8}a^2+3ab-b^2\right)x\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x
-3/2*e)+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

Maxima [A]
time = 0.49, size = 127, normalized size = 1.11

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-24\,ab+8\,b^2\right)(fx+e)+24\left(2\,ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-8\,ab\right)\tan(fx+e)^3+\left(3\,a^2-8\,ab\right)\tan(fx+e)\right)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*a
*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a*b
)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 1.25, size = 113, normalized size = 0.99

$$\frac{3\,(3\,a^2-24\,ab+8\,b^2)fx\cos\left(fx+e\right)^3+\left(6\,a^2\cos\left(fx+e\right)^6-3\,(5\,a^2-8\,ab)\cos\left(fx+e\right)^4+16\,(3\,ab-2\,b^2)\cos\left(fx+e\right)^2+8\,b^2\right)\sin\left(fx+e\right)}{24\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^6
 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 + 8
*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 123, normalized size = 1.08

$$\frac{8\,b^2\tan\left(fx+e\right)^3+48\,ab\tan\left(fx+e\right)-24\,b^2\tan\left(fx+e\right)+3\,(3\,a^2-24\,ab+8\,b^2)(fx+e)-\frac{3\left(5\,a^2\tan(fx+e)^3-8\,ab\tan(fx+e)^3+3\,a^2\tan(fx+e)-8\,ab\tan(fx+e)\right)}{\left(\tan(fx+e)^2+1\right)^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3*
(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(f*
x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)
/f
```

Mupad [B]
time = 4.33, size = 116, normalized size = 1.02

$$x\left(\frac{3\,a^2}{8}-3\,a\,b+b^2\right)+\frac{\left(a\,b-\frac{5\,a^2}{8}\right)\,\tan(e+f\,x)^3+\left(a\,b-\frac{3\,a^2}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}+\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\,\left(3\,b^2-2\,b\,(a+b)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - 3*a*b + b^2) + (tan(e + f*x)*(a*b - (3*a^2)/8) + tan(e + f*x
)^3*(a*b - (5*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b^2*t
an(e + f*x)^3)/(3*f) - (tan(e + f*x)*(3*b^2 - 2*b*(a + b)))/f
```

3.23 $\int (a+b\sec^2(e+fx))^2\sin^2(e+fx)\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}a(a-4b)x-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a*(a-4*b)*x-1/2*a*(a-4*b)*tan(f*x+e)/f+1/2*a^2*sin(f*x+e)^2*tan(f*x+e)/
f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {4217, 474, 470, 327, 209}

$$\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] (a*(a - 4*b)*x)/2 - (a*(a - 4*b)*Tan[e + f*x])/(2*f) + (a^2*Sin[e + f*x]^2*
Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{\text{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2-2b^2x^2\right)}{1+x^2}\,dx,x\right.}{2f}$$
$$=\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}-\frac{(a(a-4b))\text{Sub}}{}$$
$$=-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3}{}$$
$$=\frac{1}{2}a(a-4b)x-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{a^2\sin^2(e+fx)\tan(e+}{2f}$$

Mathematica [A]
time = 0.68, size = 126, normalized size = 1.73

$$-\frac{\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(-4b^2\sec(e)\sin(fx)-4(6a-b)b\cos^2(e+fx)\sec(e)\sin(fx)+3a\cos^3(e+fx)(-2(a-4b)fx+a\sin(2(e+fx)))-4b^2\cos(e+fx)\tan(e)\right)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] -1/3*((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*(
6*a - b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4*b
```

```
)*f*x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(f*(a + 2*b + a*C
os[2*(e + f*x)])^2)
```

Maple [A]
time = 0.08, size = 71, normalized size = 0.97

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$
default	$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$
risch	$\frac{a^2x}{2}-2xab+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^2}{8f}-\frac{2ib\left(-6a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-12a\,\mathrm{e}^{2i(fx+e)}-6a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
norman	$\left(-\frac{1}{2}a^2+2ab\right)x+\left(-\frac{1}{2}a^2+2ab\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{1}{2}a^2-2ab\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{1}{2}a^2-2ab\right)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-a^2+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-e
)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

Maxima [A]
time = 0.49, size = 72, normalized size = 0.99

$$\frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\left(a^2-4\,ab\right)(fx+e)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 7.33, size = 86, normalized size = 1.18

$$\frac{3\left(a^2-4\,ab\right)fx\cos\left(fx+e\right)^3-\left(3\,a^2\cos\left(fx+e\right)^4-2\left(6\,ab-b^2\right)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
- b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Giac [A]
time = 0.49, size = 67, normalized size = 0.92

$$\frac{2\,b^2\tan\left(fx+e\right)^3 + 12\,ab\tan\left(fx+e\right) + 3\left(a^2-4\,ab\right)(fx+e) - \frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B]
time = 4.46, size = 94, normalized size = 1.29

$$\frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f} - \frac{\tan(e+f\,x)\,\left(2\,b^2-2\,b\,(a+b)\right)}{f} - \frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a-4\,b)}{2\left(2\,a\,b-\frac{a^2}{2}\right)}\right)\,(a-4\,b)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (a^2*sin(2*e + 2*f*x))/(4*f) - (tan(e + f*x)*(
2*b^2 - 2*b*(a + b)))/f - (a*atan((a*tan(e + f*x)*(a - 4*b))/(2*(2*a*b - a^
2/2)))*(a - 4*b))/(2*f)
```

3.24 $\int (a+b\sec^2(e+fx))^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx\right.}{f}$$
$$= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(40) = 80.
time = 0.26, size = 106, normalized size = 2.65

$$\frac{4\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(3a^2fx\cos^3(e+fx)+b^2\sec(e)\sin(fx)+2b(3a+b)\cos^2(e+fx)\sec(e)\sin(fx)+b^2\cos(e+fx)\tan(e)\right)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.06, size = 48, normalized size = 1.20

method	result
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}+3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^2x+3a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))

Maxima [A]
time = 0.29, size = 47, normalized size = 1.18

$$a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)b^2}{3f}+\frac{2ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f

Fricas [A]
time = 1.65, size = 62, normalized size = 1.55

$$\frac{3a^2fx\cos\left(fx+e\right)^3+\left(2\left(3ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)**2)**2,x)

[Out] Integral((a + b*sec(e + f*x)**2)**2, x)

Giac [A]
time = 0.45, size = 49, normalized size = 1.22

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6ab\tan\left(fx+e\right)+3b^2\tan\left(fx+e\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(f*x + e))/f

Mupad [B]
time = 4.33, size = 42, normalized size = 1.05

$$\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=50

$$-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)^2*cot(f*x+e)/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {4217, 276}

$$\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Cot[e + f*x])/f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e +
 f*x]^3)/(3*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx&=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(50) = 100.
time = 0.78, size = 109, normalized size = 2.18

$$\frac{4(b+a\cos^2(e+fx))^2\sec^3(e+fx)\left(b^2\sec(e)\sin(fx)+\cos^2(e+fx)\left(3(a+b)^2\cot(e+fx)\csc(e)+b(6a+5b)\sec(e)\right)\sin(fx)+b^2\cos(e+fx)\tan(e)\right)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e + f
*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] + b
^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.07, size = 96, normalized size = 1.92

method	result
derivativedivides	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
default	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
risch	$-\frac{2i\left(3a^2e^{6i(fx+e)}+9a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+9a^2e^{2i(fx+e)}+24ab\,e^{2i(fx+e)}+16b^2e^{2i(fx+e)}+3a^2+12ab+8b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}-1\right)}$
norman	$\frac{\frac{a^2+2ab+b^2}{2f}+\frac{(a^2+2ab+b^2)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{2(a^2+4ab+3b^2)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(a^2+4ab+3b^2)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{(9a^2+42ab}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3/
sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Maxima [A]
time = 0.27, size = 57, normalized size = 1.14

$$\frac{b^2\tan(fx+e)^3+6(ab+b^2)\tan(fx+e)-\frac{3(a^2+2ab+b^2)}{\tan(fx+e)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^2
)/tan(f*x + e))/f
```

Fricas [A]
time = 0.84, size = 75, normalized size = 1.50

$$-\frac{(3\,a^2+12\,ab+8\,b^2)\cos{(fx+e)}^4-2\,(3\,ab+2\,b^2)\cos{(fx+e)}^2-b^2}{3\,f\cos{(fx+e)}^3\sin{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x +
 e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2{(e+fx)}\right)^2\csc^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Giac [A]
time = 0.51, size = 60, normalized size = 1.20

$$\frac{b^2\tan{(fx+e)}^3+6\,ab\tan{(fx+e)}+6\,b^2\tan{(fx+e)}-\frac{3\,(a^2+2\,ab+b^2)}{\tan(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
+ 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B]
time = 4.40, size = 56, normalized size = 1.12

$$\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{a^2+2\,a\,b+b^2}{f\tan{(e+f\,x)}}+\frac{2\,b\tan(e+f\,x)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (2*a*b + a^2 + b^2)/(f*tan(e + f*x)) + (2*b*ta
n(e + f*x)*(a + b))/f
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{(a+b)(a+3b)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}+\frac{b(2a+3b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a+b)*(a+3*b)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f+b*(2*a+3*b)*tan(f*x+
e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4217, 459}

$$\frac{b(2a+3b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}-\frac{(a+b)(a+3b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cot[e + f*x])/f) - ((a + b)^2*Cot[e + f*x]^3)/(3*f) +
(b*(2*a + 3*b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^2}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(a+b)(a+3b)}{x^2}+b^2x^2\right)dx, x, \tan(e+\right.}{f}$$
$$= -\frac{(a+b)(a+3b)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}+\frac{b(2a+3}{}$$

Mathematica [A]

time = 0.88, size = 151, normalized size = 1.99

$$-\frac{\csc(2e)\csc^3(2(e+fx))\left(8a(a+2b)\sin(2e)-6(a+2b)^2\sin(2fx)-3a^2\sin(2(e+fx))-6ab\sin(2(e+fx))+a^2\sin(6(e+fx))+2ab\sin(6(e+fx))+3a^2\sin(4e+2fx)+a^2\sin(4e+6fx)+8ab\sin(4e+6fx)+8b^2\sin(4e+6fx)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/6*(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*S
in[2*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(e
+ f*x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e + 6
*f*x] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/f
```

Maple [A]

time = 0.08, size = 144, normalized size = 1.89

method	result
risch	$\frac{4i\left(3a^2e^{8i(fx+e)}+8a^2e^{6i(fx+e)}+16ab\,e^{6i(fx+e)}+6a^2e^{4i(fx+e)}+24ab\,e^{4i(fx+e)}+24b^2e^{4i(fx+e)}-a^2-8ab-8b^2\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^3}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{(\csc^2(fx+e))}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3}\right.}{f}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{(\csc^2(fx+e))}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3}\right.}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{24f}+\frac{(a^2+2ab+b^2)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}+\frac{(a^2+6ab+5b^2)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{(a^2+6ab+5b^2)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{(11a^2+86ab+}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*
x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e))+b^2*(1/3/sin(f*x+e)^3/cos(f*
```

```
x+e)^3-2/3/sin(f*x+e)^3/cos(f*x+e)+8/3/sin(f*x+e)/cos(f*x+e)-16/3*cot(f*x+e
)))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.11

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+4\,ab+3\,b^2\right)\tan(fx+e)^2+a^2+2\,ab+b^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
+ 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

Fricas [A]
time = 1.95, size = 107, normalized size = 1.41

$$-\frac{2\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^6-3\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+6\left(ab+b^2\right)\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos
(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 98, normalized size = 1.29

$$\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+9\,b^2\tan\left(fx+e\right)-\frac{3\,a^2\tan(fx+e)^2+12\,ab\tan(fx+e)^2+9\,b^2\tan(fx+e)^2+a^2+2\,ab+b^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a*b
 + b^2)/tan(f*x + e)^3)/f
```

Mupad [B]
time = 4.45, size = 85, normalized size = 1.12

$$\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\frac{2\,a\,b}{3}+\tan(e+f\,x)^2\,\left(a^2+4\,a\,b+3\,b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\tan\left(e+f\,x\right)^3}+\frac{b\tan(e+f\,x)\,\left(2\,a+3\,b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/3 + tan(e + f*x)^2*(4*a*b + a^2 + 3*b
^2) + a^2/3 + b^2/3)/(f*tan(e + f*x)^3) + (b*tan(e + f*x)*(2*a + 3*b))/f
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}+\frac{2b(a+2b)\tan(e+}{f}$$

```
[Out] -(a^2+6*a*b+6*b^2)*cot(f*x+e)/f-2/3*(a+b)*(a+2*b)*cot(f*x+e)^3/f-1/5*(a+b)^
2*cot(f*x+e)^5/f+2*b*(a+2*b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]

time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4217, 459}

$$-\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x])/f) - (2*(a + b)*(a + 2*b)*Cot[e + f*
x]^3)/(3*f) - ((a + b)^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x
])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^2}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(2b(a+2b)+\frac{(a+b)^2}{x^6}+\frac{2(a+b)(a+2b)}{x^4}+\frac{a^2+6ab+6b^2}{x^2}+b^2x^2\right)\right.}{f}$$

$$= -\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f} - \frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(103) = 206.
time = 1.18, size = 353, normalized size = 3.43

$$-\frac{\csc(e)\csc^5(e+fx)\sec(e)\sec^3(e+fx)\left(20a(5a+12b)\sin(2e)-32(2a^2+9ab+12b^2)\sin(2fx)-24a^2\sin(2(e+fx))-108ab\sin(2(e+fx))-54b^2\sin(2(e+fx))+8a^2\sin(4(e+fx))+36ab\sin(4(e+fx))+18b^2\sin(4(e+fx))+8a^2\sin(6(e+fx))+36ab\sin(6(e+fx))+18b^2\sin(6(e+fx))-4a^2\sin(8(e+fx))-18ab\sin(8(e+fx))-9b^2\sin(8(e+fx))+8a^2\sin(2(e+2fx))+96ab\sin(2(e+2fx))+128b^2\sin(2(e+2fx))+40a^2\sin(4e+2fx)+8a^2\sin(4e+6fx)+96ab\sin(4e+6fx)+128b^2\sin(4e+6fx)-4a^2\sin(6e+8fx)-48ab\sin(6e+8fx)-64b^2\sin(6e+8fx)\right)}{1920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/1920*(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Sin
[2*e] - 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)] -
108*a*b*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*x)]
 + 36*a*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e + f*x
)] + 36*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(e + f
*x)] - 18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2*(e +
2*f*x)] + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 40*a^2*S
in[4*e + 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x] + 128*b^
2*Sin[4*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f*x] - 64*
b^2*Sin[6*e + 8*f*x]))/f
```

Maple [A]
time = 0.07, size = 190, normalized size = 1.84

method	result
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\,\mathrm{co}}\right.}{}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\,\mathrm{co}}\right.}{}$
risch	$-\frac{16i\left(10a^2e^{10i(fx+e)}+25a^2e^{8i(fx+e)}+60ab\,e^{8i(fx+e)}+16a^2e^{6i(fx+e)}+72ab\,e^{6i(fx+e)}+96b^2e^{6i(fx+e)}-2a^2e^{4i(fx+e)}-24\right.}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/5/
sin(f*x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+
e)-16/5*cot(f*x+e))+b^2*(-1/5/sin(f*x+e)^5/cos(f*x+e)^3+8/15/sin(f*x+e)^3/c
os(f*x+e)^3-16/15/sin(f*x+e)^3/cos(f*x+e)+64/15/sin(f*x+e)/cos(f*x+e)-128/1
5*cot(f*x+e)))
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.09

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,\left(ab+2\,b^2\right)\tan\left(fx+e\right)-\frac{15\,\left(a^2+6\,ab+6\,b^2\right)\tan(fx+e)^4+10\,\left(a^2+3\,ab+2\,b^2\right)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*a
*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Fricas [A]
time = 3.47, size = 147, normalized size = 1.43

$$-\frac{8\,\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\,\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\,\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^4-10\,\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-5\,b^2}{15\,\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^2
)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b +
4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*
cos(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.55, size = 141, normalized size = 1.37

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+60\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan(fx+e)^4+90\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2+30\,ab\tan(fx+e)^2+20\,b^2\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^2 +
 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Mupad [B]
time = 4.80, size = 108, normalized size = 1.05

$$\frac{b^2\tan(e+fx)^3}{3f}-\frac{\frac{2ab}{5}+\tan(e+fx)^4\left(a^2+6ab+6b^2\right)+\frac{a^2}{5}+\frac{b^2}{5}+\tan(e+fx)^2\left(\frac{2a^2}{3}+2ab+\frac{4b^2}{3}\right)}{f\tan\left(e+fx\right)^5}+\frac{2b\tan(e+fx)\ (a+2b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/5 + tan(e + f*x)^4*(6*a*b + a^2 + 6*b
^2) + a^2/5 + b^2/5 + tan(e + f*x)^2*(2*a*b + (2*a^2)/3 + (4*b^2)/3))/(f*ta
n(e + f*x)^5) + (2*b*tan(e + f*x)*(a + 2*b))/f
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{b}\,(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{\cos^5(e+fx)}{5af}$$

```
[Out] -(a+b)^2*cos(f*x+e)/a^3/f+1/3*(2*a+b)*cos(f*x+e)^3/a^2/f-1/5*cos(f*x+e)^5/a
/f+(a+b)^2*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(7/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4218, 472, 211}

$$\frac{\sqrt{b}\,(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{\cos^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(7/2)*f) - ((
a + b)^2*Cos[e + f*x])/(a^3*f) + ((2*a + b)*Cos[e + f*x]^3)/(3*a^2*f) - Cos
[e + f*x]^5/(5*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^3} - \frac{(2a+b)x^2}{a^2} + \frac{x^4}{a} + \frac{-a^2b-2ab^2-b^3}{a^3(b+ax^2)}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2 f} - \frac{\cos^5(e+fx)}{5af} + \frac{(b(a+b)^2)\,\text{Sub}}{}$$
$$= \frac{\sqrt{b}\,(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+f}{3a^2 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.30, size = 425, normalized size = 4.34

$$\frac{(a+2b+a\cos(2(e+fx)))\left(15(5a^3+64a^2b+128ab^2+64b^3)\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+15(5a^3+64a^2b+128ab^2+64b^3)\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)-75a^3\text{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-75a^3\text{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-8\sqrt{a}\sqrt{b}\cos(e+fx)\left(89a^2+220ab+120b^2-4a(7a+5b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\right)\sec^2(e+fx)}{1920a^{7/2}\sqrt{b}\,f\,(a+b\sec^2(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)
*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[
(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f
*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[((-Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 75
*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*Sqrt[a]*S
qrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*Cos[2*(e
+ f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*Sqrt[b]*f*
(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.24, size = 115, normalized size = 1.17

method	result
derivativedivides	$-\frac{\frac{-\frac{\left(\cos^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}-\frac{ab\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b\left(a^2+2ab+b^2\right)\arctan\left(\frac{a\,\text{co}}{\sqrt{}}\right.}{a^3\sqrt{ab}}}{f}$

default	$\frac{-\frac{\frac{(\cos^5(fx+e))a^2}{5}-\frac{2a^2(\cos^3(fx+e))}{3}-\frac{ab(\cos^3(fx+e))}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b(a^2+2ab+b^2)\arctan\left(a\right.}{a^3\sqrt{ab}}}{f}$
risch	$-\frac{5\,e^{i(fx+e)}}{16fa}-\frac{7\,e^{i(fx+e)}b}{8f\,a^2}-\frac{e^{i(fx+e)}b^2}{2f\,a^3}-\frac{5\,e^{-i(fx+e)}}{16fa}-\frac{7\,e^{-i(fx+e)}b}{8f\,a^2}-\frac{e^{-i(fx+e)}b^2}{2f\,a^3}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-1/3*a*b*cos(f*x+e)^3
+a^2*cos(f*x+e)+2*a*b*cos(f*x+e)+b^2*cos(f*x+e))+b*(a^2+2*a*b+b^2)/a^3/(a*b
)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]

time = 0.47, size = 106, normalized size = 1.08

$$\frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{3\,a^2\cos(fx+e)^5-5\,(2\,a^2+ab)\cos(fx+e)^3+15\,(a^2+2\,ab+b^2)\cos(fx+e)}{a^3}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b
)*a^3) - (3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2 +
 2*a*b + b^2)*cos(f*x + e))/a^3)/f
```

Fricas [A]

time = 3.30, size = 239, normalized size = 2.44

$$\left[-\frac{6\,a^2\cos(fx+e)^5-10\,(2\,a^2+ab)\cos(fx+e)^3-15\,(a^2+2\,ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+30\,(a^2+2\,ab+b^2)\cos(fx+e)}{30\,a^3f},\ -\frac{3\,a^2\cos(fx+e)^5-5\,(2\,a^2+ab)\cos(fx+e)^3-15\,(a^2+2\,ab+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+15\,(a^2+2\,ab+b^2)\cos(fx+e)}{15\,a^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e
) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a^3*
f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2
+ 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2 + 2*a
*b + b^2)*cos(f*x + e))/(a^3*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(86) = 172.
time = 0.48, size = 349, normalized size = 3.56

$$-\frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(8\,a^2+25\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{110\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{60\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{160\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{90\,ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{60\,b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{15\,ab(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{15\,b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}\right)}{a^3\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*c
os(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^3) - 2*(8*a^2 + 25*a*b + 15*b^2 - 40
*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 110*a*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^3*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

Mupad [**B**]
time = 4.31, size = 123, normalized size = 1.26

$$\frac{\cos(e+f\,x)^3\,\left(\frac{b}{3\,a^2}+\frac{2}{3\,a}\right)}{f}-\frac{\cos(e+f\,x)^5}{5\,a\,f}-\frac{\cos(e+f\,x)\,\left(\frac{1}{a}+\frac{b\,\left(\frac{b}{a^2}+\frac{2}{a}\right)}{a}\right)}{f}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)^2}{a^2\,b+2\,a\,b^2+b^3}\right)\,(a+b)^2}{a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (cos(e + f*x)^3*(b/(3*a^2) + 2/(3*a)))/f - cos(e + f*x)^5/(5*a*f) - (cos(e
+ f*x)*(1/a + (b*(b/a^2 + 2/a))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(
e + f*x)*(a + b)^2)/(2*a*b^2 + a^2*b + b^3))*(a + b)^2)/(a^(7/2)*f)
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\cos^3(e+fx)}{3af}$$

```
[Out] -(a+b)*cos(f*x+e)/a^2/f+1/3*cos(f*x+e)^3/a/f+(a+b)*arctan(cos(f*x+e)*a^(1/2
)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4218, 470, 327, 211}

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\cos^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(5/2)*f) - ((a
+ b)*Cos[e + f*x])/(a^2*f) + Cos[e + f*x]^3/(3*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= \frac{\cos^3(e+fx)}{3af} - \frac{(a+b)\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}\\
&= -\frac{(a+b)\cos(e+fx)}{a^2 f} + \frac{\cos^3(e+fx)}{3af} + \frac{(b(a+b))\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx\right.}{a^2 f}\\
&= \frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2} f} - \frac{(a+b)\cos(e+fx)}{a^2 f} + \frac{\cos^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.94, size = 376, normalized size = 5.30

$$\frac{(a+2b+a\cos(2(e+fx)))\left(3(a^2+8ab+8b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+3(a^2+8ab+8b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)-3a^2\text{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-3a^2\text{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+4\sqrt{a}\sqrt{b}\cos(e+fx)(-5a-6b+a\cos(2(e+fx)))\right)\sec^2(e+fx)}{48a^{5/2}\sqrt{b}f\left(a+b\sec^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] +
 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - Sqrt[a
+ b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b + a*Cos[
2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e + f*x]^2)
)
```

Maple [A]
time = 0.14, size = 67, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^{3}(fx+e)\right)}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^{2}}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{a\left(\cos^{3}(fx+e)\right)}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^{2}}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}}{f}$
risch	$-\frac{3\,e^{i(fx+e)}}{8fa}-\frac{e^{i(fx+e)}b}{2f\,a^{2}}-\frac{3\,e^{-i(fx+e)}}{8fa}-\frac{e^{-i(fx+e)}b}{2f\,a^{2}}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^{2}f}+\frac{i\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-b*cos(f*x+e))+b*(a+b)/a^2/(a*b)
^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.47, size = 66, normalized size = 0.93

$$\frac{\frac{3\,(ab+b^{2})\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{a\cos(fx+e)^{3}-3\,(a+b)\cos(fx+e)}{a^{2}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (a*co
s(f*x + e)^3 - 3*(a + b)*cos(f*x + e))/a^2)/f
```

Fricas [A]
time = 5.37, size = 162, normalized size = 2.28

$$\left[\frac{2\,a\cos\left(fx+e\right)^{3}+3\,(a+b)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^{2}+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^{2}+b}\right)-6\,(a+b)\cos\left(fx+e\right)}{6\,a^{2}f},\ \frac{a\cos\left(fx+e\right)^{3}+3\,(a+b)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-3\,(a+b)\cos\left(fx+e\right)}{3\,a^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

[Out] [1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*x + e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 85, normalized size = 1.20

$$\frac{(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2f}+\frac{a^2f^5\cos\left(fx+e\right)^3-3\,a^2f^5\cos\left(fx+e\right)-3\,abf^5\cos\left(fx+e\right)}{3\,a^3f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) - 3*a*b*f^5*cos(f*x + e))/(a^3*f^6)
```

Mupad [**B**]
time = 0.12, size = 76, normalized size = 1.07

$$\frac{\cos\left(e+f\,x\right)^3}{3\,a\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{b}{a^2}+\frac{1}{a}\right)}{f}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)}{b^2+a\,b}\right)\,(a+b)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] cos(e + f*x)^3/(3*a*f) - (cos(e + f*x)*(b/a^2 + 1/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b))/(a*b + b^2))*(a + b))/(a^(5/2)*f)
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

```
[Out] -cos(f*x+e)/a/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/f
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4218, 327, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(3/2)*f) - Cos[e + f*x]
/(a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{af} + \frac{b\mathrm{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}\\
&= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f} - \frac{\cos(e+fx)}{af}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 329, normalized size = 7.00

$$
\frac{\left((a+4b)\mathrm{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+(a+4b)\mathrm{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)-a\mathrm{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-a\mathrm{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-4\sqrt{a}\,\sqrt{b}\,\cos(e+fx)\right)(a+2b+a\cos(2(e+fx)))}{8a^{3/2}\sqrt{b}\,f\,(b+a\cos^2(e+fx))}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(Sqrt[
a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqrt[a +
b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a + 2*b +
a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

Maple [A]
time = 0.07, size = 44, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\sec(fx+e)}}{f}$

risch	$-\frac{e^{i(fx+e)}}{2fa}-\frac{e^{-i(fx+e)}}{2fa}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(f}}{a}\right.}{2a^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-b/a/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2))-1/a/sec(f*x+e))
```

Maxima [A]
time = 0.48, size = 42, normalized size = 0.89

$$\frac{\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{\cos(fx+e)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] (b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) - cos(f*x + e)/a)/f
```

Fricas [A]
time = 5.72, size = 124, normalized size = 2.64

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\cos(fx+e)}{2af},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-\cos(fx+e)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.43, size = 42, normalized size = 0.89

$$\frac{b \arctan\left(\frac{a \cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\, af} - \frac{\cos(fx+e)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

Mupad [B]
time = 0.07, size = 39, normalized size = 0.83

$$\frac{\sqrt{b}\, \mathrm{atan}\left(\frac{\sqrt{a}\, \cos(e+f\,x)}{\sqrt{b}}\right)}{a^{3/2}\, f} - \frac{\cos(e+f\,x)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(a^(3/2)*f) - cos(e + f*x)/(
a*f)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}\,(a+b)f}-\frac{\tanh^{-1}(\cos(e+fx))}{(a+b)f}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+
b)/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 492, 212, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}\,f(a+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*f) - ArcT
anh[Cos[e + f*x]]/((a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f} + \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f}\\ &= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}\,(a+b)f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.15, size = 239, normalized size = 4.35

$$\frac{\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] -
Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$

default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$
risch	$\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}-\frac{\ln(e^{i(fx+e)}+1)}{f(a+b)}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a(a+b)f}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{}}{}\right.}{2a(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(2*a+2*b)*ln(cos(f*x+e)-1)+b/(a+b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(
a*b)^(1/2))-1/(2*a+2*b)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.47, size = 67, normalized size = 1.22

$$\frac{\frac{2\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}-\frac{\log(\cos(fx+e)+1)}{a+b}+\frac{\log(\cos(fx+e)-1)}{a+b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - log(cos(f*x
 + e) + 1)/(a + b) + log(cos(f*x + e) - 1)/(a + b))/f
```

Fricas [A]
time = 7.73, size = 164, normalized size = 2.98

$$\left[\frac{\sqrt{-\frac{b}{a}}\,\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\,\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\,\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\,\cos(fx+e)+\frac{1}{2}\right)}{2\,(a+b)f},\ \frac{2\,\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}\,\cos(fx+e)}{b}\right)-\log\left(\frac{1}{2}\,\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\,\cos(fx+e)+\frac{1}{2}\right)}{2\,(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x +
e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
 - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.47, size = 85, normalized size = 1.55

$$-\frac{\frac{2\,b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}-\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b))
)/(sqrt(a*b)*(a + b)) - log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(
a + b))/f
```

Mupad [B]
time = 0.21, size = 123, normalized size = 2.24

$$-\frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\left(2\,a^3+2\,a\,b^2\right)-\frac{\cos(e+f\,x)\,\left(8\,a^5+8\,a^4\,b-8\,a^3\,b^2-8\,a^2\,b^3\right)}{4\,(a+b)^2}}{2\,a\,b\,(a+b)}\right)}{f\,\left(a+b\right)}-\frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\sqrt{-a\,b}}{b}\right)\,\sqrt{-a\,b}}{f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - atanh((cos(e + f*x)*(2*a*b^2 + 2*a^3) - (cos(e + f*x)*(8*a^4*b + 8*a^5 -
8*a^2*b^3 - 8*a^3*b^2))/(4*(a + b)^2))/(2*a*b*(a + b)))/(f*(a + b)) - (atan
h((cos(e + f*x)*(-a*b)^(1/2))/b)*(-a*b)^(1/2))/(f*(a*b + a^2))
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2f}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

```
[Out] -1/2*(a-b)*arctanh(cos(f*x+e))/(a+b)^2/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f+
arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/f
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 482, 536, 212, 211}

$$\frac{\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^2*f) - ((
a - b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x])
/(2*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-ax^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2(a+b)f} \\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^2f} + \frac{(ab)\text{Subs}}{} \\
&= \frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2f} - \frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^2f} - \frac{\cot(e+fx)\csc}{2(a+b)}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.71, size = 371, normalized size = 4.31

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-8\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)-8\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+a\csc^2\left(\frac{1}{2}(e+fx)\right)+b\csc^2\left(\frac{1}{2}(e+fx)\right)+4a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-4b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-4a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-a\sec^2\left(\frac{1}{2}(e+fx)\right)-b\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2(e+fx)}{16(a+b)^2f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/16*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*
(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]
```

```
- 8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(e + f*
x)/2]^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*a*Log[S
in[(e + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2 - b*Sec
[(e + f*x)/2]^2)*Sec[e + f*x]^2)/((a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.21, size = 115, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(a-b)\ln(\cos(fx+e)-1)}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e))}{4(a+b)^2}}{f}$
default	$\frac{\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(a-b)\ln(\cos(fx+e)-1)}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e))}{4(a+b)^2}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{f(a+b)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{2f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f(a^2+2ab+b^2)}-\frac{i\sqrt{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(4*a+4*b)/(cos(f*x+e)-1)+1/4*(a-b)/(a+b)^2*ln(cos(f*x+e)-1)+a*b/(a+b
)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/(4*a+4*b)/(1+cos(f*x+e))
+1/4/(a+b)^2*(-a+b)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.48, size = 133, normalized size = 1.55

$$\frac{\frac{4\,ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}-\frac{(a-b)\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2}+\frac{(a-b)\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2}+\frac{2\,\cos(fx+e)}{(a+b)\cos(fx+e)^2-a-b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b))
 - (a - b)*log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) + (a - b)*log(cos(f*x
+ e) - 1)/(a^2 + 2*a*b + b^2) + 2*cos(f*x + e)/((a + b)*cos(f*x + e)^2 - a
- b))/f
```

Fricas [A]
time = 2.28, size = 345, normalized size = 4.01

$$\left[\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{a\cos(fx+e)^{2}+2\sqrt{-ab}\cos(fx+e)-b}{a\cos(fx+e)^{2}+b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(\left(a^{2}+2ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2ab+b^{2}\right)f\right)}, \frac{4\sqrt{ab}\left(\cos\left(fx+e\right)^{2}-1\right)\arctan\left(\frac{\sqrt{ab}\cos(fx+e)}{b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(\left(a^{2}+2ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2ab+b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*
b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - ((a
 - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*
x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*cos(f*
x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e)^2 - a
 + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
 (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(74) = 148.
time = 0.46, size = 209, normalized size = 2.43

$$-\frac{\frac{8\,ab\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{ab}}-\frac{2\,(a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^{2}+2\,ab+b^{2}}-\frac{\left(a+b-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\left(a^{2}+2\,ab+b^{2}\right)(\cos(fx+e)-1)}+\frac{\cos(fx+e)-1}{(a+b)(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b
)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(a - b)*log(abs(-cos(f*x + e) + 1)/
abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) - (a + b - 2*a*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x
+ e) + 1)/((a^2 + 2*a*b + b^2)*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/((a
 + b)*(cos(f*x + e) + 1)))/f
```

Mupad [B]
time = 4.91, size = 392, normalized size = 4.56

$$-\frac{2\,a\,\cos\left(e+f\,x\right)+2\,b\,\cos\left(e+f\,x\right)-a\,\ln\left(\cos\left(e+f\,x\right)-1\right)+a\,\ln\left(\cos\left(e+f\,x\right)+1\right)+b\,\ln\left(\cos\left(e+f\,x\right)-1\right)-b\,\ln\left(\cos\left(e+f\,x\right)+1\right)+a\,\ln\left(\cos\left(e+f\,x\right)-1\right)\,{\cos\left(e+f\,x\right)}^2-a\,\ln\left(\cos\left(e+f\,x\right)+1\right)\,{\cos\left(e+f\,x\right)}^2-b\,\ln\left(\cos\left(e+f\,x\right)-1\right)\,{\cos\left(e+f\,x\right)}^2+b\,\ln\left(\cos\left(e+f\,x\right)+1\right)\,{\cos\left(e+f\,x\right)}^2-\mathrm{atan}\left(\frac{a^3\,\cos(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}+a\,b^2\,\cos(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}+a^2\,b\,\cos(e+f\,x)\,\sqrt{-a\,b}\,2\mathrm{i}}{a^3\,b+2\,a^2\,b^2+a\,b^3}\right)\,\sqrt{-a\,b}\,4\mathrm{i}+{\cos\left(e+f\,x\right)}^2\,\mathrm{atan}\left(\frac{a^3\,\cos(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}+a\,b^2\,\cos(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}+a^2\,b\,\cos(e+f\,x)\,\sqrt{-a\,b}\,2\mathrm{i}}{a^3\,b+2\,a^2\,b^2+a\,b^3}\right)\,\sqrt{-a\,b}\,4\mathrm{i}}{-4\,f\,a^2\,{\cos\left(e+f\,x\right)}^2+4\,f\,a^2-8\,f\,a\,b\,{\cos\left(e+f\,x\right)}^2+8\,f\,a\,b-4\,f\,b^2\,{\cos\left(e+f\,x\right)}^2+4\,f\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] -(2*a*cos(e + f*x) + 2*b*cos(e + f*x) - atan((a^3*cos(e + f*x)*(-a*b)^(1/2)
*1i + a*b^2*cos(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*
2i)/(a*b^3 + a^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i - a*log(cos(e + f*x) - 1)
+ a*log(cos(e + f*x) + 1) + b*log(cos(e + f*x) - 1) - b*log(cos(e + f*x) +
1) + cos(e + f*x)^2*atan((a^3*cos(e + f*x)*(-a*b)^(1/2)*1i + a*b^2*cos(e +
f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*2i)/(a*b^3 + a^3*b +
 2*a^2*b^2))*(-a*b)^(1/2)*4i + a*log(cos(e + f*x) - 1)*cos(e + f*x)^2 - a*l
og(cos(e + f*x) + 1)*cos(e + f*x)^2 - b*log(cos(e + f*x) - 1)*cos(e + f*x)^
2 + b*log(cos(e + f*x) + 1)*cos(e + f*x)^2)/(4*a^2*f + 4*b^2*f - 4*a^2*f*co
s(e + f*x)^2 - 4*b^2*f*cos(e + f*x)^2 + 8*a*b*f - 8*a*b*f*cos(e + f*x)^2)
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=129

$$\frac{a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3f}-\frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8(a+b)^3f}-\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f}-$$

```
[Out] -1/8*(3*a^2-6*a*b-b^2)*arctanh(cos(f*x+e))/(a+b)^3/f-1/8*(3*a-b)*cot(f*x+e)
*csc(f*x+e)/(a+b)^2/f-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f+a^(3/2)*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4218, 482, 541, 536, 212, 211}

$$\frac{a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3}-\frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^3}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)}-\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^3*f) - ((
3*a^2 - 6*a*b - b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^3*f) - ((3*a - b)*Co
t[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4
*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-3ax^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b(5a+}{(1-}\right.}{}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{(a^2b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3f} - \frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8(a+b)^3f} - \frac{(3a-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.98, size = 549, normalized size = 4.26

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-64a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)-64a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+6a^2\csc^2\left(\frac{1}{2}(e+fx)\right)+4ab\csc^2\left(\frac{1}{2}(e+fx)\right)-2b^2\csc^2\left(\frac{1}{2}(e+fx)\right)+a^2\csc^4\left(\frac{1}{2}(e+fx)\right)+2ab\csc^4\left(\frac{1}{2}(e+fx)\right)+b^2\csc^4\left(\frac{1}{2}(e+fx)\right)+24a^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-48ab\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-24a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+48ab\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-6a^2\sec^2\left(\frac{1}{2}(e+fx)\right)-4ab\sec^2\left(\frac{1}{2}(e+fx)\right)+2b^2\sec^2\left(\frac{1}{2}(e+fx)\right)-a^2\sec^4\left(\frac{1}{2}(e+fx)\right)-2ab\sec^4\left(\frac{1}{2}(e+fx)\right)-b^2\sec^4\left(\frac{1}{2}(e+fx)\right)\right)\sec^2(e+fx)}{128(a+b)^3f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
] - 64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 + 4*a*b*
Csc[(e + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]^4 + 2*
a*b*Csc[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e + f*x)/
2]] - 48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] - 24*a^2*L
og[Sin[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[Sin[(e + f*
x)/2]] - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2*b^2*Sec[(e
 + f*x)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^4 - b^2*Sec[
(e + f*x)/2]^4)*Sec[e + f*x]^2)/((a + b)^3*f*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.24, size = 181, normalized size = 1.40

method	result
derivativedivides	$\frac{-\frac{1}{2(8a+8b)(\cos(fx+e)-1)^2}-\frac{-3a+b}{16(a+b)^2(\cos(fx+e)-1)}+\frac{\left(3a^2-6ab-b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^3}+\frac{a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{}{2(8a+8b)(1+}}{f}$
default	$\frac{-\frac{1}{2(8a+8b)(\cos(fx+e)-1)^2}-\frac{-3a+b}{16(a+b)^2(\cos(fx+e)-1)}+\frac{\left(3a^2-6ab-b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^3}+\frac{a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{}{2(8a+8b)(1+}}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}-7b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-7b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{4f(a+b)^2\left(e^{2i(fx+e)}-1\right)^4}+\frac{3\ln(e^{i}}{8f(a^3+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(8*a+8*b)/(cos(f*x+e)-1)^2-1/16*(-3*a+b)/(a+b)^2/(cos(f*x+e)-1)+1
/16*(3*a^2-6*a*b-b^2)/(a+b)^3*ln(cos(f*x+e)-1)+a^2*b/(a+b)^3/(a*b)^(1/2)*ar
ctan(a*cos(f*x+e)/(a*b)^(1/2))+1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-3*a+b)
/(a+b)^2/(1+cos(f*x+e))+1/16/(a+b)^3*(-3*a^2+6*a*b+b^2)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.48, size = 238, normalized size = 1.84

$$\frac{\frac{16\,a^{2}b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{ab}}-\frac{\left(3\,a^{2}-6\,ab-b^{2}\right)\log\left(\cos\left(fx+e\right)+1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{\left(3\,a^{2}-6\,ab-b^{2}\right)\log\left(\cos\left(fx+e\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(\left(3\,a-b\right)\cos\left(fx+e\right)^{3}-\left(5\,a+b\right)\cos\left(fx+e\right)\right)}{\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*sqrt(a*b)) - (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) - 1)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 2*((3*a - b)*cos(f*x + e)^3 - (5*a + b)*cos(f*x +
e))/((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e
)^2 + a^2 + 2*a*b + b^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(121) = 242$.
time = 5.07, size = 721, normalized size = 5.59

$$\left[\frac{2\left(3\,a^{2}+2\,ab-b^{2}\right)\cos\left(fx+e\right)^{3}+8\left(a\cos\left(fx+e\right)^{4}-2\,a\cos\left(fx+e\right)^{2}+a\right)\sqrt{-ab}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{-ab}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)-2\left(5\,a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)-\left(\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3\,a^{2}-6\,ab-b^{2}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3\,a^{2}-6\,ab-b^{2}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{16\left(\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\right)},\ \frac{2\left(3\,a^{2}+2\,ab-b^{2}\right)\cos\left(fx+e\right)^{3}+16\left(a\cos\left(fx+e\right)^{4}-2\,a\cos\left(fx+e\right)^{2}+a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\cos\left(fx+e\right)}{b}\right)-2\left(5\,a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)-\left(\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3\,a^{2}-6\,ab-b^{2}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,a^{2}-6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3\,a^{2}-6\,ab-b^{2}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{16\left(\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*c
os(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) -
 ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x +
e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b -
 b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a
*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*
cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3
 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arctan(sqrt(a*b
)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a^2 - 6*a*b
- b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*
a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2)*cos(f*x + e
)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(-1/
2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(115) = 230.
time = 0.49, size = 389, normalized size = 3.02

$$-\frac{\frac{64\,a^2b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}}-\frac{4\,(3a^2-6ab-b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3a^2b+3ab^2+b^3}+\frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2+2ab+b^2}+\frac{\left(a^2+2ab+b^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{36\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{6\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(a^3+3a^2b+3ab^2+b^3)(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt
(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 4*(3*a^2 - 6*a*b - b^
2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + (8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
^2 + 2*a*b + b^2) + (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 36*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 - 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2))/f
```

Mupad [B]
time = 7.90, size = 870, normalized size = 6.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (atan((a^5*cos(e + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(
1/2)*12i + a^3*b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a
^3*b)^(1/2)*1i + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5
+ 12*a^3*b^4 + 30*a^4*b^3 + 28*a^5*b^2))*(-a^3*b)^(1/2)*8i - 5*a^2*cos(e +
f*x) - b^2*cos(e + f*x) + 3*a^2*cos(e + f*x)^3 - b^2*cos(e + f*x)^3 - 3*a^2
*atanh(cos(e + f*x)) + b^2*atanh(cos(e + f*x)) - atan((a^5*cos(e + f*x)*(-a
^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos(
e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3 +
 28*a^5*b^2))*cos(e + f*x)^2*(-a^3*b)^(1/2)*16i + atan((a^5*cos(e + f*x)*(-
a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*cos(e +
 f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^4*b*cos
```

```
(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*a^4*b^3
+ 28*a^5*b^2))*cos(e + f*x)^4*(-a^3*b)^(1/2)*8i - 6*a*b*cos(e + f*x) + 6*a^
2*cos(e + f*x)^2*atanh(cos(e + f*x)) - 3*a^2*cos(e + f*x)^4*atanh(cos(e + f
*x)) - 2*b^2*cos(e + f*x)^2*atanh(cos(e + f*x)) + b^2*cos(e + f*x)^4*atanh(
cos(e + f*x)) + 2*a*b*cos(e + f*x)^3 + 6*a*b*atanh(cos(e + f*x)) - 12*a*b*c
os(e + f*x)^2*atanh(cos(e + f*x)) + 6*a*b*cos(e + f*x)^4*atanh(cos(e + f*x)
))/(8*a^3*f + 8*b^3*f - 16*a^3*f*cos(e + f*x)^2 + 8*a^3*f*cos(e + f*x)^4 -
16*b^3*f*cos(e + f*x)^2 + 8*b^3*f*cos(e + f*x)^4 + 24*a*b^2*f + 24*a^2*b*f
- 48*a*b^2*f*cos(e + f*x)^2 - 48*a^2*b*f*cos(e + f*x)^2 + 24*a*b^2*f*cos(e
+ f*x)^4 + 24*a^2*b*f*cos(e + f*x)^4)
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=166

$$\frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4}-\frac{\sqrt{b}\,(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f}-\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin}{16a^3f}$$

```
[Out] 1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*x/a^4-1/16*(11*a^2+18*a*b+8*b^2)*cos(
f*x+e)*sin(f*x+e)/a^3/f+1/8*(3*a+2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos
(f*x+e)^3*sin(f*x+e)^3/a/f-(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))*b^(1/2)/a^4/f
```

Rubi [A]
time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4217, 481, 592, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f}+\frac{(3a+2b)\sin(e+fx)\cos^3(e+fx)}{8a^2f}-\frac{(11a^2+18ab+8b^2)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x(5a^3+30a^2b+40ab^2+16b^3)}{16a^4}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*x)/(16*a^4) - (Sqrt[b]*(a + b)^(5/2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*f) - ((11*a^2 + 18*a*b +
 8*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((3*a + 2*b)*Cos[e + f*x]^3
*Sin[e + f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-3(2a+b)x^2)}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{3(a+b)}{}\right.}{}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+f}{8a^2f}$$

$$= -\frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+f}{8a^2f}$$

$$= \frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4} - \frac{\sqrt{b}\,(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{(11a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.54, size = 357, normalized size = 2.15

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3\sqrt{b}\,(9a^4+136a^3b+384a^2b^2+384ab^3+128b^4)\,\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{b(\cos(e)-i\sin(e))^4}\left(3a^3(9a+8b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)+2\sqrt{b}\,\sqrt{a+b}\,(-12a^3e+60a^3fx+360a^2bfx+480ab^2fx+192b^3fx-3a(15a^2+32ab+16b^2)\sin(2(e+fx))+3a^2(3a+2b)\sin(4(e+fx))-a^3\sin(6(e+fx)))\right)\right)}{768a^4\sqrt{b}\,\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3*
b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2
*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4
]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sqrt[b]
*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x + 192*
b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a + 2*
b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)]))))/(768*a^4*Sqrt[b]*Sqrt[a + b]
*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.21, size = 170, normalized size = 1.02

method	result

derivativedivides	$\frac{\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{(5a^3+30a^2b+40}{}}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{(5a^3+30a^2b+40}{}}{a^4}}{f}$
risch	$\frac{5x}{16a}+\frac{15xb}{8a^2}+\frac{5x\,b^2}{2a^3}+\frac{x\,b^3}{a^4}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}b^2}{8a^3f}-\frac{15i\mathrm{e}^{-2i(fx+e)}}{128af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{4a^2f}+\frac{15i\mathrm{e}^{2i(fx+e)}}{128af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(((-9/8*a^2*b-1/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-2*a^2*b-a*b^2-
5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-7/8*a^2*b-1/2*a*b^2)*tan(f*x+e))/(tan(f*x+
e)^2+1)^3+1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*arctan(tan(f*x+e)))-b/a^4*(
a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 217, normalized size = 1.31

$$-\frac{\frac{3\,(11\,a^2+18\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\tan(fx+e)^3+3\,(5\,a^2+14\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3}-\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)(fx+e)}{a^4}+\frac{48\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^4}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/48*((3*(11*a^2 + 18*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*
b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 14*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x
 + e)^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) - 3*(5*a^3 + 3
0*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 + 48*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

Fricas [A]
time = 2.27, size = 446, normalized size = 2.69

$$\left[\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)fx+12\,(a^2+2\,ab+b^2)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)-(8\,a^3\cos(fx+e)^5-2\,(13\,a^3+6\,a^2b)\cos(fx+e)^3+3\,(11\,a^3+18\,a^2b+8\,ab^2)\cos(fx+e))\sin(fx+e)}{48\,a^4f},\ \frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)fx+24\,(a^2+2\,ab+b^2)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\,\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-(8\,a^3\cos(fx+e)^5-2\,(13\,a^3+6\,a^2b)\cos(fx+e)^3+3\,(11\,a^3+18\,a^2b+8\,ab^2)\cos(fx+e))\sin(fx+e)}{48\,a^4f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^2
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 + 3*(1
1*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48*(3*(5
*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sqrt(a*b
+ b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x +
 e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x +
 e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 237, normalized size = 1.43

$$\frac{\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)(fx+e)}{a^4}-\frac{48\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4}-\frac{33\,a^2\tan(fx+e)^5+54\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+96\,ab\tan(fx+e)^3+48\,b^2\tan(fx+e)^3+15\,a^2\tan(fx+e)+42\,ab\tan(fx+e)+24\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3a^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*
tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x + e)
^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
+ 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 42*
a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

Mupad [B]
time = 5.71, size = 1448, normalized size = 8.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((25*b^3*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*
b^3 - 5*a^4*b^2)^(1/2))/(128*((227*a*b^5)/128 + (217*b^6)/128 + (119*a^2*b^
4)/128 + (25*a^3*b^3)/128 + (13*b^7)/(16*a) + (5*b^8)/(32*a^2))) + (11*b^4*
tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2
)^(1/2))/(32*((217*a*b^6)/128 + (13*b^7)/16 + (227*a^2*b^5)/128 + (119*a^3*
b^4)/128 + (25*a^4*b^3)/128 + (5*b^8)/(32*a))) + (5*b^5*tan(e + f*x)*(- 5*a
*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2)^(1/2))/(32*((13*a
*b^7)/16 + (5*b^8)/32 + (217*a^2*b^6)/128 + (227*a^3*b^5)/128 + (119*a^4*b^
4)/128 + (25*a^5*b^3)/128)))*(-b*(a + b)^5)^(1/2))/(a^4*f) - (atan((((((51
2*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e
 + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3
*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) -
(tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^
4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^
3*5i + b^3*16i)*1i)/(32*a^4) - (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*
b^3 + 320*a^11*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)
*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*
b*30i + a^3*5i + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 +
 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/
(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i)*1i)/(32*a^4))/((992*a
*b^10 + 128*b^11 + 3344*a^2*b^9 + 6380*a^3*b^8 + 7496*a^4*b^7 + 5511*a^5*b^
6 + 2445*a^6*b^5 + 585*a^7*b^4 + 55*a^8*b^3)/(128*a^9) + (((((512*a^8*b^5 +
 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) - (tan(e + f*x)*(20
48*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(409
6*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) - (tan(e + f*
x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 183
6*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*
16i))/(32*a^4) + (((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11
*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i +
a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i
 + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7
+ 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*
b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4)))*(a*b^2*40i + a^2*b*30i
+ a^3*5i + b^3*16i)*1i)/(16*a^4*f) - ((tan(e + f*x)*(14*a*b + 5*a^2 + 8*b^2
))/(16*a^3) + (tan(e + f*x)^3*(12*a*b + 5*a^2 + 6*b^2))/(6*a^3) + (tan(e +
f*x)^5*(18*a*b + 11*a^2 + 8*b^2))/(16*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e
+ f*x)^4 + tan(e + f*x)^6 + 1))
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{(3a^2+12ab+8b^2)\,x}{8a^3}-\frac{\sqrt{b}\,(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f}+\frac{\cos^3(e+f}{}$$

```
[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/
4*cos(f*x+e)^3*sin(f*x+e)/a/f-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))*b^(1/2)/a^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{x(3a^2+12ab+8b^2)}{8a^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a^3*f) - ((5*a + 4*b)*Cos[e + f*x]*Sin[e +
f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{a+b+(b-4(a+b))x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{(a+b}{\quad}\right.}{}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{(b(a+b)^2)\,\text{Su}}{}$$

$$= \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}\,(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\cos(e}{8a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.13, size = 303, normalized size = 2.59

$$\frac{(a+2b+a\cos (2(e+fx)))\sec ^2(e+fx)\left(\sqrt{b}\left(3a^3+34a^2b+64ab^2+32b^3\right)\text{ArcTan}\left(\frac{\sec (fx)(\cos (2e)-i\sin (2e))(-((a+2b)\sin (fx))+a\sin (2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos (e)-i\sin (e))^4}}\right)(\cos (2e)-i\sin (2e))+\sqrt{b(\cos (e)-i\sin (e))^4}\left(a^2(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan (e+fx)}{\sqrt{a+b}}\right)+\sqrt{b}\sqrt{a+b}\left(-2a^2e+12a^2fx+48abfx+32b^2fx-8a(a+b)\sin (2(e+fx))+a^2\sin (4(e+fx))\right)\right)\right)}{64a^3\sqrt{b}\sqrt{a+b}f\left(a+b\sec ^2(e+fx)\right)\sqrt{b(\cos (e)-i\sin (e))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b +
 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a^2*e
+ 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*
Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.17, size = 119, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}-\frac{(a+b)^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right.}{a^3\sqrt{(a+b)\,b}}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}-\frac{(a+b)^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right.}{a^3\sqrt{(a+b)\,b}}}{f}$
risch	$\frac{3x}{8a}+\frac{3xb}{2a^2}+\frac{x\,b^2}{a^3}+\frac{i e^{2i(fx+e)}}{8af}+\frac{i e^{2i(fx+e)}b}{8a^2f}-\frac{i e^{-2i(fx+e)}}{8af}-\frac{i e^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-ab-b^2}\,\ln\left(e^{2i(fx+e)}+\right.}{2f\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*(((-1/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-1/2*a*b)*tan(f*x+e))
/(tan(f*x+e)^2+1)^2+1/8*(3*a^2+12*a*b+8*b^2)*arctan(tan(f*x+e)))-(a+b)^2*b/
a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.47, size = 143, normalized size = 1.22

$$-\frac{\frac{(5\,a+4\,b)\tan(fx+e)^3+(3\,a+4\,b)\tan(fx+e)}{a^2\tan(fx+e)^4+2\,a^2\tan(fx+e)^2+a^2}-\frac{(3\,a^2+12\,ab+8\,b^2)(fx+e)}{a^3}+\frac{8\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(((5*a + 4*b)*tan(f*x + e)^3 + (3*a + 4*b)*tan(f*x + e))/(a^2*tan(f*x
+ e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a
^3 + 8*(a^2*b + 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^3))/f
```

Fricas [A]
time = 2.57, size = 348, normalized size = 2.97

$$\left[\frac{(3a^2+12ab+8b^2)fx+2\sqrt{-ab-b^2}\,(a+b)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)+\left(2a^2\cos(fx+e)^3-(5a^2+4ab)\cos(fx+e)\right)\sin(fx+e)}{8a^3f}, \frac{(3a^2+12ab+8b^2)fx+4\sqrt{ab+b^2}\,(a+b)\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)+\left(2a^2\cos(fx+e)^3-(5a^2+4ab)\cos(fx+e)\right)\sin(fx+e)}{8a^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2 +
 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a +
 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)
/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x + e)^3
 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 + 12*a*
b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(f*x + e
)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.48, size = 152, normalized size = 1.30

$$\frac{\frac{(3a^2+12ab+8b^2)(fx+e)}{a^3}-\frac{8(a^2b+2ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^3}-\frac{5a\tan(fx+e)^3+4b\tan(fx+e)^3+3a\tan(fx+e)+4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(pi
*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))
/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*tan
(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B]
time = 4.58, size = 494, normalized size = 4.22

$$\frac{\mathrm{atanh}\left(\frac{9\,b^3\,\tan(e+f\,x)\,\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{32\,\left(\frac{13\,a\,b^4}{16}+\frac{25\,b^5}{32}+\frac{9\,a^2\,b^3}{32}+\frac{b^6}{4\,a}\right)}+\frac{b^4\,\tan(e+f\,x)\,\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{4\,\left(\frac{9\,a^3\,b^3}{32}+\frac{13\,a^2\,b^4}{16}+\frac{25\,a\,b^5}{32}+\frac{b^6}{4}\right)}\right)\,\sqrt{-b\,{(a+b)}^3}}{a^3\,f}-\frac{\frac{\tan(e+f\,x)\,(3\,a+4\,b)}{8\,a^2}+\frac{{\tan(e+f\,x)}^3\,(5\,a+4\,b)}{8\,a^2}}{f\,\left({\tan\left(e+f\,x\right)}^4+2\,{\tan\left(e+f\,x\right)}^2+1\right)}-\frac{\mathrm{atan}\left(\frac{159\,b^3\,\tan(e+f\,x)}{256\,\left(\frac{27\,a\,b^2}{256}+\frac{159\,b^3}{256}+\frac{75\,b^4}{64\,a}+\frac{29\,b^5}{32\,a^2}+\frac{b^6}{4\,a^3}\right)}+\frac{75\,b^4\,\tan(e+f\,x)}{64\,\left(\frac{159\,a\,b^3}{256}+\frac{75\,b^4}{64}+\frac{27\,a^2\,b^2}{256}+\frac{29\,b^5}{32\,a}+\frac{b^6}{4\,a^2}\right)}+\frac{29\,b^5\,\tan(e+f\,x)}{32\,\left(\frac{75\,a\,b^4}{64}+\frac{29\,b^5}{32}+\frac{159\,a^2\,b^3}{256}+\frac{27\,a^3\,b^2}{256}+\frac{b^6}{4\,a}\right)}+\frac{b^6\,\tan(e+f\,x)}{4\,\left(\frac{27\,a^4\,b^2}{256}+\frac{159\,a^3\,b^3}{256}+\frac{75\,a^2\,b^4}{64}+\frac{29\,a\,b^5}{32}+\frac{b^6}{4}\right)}+\frac{27\,b^2\,\tan(e+f\,x)}{256\,\left(\frac{27\,b^2}{256}+\frac{159\,b^3}{256\,a}+\frac{75\,b^4}{64\,a^2}+\frac{29\,b^5}{32\,a^3}+\frac{b^6}{4\,a^4}\right)}\right)\,\left(a^2\,3\mathrm{i}+a\,b\,12\mathrm{i}+b^2\,8\mathrm{i}\right)\,1\mathrm{i}}{8\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((9*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(32
*((13*a*b^4)/16 + (25*b^5)/32 + (9*a^2*b^3)/32 + b^6/(4*a))) + (b^4*tan(e +
 f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(4*((25*a*b^5)/32 + b^6/
4 + (13*a^2*b^4)/16 + (9*a^3*b^3)/32)))*(-b*(a + b)^3)^(1/2))/(a^3*f) - ((t
an(e + f*x)*(3*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(5*a + 4*b))/(8*a^2))/(f
*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((159*b^3*tan(e + f*x))/(2
56*((27*a*b^2)/256 + (159*b^3)/256 + (75*b^4)/(64*a) + (29*b^5)/(32*a^2) +
b^6/(4*a^3))) + (75*b^4*tan(e + f*x))/(64*((159*a*b^3)/256 + (75*b^4)/64 +
(27*a^2*b^2)/256 + (29*b^5)/(32*a) + b^6/(4*a^2))) + (29*b^5*tan(e + f*x))/
(32*((75*a*b^4)/64 + (29*b^5)/32 + (159*a^2*b^3)/256 + (27*a^3*b^2)/256 + b
^6/(4*a))) + (b^6*tan(e + f*x))/(4*((29*a*b^5)/32 + b^6/4 + (75*a^2*b^4)/64
 + (159*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(e + f*x))/(256*((27
*b^2)/256 + (159*b^3)/(256*a) + (75*b^4)/(64*a^2) + (29*b^5)/(32*a^3) + b^6
/(4*a^4))))*(a*b*12i + a^2*3i + b^2*8i)*1i)/(8*a^3*f)
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{(a+2b)x}{2a^2}-\frac{\sqrt{b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}-\frac{\cos(e+fx)\sin(e+fx)}{2af}$$

```
[Out] 1/2*(a+2*b)*x/a^2-1/2*cos(f*x+e)*sin(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/(
a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/f
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 482, 536, 209, 211}

$$-\frac{\sqrt{b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}+\frac{x(a+2b)}{2a^2}-\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/(2*a^2) - (Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(a^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(a -}{}\\
&= \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.92, size = 245, normalized size = 3.22

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,f} - \frac{-4(a+2b)x - \frac{(a^2+8ab+8b^2)\operatorname{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{\sqrt{a+b}\,f\sqrt{b(\cos(e)-i\sin(e))^4}} + \frac{2a\cos(2fx)\sin(2e)}{f} + \frac{2a\cos(2e)\sin(2fx)}{f}}{a^2}\right)}{16\,(a+b\sec^2(e+fx))}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b +
 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a
*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
- I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Cos[2*f
*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e + f*x
]^2))

Maple [A]
time = 0.12, size = 78, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a^2\sqrt{(a+b)\,b}}}{f}$
default	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a^2\sqrt{(a+b)\,b}}}{f}$
risch	$\frac{x}{2a}+\frac{xb}{a^2}+\frac{i e^{2i(fx+e)}}{8af}-\frac{i e^{-2i(fx+e)}}{8af}-\frac{\sqrt{-ab-b^2}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab-b^2}-a-2b}{a}\right)}{2f\,a^2}+\frac{\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^2*(-1/2*a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2*(a+2*b)*arctan(tan(f*x+e
)))-(a+b)*b/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))

Maxima [A]
time = 0.47, size = 81, normalized size = 1.07

$$\frac{\frac{(fx+e)(a+2\,b)}{a^2}-\frac{\tan(fx+e)}{a\tan(fx+e)^2+a}-\frac{2\,(ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")

[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - tan(f*x + e)/(a*tan(f*x + e)^2 + a) - 2*(a*b
 + b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2))/f

Fricas [A]
time = 3.28, size = 271, normalized size = 3.57

$$\left[\frac{2\,(a+2\,b)fx-2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab-b^2}\,\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\,\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,a^2f},\ \frac{(a+2\,b)fx-a\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{ab+b^2}\,\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\,\sqrt{ab+b^2}\,\cos(fx+e)\sin(fx+e)}\right)}{2\,a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
+ 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f), 1/2*
((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))
/(a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.47, size = 92, normalized size = 1.21

$$\frac{\frac{(fx+e)(a+2\,b)}{a^2}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2}-\frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/((t
an(f*x + e)^2 + 1)*a))/f
```

Mupad [B]
time = 4.45, size = 111, normalized size = 1.46

$$\frac{\mathrm{atanh}\left(\frac{\sin(e+f\,x)\,\sqrt{-b^2-a\,b}}{a\,\cos(e+f\,x)+b\,\cos(e+f\,x)}\right)\,\sqrt{-b^2-a\,b}-a\left(\frac{\sin(2\,e+2\,f\,x)}{4}-\frac{\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{2}\right)+b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atanh((sin(e + f*x)*(- a*b - b^2)^(1/2))/(a*cos(e + f*x) + b*cos(e + f*x))
)*(- a*b - b^2)^(1/2) - a*(sin(2*e + 2*f*x)/4 - atan(sin(e + f*x)/cos(e + f
*x))/2) + b*atan(sin(e + f*x)/cos(e + f*x)))/(a^2*f)
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{a}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4212, 3260, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}$$
$$= \frac{x}{a} + \frac{b\mathrm{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af}$$
$$= \frac{x}{a} + \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 182, normalized size = 4.04

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}\,fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\mathrm{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))\right)}{2a\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.09, size = 46, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-}}{}\right.}{2(a+b)fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*arctan(tan(f*x+e))-1/a*b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 46, normalized size = 1.02

$$-\frac{\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\, a} - \frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Fricas [A]
time = 2.37, size = 241, normalized size = 5.36

$$\left[\frac{4\,fx + \sqrt{-\frac{b}{a+b}} \log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos(fx+e)^2 + 4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3 - (ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e) + b^2}{a^2\cos(fx+e)^4 + 2\,ab\cos(fx+e)^2 + b^2}\right)}{4\,af}, \frac{2\,fx + \sqrt{\frac{b}{a+b}} \arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2 - b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sec^2(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.42, size = 65, normalized size = 1.44

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B]
time = 4.56, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}}{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{f\,(a^2+b\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}$$

```
[Out] -cot(f*x+e)/(a+b)/f-arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(3
/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4217, 331, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(3/2)*f)) -
 Cot[e + f*x]/((a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot(e+fx)}{(a+b)f} - \frac{b\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)f} \\
&= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 189, normalized size = 3.50

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\,\csc(e)\csc(e+fx)\sqrt{b(\cos(e)-i\sin(e))^4}\,\sin(fx)\right)}{2(a+b)^{3/2}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Csc[e
]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2)*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)\sqrt{(a+b)\,b}}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)\sqrt{(a+b)\,b}}}{f}$

risch	$-\frac{2i}{f(a+b)\left(\mathrm{e}^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)\,b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)\,b}-a-2b}{a}\right)}{2(a+b)^{2}f}+\frac{\sqrt{-(a+b)\,b}\,\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)/tan(f*x+e)-b/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)
*b)^(1/2)))
```

Maxima [A]
time = 0.47, size = 52, normalized size = 0.96

$$-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(a+b)}+\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)) + 1/((
a + b)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(48) = 96.
time = 3.30, size = 287, normalized size = 5.31

$$\left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a^2+3ab+2b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)-4\cos(fx+e)}{4(a+b)f\sin(fx+e)},\ \frac{\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-2\cos(fx+e)}{2(a+b)f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b +
 b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^
4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b)*f*
sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x
+ e))/((a + b)*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.48, size = 71, normalized size = 1.31

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,(a+b)}+\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

Mupad [B]
time = 4.28, size = 46, normalized size = 0.85

$$-\frac{\cot(e+f\,x)}{f\,(a+b)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{f\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - cot(e + f*x)/(f*(a + b)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/(a + b)^(
1/2)))/(f*(a + b)^(3/2))
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f}-\frac{a\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}$$

```
[Out] -a*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f-a*arctan(b^(1/2)*tan(f*x+e
)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 464, 331, 211}

$$-\frac{a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(5/2)*f))
 - (a*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{a\operatorname{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\
&= -\frac{a\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)^2 f}\\
&= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2} f} - \frac{a\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.31, size = 226, normalized size = 2.97

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3ab\operatorname{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\frac{1}{4}\sqrt{a+b}\,\csc(e)\csc^3(e+fx)\sqrt{b(\cos(e)-i\sin(e))^4}\,(6a\sin(fx)-3b\sin(2e+fx)+(-2a+b)\sin(2e+3fx))\right)}{6(a+b)^{5/2}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b*Sin
[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a + b*S
ec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.16, size = 69, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^2\sqrt{(a+b)\,b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^2\sqrt{(a+b)\,b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
risch	$\frac{2i\left(3b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-2a+b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)\,b}\;a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)\,b}+a+2b}{a}\right)}{2(a+b)^3f}-\frac{\sqrt{-(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*b/(a+b)^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/3/
(a+b)/tan(f*x+e)^3-a/(a+b)^2/tan(f*x+e))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 1.12

$$-\frac{\frac{3\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}+\frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqr
t((a + b)*b)) + (3*a*tan(f*x + e)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x +
 e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(69) = 138.
time = 3.60, size = 419, normalized size = 5.51

$$\left[-\frac{4\,(2\,a-b)\cos(fx+e)^3-3\,(a\cos(fx+e)^2-a)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)-12\,a\cos(fx+e)}{12\,\left((a^2+2\,ab+b^2)f\cos(fx+e)^2-(a^2+2\,ab+b^2)f\right)\sin(fx+e)},\ -\frac{2\,(2\,a-b)\cos(fx+e)^3-3\,(a\cos(fx+e)^2-a)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-6\,a\cos(fx+e)}{6\,\left((a^2+2\,ab+b^2)f\cos(fx+e)^2-(a^2+2\,ab+b^2)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos
(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)*cos(f
*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.49, size = 103, normalized size = 1.36

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)^2
 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Mupad [B]
time = 4.31, size = 80, normalized size = 1.05

$$-\frac{\frac{1}{3\,(a+b)}+\frac{a\tan(e+f\,x)^2}{(a+b)^2}}{f\tan\left(e+f\,x\right)^3}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^2+2\,a\,b+b^2)}{(a+b)^{5/2}}\right)}{f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
[Out] - (1/(3*(a + b)) + (a*tan(e + f*x)^2)/(a + b)^2)/(f*tan(e + f*x)^3) - (a*b^
(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(f*(a
 + b)^(5/2))
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f}-\frac{a^2\cot(e+fx)}{(a+b)^3f}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f}$$

```
[Out] -a^2*cot(f*x+e)/(a+b)^3/f-1/3*(2*a+b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(f*x+e)
^5/(a+b)/f-a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 472, 211}

$$-\frac{a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(7/2)*f
)) - (a^2*Cot[e + f*x])/((a + b)^3*f) - ((2*a + b)*Cot[e + f*x]^3)/(3*(a +
b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)x^6}+\frac{2a+b}{(a+b)^2x^4}+\frac{a^2}{(a+b)^3x^2}-\frac{a^2b}{(a+b)^3(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{(a+b)^3f}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f}-\frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+b+}\right.}{(a}\\
&= -\frac{a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f}-\frac{a^2\cot(e+fx)}{(a+b)^3f}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot}{5}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.97, size = 318, normalized size = 3.03

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(240a^2b\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\csc(e)\csc^5(e+fx)\sqrt{b(\cos(e)-i\sin(e))^4}\,(10(8a^2+b^2)\sin(fx)-30b(3a+b)\sin(2e+fx)-40a^2\sin(2e+3fx)+30ab\sin(2e+3fx)+10b^2\sin(2e+3fx)+15ab\sin(4e+3fx)+8a^2\sin(4e+5fx)-9ab\sin(4e+5fx)-2b^2\sin(4e+5fx))\right)}{480(a+b)^{7/2}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2)*Si
n[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*a*b*S
in[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] + 8*a^2
*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])))/(480
*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.19, size = 93, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{a^2 b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^3\sqrt{(a+b)\,b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3\tan(fx+e)}-\frac{2a+b}{3(a+b)^2\tan(fx+e)^3}}{f}$

default	$\frac{-\frac{a^2 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^3 \sqrt{(a+b)\,b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3\tan(fx+e)}-\frac{2a+b}{3(a+b)^2\tan(fx+e)^3}}{f}$
risch	$\frac{2i\left(15ab\,\mathrm{e}^{8i(fx+e)}-90ab\,\mathrm{e}^{6i(fx+e)}-30b^2\mathrm{e}^{6i(fx+e)}-80a^2\mathrm{e}^{4i(fx+e)}-10b^2\mathrm{e}^{4i(fx+e)}+40a^2\mathrm{e}^{2i(fx+e)}-30ab\,\mathrm{e}^{2i(fx+e)}-10b^2\mathrm{e}\right.}{15f(a+b)^3\left(\mathrm{e}^{2i(fx+e)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/
5/(a+b)/tan(f*x+e)^5-a^2/(a+b)^3/tan(f*x+e)-1/3*(2*a+b)/(a+b)^2/tan(f*x+e)^
3)
```

Maxima [A]
time = 0.48, size = 141, normalized size = 1.34

$$-\frac{\frac{15\,a^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}}+\frac{15\,a^2\tan(fx+e)^4+5\,(2\,a^2+3\,ab+b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*a^2*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sqrt((a + b)*b)) + (15*a^2*tan(f*x + e)^4 + 5*(2*a^2 + 3*a*b +
 b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(97) = 194.
time = 4.30, size = 615, normalized size = 5.86

$$\left[-\frac{4\,(8\,a^2-9\,ab-2\,b^2)\cos(fx+e)^5-20\,(4\,a^2-3\,ab-b^2)\cos(fx+e)^3-15\,(a^2\cos(fx+e)^4-2\,a^2\cos(fx+e)^2+a^2)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)+60\,a^2\cos(fx+e)}{60\,\left((a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^4-2\,(a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^2+(a^3+3\,a^2b+3\,ab^2+b^3)f\right)\sin(fx+e)},\ -\frac{2\,(8\,a^2-9\,ab-2\,b^2)\cos(fx+e)^5-10\,(4\,a^2-3\,ab-b^2)\cos(fx+e)^3-15\,(a^2\cos(fx+e)^4-2\,a^2\cos(fx+e)^2+a^2)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{((a+2\,b)\cos(fx+e)^2-b)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+30\,a^2\cos(fx+e)}{30\,\left((a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^4-2\,(a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^2+(a^3+3\,a^2b+3\,ab^2+b^3)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2)
*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
```

```
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1/30*(2*(
8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*cos(f*x +
e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x +
e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [**A**]
time = 0.51, size = 173, normalized size = 1.65

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{15a^2\tan(fx+e)^4+10a^2\tan(fx+e)^2+15ab\tan(fx+e)^2+5b^2\tan(fx+e)^2+3a^2+6ab+3b^2}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (15
*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*b^2
*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(f*x + e)^5))/f
```

Mupad [**B**]
time = 5.05, size = 112, normalized size = 1.07

$$-\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(2\,a+b)}{3\,(a+b)^2}+\frac{a^2\tan(e+f\,x)^4}{(a+b)^3}}{f\tan\left(e+f\,x\right)^5}-\frac{a^2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{(a+b)^{7/2}}\right)}{f\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(2*a + b))/(3*(a + b)^2) + (a^2*tan(e +
f*x)^4)/(a + b)^3)/(f*tan(e + f*x)^5) - (a^2*b^(1/2)*atan((b^(1/2)*tan(e +
f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(f*(a + b)^(7/2))
```

3.41 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=161

$$\frac{\sqrt{b}\,(a+b)(3a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}$$

```
[Out] -1/2*(a+b)*(3*a+7*b)*cos(f*x+e)/a^4/f+1/6*(a+b)*(3*a+7*b)*cos(f*x+e)^3/a^3/
b/f-1/5*cos(f*x+e)^5/a^2/f-1/2*(a+b)^2*cos(f*x+e)^5/a^2/b/f/(b+a*cos(f*x+e)
^2)+1/2*(a+b)*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/
f
```

Rubi [A]

time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 474, 470, 308, 211}

$$\frac{\sqrt{b}\,(a+b)(3a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\cos^5(e+fx)}{5a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(a + b)*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(
9/2)*f) - ((a + b)*(3*a + 7*b)*Cos[e + f*x])/(2*a^4*f) + ((a + b)*(3*a + 7*
b)*Cos[e + f*x]^3)/(6*a^3*b*f) - Cos[e + f*x]^5/(5*a^2*f) - ((a + b)^2*Cos[
e + f*x]^5)/(2*a^2*b*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)^2}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{x^4\left(-2a^2+5(a+b)^2-2abx^2\right)}{b+ax^2}\,dx, x, \cos(e+fx\right.}{2a^2bf}\\
&= -\frac{\cos^5(e+fx)}{5a^2f}-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)}+\frac{((a+b)(3a+7b))\text{Subst}\left(\int \frac{x^4}{b+a}\right.}{2a^2bf}\\
&= -\frac{\cos^5(e+fx)}{5a^2f}-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)}+\frac{((a+b)(3a+7b))\text{Subst}\left(\int \left(-\right.\right.}{}\\
&= -\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{\cos^5(e+fx}{5a^2f}\\
&= \frac{\sqrt{b}\,(a+b)(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.25, size = 454, normalized size = 2.82

$$\frac{\frac{15\left(3a^4+384a^2b^2+1280ab^3+896b^4\right)\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}+\frac{15\left(3a^4+384a^2b^2+1280ab^3+896b^4\right)\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{45a^4\text{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{45a^4\text{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{16\sqrt{a}\cos(e+fx)\left(150a^3+1436a^2b+2960ab^2+1680b^3+a\left(125a^2+688ab+560b^2\right)\cos(2(e+fx))-2a^2(11a+14b)\cos(4(e+fx))+3a^3\cos(6(e+fx))\right)}{a+2b+a\cos(2(e+fx))}}{3840a^{9/2}f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
 - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2)
 + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/
2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3
/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(
3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2 + 1680*b
^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*a + 14*b)
*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])
)/(3840*a^(9/2)*f)
```

Maple [A]

time = 0.30, size = 159, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}-\frac{2ab\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)c}{b+a\left(\cos^2(fx+\right.}\right.}{}}{f}$
default	$\frac{-\frac{\frac{\left(\cos^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}-\frac{2ab\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)c}{b+a\left(\cos^2(fx+\right.}\right.}{}}{f}$
risch	$\frac{5\,e^{3i(fx+e)}}{96a^2f}+\frac{e^{3i(fx+e)}b}{12a^3f}-\frac{5\,e^{i(fx+e)}}{16a^2f}-\frac{7\,e^{i(fx+e)}b}{4a^3f}-\frac{3\,e^{i(fx+e)}b^2}{2a^4f}-\frac{5\,e^{-i(fx+e)}}{16a^2f}-\frac{7\,e^{-i(fx+e)}b}{4a^3f}-\frac{3\,e^{-i(f}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^4*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-2/3*a*b*cos(f*x+e)^3
+a^2*cos(f*x+e)+4*a*b*cos(f*x+e)+3*b^2*cos(f*x+e))+b/a^4*((-1/2*a^2-a*b-1/2
*b^2)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a^2+10*a*b+7*b^2)/(a*b)^(1/2)*ar
ctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]

time = 0.48, size = 154, normalized size = 0.96

$$-\frac{\frac{15\,(a^2b+2\,ab^2+b^3)\cos(fx+e)}{a^5\cos(fx+e)^2+a^4b}-\frac{15\,(3\,a^2b+10\,ab^2+7\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{2\left(3\,a^2\cos(fx+e)^5-10\,(a^2+ab)\cos(fx+e)^3+15\,(a^2+4\,ab+3\,b^2)\cos(fx+e)\right)}{a^4}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)/(a^5*cos(f*x + e)^2 + a^4*b)
 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a
*b)*a^4) + 2*(3*a^2*cos(f*x + e)^5 - 10*(a^2 + a*b)*cos(f*x + e)^3 + 15*(a^
2 + 4*a*b + 3*b^2)*cos(f*x + e))/a^4)/f
```

Fricas [**A**]
time = 3.47, size = 421, normalized size = 2.61

$$\left[-\frac{12\,a^3\cos(fx+e)^7-4\,(10\,a^3+7\,a^2b)\cos(fx+e)^5+20\,(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^3-15\,(3\,a^2b+10\,ab^2+7\,b^3+(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+30\,(3\,a^2b+10\,ab^2+7\,b^3)\cos(fx+e)}{60\,(a^5f\cos(fx+e)^2+a^4bf)},\ -\frac{6\,a^3\cos(fx+e)^7-2\,(10\,a^3+7\,a^2b)\cos(fx+e)^5+10\,(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^3-15\,(3\,a^2b+10\,ab^2+7\,b^3+(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+15\,(3\,a^2b+10\,ab^2+7\,b^3)\cos(fx+e)}{30\,(a^5f\cos(fx+e)^2+a^4bf)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(3
*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3
+ (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(3*a^
2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/
30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(3*a^3
+ 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3 + (3*
a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(
f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x
+ e)^2 + a^4*b*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(143) = 286.
time = 0.52, size = 509, normalized size = 3.16

$$-\frac{\frac{15\,(3\,a^2b+10\,ab^2+7\,b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{30\left(a^2b+2\,ab^2+b^3+\frac{a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)a^4}-\frac{4\left(8\,a^2+50\,ab+45\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{220\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{180\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{320\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{270\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{180\,ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{180\,b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{30\,ab(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{45\,b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}\right)}{a^4\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a
*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^4) + 30*(a^2*b + 2*a*b^2 + b^3
+ a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1))/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^4) - 4*(8*a^2 + 50
*a*b + 45*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 220*a*b*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - 180*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 270*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 180*b^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x + e) - 1)^4/(cos
(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^4*(
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

Mupad [B]
time = 0.16, size = 195, normalized size = 1.21

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{2\,b}{3\,a^3}+\frac{2}{3\,a^2}\right)}{f}-\frac{\cos\left(e+f\,x\right)^5}{5\,a^2\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{1}{a^2}-\frac{b^2}{a^4}+\frac{2\,b\,\left(\frac{2\,b}{a^3}+\frac{2}{a^2}\right)}{a}\right)}{f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{a^2\,b}{2}+a\,b^2+\frac{b^3}{2}\right)}{f\,\left(a^5\,{\cos\left(e+f\,x\right)}^2+b\,a^4\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)\,(3\,a+7\,b)}{3\,a^2\,b+10\,a\,b^2+7\,b^3}\right)\,\left(a+b\right)\,\left(3\,a+7\,b\right)}{2\,a^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (cos(e + f*x)^3*((2*b)/(3*a^3) + 2/(3*a^2)))/f - cos(e + f*x)^5/(5*a^2*f) -
 (cos(e + f*x)*(1/a^2 - b^2/a^4 + (2*b*((2*b)/a^3 + 2/a^2))/a))/f - (cos(e
+ f*x)*(a*b^2 + (a^2*b)/2 + b^3/2))/(f*(a^4*b + a^5*cos(e + f*x)^2)) + (b^(
1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b)*(3*a + 7*b))/(10*a*b^2 + 3*
a^2*b + 7*b^3))*(a + b)*(3*a + 7*b))/(2*a^(9/2)*f)
```

3.42 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{b}\,(3a+5b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\cos^3(e+fx)}{3a^2f}-\frac{b(a+b)\cos(e+fx)}{2a^3f\,(b+a\cos^2(e+fx))}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^3/f+1/3*cos(f*x+e)^3/a^2/f-1/2*b*(a+b)*cos(f*x+e)/a^3
/f/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1
/2)/a^(7/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4218, 466, 1167, 211}

$$\frac{\sqrt{b}\,(3a+5b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}-\frac{b(a+b)\cos(e+fx)}{2a^3f\,(a\cos^2(e+fx)+b)}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\cos^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(7/2)*f)
- ((a + 2*b)*Cos[e + f*x])/(a^3*f) + Cos[e + f*x]^3/(3*a^2*f) - (b*(a + b)*
Cos[e + f*x])/(2*a^3*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cos(e+fx\right.}{2a^3f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x\right.}{2a^3f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(b(3a\ldots}{}$$

$$= \frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.47, size = 403, normalized size = 3.54

$$\frac{\frac{3(3a^3+192ab^2+320b^3)\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}+\frac{3(3a^3+192ab^2+320b^3)\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{9a^3\text{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{9a^3\text{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{32\sqrt{a}\cos(e+fx)\left(9a^2+56ab+60b^2+4a(2a+5b)\cos(2(e+fx))-a^2\cos(4(e+fx))\right)}{a+2b+a\cos(2(e+fx))}}{384a^{7/2}f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3 + 1
92*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] - Sq
```

rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] + Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e + f*x]*(9
*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)

Maple [A]
time = 0.21, size = 102, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^2f}-\frac{3\,e^{i(fx+e)}}{8a^2f}-\frac{e^{i(fx+e)}b}{a^3f}-\frac{3\,e^{-i(fx+e)}}{8a^2f}-\frac{e^{-i(fx+e)}b}{a^3f}+\frac{e^{-3i(fx+e)}}{24a^2f}-\frac{(a+b)b\left(e^{3i(fx+e)}+e\right.}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^3*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-2*b*cos(f*x+e))+b/a^3*((-1/2*a-
1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)/(a*b)^(1/2)*arctan(a*cos
(f*x+e)/(a*b)^(1/2))))

Maxima [A]
time = 0.47, size = 109, normalized size = 0.96

$$-\frac{\frac{3\left(ab+b^2\right)\cos(fx+e)}{a^4\cos(fx+e)^2+a^3b}-\frac{3\left(3\,ab+5\,b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(a\cos(fx+e)^3-3\,(a+2\,b)\cos(fx+e)\right)}{a^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/6*(3*(a*b + b^2)*cos(f*x + e)/(a^4*cos(f*x + e)^2 + a^3*b) - 3*(3*a*b +
5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(a*cos(f*x + e)
^3 - 3*(a + 2*b)*cos(f*x + e))/a^3)/f

Fricas [A]
time = 5.02, size = 311, normalized size = 2.73

$$\left[\frac{4 a^{2} \cos \left(f x+e\right)^{5}-4 \left(3 a^{2}+5 a b\right) \cos \left(f x+e\right)^{3}+3 \left(\left(3 a^{2}+5 a b\right) \cos \left(f x+e\right)^{2}+3 a b+5 b^{2}\right) \sqrt{-\frac{b}{a}} \log \left(-\frac{a \cos \left(f x+e\right)^{2}+2 a \sqrt{-\frac{b}{a}} \cos \left(f x+e\right)-b}{a \cos \left(f x+e\right)^{2}+b}\right)-6 \left(3 a b+5 b^{2}\right) \cos \left(f x+e\right)}{12 \left(a^{4} f \cos \left(f x+e\right)^{2}+a^{3} b f\right)}, \frac{2 a^{2} \cos \left(f x+e\right)^{5}-2 \left(3 a^{2}+5 a b\right) \cos \left(f x+e\right)^{3}+3 \left(\left(3 a^{2}+5 a b\right) \cos \left(f x+e\right)^{2}+3 a b+5 b^{2}\right) \sqrt{\frac{b}{a}} \arctan \left(\frac{a \sqrt{\frac{b}{a}} \cos \left(f x+e\right)}{b}\right)-3 \left(3 a b+5 b^{2}\right) \cos \left(f x+e\right)}{6 \left(a^{4} f \cos \left(f x+e\right)^{2}+a^{3} b f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
+ 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2
+ 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b + 5*b
^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x + e)
^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e)^2 +
 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a*b + 5
*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 136, normalized size = 1.19

$$\frac{\left(3 \, a b + 5 \, b^{2}\right) \arctan \left(\frac{a \cos\left(f x + e\right)}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a^{3} f} - \frac{\frac{a b \cos\left(f x + e\right)}{f} + \frac{b^{2} \cos\left(f x + e\right)}{f}}{2 \, {\left(a \cos\left(f x + e\right)^{2} + b\right)} a^{3}} + \frac{a^{4} f^{11} \cos\left(f x + e\right)^{3} - 3 \, a^{4} f^{11} \cos\left(f x + e\right) - 6 \, a^{3} b f^{11} \cos\left(f x + e\right)}{3 \, a^{6} f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1/
2*(a*b*cos(f*x + e)/f + b^2*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)*a^3) +
1/3*(a^4*f^11*cos(f*x + e)^3 - 3*a^4*f^11*cos(f*x + e) - 6*a^3*b*f^11*cos(f
*x + e))/(a^6*f^12)
```

Mupad [B]
time = 0.14, size = 130, normalized size = 1.14

$$\frac{{\cos\left(e+f\,x\right)}^3}{3\,a^2\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{2\,b}{a^3}+\frac{1}{a^2}\right)}{f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{b^2}{2}+\frac{a\,b}{2}\right)}{f\,\left(a^4\,{\cos\left(e+f\,x\right)}^2+b\,a^3\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos\left(e+f\,x\right)\,\left(3\,a+5\,b\right)}{5\,b^2+3\,a\,b}\right)\,\left(3\,a+5\,b\right)}{2\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] cos(e + f*x)^3/(3*a^2*f) - (cos(e + f*x)*((2*b)/a^3 + 1/a^2))/f - (cos(e +
f*x)*((a*b)/2 + b^2/2))/(f*(a^3*b + a^4*cos(e + f*x)^2)) + (b^(1/2)*atan((a
^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 5*b))/(3*a*b + 5*b^2))*(3*a + 5*b))/(2*a
^(7/2)*f)
```

3.43 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))}$$

```
[Out] -3/2*cos(f*x+e)/a^2/f+1/2*cos(f*x+e)^3/a/f/(b+a*cos(f*x+e)^2)+3/2*arctan(co
s(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 294, 327, 211}

$$\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\,(a\cos^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(5/2)*f) - (3*Cos[e
 + f*x])/(2*a^2*f) + Cos[e + f*x]^3/(2*a*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))} - \frac{3\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2af}$$

$$= -\frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))} + \frac{(3b)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+\right.}{2a^2f}$$

$$= \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\,(b+a\cos^2(e+fx))}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.09, size = 393, normalized size = 4.68

$$\frac{(a+2b+a\cos(2(e+fx)))^2\left(\frac{(a^2+24b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}+\frac{(a^2+24b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{a}-\sqrt{a+b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{a}+\sqrt{a+b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{16\sqrt{a}\,\cos(e+fx)(a+3b+a\cos(2(e+fx)))}{a+2b+a\cos(2(e+fx))}\right)\sec^4(e+fx)}{64a^{5/2}f\,(a+b\sec^2(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2
) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] - Sqrt
[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] + Sqrt[a
 + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(a + 3
*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e + f*x]^4)/(
64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.10, size = 69, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\left(\sec^2(fx+e)\right)}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\left(\sec^2(fx+e)\right)}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2a^2f}-\frac{e^{-i(fx+e)}}{2a^2f}-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{f\,a^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{3i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e}{a}\right.}{4a^3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*b*(1/2*sec(f*x+e)/(a+b*sec(f*x+e)^2)+3/2/(a*b)^(1/2)*arctan(sec
(f*x+e)*b/(a*b)^(1/2)))-1/a^2/sec(f*x+e))
```

Maxima [A]
time = 0.48, size = 74, normalized size = 0.88

$$-\frac{\frac{b\cos(fx+e)}{a^3\cos(fx+e)^2+a^2b}-\frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\cos(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^3*cos(f*x + e)^2 + a^2*b) - 3*b*arctan(a*cos(f*x +
e)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*cos(f*x + e)/a^2)/f
```

Fricas [A]
time = 3.02, size = 213, normalized size = 2.54

$$\left[-\frac{4\,a\cos(fx+e)^3-3\left(a\cos(fx+e)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+6\,b\cos(fx+e)}{4\left(a^3f\cos(fx+e)^2+a^2bf\right)},\ -\frac{2\,a\cos(fx+e)^3-3\left(a\cos(fx+e)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+3\,b\cos(fx+e)}{2\left(a^3f\cos(fx+e)^2+a^2bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*cos
(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 6*
b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)^3
- 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 3
*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{(e+fx)}}{\left(a+b\sec^2{(e+fx)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.47, size = 72, normalized size = 0.86

$$\frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2f}-\frac{\cos{(fx+e)}}{a^2f}-\frac{b\cos{(fx+e)}}{2\left(a\cos{(fx+e)}^2+b\right)a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a^
2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

Mupad [B]
time = 4.56, size = 72, normalized size = 0.86

$$\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,f}-\frac{b\,\cos{(e+f\,x)}}{2\,f\,\left(a^3\cos{(e+f\,x)}^2+b\,a^2\right)}-\frac{\cos{(e+f\,x)}}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(2*a^(5/2)*f) - (b*cos(e +
 f*x))/(2*f*(a^2*b + a^3*cos(e + f*x)^2)) - cos(e + f*x)/(a^2*f)
```

3.44 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f}-\frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^2 f}-\frac{b\cos(e+fx)}{2a(a+b)f\,(b+a\cos^2(e+fx))}$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^2/f-1/2*b*cos(f*x+e)/a/(a+b)/f/(b+a*cos(f*x+e)^2
)+1/2*(3*a+b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4218, 481, 536, 212, 211}

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2}-\frac{b\cos(e+fx)}{2af(a+b)\,(a\cos^2(e+fx)+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(3/2)*(a +
b)^2*f) - ArcTanh[Cos[e + f*x]]/((a + b)^2*f) - (b*Cos[e + f*x])/(2*a*(a +
b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{2a(a+b)f}\\ &= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}-\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^2 f}+\frac{(b(3a\\ &= \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f}-\frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^2 f}-\frac{b\cos(e}{2a(a+b)f\,(b+}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.53, size = 384, normalized size = 3.88

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-\frac{2b(a+b)}{a}+\frac{\sqrt{b}\,(3a+b)\operatorname{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{a^{3/2}}+\frac{\sqrt{b}\,(3a+b)\operatorname{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{a^{3/2}}-2(a+2b+a\cos(2(e+fx)))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\sec(e+fx)+2(a+2b+a\cos(2(e+fx)))\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sec(e+fx)\right)}{8(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b]
*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
```

```
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/a^
(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/
2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]]*S
ec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]

time = 0.22, size = 103, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{f(a^2+2ab+b^2)}+\frac{3i\sqrt{ab}\,\ln\left(e^{2i(f\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)^2*ln(cos(f*x+e)-1)+b/(a+b)^2*(-1/2*(a+b)/a*cos(f*x+e)/(b+a*c
os(f*x+e)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))-1/
2/(a+b)^2*ln(1+cos(f*x+e)))
```

Maxima [A]

time = 0.48, size = 143, normalized size = 1.44

$$-\frac{\frac{b\cos(fx+e)}{a^2b+ab^2+(a^3+a^2b)\cos(fx+e)^2}-\frac{(3\,ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}}+\frac{\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2}-\frac{\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*cos(f*x + e)/(a^2*b + a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2) - (3*a*
b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b
)) + log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) - log(cos(f*x + e) - 1)/(a^2
 + 2*a*b + b^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(91) = 182$.
time = 5.66, size = 408, normalized size = 4.12

$$\left[\frac{\left((3a^2+ab)\cos\left(fx+e\right)^2+3ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\left(ab+b^2\right)\cos\left(fx+e\right)-2\left(a^2\cos\left(fx+e\right)^2+ab\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+2\left(a^2\cos\left(fx+e\right)^2+ab\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left((a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f\right)}, \frac{\left((3a^2+ab)\cos\left(fx+e\right)^2+3ab+b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-\left(ab+b^2\right)\cos\left(fx+e\right)-\left(a^2\cos\left(fx+e\right)^2+ab\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(a^2\cos\left(fx+e\right)^2+ab\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\left((a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f
*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(a
*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x + e)
+ 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a^4 +
 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f), 1/2*
(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*
cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a*b)*log
(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 +
 a*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(87) = 174$.
time = 0.56, size = 272, normalized size = 2.75

$$-\frac{\frac{(3\,ab+b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\,\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}}-\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2\,ab+b^2}+\frac{2\left(ab+b^2+\frac{ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^3+2\,a^2b+ab^2)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + b^2)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) +
sqrt(a*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) - log(abs(-cos(f*x + e) + 1
)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) + 2*(a*b + b^2 + a*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(
(a^3 + 2*a^2*b + a*b^2)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

Mupad [B]
time = 5.95, size = 2188, normalized size = 22.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*
a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b
 + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 +
 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 1
6*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^
3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^
4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) - ((3*a + b)*(-a^3*b
)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a
^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b
^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^
2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a
*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x
)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a
+ b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*
b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)
^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b
^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b +
a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)
*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b +
a^3)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^
4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a
 + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*
b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/
(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))*(3*a + b)*(
-a^3*b)^(1/2)*1i)/(2*f*(2*a^4*b + a^5 + a^3*b^2)) - (atan((((((2*a^6*b + 2*
a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3
```

```
*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 3
2*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a +
 b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 +
2*a^2*b + a^3)))/(a + b)^2 - ((((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b
^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(48
*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a
 + b)^2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^
3 + 4*a^4 + b^4 + 9*a^2*b^2)*1i)/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2)/((
((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a
^3*b + a^4 + 3*a^2*b^2)) - (cos(e + f*x)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 -
48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a + b)^2*(a*b^2 + 2*a^2*b + a^3)
))/(2*(a + b)^2) + (cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a
*b^2 + 2*a^2*b + a^3)))/(a + b)^2 - ((5*a*b^2)/2 + 3*a^2*b + b^3/2)/(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2) + (((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*
b^3 + 8*a^5*b^2)/(2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) + (cos(e + f*x)*(4
8*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(
a + b)^2*(a*b^2 + 2*a^2*b + a^3)))/(2*(a + b)^2) - (cos(e + f*x)*(6*a*b^3 +
 4*a^4 + b^4 + 9*a^2*b^2))/(4*(a*b^2 + 2*a^2*b + a^3)))/(a + b)^2))*1i)/(f*
(a + b)^2) - (b*cos(e + f*x))/(2*a*f*(a + b)*(b + a*cos(e + f*x)^2))
```

3.45 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(3a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3 f}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^3 f}+\frac{(a-b)\cos(e+fx)}{2(a+b)^2 f\,(b+a\cos^2(e+fx))}-\frac{c}{2(a}$$

```
[Out] -1/2*(a-3*b)*arctanh(cos(f*x+e))/(a+b)^3/f+1/2*(a-b)*cos(f*x+e)/(a+b)^2/f/(
b+a*cos(f*x+e)^2)-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*
(3*a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f/a^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4218, 481, 541, 536, 212, 211}

$$\frac{\sqrt{b}\,(3a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}\,f(a+b)^3}+\frac{(a-b)\cos(e+fx)}{2f(a+b)^2\,(a\cos^2(e+fx)+b)}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^3}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\,(a\cos^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*Sqrt[a]*(a +
b)^3*f) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^3*f) + ((a - b)*Cos[
e + f*x])/(2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*
x])/(2*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))} - \frac{(a-3b)\text{S}}{}$$

$$= \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3f} - \frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^3f} + \frac{}{2(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.29, size = 468, normalized size = 3.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*(
-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/Sqr
t[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[(e + f
*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[
(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log
[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.28, size = 148, normalized size = 1.01

method	result

derivativedivides	$\frac{\frac{1}{4(a+b)^2(\cos(fx+e)-1)}+\frac{(a-3b)\ln(\cos(fx+e)-1)}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(fx+}}{f}$
default	$\frac{\frac{1}{4(a+b)^2(\cos(fx+e)-1)}+\frac{(a-3b)\ln(\cos(fx+e)-1)}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(fx+}}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+5b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+5b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{f(a+b)^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+}\right.}{2f(a^3+3a^2b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/(a+b)^2/(cos(f*x+e)-1)+1/4*(a-3*b)/(a+b)^3*ln(cos(f*x+e)-1)+b/(a+b
)^3*((-1/2*a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a-b)/(a*b)^(1/2)*a
rctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^2/(1+cos(f*x+e))+1/4/(a+b)^3*(-a
+3*b)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.49, size = 238, normalized size = 1.62

$$-\frac{\frac{(a-3\,b)\log(\cos(fx+e)+1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{(a-3\,b)\log(\cos(fx+e)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,(3\,ab-b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}-\frac{2\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)}{(a^3+2\,a^2b+ab^2)\cos(fx+e)^4-a^2b-2\,ab^2-b^3-(a^3+a^2b-ab^2-b^3)\cos(fx+e)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a - 3*b)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a
- 3*b)*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(3*a*b - b
^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(
a*b)) - 2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a^3 + 2*a^2*b + a*b
^2)*cos(f*x + e)^4 - a^2*b - 2*a*b^2 - b^3 - (a^3 + a^2*b - a*b^2 - b^3)*co
s(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(138) = 276.
time = 4.28, size = 726, normalized size = 4.94

$$\left[\frac{2\,(a^2-b^2)\cos(fx+e)^3-\left((3\,a^2-ab)\cos(fx+e)^4-(3\,a^2-4\,ab+b^2)\cos(fx+e)^2-3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{a\cos(fx+e)^2-2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+4\,(ab+b^2)\cos(fx+e)-\left((a^2-3\,ab)\cos(fx+e)^4-(a^2-4\,ab+3\,b^2)\cos(fx+e)^2-ab+3\,b^2\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((a^2-3\,ab)\cos(fx+e)^4-(a^2-4\,ab+3\,b^2)\cos(fx+e)^2-ab+3\,b^2\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{4\left((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2\,a^3b-2\,ab^3-b^4)f\cos(fx+e)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)f\right)},\ \frac{2\,(a^2-b^2)\cos(fx+e)^3+2\left((3\,a^2-ab)\cos(fx+e)^4-(3\,a^2-4\,ab+b^2)\cos(fx+e)^2-3\,ab+b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+4\,(ab+b^2)\cos(fx+e)-\left((a^2-3\,ab)\cos(fx+e)^4-(a^2-4\,ab+3\,b^2)\cos(fx+e)^2-ab+3\,b^2\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((a^2-3\,ab)\cos(fx+e)^4-(a^2-4\,ab+3\,b^2)\cos(fx+e)^2-ab+3\,b^2\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{4\left((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2\,a^3b-2\,ab^3-b^4)f\cos(fx+e)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^2
 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x + e
)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b + b
^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*co
s(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*co
s(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2
*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4
 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 - a*b)*cos
(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(b/a)
*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e) - ((a^2 -
3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b
 + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3
 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(131) = 262.
time = 0.57, size = 550, normalized size = 3.74

$$\frac{\frac{6\,(a-3\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{12\,(3\,ab-b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}+\frac{3\,a^2+6\,ab+3\,b^2+\frac{4\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{20\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{15\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,a^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{4\,ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{6\,b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}-\frac{3\,(\cos(fx+e)-1)}{(a^2+2\,ab+b^2)(\cos(fx+e)+1)}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(a - 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 12*(3*a*b - b^2)*arctan(-(a*cos(f*x + e) - b)/(s
qrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a
*b)) + (3*a^2 + 6*a*b + 3*b^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 20*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)
```

```
/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 15*b^2*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 4*a*
b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 6*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b + b^2)*(cos(
f*x + e) + 1)))/f
```

Mupad [B]
time = 5.65, size = 1845, normalized size = 12.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - ((cos(e + f*x)^3*(a - b))/(2*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x))/(2*a
*b + a^2 + b^2))/(f*(b - a*cos(e + f*x)^4 + cos(e + f*x)^2*(a - b))) - (log
(cos(e + f*x) - 1)*(b/(a + b)^3 - 1/(4*(a + b)^2)))/f - (log(cos(e + f*x) +
 1)*(a - 3*b))/(4*f*(a + b)^3) - (atan((((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4
 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b
^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^
4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^
6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a
 - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b
^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a
*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b +
a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) +
((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^
2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8
*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7
*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2
) + (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 -
80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^
3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
)*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b
^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/((((3*a*b^4)/4 - (3*a^4*b)/4 - (13*a^2*b^3
)/4 + (13*a^3*b^2)/4)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*
b^3 + 15*a^4*b^2) + ((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6
*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (
(-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 +
```

```
 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16
*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 14
4*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))*(
3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)) - ((-a*b)^(1/2)*((cos(e +
 f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3
*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3
*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*
b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) + (cos(e + f*x)*(-a*b
)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^
5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a
^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3
+ 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2
*b^2))))*(-a*b)^(1/2)*(3*a - b)*1i)/(2*f*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2
))
```

3.46 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=197

$$\frac{3\sqrt{a}\,(a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4 f}-\frac{3(a^2-6ab+b^2)\tanh^{-1}(\cos(e+fx))}{8(a+b)^4 f}+\frac{3a(a-3b)\cos(e+fx}{8(a+b)^3 f\,(b+a\cos^2(e+}$$

[Out] -3/8*(a^2-6*a*b+b^2)*arctanh(cos(f*x+e))/(a+b)^4/f+3/8*a*(a-3*b)*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)-1/8*(a-5*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f/(b+a*cos(f*x+e)^2)+3/2*(a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^4/f

Rubi [A]
time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4218, 481, 541, 536, 212, 211}

$$-\frac{3(a^2-6ab+b^2)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^4}+\frac{3\sqrt{a}\,\sqrt{b}\,(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4}+\frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\,(a\cos^2(e+fx)+b)}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)\,(a\cos^2(e+fx)+b)}-\frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\,(a\cos^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*(a + b)^4*f) - (3*(a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^4*f) + (3*a*(a - 3*b)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) - ((a - 5*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{b+(-a+4b)x^2}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\,(b+a\cos^2(e+fx))} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e}{4(a+b)}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e}{4(a+b)}$$

$$= \frac{3\sqrt{a}\,(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4f} - \frac{3(a^2-6ab+b^2)\tanh^{-1}(\cos(e+fx}{8(a+b)^4f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.76, size = 450, normalized size = 2.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*b^2 + 4
*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e + f*x)])
*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2*(e
 + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2
*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)^4*f*(a + b
*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.35, size = 213, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^2(\cos(fx+e)-1)^2}-\frac{-3a+5b}{16(a+b)^3(\cos(fx+e)-1)}+\frac{\left(3a^2-18ab+3b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^4}+\frac{ba\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{3(a-b)\,\mathrm{ar}}{}\right.}{(a+b)^4}}{f}$
default	$\frac{-\frac{1}{16(a+b)^2(\cos(fx+e)-1)^2}-\frac{-3a+5b}{16(a+b)^3(\cos(fx+e)-1)}+\frac{\left(3a^2-18ab+3b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^4}+\frac{ba\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\left(\cos^2(fx+e)\right)}+\frac{3(a-b)\,\mathrm{ar}}{}\right.}{(a+b)^4}}{f}$
risch	$-\frac{-3a^2e^{11i(fx+e)}+9ab\,e^{11i(fx+e)}+5a^2e^{9i(fx+e)}-11ab\,e^{9i(fx+e)}+20b^2e^{9i(fx+e)}+30a^2e^{7i(fx+e)}+66ab\,e^{7i(fx+e)}+12b}{4f(a+b)^3\left(e^{2i(fx+e)}-1\right)^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/(a+b)^2/(cos(f*x+e)-1)^2-1/16*(-3*a+5*b)/(a+b)^3/(cos(f*x+e)-1)+
1/16/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(cos(f*x+e)-1)+1/(a+b)^4*b*a*((-1/2*a-1
/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2*(a-b)/(a*b)^(1/2)*arctan(a*cos(f*x+
e)/(a*b)^(1/2)))+1/16/(a+b)^2/(1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3/(1+c
os(f*x+e))+1/16/(a+b)^4*(-3*a^2+18*a*b-3*b^2)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.49, size = 378, normalized size = 1.92

$$-\frac{\frac{3\left(a^2-6ab+b^2\right)\log(\cos(fx+e)+1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{3\left(a^2-6ab+b^2\right)\log(\cos(fx+e)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{24\left(a^2b-ab^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab}}-\frac{2\left(3\left(a^2-3ab\right)\cos(fx+e)^5-\left(5a^2-14ab+5b^2\right)\cos(fx+e)^3-3\left(3ab-b^2\right)\cos(fx+e)\right)}{\left(a^4+3a^3b+3a^2b^2+ab^3\right)\cos(fx+e)^6-\left(2a^4+5a^3b+3a^2b^2-ab^3-b^4\right)\cos(fx+e)^4+a^3b+3a^2b^2+3ab^3+b^4+\left(a^4+a^3b-3a^2b^2-5ab^3-2b^4\right)\cos(fx+e)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - 3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) - 1)/(a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 24*(a^2*b - a*b^2)*arctan(a*cos(f*x +
e)/sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - 2*(
3*(a^2 - 3*a*b)*cos(f*x + e)^5 - (5*a^2 - 14*a*b + 5*b^2)*cos(f*x + e)^3 -
3*(3*a*b - b^2)*cos(f*x + e))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x
+ e)^6 - (2*a^4 + 5*a^3*b + 3*a^2*b^2 - a*b^3 - b^4)*cos(f*x + e)^4 + a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*
cos(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(189) = 378$.

time = 3.74, size = 1240, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*
a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3
*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b
^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(a*co
s(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^3 - 6
*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x
 + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x
+ e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^
2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x
+ e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*
x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f), 1/16*(6*(a
^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*a*b^2 + 5*b
^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3*a*b + b^2)
*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a*
b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x +
 e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b
^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^
2 - 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4
 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^
2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(179) = 358.
time = 0.59, size = 669, normalized size = 3.40

$$\frac{\frac{12\,(a^2-6\,ab+b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{96\,(a^2b-ab^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab}}-\frac{\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(a^2+2\,ab+b^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{108\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{18\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(fx+e)-1)^2}-\frac{64\left(a^2b+ab^2+\frac{a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 6*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) +
1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 96*(a^2*b - a*b^2)*arctan
(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - (8*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 - b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f
*x + e) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) -
 1)^2) - 64*(a^2*b + a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

Mupad [B]
time = 9.33, size = 2500, normalized size = 12.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*
a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 2
0*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*
b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b
^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9
+ b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 +
36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11
- 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 +
 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4
 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^
```

3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)
3i)/(4(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((cos(
e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32
*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)) -
(3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 -
6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^9*
b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a^3
*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e + f*
x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^
8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^
3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^
5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*
b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b)*3i)/(4*(4*a*b^3 + 4*a^3*b
+ a^4 + b^4 + 6*a^2*b^2)))/(((27*a^7*b)/64 + (81*a^3*b^5)/64 - (297*a^4*b^4
)/32 + (189*a^5*b^3)/16 - (135*a^6*b^2)/32)/(9*a*b^8 + 9*a^8*b + a^9 + b^9
+ 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7
b^2) - (3(-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 3
96*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4
+ 20*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a
^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^
7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a
^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3
 + 36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^
11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^
5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b +
a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3
*b^3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a -
 b))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (3*(-a*b)^(1/2)*((co
s(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(
32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))
- (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2
- 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4 + 114*a^
9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^7 + 84*a
^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) + (3*cos(e +
f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*
b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*
b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*
b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*
a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(a - b))/(4*(4*a*b^3 + 4*a^3*b +
 a^4 + b^4 + 6*a^2*b^2))))*(-a*b)^(1/2)*(a - b)*3i)/(2*f*(4*a*b^3 + 4*a^3*b
 + a^4 + b^4 + 6*a^2*b^2)) - ((3*cos(e + f*x)^5*(3*a*b - a^2))/(8*(3*a*b^2
+ 3*a^2*b + a^3 + b^3)) + (3*b*cos(e + f*x)*(3*a - b))/(8*(3*a*b^2 + 3*a^2*
b + a^3 + b^3)) + (cos(e + f*x)^3*(5*a^2 - 14*a*b + 5*b^2))/(8*(a + b)*(2*a
b + a^2 + b^2)))/(f(b - cos(e + f*x)^4*(2*a - b) + a*cos(e + f*x)^6 + cos
(e + f*x)^2*(a - 2*b))) - (atan((((((9*a^11*b)/2 - (3*a^2*b^10)/2 - (15*a^3

```
*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^8*b^4
+ 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36*a^2*b^
7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^2) - (co
s(e + f*x)*(3/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))
*(1792*a^10*b + 256*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168
*a^5*b^6 - 3584*a^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(32*
(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3
/(16*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4)) + (cos(e + f
*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*a^5*b^2))/(32*(6*a
*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)))*(3/(16
*(a + b)^2) - (3*b)/(2*(a + b)^3) + (3*b^2)/(2*(a + b)^4))*1i - ((((9*a^11*
b)/2 - (3*a^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*...
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=267

$$\frac{(5a^3+60a^2b+120ab^2+64b^3)\,x}{16a^5}-\frac{\sqrt{b}\,(a+b)^{3/2}(3a+8b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f}-\frac{(33a^2+82ab+48b^2)\cos}{48a^3f\,(a+b+b\,\text{t}}$$

```
[Out] 1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*x/a^5-1/2*(a+b)^(3/2)*(3*a+8*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f-1/48*(33*a^2+82*a*b+48*b^2
)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(9*a+8*b)*cos(f*x+e
)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f
/(a+b+b*tan(f*x+e)^2)-1/16*b*(19*a^2+52*a*b+32*b^2)*tan(f*x+e)/a^4/f/(a+b+b
*tan(f*x+e)^2)
```

Rubi [A]
time = 0.29, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4217, 481, 592, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(a+b)^{3/2}(3a+8b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f}+\frac{(9a+8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\,(a+b\tan^2(e+fx)+b)}-\frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\,(a+b\tan^2(e+fx)+b)}-\frac{(33a^2+82ab+48b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\,(a+b\tan^2(e+fx)+b)}+\frac{x(5a^3+60a^2b+120ab^2+64b^3)}{16a^5}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*x)/(16*a^5) - (Sqrt[b]*(a + b)^(3/
2)*(3*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*f) - ((33
*a^2 + 82*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan
[e + f*x]^2)) + ((9*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b
+ b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Ta
n[e + f*x]^2)) - (b*(19*a^2 + 52*a*b + 32*b^2)*Tan[e + f*x])/(16*a^4*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(b-6(a+b))x^2)}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+f\right.}{6af}$$

$$= \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))} + \frac{(9a+8b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))} + \frac{(9a+8b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= -\frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))} + \frac{(9a+8b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= \frac{(5a^3+60a^2b+120ab^2+64b^3)\,x}{16a^5} - \frac{\sqrt{b}\,(a+b)^{3/2}(3a+8b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx}{\sqrt{a+b}}\right.}{2a^5f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.71, size = 2738, normalized size = 10.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/512*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3
- 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x])
+ a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*
e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a
*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b +
 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a +
```

2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(4096*a^3*
(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x
]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) -
 (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(2
048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2
*Sec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(
3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e
 + f*x)]))))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[
2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2
*b^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Si
n[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^
4)*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f
*x] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 76
8*a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2
*e] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640
*a*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] -
 318*a^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 1
2*a^4*b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin
[2*(e + 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x]
- 256*a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[
6*e + 4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] +
2*a^4*b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6
*f*x] + 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(2048
*a^4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)^2*Sec[e + f*x]^4*(-(((a^6 - 48*a^5*b - 1200*a^4*b^2 - 6400*a^3*b^3 - 1344
0*a^2*b^4 - 12288*a*b^5 - 4096*b^6)*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a +
 b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b
*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x
])]*Cos[2*e])/(8*a^5*b*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I
/8)*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*S
in[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*b*Sqrt[a + b]*f
*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/(a + b)) - (Sec[2*e]*(-960*a^5*b*f*x*Co
s[2*e] - 10944*a^4*b^2*f*x*Cos[2*e] - 44544*a^3*b^3*f*x*Cos[2*e] - 83712*a^
2*b^4*f*x*Cos[2*e] - 73728*a*b^5*f*x*Cos[2*e] - 24576*b^6*f*x*Cos[2*e] - 48
0*a^5*b*f*x*Cos[2*f*x] - 4512*a^4*b^2*f*x*Cos[2*f*x] - 13248*a^3*b^3*f*x*Co
s[2*f*x] - 15360*a^2*b^4*f*x*Cos[2*f*x] - 6144*a*b^5*f*x*Cos[2*f*x] - 480*a
^5*b*f*x*Cos[4*e + 2*f*x] - 4512*a^4*b^2*f*x*Cos[4*e + 2*f*x] - 13248*a^3*b
^3*f*x*Cos[4*e + 2*f*x] - 15360*a^2*b^4*f*x*Cos[4*e + 2*f*x] - 6144*a*b^5*f
*x*Cos[4*e + 2*f*x] - 3*a^6*Sin[2*e] - 156*a^5*b*Sin[2*e] - 1500*a^4*b^2*Si
n[2*e] - 5760*a^3*b^3*Sin[2*e] - 10560*a^2*b^4*Sin[2*e] - 9216*a*b^5*Sin[2*
e] - 3072*b^6*Sin[2*e] + 3*a^6*Sin[2*f*x] + 366*a^5*b*Sin[2*f*x] + 3000*a^4
*b^2*Sin[2*f*x] + 8400*a^3*b^3*Sin[2*f*x] + 9600*a^2*b^4*Sin[2*f*x] + 3840*

```
a*b^5*Sin[2*f*x] + 216*a^5*b*Sin[4*e + 2*f*x] + 1800*a^4*b^2*Sin[4*e + 2*f*
x] + 5040*a^3*b^3*Sin[4*e + 2*f*x] + 5760*a^2*b^4*Sin[4*e + 2*f*x] + 2304*a
*b^5*Sin[4*e + 2*f*x] + 76*a^5*b*Sin[2*e + 4*f*x] + 460*a^4*b^2*Sin[2*e + 4
*f*x] + 768*a^3*b^3*Sin[2*e + 4*f*x] + 384*a^2*b^4*Sin[2*e + 4*f*x] + 76*a^
5*b*Sin[6*e + 4*f*x] + 460*a^4*b^2*Sin[6*e + 4*f*x] + 768*a^3*b^3*Sin[6*e +
 4*f*x] + 384*a^2*b^4*Sin[6*e + 4*f*x] - 16*a^5*b*Sin[4*e + 6*f*x] - 48*a^4
*b^2*Sin[4*e + 6*f*x] - 32*a^3*b^3*Sin[4*e + 6*f*x] - 16*a^5*b*Sin[8*e + 6*
f*x] - 48*a^4*b^2*Sin[8*e + 6*f*x] - 32*a^3*b^3*Sin[8*e + 6*f*x] + 4*a^5*b*
Sin[6*e + 8*f*x] + 4*a^4*b^2*Sin[6*e + 8*f*x] +...
```

Maple [A]
time = 0.17, size = 204, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-4a^2b-3a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{7}{4}a^2b-\frac{3}{2}a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{\left(5a^3+60a^2b+120\right.}{}}{a^5}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-4a^2b-3a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{7}{4}a^2b-\frac{3}{2}a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{\left(5a^3+60a^2b+120\right.}{}}{a^5}}{f}$
risch	$\frac{5x}{16a^2}+\frac{15xb}{4a^3}+\frac{15x\,b^2}{2a^4}+\frac{4x\,b^3}{a^5}-\frac{i\mathrm{e}^{4i(fx+e)}b}{32a^3f}+\frac{3i\mathrm{e}^{-4i(fx+e)}}{128a^2f}+\frac{i\mathrm{e}^{-4i(fx+e)}b}{32a^3f}-\frac{ib\left(a^3\mathrm{e}^{2i(fx+e)}+4a^2b\,\mathrm{e}^{2i(fx+e)}\right.}{a^5f\left(a\,\mathrm{e}^{4i(fx+e}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^5*(((-9/4*a^2*b-3/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-4*a^2*b-3*a*b^
2-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-7/4*a^2*b-3/2*a*b^2)*tan(f*x+e))/(tan(f*
x+e)^2+1)^3+1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*arctan(tan(f*x+e)))-(a+b
)^2*b/a^5*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+8*b)/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 312, normalized size = 1.17

$$-\frac{\frac{3\left(19a^2b+52ab^2+32b^3\right)\tan(fx+e)^7+\left(33a^3+253a^2b+516ab^2+288b^3\right)\tan(fx+e)^5+\left(40a^3+319a^2b+564ab^2+288b^3\right)\tan(fx+e)^3+3\left(5a^3+41a^2b+68ab^2+32b^3\right)\tan(fx+e)}{a^4b\tan(fx+e)^8+\left(a^5+4a^4b\right)\tan(fx+e)^6+a^5+a^4b+3\left(a^5+2a^4b\right)\tan(fx+e)^4+\left(3a^5+4a^4b\right)\tan(fx+e)^2}-\frac{3\left(5a^3+60a^2b+120ab^2+64b^3\right)(fx+e)}{a^5}+\frac{24\left(3a^3b+14a^2b^2+19ab^3+8b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^5}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/48*((3*(19*a^2*b + 52*a*b^2 + 32*b^3)*tan(f*x + e)^7 + (33*a^3 + 253*a^2*b + 516*a*b^2 + 288*b^3)*tan(f*x + e)^5 + (40*a^3 + 319*a^2*b + 564*a*b^2 + 288*b^3)*tan(f*x + e)^3 + 3*(5*a^3 + 41*a^2*b + 68*a*b^2 + 32*b^3)*tan(f*x + e))/(a^4*b*tan(f*x + e)^8 + (a^5 + 4*a^4*b)*tan(f*x + e)^6 + a^5 + a^4*b + 3*(a^5 + 2*a^4*b)*tan(f*x + e)^4 + (3*a^5 + 4*a^4*b)*tan(f*x + e)^2) - 3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 + 24*(3*a^3*b + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))/f

Fricas [**A**]
time = 4.31, size = 700, normalized size = 2.62

$$\left[\frac{3\left(5a^4+60a^3b+120a^2b^2+64ab^3\right)fx\cos\left(fx+e\right)^2+3\left(5a^3b+60a^2b^2+120ab^3+64b^4\right)fx+6\left(3a^2b+11ab^2+8b^3+\left(3a^3+11a^2b+8ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+2b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2}\right)-\left(8a^4\cos\left(fx+e\right)^7-2\left(13a^4+8a^3b\right)\cos\left(fx+e\right)^5+\left(33a^4+82a^3b+48a^2b^2\right)\cos\left(fx+e\right)^3+3\left(19a^3b+52a^2b^2+32ab^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\left(a^6f\cos\left(fx+e\right)^2+a^5bf\right)}, \frac{3\left(5a^4+60a^3b+120a^2b^2+64ab^3\right)fx\cos\left(fx+e\right)^2+3\left(5a^3b+60a^2b^2+120ab^3+64b^4\right)fx+12\left(3a^2b+11ab^2+8b^3+\left(3a^3+11a^2b+8ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{ab+b^2}\arctan\left(\frac{\left(a+2b\right)\cos\left(fx+e\right)^2-b}{2\sqrt{ab+b^2}\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)-\left(8a^4\cos\left(fx+e\right)^7-2\left(13a^4+8a^3b\right)\cos\left(fx+e\right)^5+\left(33a^4+82a^3b+48a^2b^2\right)\cos\left(fx+e\right)^3+3\left(19a^3b+52a^2b^2+32ab^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\left(a^6f\cos\left(fx+e\right)^2+a^5bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)

[Out] Timed out

Giac [A]
time = 0.51, size = 294, normalized size = 1.10

$$\frac{\frac{3\,(5\,a^3+60\,a^2b+120\,ab^2+64\,b^3)(fx+e)}{a^5}-\frac{24\,(3\,a^3b+14\,a^2b^2+19\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{24\,(a^2b\tan(fx+e)+2\,ab^2\tan(fx+e)+b^3\tan(fx+e))}{\left(b\tan(fx+e)^2+a+b\right)a^4}-\frac{33\,a^2\tan(fx+e)^5+108\,ab\tan(fx+e)^5+72\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+192\,ab\tan(fx+e)^3+144\,b^2\tan(fx+e)^3+15\,a^2\tan(fx+e)+84\,ab\tan(fx+e)+72\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3a^4}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*tan
(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a
 + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*tan(f*
x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*x + e))
/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B]
time = 6.66, size = 1461, normalized size = 5.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atanh((75*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(2
56*((211*a*b^4)/128 + (811*b^5)/256 + (75*a^2*b^3)/256 + (41*b^6)/(16*a) +
(3*b^7)/(4*a^2))) + (17*b^4*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b
^2)^(1/2))/(16*((811*a*b^5)/256 + (41*b^6)/16 + (211*a^2*b^4)/128 + (75*a^3
*b^3)/256 + (3*b^7)/(4*a))) + (3*b^5*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4
- 3*a^2*b^2)^(1/2))/(4*((41*a*b^6)/16 + (3*b^7)/4 + (811*a^2*b^5)/256 + (21
1*a^3*b^4)/128 + (75*a^4*b^3)/256)))*(-b*(a + b)^3)^(1/2)*(3*a + 8*b))/(2*a
^5*f) - (atan((((((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^
2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^
2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i
+ b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^
7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))
*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5) - (((((8*a^10*b^5
 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (tan(e + f*x)*(20
48*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(
4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) + (tan(e
+ f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*
b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3
*5i + b^3*64i)*1i)/(32*a^5))/((376*a*b^10 + 64*b^11 + 937*a^2*b^9 + (10285*
a^3*b^8)/8 + (33701*a^4*b^7)/32 + (8333*a^5*b^6)/16 + (38085*a^6*b^5)/256 +
```

(2765*a^7*b^4)/128 + (285*a^8*b^3)/256)/a^12 + (((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) + (((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5) + (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(32*a^5)))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(16*a^5*f) - ((tan(e + f*x)*(68*a*b^2 + 41*a^2*b + 5*a^3 + 32*b^3))/(16*a^4) + (tan(e + f*x)^5*(516*a*b^2 + 253*a^2*b + 33*a^3 + 288*b^3))/(48*a^4) + (tan(e + f*x)^3*(564*a*b^2 + 319*a^2*b + 40*a^3 + 288*b^3))/(48*a^4) + (b*tan(e + f*x)^7*(52*a*b + 19*a^2 + 32*b^2))/(16*a^4))/(f*(a + b + tan(e + f*x)^2*(3*a + 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)^6*(a + 4*b)))

3.48 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{3(a^2+8ab+8b^2)\,x}{8a^4}-\frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))}+\frac{\text{co}}{4af}$$

```
[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))*b^(1/2)*(a+b)^(1/2)/a^4/f-1/8*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+
b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3/8*
b*(3*a+4*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 209, 211}

$$-\frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{3b(3a+4b)\tan(e+fx)}{8a^3f\,(a+b\tan^2(e+fx)+b)}-\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{8a^2f\,(a+b\tan^2(e+fx)+b)}+\frac{3x(a^2+8ab+8b^2)}{8a^4}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*f) - ((5*a + 6*b)*Cos[e + f*
x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) - (3*b*(3*a + 4*b)*Tan[e + f*
x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-5b)x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\big(\int}{}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{3b(3a}{8a^3f\,(a}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{3b(3a}{8a^3f\,(a}$$

$$= \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b}{8a^2f\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.50, size = 1105, normalized size = 5.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Se
c[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b
)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x
)]))))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e +
2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 -
 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]))
+ (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)*f*x
*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f*x] +
```

```
80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 768*a^2*
b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2*e] +
34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640*a*b^4
*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] - 318*a
^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 12*a^4*
b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin[2*(e
+ 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x] - 256*
a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[6*e +
4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] + 2*a^4*
b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6*f*x]
+ 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(1024*a^4*b
*(a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]

time = 0.22, size = 147, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{3\left(a^2+8ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{3(}{}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{3\left(a^2+8ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{3(}{}\right.}{a^4}}{f}$
risch	$\frac{3x}{8a^2}+\frac{3xb}{a^3}+\frac{3x\,b^2}{a^4}-\frac{i e^{4i(fx+e)}}{64a^2f}+\frac{i e^{2i(fx+e)}}{8a^2f}+\frac{i e^{2i(fx+e)}b}{4a^3f}-\frac{i e^{-2i(fx+e)}}{8a^2f}-\frac{i e^{-2i(fx+e)}b}{4a^3f}+\frac{i e^{-4i(fx+e)}}{64a^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(((-a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-a*b)*tan(f*x+e))/(tan(f*
x+e)^2+1)^2+3/8*(a^2+8*a*b+8*b^2)*arctan(tan(f*x+e)))-(a+b)*b/a^4*(1/2*a*ta
n(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2*(a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+
e)/((a+b)*b)^(1/2))))
```

Maxima [A]

time = 0.49, size = 213, normalized size = 1.12

$$-\frac{\frac{3\,(3\,ab+4\,b^2)\tan(fx+e)^5+(5\,a^2+24\,ab+24\,b^2)\tan(fx+e)^3+3\,(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^3b\tan(fx+e)^6+(a^4+3\,a^3b)\tan(fx+e)^4+a^4+a^3b+(2\,a^4+3\,a^3b)\tan(fx+e)^2}-\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}+\frac{12\,(a^2b+3\,ab^2+2\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^4}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/8*((3*(3*a*b + 4*b^2)*tan(f*x + e)^5 + (5*a^2 + 24*a*b + 24*b^2)*tan(f*x + e)^3 + 3*(a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a^3*b*tan(f*x + e)^6 + (a^4 + 3*a^3*b)*tan(f*x + e)^4 + a^4 + a^3*b + (2*a^4 + 3*a^3*b)*tan(f*x + e)^2) - 3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 + 12*(a^2*b + 3*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f

Fricas [A]
time = 2.47, size = 546, normalized size = 2.86

$$\left[\frac{3\,(a^3+8\,a^2b+8\,ab^2)fx\cos{(fx+e)}^2+3\,(a^2b+8\,ab^2+8\,b^3)fx+3\,((a^2+2\,ab)\cos{(fx+e)}^2+ab+2\,b^2)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,((a+2\,b)\cos(fx+e)^3-b\cos(fx+e))\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+(2\,a^3\cos{(fx+e)}^5-(5\,a^3+6\,a^2b)\cos{(fx+e)}^3-3\,(3\,a^2b+4\,ab^2)\cos{(fx+e)})\sin{(fx+e)}}{8\,(a^5f\cos{(fx+e)}^2+a^4bf)}, \frac{3\,(a^3+8\,a^2b+8\,ab^2)fx\cos{(fx+e)}^2+3\,(a^2b+8\,ab^2+8\,b^3)fx+6\,((a^2+2\,ab)\cos{(fx+e)}^2+ab+2\,b^2)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\,\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)+(2\,a^3\cos{(fx+e)}^5-(5\,a^3+6\,a^2b)\cos{(fx+e)}^3-3\,(3\,a^2b+4\,ab^2)\cos{(fx+e)})\sin{(fx+e)}}{8\,(a^5f\cos{(fx+e)}^2+a^4bf)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)*f*x + 6*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)

[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)

Giac [A]
time = 0.53, size = 193, normalized size = 1.01

$$\frac{\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}-\frac{12\,(a^2b+3\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4}-\frac{4\,(ab\tan(fx+e)+b^2\tan(fx+e))}{(b\tan(fx+e)^2+a+b)a^3}-\frac{5\,a\tan(fx+e)^3+8\,b\tan(fx+e)^3+3\,a\tan(fx+e)+8\,b\tan(fx+e)}{(\tan(fx+e)^2+1)^2a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*(
pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*tan(
f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3*a*t
an(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B]
time = 5.77, size = 435, normalized size = 2.28

$$\frac{3\,\mathrm{atanh}\left(\frac{27\,b^3\,\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{64\,\left(\frac{27\,a\,b^3}{64}+\frac{81\,b^4}{64}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^4\,\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{32\,\left(\frac{27\,a^2\,b^3}{64}+\frac{81\,a\,b^4}{64}+\frac{27\,b^5}{32}\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{2\,a^4\,f}-\frac{\frac{3\tan(e+f\,x)^5\,(4\,b^2+3\,a\,b)}{8\,a^3}+\frac{\tan(e+f\,x)^3\,(5\,a^2+24\,a\,b+24\,b^2)}{8\,a^3}+\frac{3\tan(e+f\,x)\,(a^2+5\,a\,b+4\,b^2)}{8\,a^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+(a+3\,b)\,\tan\left(e+f\,x\right)^4+(2\,a+3\,b)\,\tan\left(e+f\,x\right)^2+a+b\right)}-\frac{\mathrm{atan}\left(\frac{27\,b^2\,\tan(e+f\,x)}{256\,\left(\frac{27\,b^2}{256}+\frac{243\,b^3}{256\,a}+\frac{27\,b^4}{16\,a^2}+\frac{27\,b^5}{32\,a^3}\right)}+\frac{243\,b^3\,\tan(e+f\,x)}{256\,\left(\frac{27\,a\,b^2}{256}+\frac{243\,b^3}{256}+\frac{27\,b^4}{16\,a}+\frac{27\,b^5}{32\,a^2}\right)}+\frac{27\,b^4\,\tan(e+f\,x)}{16\,\left(\frac{243\,a\,b^3}{256}+\frac{27\,b^4}{16}+\frac{27\,a^2\,b^2}{256}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^5\,\tan(e+f\,x)}{32\,\left(\frac{27\,a^3\,b^2}{256}+\frac{243\,a^2\,b^3}{256}+\frac{27\,a\,b^4}{16}+\frac{27\,b^5}{32}\right)}\right)\,\left(a^2\,1{}\mathrm{i}+a\,b\,8{}\mathrm{i}+b^2\,8{}\mathrm{i}\right)\,3{}\mathrm{i}}{8\,a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (3*atanh((27*b^3*tan(e + f*x)*(- a*b - b^2)^(1/2))/(64*((27*a*b^3)/64 + (81
*b^4)/64 + (27*b^5)/(32*a))) + (27*b^4*tan(e + f*x)*(- a*b - b^2)^(1/2))/(3
2*((81*a*b^4)/64 + (27*b^5)/32 + (27*a^2*b^3)/64)))*(a + 2*b)*(-b*(a + b))^
(1/2))/(2*a^4*f) - (atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (243*b^
3)/(256*a) + (27*b^4)/(16*a^2) + (27*b^5)/(32*a^3))) + (243*b^3*tan(e + f*x
))/(256*((27*a*b^2)/256 + (243*b^3)/256 + (27*b^4)/(16*a) + (27*b^5)/(32*a^
2))) + (27*b^4*tan(e + f*x))/(16*((243*a*b^3)/256 + (27*b^4)/16 + (27*a^2*b
^2)/256 + (27*b^5)/(32*a))) + (27*b^5*tan(e + f*x))/(32*((27*a*b^4)/16 + (2
7*b^5)/32 + (243*a^2*b^3)/256 + (27*a^3*b^2)/256)))*(a*b*8i + a^2*1i + b^2*
8i)*3i)/(8*a^4*f) - ((3*tan(e + f*x)^5*(3*a*b + 4*b^2))/(8*a^3) + (tan(e +
f*x)^3*(24*a*b + 5*a^2 + 24*b^2))/(8*a^3) + (3*tan(e + f*x)*(5*a*b + a^2 +
4*b^2))/(8*a^3))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^6
+ tan(e + f*x)^4*(a + 3*b)))
```

3.49 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{(a+4b)x}{2a^3}-\frac{\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,f}-\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))}-\frac{b\tan(e+fx)}{a^2f\,(a+b+b\tan^2(e+fx$$

```
[Out] 1/2*(a+4*b)*x/a^3-1/2*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/f/(a+b)^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-b*
tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {4217, 482, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}}+\frac{x(a+4b)}{2a^3}-\frac{b\tan(e+fx)}{a^2f\,(a+b\tan^2(e+fx)+b)}-\frac{\sin(e+fx)\cos(e+fx)}{2af\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*x)/(2*a^3) - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/(a^2*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+4b)\text{Sub}\cdots}{\cdots}$$

$$= \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,f} - \frac{\cos(e+fx)\sin(e+f\cdots}{2af\left(a+b+b\tan^2(e+\cdots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.97, size = 825, normalized size = 6.35

$$-\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(16x+\frac{(-a^3+6a^2b+24ab^2+16b^3)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{(a^2+8ab+8b^2)((a+2b)\sin(2e)-a\sin(2fx))}{b(a+b)f(a+2b+a\cos(2(e+fx)))(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{128a^2(a+b\sec^2(e+fx))^2}-\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(-64(a+2b)x+\frac{(a^4-16a^3b-144a^2b^2-256ab^3-128b^4)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{16a\cos(2fx)\sin(2e)}{f}+\frac{16a\cos(2e)\sin(2fx)}{f}-\frac{(a^3+18a^2b+48ab^2+32b^3)((a+2b)\sin(2e)-a\sin(2fx))}{b(a+b)f(a+2b+a\cos(2(e+fx)))(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{256a^3(a+b\sec^2(e+fx))^2}+\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)}{128b^{3/2}f(a+b\sec^2(e+fx))^2}+\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(-\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{\sqrt{b}(a+2b)\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)}{256b^{3/2}f(a+b\sec^2(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/128*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*
a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(
a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(a^2*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[
e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3 -
128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
 - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a*C
os[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b + 4
8*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*
b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(256*a^3*(a
+ b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(
((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*S
qrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(128*b^
(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e
 + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2))
+ (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x
)]))))/(256*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.18, size = 109, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}}{f}$

risch	$\frac{x}{2a^2}+\frac{2xb}{a^3}+\frac{i e^{2i(fx+e)}}{8a^2 f}-\frac{i e^{-2i(fx+e)}}{8a^2 f}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^3 f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{3\sqrt{-(a+b)\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*(-1/2*a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2*(a+4*b)*arctan(tan(f*x+e
)))-1/a^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+4*b)/((a+b)*b)^
(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.48, size = 132, normalized size = 1.02

$$-\frac{\frac{2\,b\tan(fx+e)^3+(a+2\,b)\tan(fx+e)}{a^2b\tan(fx+e)^4+a^3+a^2b+(a^3+2\,a^2b)\tan(fx+e)^2}-\frac{(fx+e)(a+4\,b)}{a^3}+\frac{(3\,ab+4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*tan(f*x + e)^3 + (a + 2*b)*tan(f*x + e))/(a^2*b*tan(f*x + e)^4 +
 a^3 + a^2*b + (a^3 + 2*a^2*b)*tan(f*x + e)^2) - (f*x + e)*(a + 4*b)/a^3 +
(3*a*b + 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3
))/f
```

Fricas [A]
time = 3.08, size = 463, normalized size = 3.56

$$\left[\frac{4\,(a^2+4\,ab)fx\cos(fx+e)^2+4\,(ab+4\,b^2)fx+\left((3\,a^2+4\,ab)\cos(fx+e)^2+3\,ab+4\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)-4\left(a^2\cos(fx+e)^3+2\,ab\cos(fx+e)\right)\sin(fx+e)}{8\left(a^4f\cos(fx+e)^2+a^3bf\right)},\ \frac{2\,(a^2+4\,ab)fx\cos(fx+e)^2+2\,(ab+4\,b^2)fx+\left((3\,a^2+4\,ab)\cos(fx+e)^2+3\,ab+4\,b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-2\left(a^2\cos(fx+e)^3+2\,ab\cos(fx+e)\right)\sin(fx+e)}{4\left(a^4f\cos(fx+e)^2+a^3bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b
 + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a^2*
cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 +
a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*f*x + (
(3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)
```

```
)) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f
*x + e)^2 + a^3*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.53, size = 149, normalized size = 1.15

$$\frac{\frac{(fx+e)(a+4b)}{a^3} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab+4b^2)}{\sqrt{ab+b^2}\,a^3} - \frac{2b\tan(fx+e)^3+a\tan(fx+e)+2b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2b\tan(fx+e)^2+a+b\right)a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arcta
n(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3) -
(2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e)^4
 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

Mupad [B]
time = 5.21, size = 816, normalized size = 6.28

$$-\frac{\frac{\tan(e+fx)(a+2b)}{2a^2}+\frac{b\tan(e+fx)^3}{a^2}}{f\left(b\tan(e+fx)^4+(a+2b)\tan(e+fx)^2+a+b\right)} - \frac{\operatorname{atan}\left(\frac{b^2\tan(e+fx)}{4\left(\frac{b^2}{4}+\frac{b^3}{a}\right)}+\frac{b^3\tan(e+fx)}{b^3+\frac{ab^2}{4}}\right)(a\,1i+b\,4i)\,1i}{2a^3f} + \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{\tan(e+fx)(5a^2b^3+16ab^4+16b^5)}{a^4}+\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}+\frac{\tan(e+fx)(8a^7b^2+16a^6b^3)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4(a^4+ba^3)}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}\,1i}{a^4+ba^3}+\frac{\left(\frac{\tan(e+fx)(5a^2b^3+16ab^4+16b^5)}{a^4}-\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}-\frac{\tan(e+fx)(8a^7b^2+16a^6b^3)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4(a^4+ba^3)}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}\,1i}{a^4+ba^3}}{\frac{\frac{3a^2b^3}{2}+8ab^4+8b^5}{a^6}+\frac{\left(\frac{\tan(e+fx)(5a^2b^3+16ab^4+16b^5)}{a^4}+\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}+\frac{\tan(e+fx)(8a^7b^2+16a^6b^3)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4(a^4+ba^3)}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}-\frac{\left(\frac{\tan(e+fx)(5a^2b^3+16ab^4+16b^5)}{a^4}-\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}-\frac{\tan(e+fx)(8a^7b^2+16a^6b^3)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4(a^4+ba^3)}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}\,2i}{f(a^4+ba^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^6*b^3
+ 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b
*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^
3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4) + (((tan(e +
 f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a^6
- (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2))/
(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((3*a
```

```
)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4))/((8*a*b^4 + 8*b^5 + (3*a^2*b
^3)/2)/a^6 + (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^
6*b^3 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 +
b)*(-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4) - (((tan(
e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)/a
^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(1/2
))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4))*((
3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^
(1/2)*2i)/(f*(a^3*b + a^4)) - (atan((b^2*tan(e + f*x))/(4*(b^2/4 + b^3/a))
+ (b^3*tan(e + f*x))/((a*b^2)/4 + b^3))*(a*1i + b*4i)*1i)/(2*a^3*f) - ((tan
(e + f*x)*(a + 2*b))/(2*a^2) + (b*tan(e + f*x)^3)/a^2)/(f*(a + b + b*tan(e
+ f*x)^4 + tan(e + f*x)^2*(a + 2*b)))
```

3.50 $\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{x}{a^2}-\frac{\sqrt{b}\,(3a+2b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4213, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a+2b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e\right.}{2a(a+b)f}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} -\\
&= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+f}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.17, size = 240, normalized size = 2.61

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
```

)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)

Maple [A]
time = 0.15, size = 90, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2} - \frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)} + \frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2} - \frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)} + \frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}}{f}$
risch	$\frac{x}{a^2} - \frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)} + \frac{3\sqrt{-(a+b)\,b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)}}{a}\right.}{4(a+b)^2 fa}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^2*arctan(tan(f*x+e))-1/a^2*b*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)+1/2*(3*a+2*b)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(
1/2))))

Maxima [A]
time = 0.48, size = 110, normalized size = 1.20

$$-\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(83) = 166$.
time = 3.14, size = 455, normalized size = 4.95

$$\left[\frac{8\,(a^2+ab)fx\cos\left(fx+e\right)^2-4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\,(ab+b^2)fx+\left((3\,a^2+2\,ab)\cos\left(fx+e\right)^2+3\,ab+2\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{8\left((a^4+a^3b)f\cos\left(fx+e\right)^2+(a^3b+a^2b^2)f\right)},\ \frac{4\,(a^2+ab)fx\cos\left(fx+e\right)^2-2\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+4\,(ab+b^2)fx+\left((3\,a^2+2\,ab)\cos\left(fx+e\right)^2+3\,ab+2\,b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{4\left((a^4+a^3b)f\cos\left(fx+e\right)^2+(a^3b+a^2b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A]
time = 0.47, size = 114, normalized size = 1.24

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B]
time = 6.35, size = 2056, normalized size = 22.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
```

```
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.51 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f}-\frac{3\cot(e+fx)}{2(a+b)^2f}+\frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -3/2*cot(f*x+e)/(a+b)^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2
)/(a+b)^(5/2)/f+1/2*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}-\frac{3\cot(e+fx)}{2f(a+b)^2}+\frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*f)
 - (3*Cot[e + f*x])/(2*(a + b)^2*f) + Cot[e + f*x]/(2*(a + b)*f*(a + b + b*
Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right.}{2(a+b)f} \\
&= -\frac{3\cot(e+fx)}{2(a+b)^2f} + \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\right.}{2(a+} \\
&= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{3\cot(e+fx)}{2(a+b)^2f} + \frac{\cot(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.55, size = 242, normalized size = 2.66

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{3b\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}+2(a+2b+a\cos(2(e+fx)))\csc(e)\csc(e+fx)\sin(fx)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e]
 - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*b +
a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^2*f*(a
 + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.17, size = 78, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2\sqrt{(a+b)\,b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2\sqrt{(a+b)\,b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
risch	$-\frac{i\left(2a^2 e^{4i(fx+e)}+ab\,e^{4i(fx+e)}+2b^2 e^{4i(fx+e)}+4a^2 e^{2i(fx+e)}+8ab\,e^{2i(fx+e)}-2b^2 e^{2i(fx+e)}+2a^2-ab\right)}{a(a+b)^2 f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^2*b*(1/2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2/((a+b)*b)^(1/2)*
arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(a+b)^2/tan(f*x+e))
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.33

$$-\frac{\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}+\frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{(a^2b+2\,ab^2+b^3)\tan(fx+e)^3+(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt(
(a + b)*b)) + (3*b*tan(f*x + e)^2 + 2*a + 2*b)/((a^2*b + 2*a*b^2 + b^3)*tan
(f*x + e)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(81) = 162$.
time = 2.96, size = 429, normalized size = 4.71

$$\left[-\frac{4\,(2\,a-b)\cos(fx+e)^3-3\left(a\cos(fx+e)^2+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)+12\,b\cos(fx+e)}{8\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)f\right)\sin(fx+e)},\ -\frac{2\,(2\,a-b)\cos(fx+e)^3-3\left(a\cos(fx+e)^2+b\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+6\,b\cos(fx+e)}{4\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*
cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a - b
)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*s
in(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2
+ (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [**A**]
time = 0.58, size = 127, normalized size = 1.40

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^2+2ab+b^2)\sqrt{ab+b^2}}+\frac{3b\tan(fx+e)^2+2a+2b}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^2+2ab+b^2)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2 +
 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 + 2*
a*b + b^2)))/f
```

Mupad [**B**]
time = 4.40, size = 91, normalized size = 1.00

$$-\frac{\frac{1}{a+b}+\frac{3b\tan(e+fx)^2}{2(a+b)^2}}{f\left(b\tan(e+fx)^3+(a+b)\tan(e+fx)\right)}-\frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)\,(a^2+2ab+b^2)}{(a+b)^{5/2}}\right)}{2f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(a + b) + (3*b*tan(e + f*x)^2)/(2*(a + b)^2))/(f*(b*tan(e + f*x)^3 + t
an(e + f*x)*(a + b))) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2
+ b^2))/(a + b)^(5/2)))/(2*f*(a + b)^(5/2))
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{(3a-2b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f}-\frac{(a-b)\cot(e+fx)}{(a+b)^3f}-\frac{\cot^3(e+fx)}{3(a+b)^2f}-\frac{ab\tan(e+fx)}{2(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -(a-b)*cot(f*x+e)/(a+b)^3/f-1/3*cot(f*x+e)^3/(a+b)^2/f-1/2*(3*a-2*b)*arctan
(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f-1/2*a*b*tan(f*x+e)/(
a+b)^3/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 467, 1275, 211}

$$-\frac{\sqrt{b}\,(3a-2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}-\frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}-\frac{\cot^3(e+fx)}{3f(a+b)^2}-\frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a +
 b)^(7/2)*f) - ((a - b)*Cot[e + f*x])/((a + b)^3*f) - Cot[e + f*x]^3/(3*(a
+ b)^2*f) - (a*b*Tan[e + f*x])/(2*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4(a+b+bx^2)}\,dx, x, \mathrm{t}\right.}{2f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2x^4} - \frac{2(a-b)}{b(a+b)^3x^2} + \frac{}{(a}\right.\right.}{2f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} -$$

$$= -\frac{(3a-2b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.87, size = 637, normalized size = 5.18

$$-\frac{(a+2b+a\cos(2e+2fx))^2\cot(e)\csc^2(e+fx)\sec^4(e+fx)}{12(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^2} + \frac{(3a-2b)(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(\frac{b\text{ArcTan}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(fx)+a\sin(2e+fx))\right)\cos(2e)}{8\sqrt{a+b}\,f\sqrt{b\cos(4e)-ib\sin(4e)}} - \frac{ib\text{ArcTan}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(fx)+a\sin(2e+fx))\right)\sin(2e)}{8\sqrt{a+b}\,f\sqrt{b\cos(4e)-ib\sin(4e)}}\right)}{(a+b)^3\left(a+b\sec^2(e+fx)\right)^2} + \frac{(a+2b+a\cos(2e+2fx))^2\csc(e)\csc^3(e+fx)\sec^4(e+fx)\sin(fx)}{12(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^2} + \frac{(a+2b+a\cos(2e+2fx))^2\csc(e)\csc(e+fx)\sec^4(e+fx)(a\sin(fx)-2b\sin(fx))}{6(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^2} + \frac{(a+2b+a\cos(2e+2fx))\sec^4(e+fx)\left(ab\sin(2e)+2b^2\sin(2e)-ab\sin(2fx)\right)}{8(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^2(\cos(e)-\sin(e))(\cos(e)+\sin(e))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/12*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Cot[e]*Csc[e + f*x]^2*Sec[e + f*x]^
4)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^2) + ((3*a - 2*b)*(a + 2*b + a*Cos[2
*e + 2*f*x])^2*Sec[e + f*x]^4*((b*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*
Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos
[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*
```

```
Cos[2*e])/(8*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/8)*b*ArcT
an[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I
/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x])
 - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(Sqrt[a + b]*f*Sqrt[b*Cos[4*
e] - I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*
Cos[2*e + 2*f*x])^2*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^4*Sin[f*x])/(12*(a +
 b)^2*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Csc[e
]*Csc[e + f*x]*Sec[e + f*x]^4*(a*Sin[f*x] - 2*b*Sin[f*x]))/(6*(a + b)^3*f*(
a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^4*(
a*b*Sin[2*e] + 2*b^2*Sin[2*e] - a*b*Sin[2*f*x]))/(8*(a + b)^3*f*(a + b*Sec[
e + f*x]^2)^2*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
```

Maple [A]

time = 0.24, size = 106, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
risch	$\frac{i\left(9ab\,e^{8i(fx+e)}-6b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+18ab\,e^{6i(fx+e)}+66b^2e^{6i(fx+e)}+20a^2e^{4i(fx+e)}+44ab\,e^{4i(fx+e)}-66b^2e^{4i}\right.}{3f(a+b)^3\left(e^{2i(fx+e)}-1\right)^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(f}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-2*b)/((a+
b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^2/tan(f*x+e)^3-
(a-b)/(a+b)^3/tan(f*x+e))
```

Maxima [A]

time = 0.49, size = 198, normalized size = 1.61

$$-\frac{\frac{3\left(3ab-2b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{3\left(3ab-2b^2\right)\tan(fx+e)^4+2\left(3a^2+ab-2b^2\right)\tan(fx+e)^2+2a^2+4ab+2b^2}{\left(a^3b+3a^2b^2+3ab^3+b^4\right)\tan(fx+e)^5+\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\tan(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)) + (3*(3*a*b - 2*b^2)*tan(f*x + e)^4 +
 2*(3*a^2 + a*b - 2*b^2)*tan(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)/((a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*tan(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(114) = 228$.
time = 3.09, size = 691, normalized size = 5.62

$$\left[-\frac{4\,(4\,a^2-11\,ab)\cos(fx+e)^5-8\,(3\,a^2-8\,ab+4\,b^2)\cos(fx+e)^3+3\left((3\,a^2-2\,ab)\cos(fx+e)^4-(3\,a^2-5\,ab+2\,b^2)\cos(fx+e)^2-3\,ab+2\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)-12\,(3\,ab-2\,b^2)\cos(fx+e)}{24\left((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2\,a^3b-2\,ab^3-b^4)f\cos(fx+e)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)f\right)\sin(fx+e)},\ -\frac{2\,(4\,a^2-11\,ab)\cos(fx+e)^5-4\,(3\,a^2-8\,ab+4\,b^2)\cos(fx+e)^3-3\left((3\,a^2-2\,ab)\cos(fx+e)^4-(3\,a^2-5\,ab+2\,b^2)\cos(fx+e)^2-3\,ab+2\,b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-6\,(3\,ab-2\,b^2)\cos(fx+e)}{12\left((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2\,a^3b-2\,ab^3-b^4)f\cos(fx+e)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(f
*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos
(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(
3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)*cos(f*x
+ e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2*a*b)*cos(
f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(b
/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(
f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f*x + e))/(((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x
 + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.56, size = 185, normalized size = 1.50

$$-\frac{\frac{3\,ab\tan(fx+e)}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)}+\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-2\,b^2)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab+b^2}}+\frac{2\left(3\,a\tan(fx+e)^2-3\,b\tan(fx+e)^2+a+b\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2
 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*
b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B]
time = 5.49, size = 141, normalized size = 1.15

$$-\frac{\frac{1}{3\,(a+b)}+\frac{\tan(e+f\,x)^2\,(3\,a-2\,b)}{3\,(a+b)^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-2\,b)}{2\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+(a+b)\,\tan\left(e+f\,x\right)^3\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{(a+b)^{7/2}}\right)\,(3\,a-2\,b)}{2\,f\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] - (1/(3*(a + b)) + (tan(e + f*x)^2*(3*a - 2*b))/(3*(a + b)^2) + (b*tan(e +
f*x)^4*(3*a - 2*b))/(2*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) + b*tan(e + f
*x)^5)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))/(a + b)^(7/2))*(3*a - 2*b))/(2*f*(a + b)^(7/2))
```

3.53 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} \, dx$

Optimal. Leaf size=188

$$-\frac{a(3a-4b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f}-\frac{(5a^2-10ab-b^2)\cot(e+fx)}{5(a+b)^4f}-\frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f}-\frac{}{5(a+b)}$$

```
[Out] -1/5*(5*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^4/f-1/15*(10*a+3*b)*cot(f*x+e)^3/(
a+b)^3/f-1/2*a*(3*a-4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+
b)^(9/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)-1/10*b*(5*a^2+2*b^
2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 473, 467, 1275, 211}

$$-\frac{b(5a^2+2b^2)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{(5a^2-10ab-b^2)\cot(e+fx)}{5f(a+b)^4}-\frac{a\sqrt{b}\,(3a-4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}}-\frac{(10a+3b)\cot^3(e+fx)}{15f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a
 + b)^(9/2)*f) - ((5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(5*(a + b)^4*f) - ((
10*a + 3*b)*Cot[e + f*x]^3)/(15*(a + b)^3*f) - Cot[e + f*x]^5/(5*(a + b)*f*
(a + b + b*Tan[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/(10*(a + b)^
4*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a+3b+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+\right.}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f} - \frac{\cot^5}{5(a+b)f\,(a+}$$

$$= -\frac{a(3a-4b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f} - \frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.79, size = 777, normalized size = 4.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTan
[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Sin[f*x
] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*x] + 6
40*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*e - f*x
] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*Sin[2*e +
 f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*b*Sin[4*e
 + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x] - 30*a*b^2
*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e + 3*f*x] + 20
8*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^2*b*Sin[6*e
+ 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*x] + 48*a^3*S
in[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[2*e + 5*f*x] -
 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2*b*Sin[6*e + 5*
f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x] - 45*a^2*b*Sin[
8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e + 7*f*x] + 83*a^2
*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b*Sin[6*e + 7*f*x]
- 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] - 6*a*b^2*Sin[8*e + 7
*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.22, size = 124, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^{2}(fx+e)\right)}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^{4}}-\frac{1}{5(a+b)^{2}\tan(fx+e)^{5}}-\frac{a(a-2b)}{(a+b)^{4}\tan(fx+e)}-\frac{2a}{3(a+b)^{3}\tan(f}}{f}$
default	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^{2}(fx+e)\right)}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^{4}}-\frac{1}{5(a+b)^{2}\tan(fx+e)^{5}}-\frac{a(a-2b)}{(a+b)^{4}\tan(fx+e)}-\frac{2a}{3(a+b)^{3}\tan(f}}{f}$

risch	$-\frac{i(-45a^2b\,e^{12i(fx+e)}+60a\,b^2e^{12i(fx+e)}+180a^2b\,e^{10i(fx+e)}-330a\,b^2e^{10i(fx+e)}+120b^3e^{10i(fx+e)}+160a^3e^{8i(fx+e)}+4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b*a/(a+b)^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-4*b)/((a+
b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^2/tan(f*x+e)^5-
a*(a-2*b)/(a+b)^4/tan(f*x+e)-2/3*a/(a+b)^3/tan(f*x+e)^3)
```

Maxima [A]
time = 0.49, size = 274, normalized size = 1.46

$$-\frac{\frac{15\,(3\,a^2b-4\,ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}}+\frac{15\,(3\,a^2b-4\,ab^2)\tan(fx+e)^6+10\,(3\,a^3-a^2b-4\,ab^2)\tan(fx+e)^4+6\,a^3+18\,a^2b+18\,ab^2+6\,b^3+2\,(10\,a^3+23\,a^2b+16\,ab^2+3\,b^3)\tan(fx+e)^2}{(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\tan(fx+e)^7+(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\tan(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*a^2*b - 4*a*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a^2*b - 4*
a*b^2)*tan(f*x + e)^6 + 10*(3*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^4 + 6*a^3
 + 18*a^2*b + 18*a*b^2 + 6*b^3 + 2*(10*a^3 + 23*a^2*b + 16*a*b^2 + 3*b^3)*t
an(f*x + e)^2)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(177) = 354$.
time = 2.95, size = 1021, normalized size = 5.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*a
^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2)*
cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*b +
 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*b^2)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
```

```
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 4*
a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*co
s(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f
*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 2*(40*a^3
 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 29*a^2*b + 28
*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11
*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b +
8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3
*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 254, normalized size = 1.35

$$-\frac{\frac{15\,a^2b\tan(fx+e)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)}+\frac{15\,(3\,a^2b-4\,ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}}+\frac{2\left(15\,a^2\tan(fx+e)^4-30\,ab\tan(fx+e)^4+10\,a^2\tan(fx+e)^2+10\,ab\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/pi
+ 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 - 30*
a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2
+ 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
5))/f
```

Mupad [B]
time = 6.31, size = 198, normalized size = 1.05

$$\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\tan(e+f\,x)\,(3\,a-4\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{(a+b)^{9/2}\,(4\,a\,b-3\,a^2)}\right)\,(3\,a-4\,b)}{2\,f\,(a+b)^{9/2}}-\frac{\frac{1}{5\,(a+b)}-\frac{\tan(e+f\,x)^4\,(4\,a\,b-3\,a^2)}{3\,(a+b)^3}+\frac{\tan(e+f\,x)^2\,(10\,a+3\,b)}{15\,(a+b)^2}-\frac{b\tan(e+f\,x)^6\,(4\,a\,b-3\,a^2)}{2\,(a+b)^4}}{f\,\left(b\tan\left(e+f\,x\right)^7+(a+b)\,\tan\left(e+f\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (a*b^(1/2)*atan((a*b^(1/2)*tan(e + f*x)*(3*a - 4*b)*(4*a*b^3 + 4*a^3*b + a^
4 + b^4 + 6*a^2*b^2))/((a + b)^(9/2)*(4*a*b - 3*a^2)))*(3*a - 4*b))/(2*f*(a
 + b)^(9/2)) - (1/(5*(a + b)) - (tan(e + f*x)^4*(4*a*b - 3*a^2))/(3*(a + b)
^3) + (tan(e + f*x)^2*(10*a + 3*b))/(15*(a + b)^2) - (b*tan(e + f*x)^6*(4*a
*b - 3*a^2))/(2*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x)^7))
```

3.54 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=214

$$\frac{\sqrt{b}\,(15a^2+70ab+63b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f}-\frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f}+\frac{(a+3b)(3a+5b)\cos^3(}{12a^4bf}$$

```
[Out] -1/2*(3*a^2+14*a*b+13*b^2)*cos(f*x+e)/a^5/f+1/12*(a+3*b)*(3*a+5*b)*cos(f*x+
e)^3/a^4/b/f-1/5*cos(f*x+e)^5/a^3/f-1/4*(a+b)^2*cos(f*x+e)^7/a^2/b/f/(b+a*c
os(f*x+e)^2)^2-1/8*b*(a+b)*(3*a+11*b)*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2)+1
/8*(15*a^2+70*a*b+63*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(11/
2)/f
```

Rubi [A]
time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4218, 474, 466, 1824, 211}

$$-\frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\,(a\cos^2(e+fx)+b)}+\frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf}-\frac{\cos^5(e+fx)}{5a^3f}-\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\,(a\cos^2(e+fx)+b)^2}+\frac{\sqrt{b}\,(15a^2+70ab+63b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f}-\frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])
/(8*a^(11/2)*f) - ((3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x])/(2*a^5*f) + ((a
+ 3*b)*(3*a + 5*b)*Cos[e + f*x]^3)/(12*a^4*b*f) - Cos[e + f*x]^5/(5*a^3*f)
- ((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*f*(b + a*Cos[e + f*x]^2)^2) - (b*(a +
 b)*(3*a + 11*b)*Cos[e + f*x])/(8*a^5*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\,(b+a\cos^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{x^6(-4a^2+7(a+b)^2-4abx^2)}{(b+ax^2)^2}\,dx, x, \cos(e+\right.}{4a^2bf}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\,(b+a\cos^2(e+fx))^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\,(b+a\cos^2(e+fx))} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\,(b+a\cos^2(e+fx))^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\,(b+a\cos^2(e+fx))} - \frac{\text{Subst}\left(\int (\right.}{}$$

$$= -\frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f} + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(}{5}$$

$$= \frac{\sqrt{b}\,(15a^2+70ab+63b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{(3a^2+14ab+13b^2)\cos(e}{2a^5f}$$

Mathematica [C] Result contains complex when optimal does not.

time = 12.90, size = 1641, normalized size = 7.67

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[e
 + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Cos[
e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/2)*
Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(3/2)
*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e +
 f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 11520
0*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 537600*a*b^4*
ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*
x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*b^5*ArcTan[((-Sq
rt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[((-Sqrt[a] + I*Sqrt[
a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
 Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b +
 a*Cos[2*(e + f*x)])^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + 537600*a*b^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)
])^2 + 483840*b^5*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 225*a
^5*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 - 225*a^5*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 19200*a^(5/2)*b^(5/2)*Cos[e]*C
os[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 20352*a^(9/2)*b^(5/2)*Cos[e + f*
x]*Cos[4*(e + f*x)] - 115712*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[4*(e + f*x)]
- 129024*a^(5/2)*b^(9/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 2048*a^(9/2)*b^(5/
2)*Cos[e + f*x]*Cos[6*(e + f*x)] + 4608*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[6*
(e + f*x)] - 384*a^(9/2)*b^(5/2)*Cos[e + f*x]*Cos[8*(e + f*x)] - 19200*a^(5
```

```
/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] - 32496*a^(9/2
)*b^(5/2)*Csc[e + f*x]*Sin[4*(e + f*x)] - 252080*a^(7/2)*b^(7/2)*Csc[e + f*
x]*Sin[4*(e + f*x)] - 577024*a^(5/2)*b^(9/2)*Csc[e + f*x]*Sin[4*(e + f*x)]
- 403200*a^(3/2)*b^(11/2)*Csc[e + f*x]*Sin[4*(e + f*x)]))/(491520*a^(11/2)*
b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.19, size = 190, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^{5}(fx+e)\right)a^{2}}{5}-\frac{2a^{2}\left(\cos^{3}(fx+e)\right)}{3}-ab\left(\cos^{3}(fx+e)\right)+a^{2}\cos(fx+e)+6ab\cos(fx+e)+6b^{2}\cos(fx+e)}{a^{5}}+\frac{b\left(\frac{\left(-\frac{9}{8}a^{3}-\frac{13}{4}a^{2}b-\frac{17}{8}a\right.}{}\right.}{}}{f}$
default	$\frac{-\frac{\frac{\left(\cos^{5}(fx+e)\right)a^{2}}{5}-\frac{2a^{2}\left(\cos^{3}(fx+e)\right)}{3}-ab\left(\cos^{3}(fx+e)\right)+a^{2}\cos(fx+e)+6ab\cos(fx+e)+6b^{2}\cos(fx+e)}{a^{5}}+\frac{b\left(\frac{\left(-\frac{9}{8}a^{3}-\frac{13}{4}a^{2}b-\frac{17}{8}a\right.}{}\right.}{}}{f}$
risch	$-\frac{e^{5i(fx+e)}}{160a^{3}f}+\frac{5\,e^{3i(fx+e)}}{96a^{3}f}+\frac{e^{3i(fx+e)}b}{8a^{4}f}-\frac{5\,e^{i(fx+e)}}{16a^{3}f}-\frac{21\,e^{i(fx+e)}b}{8a^{4}f}-\frac{3\,e^{i(fx+e)}b^{2}}{a^{5}f}-\frac{5\,e^{-i(fx+e)}}{16a^{3}f}-\frac{21\,e^{-i(}}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^5*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-a*b*cos(f*x+e)^3+a^2
*cos(f*x+e)+6*a*b*cos(f*x+e)+6*b^2*cos(f*x+e))+b/a^5*(((-9/8*a^3-13/4*a^2*b
-17/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+22*a*b+15*b^2)*cos(f*x+e))/(b+a*cos(
f*x+e)^2)^2+1/8*(15*a^2+70*a*b+63*b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b
)^(1/2))))
```

Maxima [A]
time = 0.48, size = 212, normalized size = 0.99

$$-\frac{\frac{15\left((9a^{3}b+26a^{2}b^{2}+17ab^{3})\cos(fx+e)^{3}+(7a^{2}b^{2}+22ab^{3}+15b^{4})\cos(fx+e)\right)}{a^{7}\cos(fx+e)^{4}+2a^{6}b\cos(fx+e)^{2}+a^{5}b^{2}}-\frac{15\,(15a^{2}b+70ab^{2}+63b^{3})\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{5}}+\frac{8\left(3a^{2}\cos(fx+e)^{5}-5\,(2a^{2}+3ab)\cos(fx+e)^{3}+15\,(a^{2}+6ab+6b^{2})\cos(fx+e)\right)}{a^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*((9*a^3*b + 26*a^2*b^2 + 17*a*b^3)*cos(f*x + e)^3 + (7*a^2*b^2 +
 22*a*b^3 + 15*b^4)*cos(f*x + e))/(a^7*cos(f*x + e)^4 + 2*a^6*b*cos(f*x + e
)^2 + a^5*b^2) - 15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)/sq
```

rt(a*b))/(sqrt(a*b)*a^5) + 8*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + 3*a*b)*cos(f*x + e)^3 + 15*(a^2 + 6*a*b + 6*b^2)*cos(f*x + e))/a^5)/f

Fricas [A]
time = 3.50, size = 601, normalized size = 2.81

$$\left[-\frac{48\,a^{4}\cos\left(fx+e\right)^{9}-16\,(10\,a^{4}+9\,a^{3}b)\cos\left(fx+e\right)^{7}+16\,(15\,a^{4}+70\,a^{3}b+63\,a^{2}b^{2})\cos\left(fx+e\right)^{5}+50\,(15\,a^{3}b+70\,a^{2}b^{2}+63\,ab^{3})\cos\left(fx+e\right)^{3}-15\,\left((15\,a^{4}+70\,a^{3}b+63\,a^{2}b^{2})\cos\left(fx+e\right)^{4}+15\,a^{2}b^{2}+70\,ab^{3}+63\,b^{4}+2\,(15\,a^{3}b+70\,a^{2}b^{2}+63\,ab^{3})\cos\left(fx+e\right)^{2}\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)+30\,(15\,a^{2}b^{2}+70\,ab^{3}+63\,b^{4})\cos\left(fx+e\right)}{240\,\left(a^{7}f\cos\left(fx+e\right)^{4}+2\,a^{6}bf\cos\left(fx+e\right)^{2}+a^{5}b^{2}f\right)},\ -\frac{24\,a^{4}\cos\left(fx+e\right)^{9}-8\,(10\,a^{4}+9\,a^{3}b)\cos\left(fx+e\right)^{7}+8\,(15\,a^{4}+70\,a^{3}b+63\,a^{2}b^{2})\cos\left(fx+e\right)^{5}+25\,(15\,a^{3}b+70\,a^{2}b^{2}+63\,ab^{3})\cos\left(fx+e\right)^{3}-15\,\left((15\,a^{4}+70\,a^{3}b+63\,a^{2}b^{2})\cos\left(fx+e\right)^{4}+15\,a^{2}b^{2}+70\,ab^{3}+63\,b^{4}+2\,(15\,a^{3}b+70\,a^{2}b^{2}+63\,ab^{3})\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos\left(fx+e\right)}{b}\right)+15\,(15\,a^{2}b^{2}+70\,ab^{3}+63\,b^{4})\cos\left(fx+e\right)}{120\,\left(a^{7}f\cos\left(fx+e\right)^{4}+2\,a^{6}bf\cos\left(fx+e\right)^{2}+a^{5}b^{2}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 8*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 70*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(194) = 388.
time = 0.64, size = 781, normalized size = 3.65

$$-\frac{\frac{15\,(15\,a^{2}b+70\,ab^{2}+63\,b^{3})\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\sqrt{ab}\,a^{5}}+\frac{30\left(9\,a^{3}b+33\,a^{2}b^{2}+39\,ab^{3}+15\,b^{4}+\frac{27\,a^{3}b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{49\,a^{2}b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{23\,ab^{3}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{45\,b^{4}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{27\,a^{3}b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{27\,a^{2}b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{3\,ab^{3}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{45\,b^{4}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{9\,a^{3}b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{11\,a^{2}b^{2}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{13\,ab^{3}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{15\,b^{4}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)^{2}a^{5}}-\frac{16\left(8\,a^{2}+75\,ab+90\,b^{2}-\frac{40\,a^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{330\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{360\,b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{480\,ab(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{540\,b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{270\,ab(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{360\,b^{2}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{45\,ab(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{90\,b^{2}(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}\right)}{a^{5}\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")

```
[Out] -1/120*(15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqr
t(a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^5) + 30*(9*a^3*b + 33*a^2*b^
2 + 39*a*b^3 + 15*b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 49
*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 45*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^
3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2 + 45*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 + 11*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b^4*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2*a^5) - 16*(8*a^2 + 75*a*b + 90*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 330*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 360*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 480*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 540*b^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 360*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 90*b^2*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4)/(a^5*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/
f
```

Mupad [B]
time = 4.51, size = 255, normalized size = 1.19

$$\frac{\cos\left(e+f\,x\right)^3\,\left(\frac{b}{a^4}+\frac{2}{3\,a^3}\right)}{f}-\frac{\left(\frac{9\,a^3\,b}{8}+\frac{13\,a^2\,b^2}{4}+\frac{17\,a\,b^3}{8}\right)\,\cos\left(e+f\,x\right)^3+\left(\frac{7\,a^2\,b^2}{8}+\frac{11\,a\,b^3}{4}+\frac{15\,b^4}{8}\right)\,\cos\left(e+f\,x\right)}{f\,\left(a^7\,\cos\left(e+f\,x\right)^4+2\,a^6\,b\,\cos\left(e+f\,x\right)^2+a^5\,b^2\right)}-\frac{\cos\left(e+f\,x\right)^5}{5\,a^3\,f}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{1}{a^3}-\frac{3\,b^2}{a^5}+\frac{3\,b\,\left(\frac{3\,b}{a^4}+\frac{2}{a^3}\right)}{a}\right)}{f}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos\left(e+f\,x\right)\,\left(15\,a^2+70\,a\,b+63\,b^2\right)}{15\,a^2\,b+70\,a\,b^2+63\,b^3}\right)\,\left(15\,a^2+70\,a\,b+63\,b^2\right)}{8\,a^{11/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (cos(e + f*x)^3*(b/a^4 + 2/(3*a^3)))/f - (cos(e + f*x)^3*((17*a*b^3)/8 + (9
*a^3*b)/8 + (13*a^2*b^2)/4) + cos(e + f*x)*((11*a*b^3)/4 + (15*b^4)/8 + (7*
a^2*b^2)/8))/(f*(a^5*b^2 + a^7*cos(e + f*x)^4 + 2*a^6*b*cos(e + f*x)^2)) -
cos(e + f*x)^5/(5*a^3*f) - (cos(e + f*x)*(1/a^3 - (3*b^2)/a^5 + (3*b*((3*b)
/a^4 + 2/a^3))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(70*a*b
+ 15*a^2 + 63*b^2))/(70*a*b^2 + 15*a^2*b + 63*b^3))*(70*a*b + 15*a^2 + 63*b
^2))/(8*a^(11/2)*f)
```

3.55 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{5\sqrt{b}\,(3a+7b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f}-\frac{(a+3b)\cos(e+fx)}{a^4f}+\frac{\cos^3(e+fx)}{3a^3f}+\frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2}-\frac{b}{8a}$$

```
[Out] -(a+3*b)*cos(f*x+e)/a^4/f+1/3*cos(f*x+e)^3/a^3/f+1/4*b^2*(a+b)*cos(f*x+e)/a
^4/f/(b+a*cos(f*x+e)^2)^2-1/8*b*(9*a+13*b)*cos(f*x+e)/a^4/f/(b+a*cos(f*x+e)
^2)+5/8*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 466, 1828, 1167, 211}

$$\frac{5\sqrt{b}\,(3a+7b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f}+\frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(a\cos^2(e+fx)+b)^2}-\frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(a\cos^2(e+fx)+b)}-\frac{(a+3b)\cos(e+fx)}{a^4f}+\frac{\cos^3(e+fx)}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(9/2)*f
) - ((a + 3*b)*Cos[e + f*x])/(a^4*f) + Cos[e + f*x]^3/(3*a^3*f) + (b^2*(a +
 b)*Cos[e + f*x])/(4*a^4*f*(b + a*Cos[e + f*x]^2)^2) - (b*(9*a + 13*b)*Cos[
e + f*x])/(8*a^4*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx, x \right.}{4a^4f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b}{}\right.}{}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(8b(a+3\right.\right.}{}$$

$$= -\frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a}{8a^4f\,(}$$

$$= \frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.18, size = 1392, normalized size = 9.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*((-3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/Sqrt[a] -
 (3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/Sqrt[a] - (2*
Sqrt[b]*Cos[e + f*x]*(3*a + 10*b + 3*a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[
2*(e + f*x)])^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(8192*b^(
5/2)*f*(a + b*Sec[e + f*x]^2)^3) + (((3*a - 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
- Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (3*a -
4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Ta
n[(f*x)/2]))/Sqrt[b]] + (2*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(3*a^2 + 6*a*b + 8*
b^2 + a*(3*a - 4*b)*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2)*(a
 + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(2048*a^(3/2)*b^(5/2)*f*(a +
 b*Sec[e + f*x]^2)^3) - ((-3*(3*a^4 - 40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 +
 8960*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*(3*a^4 - 40*a^3*b + 720*a^2*b^2 + 6720*a*b^
3 + 8960*b^4)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - (2*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(9*a^5 - 9
0*a^4*b - 10144*a^3*b^2 - 48672*a^2*b^3 - 85120*a*b^4 - 53760*b^5 + a*(9*a^
4 - 120*a^3*b - 12432*a^2*b^2 - 47936*a*b^3 - 44800*b^4)*Cos[2*(e + f*x)] -
 128*a^2*b^2*(15*a + 28*b)*Cos[4*(e + f*x)] + 128*a^3*b^2*Cos[6*(e + f*x)])
)/(a + 2*b + a*Cos[2*(e + f*x)])^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6)/(49152*a^(9/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b
+ a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*
b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Ta
n[(f*x)/2]))/Sqrt[b]])/b^(5/2) + (3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*Ar
cTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*
x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)
/2]))/Sqrt[b]])/b^(5/2) - 512*Sqrt[a]*Cos[e]*Cos[f*x] + (8*Sqrt[a]*(a^3 + 2
4*a^2*b + 80*a*b^2 + 64*b^3)*Cos[e + f*x])/(b*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (2*Sqrt[a]*(3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3)*Cos[e + f*x])/(b
^2*(a + 2*b + a*Cos[2*(e + f*x)])) + 512*Sqrt[a]*Sin[e]*Sin[f*x]))/(16384*a
^(7/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.36, size = 125, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\left(\cos^3(fx+e)\right)-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\left(\cos^3(fx+e)\right)-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^3f}-\frac{3\,e^{i(fx+e)}}{8a^3f}-\frac{3\,e^{i(fx+e)}b}{2a^4f}-\frac{3\,e^{-i(fx+e)}}{8a^3f}-\frac{3\,e^{-i(fx+e)}b}{2a^4f}+\frac{e^{-3i(fx+e)}}{24a^3f}-\frac{b\left(9a^2e^{7i(fx+e)}+13ab\,e^{7i\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-3*b*cos(f*x+e))+b/a^4*(((-9/8*a
^2-13/8*a*b)*cos(f*x+e)^3-1/8*b*(7*a+11*b)*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
+5/8*(3*a+7*b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]

time = 0.48, size = 156, normalized size = 1.01

$$-\frac{\frac{3\left(\left(9\,a^2b+13\,ab^2\right)\cos(fx+e)^3+\left(7\,ab^2+11\,b^3\right)\cos(fx+e)\right)}{a^6\cos(fx+e)^4+2\,a^5b\cos(fx+e)^2+a^4b^2}-\frac{15\left(3\,ab+7\,b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}-\frac{8\left(a\cos(fx+e)^3-3\,(a+3\,b)\cos(fx+e)\right)}{a^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*((9*a^2*b + 13*a*b^2)*cos(f*x + e)^3 + (7*a*b^2 + 11*b^3)*cos(f*x
+ e))/(a^6*cos(f*x + e)^4 + 2*a^5*b*cos(f*x + e)^2 + a^4*b^2) - 15*(3*a*b +
 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) - 8*(a*cos(f*x + e
)^3 - 3*(a + 3*b)*cos(f*x + e))/a^4)/f
```

Fricas [A]

time = 3.13, size = 459, normalized size = 2.98

$$\left[\frac{16\,a^3\cos(fx+e)^7-16\,(3\,a^3+7\,a^2b)\cos(fx+e)^5-50\,(3\,a^2b+7\,ab^2)\cos(fx+e)^3+15\,((3\,a^3+7\,a^2b)\cos(fx+e)^4+3\,ab^2+7\,b^3+2\,(3\,a^2b+7\,ab^2)\cos(fx+e)^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-30\,(3\,ab^2+7\,b^3)\cos(fx+e)}{48\,(a^6f\cos(fx+e)^4+2\,a^5bf\cos(fx+e)^2+a^4b^2f)},\ \frac{8\,a^3\cos(fx+e)^7-8\,(3\,a^3+7\,a^2b)\cos(fx+e)^5-25\,(3\,a^2b+7\,ab^2)\cos(fx+e)^3+15\,((3\,a^3+7\,a^2b)\cos(fx+e)^4+3\,ab^2+7\,b^3+2\,(3\,a^2b+7\,ab^2)\cos(fx+e)^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-15\,(3\,ab^2+7\,b^3)\cos(fx+e)}{24\,(a^6f\cos(fx+e)^4+2\,a^5bf\cos(fx+e)^2+a^4b^2f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3*
a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 + 3*
a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 3
0*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x
 + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b)*cos(
f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*
cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sq
rt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*cos(f*x +
 e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 174, normalized size = 1.13

$$\frac{5\left(3ab+7b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^4f}-\frac{\frac{9a^2b\cos(fx+e)^3}{f}+\frac{13ab^2\cos(fx+e)^3}{f}+\frac{7ab^2\cos(fx+e)}{f}+\frac{11b^3\cos(fx+e)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^4}+\frac{a^6f^{17}\cos\left(fx+e\right)^3-3a^6f^{17}\cos\left(fx+e\right)-9a^5bf^{17}\cos\left(fx+e\right)}{3a^9f^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1/
8*(9*a^2*b*cos(f*x + e)^3/f + 13*a*b^2*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x +
 e)/f + 11*b^3*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)^2*a^4) + 1/3*(a^6*f^
17*cos(f*x + e)^3 - 3*a^6*f^17*cos(f*x + e) - 9*a^5*b*f^17*cos(f*x + e))/(a
^9*f^18)
```

Mupad [B]
time = 0.17, size = 172, normalized size = 1.12

$$\frac{\cos\left(e+f\,x\right)^3}{3\,a^3\,f}-\frac{\left(\frac{9\,a^2\,b}{8}+\frac{13\,a\,b^2}{8}\right)\,\cos\left(e+f\,x\right)^3+\left(\frac{11\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)\,\cos\left(e+f\,x\right)}{f\,\left(a^6\cos\left(e+f\,x\right)^4+2\,a^5\,b\cos\left(e+f\,x\right)^2+a^4\,b^2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{3\,b}{a^4}+\frac{1}{a^3}\right)}{f}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+7\,b)}{7\,b^2+3\,a\,b}\right)\,\left(3\,a+7\,b\right)}{8\,a^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] cos(e + f*x)^3/(3*a^3*f) - (cos(e + f*x)^3*((13*a*b^2)/8 + (9*a^2*b)/8) + c
os(e + f*x)*((7*a*b^2)/8 + (11*b^3)/8))/(f*(a^4*b^2 + a^6*cos(e + f*x)^4 +
2*a^5*b*cos(e + f*x)^2)) - (cos(e + f*x)*((3*b)/a^4 + 1/a^3))/f + (5*b^(1/2
)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 7*b))/(3*a*b + 7*b^2))*(3*a + 7
*b))/(8*a^(9/2)*f)
```

3.56 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal. Leaf size=116

$$\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{\cos^5(e+fx)}{4af\,(b+a\cos^2(e+fx))^2}+\frac{5\cos^3(e+fx)}{8a^2f\,(b+a\cos^2(e+fx))}$$

```
[Out] -15/8*cos(f*x+e)/a^3/f+1/4*cos(f*x+e)^5/a/f/(b+a*cos(f*x+e)^2)^2+5/8*cos(f*
x+e)^3/a^2/f/(b+a*cos(f*x+e)^2)+15/8*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(
1/2)/a^(7/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 294, 327, 211}

$$\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{5\cos^3(e+fx)}{8a^2f\,(a\cos^2(e+fx)+b)}+\frac{\cos^5(e+fx)}{4af\,(a\cos^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(7/2)*f) - (15*Cos
[e + f*x])/(8*a^3*f) + Cos[e + f*x]^5/(4*a*f*(b + a*Cos[e + f*x]^2)^2) + (5
*Cos[e + f*x]^3)/(8*a^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)}{4af\,(b+a\cos^2(e+fx))^2} - \frac{5\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4af}$$

$$= \frac{\cos^5(e+fx)}{4af\,(b+a\cos^2(e+fx))^2} + \frac{5\cos^3(e+fx)}{8a^2f\,(b+a\cos^2(e+fx))} - \frac{15\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx\right.}{8a^2}$$

$$= -\frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\,(b+a\cos^2(e+fx))^2} + \frac{5\cos^3(e+fx)}{8a^2f\,(b+a\cos^2(e+fx))} +$$

$$= \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\,(b+a\cos^2(e+fx))^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.67, size = 656, normalized size = 5.66

$$\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(15(a^3+64b^3)\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+15(a^3+64b^3)\,\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+\ldots\right)}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
```

```
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqrt[b]*C
os[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e + f*x] +
512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] - 2
4*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2*b^(3/2)*
Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e + f*x]*(a
+ 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[
(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^(5/2)*ArcTan
[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 5
12*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] + 6*a^4*Sqrt[b]
*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4096*a
^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.17, size = 83, normalized size = 0.72

method	result
derivativedivides	$-\frac{\frac{b\left(\frac{\frac{7b\left(\sec^3(fx+e)\right)}{8}+\frac{9a\sec(fx+e)}{8}}{\left(a+b\left(\sec^2(fx+e)\right)\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
default	$-\frac{\frac{b\left(\frac{\frac{7b\left(\sec^3(fx+e)\right)}{8}+\frac{9a\sec(fx+e)}{8}}{\left(a+b\left(\sec^2(fx+e)\right)\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2a^3f}-\frac{e^{-i(fx+e)}}{2a^3f}-\frac{b\left(9a\,e^{7i(fx+e)}+27a\,e^{5i(fx+e)}+28b\,e^{5i(fx+e)}+27a\,e^{3i(fx+e)}+28b\,e^{3i(fx+e)}+9a\,e^{i(fx+e)}\right)}{4a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/a^3*b*((7/8*b*sec(f*x+e)^3+9/8*a*sec(f*x+e))/(a+b*sec(f*x+e)^2)^2+1
5/8/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2)))-1/a^3/sec(f*x+e))
```

Maxima [A]
time = 0.48, size = 109, normalized size = 0.94

$$-\frac{\frac{9\,ab\cos(fx+e)^3+7\,b^2\cos(fx+e)}{a^5\cos(fx+e)^4+2\,a^4b\cos(fx+e)^2+a^3b^2}-\frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{8\,\cos(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/(a^5*cos(f*x + e)^4 + 2*a
^4*b*cos(f*x + e)^2 + a^3*b^2) - 15*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqr
t(a*b)*a^3) + 8*cos(f*x + e)/a^3)/f
```

Fricas [A]
time = 3.36, size = 317, normalized size = 2.73

$$\left[-\frac{16\,a^{2}\cos\left(fx+e\right)^{5}+50\,ab\cos\left(fx+e\right)^{3}+30\,b^{2}\cos\left(fx+e\right)-15\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-\frac{b}{a}}\,\log\left(-\frac{a\cos(fx+e)^{2}+2\,a\sqrt{-\frac{b}{a}}\,\cos(fx+e)-b}{a\cos(fx+e)^{2}+b}\right)}{16\left(a^{5}f\cos\left(fx+e\right)^{4}+2\,a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)},\ -\frac{8\,a^{2}\cos\left(fx+e\right)^{5}+25\,ab\cos\left(fx+e\right)^{3}+15\,b^{2}\cos\left(fx+e\right)-15\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}\,\cos(fx+e)}{b}\right)}{8\left(a^{5}f\cos\left(fx+e\right)^{4}+2\,a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e)
 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(a*
cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)))/
(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*a^2*
cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2*cos(
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f
*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 92, normalized size = 0.79

$$\frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}f}-\frac{\cos\left(fx+e\right)}{a^{3}f}-\frac{\frac{9\,ab\cos(fx+e)^{3}}{f}+\frac{7\,b^{2}\cos(fx+e)}{f}}{8\left(a\cos\left(fx+e\right)^{2}+b\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(a
^3*f) - 1/8*(9*a*b*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a*cos(f*x + e
)^2 + b)^2*a^3)
```

Mupad [B]
time = 0.15, size = 105, normalized size = 0.91

$$\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,f}-\frac{\frac{7\,b^2\,\cos(e+f\,x)}{8}+\frac{9\,a\,b\cos(e+f\,x)^3}{8}}{f\,\left(a^5\cos\left(e+f\,x\right)^4+2\,a^4\,b\cos\left(e+f\,x\right)^2+a^3\,b^2\right)}-\frac{\cos\left(e+f\,x\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (15*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(8*a^(7/2)*f) - ((7*b^2*c
os(e + f*x))/8 + (9*a*b*cos(e + f*x)^3)/8)/(f*(a^3*b^2 + a^5*cos(e + f*x)^4
 + 2*a^4*b*cos(e + f*x)^2)) - cos(e + f*x)/(a^3*f)
```

3.57 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f}-\frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^3f}-\frac{b\cos^3(e+fx)}{4a(a+b)f\,(b+a\cos^2(e+fx))^2}-$$

```
[Out] -arctanh(cos(f*x+e))/(a+b)^3/f-1/4*b*cos(f*x+e)^3/a/(a+b)/f/(b+a*cos(f*x+e)
^2)^2-1/8*b*(7*a+3*b)*cos(f*x+e)/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)+1/8*(15*a
^2+10*a*b+3*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(a+b)^3
/f
```

Rubi [A]
time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4218, 481, 592, 536, 212, 211}

$$-\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\,(a\cos^2(e+fx)+b)}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3}-\frac{b\cos^3(e+fx)}{4af(a+b)\,(a\cos^2(e+fx)+b)^2}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/
(8*a^(5/2)*(a + b)^3*f) - ArcTanh[Cos[e + f*x]]/((a + b)^3*f) - (b*Cos[e +
f*x]^3)/(4*a*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) - (b*(7*a + 3*b)*Cos[e + f
*x])/(8*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\,(b+a\cos^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+\right.}{4a(a+b)f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\,(b+a\cos^2(e+fx))^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\,(b+a\cos^2(e+fx))} + \frac{\text{Su}}{}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\,(b+a\cos^2(e+fx))^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\text{Su}}{}$$

$$= \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^3f} - \frac{}{4a(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.67, size = 447, normalized size = 2.90

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(a+b)(9a+5b)(a+2b+a\cos(2(e+fx)))}{a^2} + \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cos(2(e+fx)))^2\sec(e+fx)}{a^{5/2}} + \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\text{ArcTan}\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}+\sqrt{a+b}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cos(2(e+fx)))^2\sec(e+fx)}{a^{5/2}} - 8(a+2b+a\cos(2(e+fx)))^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\sec(e+fx) + 8(a+2b+a\cos(2(e+fx)))^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sec(e+fx)\right)}{64(a+b)^3f\,(a+b\sec^2(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2*
b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*a^
2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e
 + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqr
t[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b + a*Cos[2*(
e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(a + b*Sec[e
 + f*x]^2)^3)
```

Maple [A]
time = 0.34, size = 156, normalized size = 1.01

method	result

derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2(a+b)^3}+\frac{b\left(\frac{-\frac{(9a^2+14ab+5b^2)(\cos^3(fx+e))}{8a}-\frac{b(7a^2+10ab+3b^2)\cos(fx+e)}{8a^2}}{(b+a(\cos^2(fx+e)))^2}+\frac{(15a^2+10ab+3b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right.}{(a+b)^3}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2(a+b)^3}+\frac{b\left(\frac{-\frac{(9a^2+14ab+5b^2)(\cos^3(fx+e))}{8a}-\frac{b(7a^2+10ab+3b^2)\cos(fx+e)}{8a^2}}{(b+a(\cos^2(fx+e)))^2}+\frac{(15a^2+10ab+3b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right.}{(a+b)^3}}{f}$
risch	$-\frac{b(9a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+27a^2e^{5i(fx+e)}+43ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}+43ab\,e^{3i(fx+e)}+12b^2e^{3}}{4a^2f(a+b)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)^3*ln(cos(f*x+e)-1)+b/(a+b)^3*((-1/8*(9*a^2+14*a*b+5*b^2)/a*c
os(f*x+e)^3-1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
)-1/2/(a+b)^3*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.49, size = 268, normalized size = 1.74

$$\frac{\frac{(15a^2b+10ab^2+3b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}}-\frac{(9a^2b+5ab^2)\cos(fx+e)^3+(7ab^2+3b^3)\cos(fx+e)}{a^4b^2+2a^3b^3+a^2b^4+(a^6+2a^5b+a^4b^2)\cos(fx+e)^4+2(a^5b+2a^4b^2+a^3b^3)\cos(fx+e)^2}-\frac{4\log(\cos(fx+e)+1)}{a^3+3a^2b+3ab^2+b^3}+\frac{4\log(\cos(fx+e)-1)}{a^3+3a^2b+3ab^2+b^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) - ((9*a^2*b + 5*a*b^2)*cos(f*x +
 e)^3 + (7*a*b^2 + 3*b^3)*cos(f*x + e))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + (a
^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^4 + 2*(a^5*b + 2*a^4*b^2 + a^3*b^3)*co
s(f*x + e)^2) - 4*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 4
*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(146) = 292$.
time = 3.23, size = 807, normalized size = 5.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*a
^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*a^
3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)
^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^
2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*
x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)^4 + 2
*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^
5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 1
0*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15
*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 4*(a^4*cos
(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2)
 - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2
*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(140) = 280.
time = 0.61, size = 592, normalized size = 3.84

$$-\frac{\frac{\left(15\,a^{2}b+10\,ab^{2}+3\,b^{3}\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\left(a^{5}+3\,a^{4}b+3\,a^{3}b^{2}+a^{2}b^{3}\right)\sqrt{ab}}-\frac{4\log\left(\frac{\left|-\cos\left(fx+e\right)+1\right|}{\left|\cos\left(fx+e\right)+1\right|}\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(9\,a^{3}b+21\,a^{2}b^{2}+15\,ab^{3}+3\,b^{4}+\frac{27\,a^{3}b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{13\,a^{2}b^{2}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{23\,ab^{3}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{9\,b^{4}\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{27\,a^{3}b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{9\,a^{2}b^{2}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{21\,ab^{3}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{9\,b^{4}\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{9\,a^{3}b\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{a^{2}b^{2}\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{13\,ab^{3}\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{3\,b^{4}\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{\left(a^{5}+3\,a^{4}b+3\,a^{3}b^{2}+a^{2}b^{3}\right)\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)
*cos(f*x + e) + sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b
)) - 4*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 2*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + 27*a^3*b*(cos(
```

```
f*x + e) - 1)/(cos(f*x + e) + 1) + 13*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 23*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 9*b^4*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 - 9*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 21*a*b^3*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^4*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - a^2*b^2
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/
(cos(f*x + e) + 1)^3 - 3*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a
^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2
))/f
```

Mupad [B]
time = 8.60, size = 2500, normalized size = 16.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*
b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3
+ 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5
280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^
9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e +
 f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 25
6*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 230
4*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^
3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/
(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(
16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*
(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*
(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3
*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*
b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 +
 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2
+ 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^
5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5
*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5
*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b
^2)))*(10*a*b + 15*a^2 + 3*b^2)*1i)/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^
2)))/((51*a*b^4 + 120*a^4*b + 9*b^5 + 139*a^2*b^3 + 185*a^3*b^2)/(32*(6*a^8
*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (
(-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 30
```

0*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*
b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*
b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*
b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(-
a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^
7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b
^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 +
4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a
^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b
 + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 +
 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b +
a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^10*b + 96*a^3*b^8 + 800*
a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*
a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3
 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1
280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^
8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^
6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(
10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a
*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2))))*(-a^5*b)
^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*1i)/(8*f*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*
b^2)) - ((cos(e + f*x)^3*(9*a*b + 5*b^2))/(8*a*(2*a*b + a^2 + b^2)) + (b*co
s(e + f*x)*(7*a*b + 3*b^2))/(8*a^2*(2*a*b + a^2 + b^2)))/(f*(b^2 + a^2*cos(
e + f*x)^4 + 2*a*b*cos(e + f*x)^2)) - (atan((((((224*a^10*b + 96*a^3*b^8 +
800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1
440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6
*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 -
 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2)
)/(64*(a + b)^3*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a +
 b)^3) + (cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b
^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*
1i)/(2*(a + b)^3) - ((((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^
6 + 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b
 + a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (co
s(e + f*x)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*
b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(64*(a + b)^3*(4*a^6*b
+ a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(2*(a + b)^3) - (cos(e + f*x)*(6
0*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4
*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*1i)/(2*(a + b)^3))/((((22
4*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^
7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b
^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (cos(e + f*x)*(1280*a^11*b +
256*a^12 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^...

3.58 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{b}\,(15a^2-10ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f}-\frac{(a-5b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^4f}-\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\,(b+a\cos^2(e+$$

```
[Out] -1/2*(a-5*b)*arctanh(cos(f*x+e))/(a+b)^4/f-1/4*(2*a-b)*b*cos(f*x+e)/a/(a+b)
^2/f/(b+a*cos(f*x+e)^2)^2+1/8*(4*a^2-9*a*b-b^2)*cos(f*x+e)/a/(a+b)^3/f/(b+a
*cos(f*x+e)^2)-1/2*cos(f*x+e)*cot(f*x+e)^2/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/8
*(15*a^2-10*a*b-b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+
b)^4/f
```

Rubi [A]
time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4218, 481, 592, 541, 536, 212, 211}

$$\frac{(4a^2-9ab-b^2)\cos(e+fx)}{8af(a+b)^3\,(a\cos^2(e+fx)+b)}+\frac{\sqrt{b}\,(15a^2-10ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4}-\frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\,(a\cos^2(e+fx)+b)^2}-\frac{\cos(e+fx)\cot^2(e+fx)}{2f(a+b)\,(a\cos^2(e+fx)+b)^2}-\frac{(a-5b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8
*a^(3/2)*(a + b)^4*f) - ((a - 5*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^4*f) -
 ((2*a - b)*b*Cos[e + f*x])/(4*a*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) + ((
4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(8*a*(a + b)^3*f*(b + a*Cos[e + f*x]^2))
 - (Cos[e + f*x]*Cot[e + f*x]^2)/(2*(a + b)*f*(b + a*Cos[e + f*x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx\right.}{2(a+b)f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\,(b+a\cos^2(e+fx))^2} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))^2} + \text{Subst}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\,(b+a\cos^2(e+fx))^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{c}{2(a}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\,(b+a\cos^2(e+fx))^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{c}{2(a}$$

$$= \frac{\sqrt{b}\,(15a^2-10ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^4f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.10, size = 532, normalized size = 2.50

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b*
(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 + 1
0*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]
)/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*Cos[2*(e
+ f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*(a + 2*b +
a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.41, size = 198, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(\cos(fx+e)-1)}+\frac{(a-5b)\ln(\cos(fx+e)-1)}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\left(\cos^3(fx+e)\right)-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-1\cdots\right.}{\cdots}\right.}{(a+b)^4}\cdots}{f}$
default	$\frac{\frac{1}{4(a+b)^3(\cos(fx+e)-1)}+\frac{(a-5b)\ln(\cos(fx+e)-1)}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\left(\cos^3(fx+e)\right)-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-1\cdots\right.}{\cdots}\right.}{(a+b)^4}\cdots}{f}$
risch	$-\frac{-4a^3e^{11i(fx+e)}+9a^2b\,e^{11i(fx+e)}+a\,b^2e^{11i(fx+e)}-20a^3e^{9i(fx+e)}-23a^2b\,e^{9i(fx+e)}+29a\,b^2e^{9i(fx+e)}-4b^3e^{9i(fx+e)}-\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/(a+b)^3/(cos(f*x+e)-1)+1/4*(a-5*b)/(a+b)^4*ln(cos(f*x+e)-1)+1/(a+b
)^4*b*(((-9/8*a^2-5/4*a*b-1/8*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+6*a*b-b^2)/a*c
os(f*x+e))/(b+a*cos(f*x+e)^2)^2+1/8*(15*a^2-10*a*b-b^2)/a/(a*b)^(1/2)*arcta
n(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^3/(1+cos(f*x+e))+1/4/(a+b)^4*(-a+5*b
)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.48, size = 408, normalized size = 1.92

$$-\frac{\frac{2\,(a-5\,b)\log(\cos(fx+e)+1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\,(a-5\,b)\log(\cos(fx+e)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{(15\,a^2b-10\,ab^2-b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\sqrt{ab}}-\frac{(4\,a^3-9\,a^2b-ab^2)\cos(fx+e)^5+(17\,a^2b-6\,ab^2+b^3)\cos(fx+e)^3+(11\,ab^2-b^3)\cos(fx+e)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\cos(fx+e)^6-a^4b^2-3\,a^3b^3-3\,a^2b^4-ab^5-(a^6+a^5b-3\,a^4b^2-5\,a^3b^3-2\,a^2b^4)\cos(fx+e)^4-(2\,a^5b+5\,a^4b^2+3\,a^3b^3-a^2b^4-ab^5)\cos(fx+e)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(a - 5*b)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) - 2*(a - 5*b)*log(cos(f*x + e) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(a*cos(f*x + e)/sqrt(a*b)
)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((4*a^3 - 9
*a^2*b - a*b^2)*cos(f*x + e)^5 + (17*a^2*b - 6*a*b^2 + b^3)*cos(f*x + e)^3
+ (11*a*b^2 - b^3)*cos(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos
(f*x + e)^6 - a^4*b^2 - 3*a^3*b^3 - 3*a^2*b^4 - a*b^5 - (a^6 + a^5*b - 3*a^
4*b^2 - 5*a^3*b^3 - 2*a^2*b^4)*cos(f*x + e)^4 - (2*a^5*b + 5*a^4*b^2 + 3*a^
3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(204) = 408$.

time = 2.43, size = 1370, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3*
b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^2*
b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4
)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*cos(f*
x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*c
os(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 -
 (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b
 - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*
a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (
2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x
+ e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/8*((4*
a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^3*b + 11*a^2*b^2
 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a^2*b^2)*cos(f*x +
 e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 15*a^2*
b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)
^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (11*a^2*b^2 + 10*a*b^3 -
 b^4)*cos(f*x + e) - 2*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 1
0*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a
*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 - 5*a^3*b)*cos(
f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^
3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
+ 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*
x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6
)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*
f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. 2(195) = 390.
time = 0.70, size = 750, normalized size = 3.52

$$\frac{\frac{2(a-5b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{(15a^2b-10ab^2-b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\sqrt{ab}}+\frac{\left(a+b-\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(fx+e)-1)}-\frac{\cos(fx+e)-1}{(a^3+3a^2b+3ab^2+b^3)(\cos(fx+e)+1)}-\frac{2\left(9a^3b+17a^2b^2+7ab^3-b^4+\frac{27a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{23ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3b^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{27a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{21a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{29ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{9a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{5a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{13ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{b^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(-(
a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^5 + 4*a^4*b +
 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + (a + b - 2*a*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 10*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x
 + e) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1))
 - (cos(f*x + e) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1))
- 2*(9*a^3*b + 17*a^2*b^2 + 7*a*b^3 - b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^4*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 21*a^2*b^2*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 29*a*b^3*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 - 3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a
^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 5*a^2*b^2*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 + b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

Mupad [B]
time = 6.92, size = 2728, normalized size = 12.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - ((cos(e + f*x)^3*(17*a^2*b - 6*a*b^2 + b^3))/(8*(a*b^3 + 3*a^3*b + a^4 +
3*a^2*b^2)) - (cos(e + f*x)^5*(9*a*b - 4*a^2 + b^2))/(8*(3*a*b^2 + 3*a^2*b
+ a^3 + b^3)) + (b^2*cos(e + f*x)*(11*a - b))/(8*(a*b^3 + 3*a^3*b + a^4 + 3
*a^2*b^2)))/(f*(b^2 - cos(e + f*x)^4*(2*a*b - a^2) + cos(e + f*x)^2*(2*a*b
- b^2) - a^2*cos(e + f*x)^6)) - (log(cos(e + f*x) - 1)*((3*b)/(2*(a + b)^4)
 - 1/(4*(a + b)^3)))/f - (log(cos(e + f*x) + 1)*(a - 5*b))/(4*f*(a + b)^4)
```

- (atan(((((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))/(((47*a*b^5)/32 - (15*a^5*b)/16 + (5*b^6)/64 + (21*a^2*b^4)/4 - (473*a^3*b^3)/32 + (475*a^4*b^2)/64)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) - ((-a^3*b)^(1/2)*(((11*a^11*b)/2 - (a^2*b^10)/2 + (

```
3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6 + 357*a^7*b^5 + 294*a
^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a^9*b + a^10 + 9*a^2*b^8
 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*b^4 + 84*a^7*b^3 + 36*a^
8*b^2) + (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*(1792*a^11*b
+ 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^5*b^7 - 7168*a^6*b^6 - 358
4*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a^10*b^2))/(512*(4*a^6*b + a
^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^5 +
15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*b - 15*a^2 + b^2))/(16*(4*a^6
*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b - 15*a
^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2))))*(-a^3*b
)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(8*f*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b
^3 + 6*a^5*b^2))
```

3.59 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=257

$$\frac{3\sqrt{b}\,(5a^2-10ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}\,(a+b)^5 f}-\frac{3(a^2-10ab+5b^2)\tanh^{-1}(\cos(e+fx))}{8(a+b)^5 f}+\frac{(a^2-9ab+2b^2}{8(a+b)^3 f\,(b+a}$$

```
[Out] -3/8*(a^2-10*a*b+5*b^2)*arctanh(cos(f*x+e))/(a+b)^5/f+1/8*(a^2-9*a*b+2*b^2)
*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/8*(a^2-6*a*b+b^2)*cos(f*x+e)/(
a+b)^4/f/(b+a*cos(f*x+e)^2)-1/8*(a-7*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(b+
a*cos(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)^2+
3/8*(5*a^2-10*a*b+b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^5/f
/a^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4218, 481, 592, 541, 536, 212, 211}

$$\frac{3\sqrt{b}\,(5a^2-10ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}\,f(a+b)^5}+\frac{3(a^2-6ab+b^2)\cos(e+fx)}{8f(a+b)^4\,(a\cos^2(e+fx)+b)}+\frac{(a^2-9ab+2b^2)\cos(e+fx)}{8f(a+b)^3\,(a\cos^2(e+fx)+b)^2}-\frac{3(a^2-10ab+5b^2)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^5}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\,(a\cos^2(e+fx)+b)^2}-\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\,(a\cos^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(
8*Sqrt[a]*(a + b)^5*f) - (3*(a^2 - 10*a*b + 5*b^2)*ArcTanh[Cos[e + f*x]])/(
8*(a + b)^5*f) + ((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a
*Cos[e + f*x]^2)^2) + (3*(a^2 - 6*a*b + b^2)*Cos[e + f*x])/(8*(a + b)^4*f*(
b + a*Cos[e + f*x]^2)) - ((a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2
*f*(b + a*Cos[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*(
b + a*Cos[e + f*x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-a+4b)x^2)}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx\right.}{4(a+b)f}$$

$$= -\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}}{}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3}{4(a+b}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)}{8(a+b)}$$

$$= \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)}{8(a+b)}$$

$$= \frac{3\sqrt{b}\,(5a^2-10ab+b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}\,(a+b)^5f} - \frac{3(a^2-10ab+5b^2)\tanh^{-1}(\text{co}}{8(a+b)^5f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.04, size = 549, normalized size = 2.14

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^2 -
10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] -
2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*a^2*b -
93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2 - 10*b^3
)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e + f*x)] - 3
*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 - 10*a*b + 5
*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] + 48*(a^2 - 10
```

```
*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]])*Sec[e
 + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.38, size = 259, normalized size = 1.01

method	result
derivativedivides	$-\frac{1}{16(a+b)^3(\cos(fx+e)-1)^2}-\frac{-3a+9b}{16(a+b)^4(\cos(fx+e)-1)}+\frac{\left(3a^2-30ab+15b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\left(\cos^3(fx\right.}{\left(b+a\left(\cos\right.\right.}\right.}{}$
default	$-\frac{1}{16(a+b)^3(\cos(fx+e)-1)^2}-\frac{-3a+9b}{16(a+b)^4(\cos(fx+e)-1)}+\frac{\left(3a^2-30ab+15b^2\right)\ln(\cos(fx+e)-1)}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\left(\cos^3(fx\right.}{\left(b+a\left(\cos\right.\right.}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/16/(a+b)^3/(cos(f*x+e)-1)^2-1/16*(-3*a+9*b)/(a+b)^4/(cos(f*x+e)-1)+
1/16/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(cos(f*x+e)-1)+b/(a+b)^5*(((-9/8*a^3-3
/4*a^2*b+3/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+2*a*b-5*b^2)*cos(f*x+e))/(b+a
*cos(f*x+e)^2)^2+3/8*(5*a^2-10*a*b+b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*
b)^(1/2)))+1/16/(a+b)^3/(1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(1+cos(f*x
+e))+1/16/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(1+cos(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(249) = 498.
time = 0.49, size = 539, normalized size = 2.10

$$-\frac{\frac{3\left(a^2-10ab+5b^2\right)\log(\cos(fx+e)+1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}-\frac{3\left(a^2-10ab+5b^2\right)\log(\cos(fx+e)-1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}-\frac{6\left(5a^2b-10ab^2+b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\sqrt{ab}}-\frac{2\left(3\left(a^3-6a^2b+ab^2\right)\cos(fx+e)^7-\left(5a^3-31a^2b+31ab^2-5b^3\right)\cos(fx+e)^5-\left(19a^2b-34ab^2+19b^3\right)\cos(fx+e)^3-12\left(ab^2-b^3\right)\cos(fx+e)\right)}{\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)\cos(fx+e)^8-2\left(a^6+3a^5b+2a^4b^2-2a^3b^3-3a^2b^4-ab^5\right)\cos(fx+e)^6+a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6+\left(a^6-9a^4b^2-16a^3b^3-9a^2b^4+b^6\right)\cos(fx+e)^4+2\left(a^5b+3a^4b^2+2a^3b^3-2a^2b^4-3ab^5-b^6\right)\cos(fx+e)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] -1/16*(3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x + e) + 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x
 + e) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 6*(5
*a^2*b - 10*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) - 2*(3*(a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^7 - (5*a^3 - 31*a^2*b + 31*a*b^2 - 5*b^3)*cos(f*x + e
)^5 - (19*a^2*b - 34*a*b^2 + 19*b^3)*cos(f*x + e)^3 - 12*(a*b^2 - b^3)*cos(
```

```
f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^8
 - 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cos(f*x +
e)^6 + a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6 + (a^6 - 9*a^4*b^2 -
 16*a^3*b^3 - 9*a^2*b^4 + b^6)*cos(f*x + e)^4 + 2*(a^5*b + 3*a^4*b^2 + 2*a^
3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos(f*x + e)^2))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(249) = 498$.
time = 3.21, size = 1881, normalized size = 7.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26
*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a
*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e
)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 3
0*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^
3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-b
/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)
^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*
b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x
+ e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^6 + (a^4
 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*b^2 - 10*
a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x + e)^2)*l
og(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*
b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b
+ 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^
5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/16*(6*(a
^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26*a^3*b + 26
*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a*b^3 + 19*b
^4)*cos(f*x + e)^3 + 6*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e)^8 - 2*(5*
a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 30*a^3*b + 4
6*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^3 + b^4 + 2
*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a
*sqrt(b/a)*cos(f*x + e)/b) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b
^3)*cos(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f
```

```
*x + e)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 -
 5*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b +
5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f
*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4
 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*c
os(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*
a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b -
9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x +
e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(
f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7
)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. 2(237) = 474.
time = 0.61, size = 1316, normalized size = 5.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 10*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e)
 + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 24*(5*a^
2*b - 10*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e)
+ sqrt(a*b)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sq
rt(a*b)) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a*b^2*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 16*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a^2*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 - b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6*a^5*b + 15*a^
4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b
^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 136*a^3*b*(cos(f*x + e) - 1)^2/(cos
```

```
(f*x + e) + 1)^2 + 224*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
40*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*b^4*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 + 280*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 64*a^2*b^2*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 152*a*b^3*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 + 212*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^
4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 84*a^3*b*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4 + 30*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^
4 + 84*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 123*b^4*(cos(f*x +
 e) - 1)^4/(cos(f*x + e) + 1)^4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e)
 + 1)^5 - 104*a^3*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 24*a^2*b^2*
(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 72*a*b^3*(cos(f*x + e) - 1)^5/(
cos(f*x + e) + 1)^5 - 24*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 6*
a^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 48*a^3*b*(cos(f*x + e) - 1)
^6/(cos(f*x + e) + 1)^6 - 84*a^2*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1
)^6 + 30*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3)^2))/f
```

Mupad [B]
time = 9.60, size = 2500, normalized size = 9.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan(((((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^
3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 +
28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b
)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9
- 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 21
0*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^1
0 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 4
95*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a
^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 -
 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b
^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b
 + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^
8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^
2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10
*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b +
```

a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (((cos(e + f*x)*(9*a*b^6 - 18
0*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2)
)/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4
 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54
*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^
8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11
 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*
a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b
^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 25
6*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 2304
0*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3
+ 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10
*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4
*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^
5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10
*a*b + b^2)*3i)/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^
4*b^2)))/(((135*a*b^7)/256 + (135*a^7*b)/256 - (1215*a^2*b^6)/128 + (13257*
a^3*b^5)/256 - (5913*a^4*b^4)/64 + (13257*a^5*b^3)/256 - (1215*a^6*b^2)/128
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) - (3*((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*
b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b +
 a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2
)) + (3*(-a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 -
 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*
a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12
 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 79
2*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*
b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 23
04*a^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6
+ 10752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*
b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7
*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6
b^2)))(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10
*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 +
5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (3*((cos(e + f*x)*(
9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^3 + 1
215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5
+ 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*b - 6*a
^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8 - 252*a^7*
b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4 + 210*a^11*b^3 + 54*a^12*b^2
)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a^2*b^10 + 220*a^3*b^9 + 495*a^
4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*b^5 + 495*a^8*b^4 + 220*a^9*b^3
 + 66*a^10*b^2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*(2304
*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a^3*b^10 - 8960*a^4*b^9 - 19200*a^

```
5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 10752*a^8*b^5 + 23040*a^9*b^4 + 192
00*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*
a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*
b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(
a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)
*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^
3 + 10*a^4*b^2))))*(-a*b)^(1/2)*(5*a^2 - 10*a*b...
```

3.60 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=314

$$\frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6}-\frac{5\sqrt{b}\,\sqrt{a+b}\,(a+4b)(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f}-\frac{\left(33a^2+110ab+8\right.}{48a^3f\,(a+\right.}$$

```
[Out] 5/16*(a+2*b)*(a^2+16*a*b+16*b^2)*x/a^6-5/8*(a+4*b)*(3*a+4*b)*arctan(b^(1/2)
*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^6/f-1/48*(33*a^2+110*a*b+80*
b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+1/24*(9*a+10*b)*cos
(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^3*sin(f*x+
e)^3/a/f/(a+b+b*tan(f*x+e)^2)^2-5/48*b*(9*a^2+32*a*b+24*b^2)*tan(f*x+e)/a^4
/f/(a+b+b*tan(f*x+e)^2)^2-5/16*b*(5*a^2+20*a*b+16*b^2)*tan(f*x+e)/a^5/f/(a+
b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.34, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4217, 481, 592, 541, 536, 209, 211}

$$-\frac{5\sqrt{b}\,\sqrt{a+b}\,(a+4b)(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f}+\frac{(9a+10b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{5x(a+2b)\left(a^2+16ab+16b^2\right)}{16a^6}-\frac{5b\left(5a^2+20ab+16b^2\right)\tan(e+fx)}{16a^5f\left(a+b\tan^2(e+fx)+b\right)}-\frac{5b\left(9a^2+32ab+24b^2\right)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\left(33a^2+110ab+80b^2\right)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b)*(a^2 + 16*a*b + 16*b^2)*x)/(16*a^6) - (5*Sqrt[b]*Sqrt[a + b]*(
a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*f)
- ((33*a^2 + 110*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b
+ b*Tan[e + f*x]^2)^2) + ((9*a + 10*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2
*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(
a + b + b*Tan[e + f*x]^2)^2) - (5*b*(9*a^2 + 32*a*b + 24*b^2)*Tan[e + f*x])
/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (5*b*(5*a^2 + 20*a*b + 16*b^2)*T
an[e + f*x])/(16*a^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-6a-7b)x^2)}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+ \right.}{6af}$$

$$= \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subs}}{}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\text{si}}{24a^2f\,(a+b+b\tan^2(e}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\text{si}}{24a^2f\,(a+b+b\tan^2(e}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\text{si}}{24a^2f\,(a+b+b\tan^2(e}$$

$$= -\frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{(9a+10b)\cos^3(e+fx)\text{si}}{24a^2f\,(a+b+b\tan^2(e}$$

$$= \frac{5(a+2b)(a^2+16ab+16b^2)\,x}{16a^6} - \frac{5\sqrt{b}\,\sqrt{a+b}\,(a+4b)(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{8a^6f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.33, size = 1639, normalized size = 5.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a
^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*S
in[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^
4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*
```

```
a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a
*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2
*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2
)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos
[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6 - 8*a^5*b + 12
0*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTan[(Se
c[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^4 + 32*a^3*b + 160
*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (256*a*Sin[2*(e + f*x)]
)/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2624*a^2*b^3 + 3200*a*b^4 + 128
0*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3
 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2
*b + a*Cos[2*(e + f*x)]))))/(65536*a^4*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*
b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-6144*(7*a^3 + 54*a^2*b + 120*a*b
^2 + 80*b^3)*x - (3*(3*a^8 - 64*a^7*b + 2240*a^6*b^2 + 53760*a^5*b^3 + 3136
00*a^4*b^4 + 802816*a^3*b^5 + 1032192*a^2*b^6 + 655360*a*b^7 + 163840*b^8)*
ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[
2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (12*(a^6 + 72*
a^5*b + 840*a^4*b^2 + 3584*a^3*b^3 + 6912*a^2*b^4 + 6144*a*b^5 + 2048*b^6)*
Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos
[2*(e + f*x)])^2) + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*((-I)*Cos[2*(e + f*x)
] + Sin[2*(e + f*x)]))/f + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*(I*Cos[2*(e +
f*x)] + Sin[2*(e + f*x)]))/f + (192*a^2*(a + 2*b)*((-6*I)*Cos[4*(e + f*x)]
- 6*Sin[4*(e + f*x)]))/f + ((1152*I)*a^2*(a + 2*b)*(Cos[4*(e + f*x)] + I*Si
n[4*(e + f*x)]))/f + (256*a^3*Sin[6*(e + f*x)])/f + (3*(3*a*(-a^7 + 22*a^6*
b + 1352*a^5*b^2 + 11312*a^4*b^3 + 37120*a^3*b^4 + 57856*a^2*b^5 + 43008*a*
b^6 + 12288*b^7)*Sec[2*e]*Sin[2*f*x] + (3*a^8 - 64*a^7*b - 4480*a^6*b^2 - 4
5696*a^5*b^3 - 196928*a^4*b^4 - 438272*a^3*b^5 - 528384*a^2*b^6 - 327680*a*
b^7 - 81920*b^8)*Tan[2*e]))/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
)))/(393216*a^6*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.19, size = 247, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{\left(-\frac{27}{8}a^2b-3a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-6a^2b-6a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{5\left(a^3+18a^2b+48\right.}{}}{a^6}$

default	$\frac{\frac{\left(-\frac{27}{8}a^2b-3a\,b^2-\frac{11}{16}a^3\right)\left(\tan^5(fx+e)\right)+\left(-6a^2b-6a\,b^2-\frac{5}{6}a^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3a\,b^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^3}+\frac{5\left(a^3+18a^2b+\right.}{}}{a^6}$
risch	$\frac{5x}{16a^3}+\frac{45xb}{8a^4}+\frac{15x\,b^2}{a^5}+\frac{10x\,b^3}{a^6}-\frac{3i\mathrm{e}^{4i(fx+e)}}{128a^3f}+\frac{3i\mathrm{e}^{-4i(fx+e)}b}{64a^4f}-\frac{ib\left(9a^4\mathrm{e}^{6i(fx+e)}+49a^3b\,\mathrm{e}^{6i(fx+e)}+80a^2b^2\mathrm{e}^{6}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^6*(((-27/8*a^2*b-3*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-6*a^2*b-6*a*b^2
-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-21/8*a^2*b-3*a*b^2)*tan(f*x+e))/(tan(f*x+
e)^2+1)^3+5/16*(a^3+18*a^2*b+48*a*b^2+32*b^3)*arctan(tan(f*x+e)))-(a+b)*b/a
^6*(((7/8*a^2*b+2*a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+25*a*b+16*b^2)*tan(f*x+e
))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a^2+16*a*b+16*b^2)/((a+b)*b)^(1/2)*arctan(
b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 430, normalized size = 1.37

$$-\frac{\frac{15\,(5\,a^2b^2+20\,ab^3+16\,b^4)\tan(fx+e)^9+40\,(3\,a^3b+19\,a^2b^2+39\,ab^3+24\,b^4)\tan(fx+e)^7+(33\,a^4+470\,a^3b+1910\,a^2b^2+2880\,ab^3+1440\,b^4)\tan(fx+e)^5+40\,(a^4+14\,a^3b+46\,a^2b^2+57\,ab^3+24\,b^4)\tan(fx+e)^3+15\,(a^4+14\,a^3b+41\,a^2b^2+44\,ab^3+16\,b^4)\tan(fx+e)}{a^5b^2\tan(fx+e)^{10}+(2\,a^6b+5\,a^5b^2)\tan(fx+e)^8+a^7+2\,a^6b+a^5b^2+(a^7+8\,a^6b+10\,a^5b^2)\tan(fx+e)^6+(3\,a^7+12\,a^6b+10\,a^5b^2)\tan(fx+e)^4+(3\,a^7+8\,a^6b+5\,a^5b^2)\tan(fx+e)^2}-\frac{15\,(a^3+18\,a^2b+48\,ab^2+32\,b^3)(fx+e)}{a^6}+\frac{30\,(3\,a^3b+19\,a^2b^2+32\,ab^3+16\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^6}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*((15*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*tan(f*x + e)^9 + 40*(3*a^3*b + 1
9*a^2*b^2 + 39*a*b^3 + 24*b^4)*tan(f*x + e)^7 + (33*a^4 + 470*a^3*b + 1910*
a^2*b^2 + 2880*a*b^3 + 1440*b^4)*tan(f*x + e)^5 + 40*(a^4 + 14*a^3*b + 46*a
^2*b^2 + 57*a*b^3 + 24*b^4)*tan(f*x + e)^3 + 15*(a^4 + 14*a^3*b + 41*a^2*b^
2 + 44*a*b^3 + 16*b^4)*tan(f*x + e))/(a^5*b^2*tan(f*x + e)^10 + (2*a^6*b +
5*a^5*b^2)*tan(f*x + e)^8 + a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 8*a^6*b + 10*a
^5*b^2)*tan(f*x + e)^6 + (3*a^7 + 12*a^6*b + 10*a^5*b^2)*tan(f*x + e)^4 + (
3*a^7 + 8*a^6*b + 5*a^5*b^2)*tan(f*x + e)^2) - 15*(a^3 + 18*a^2*b + 48*a*b^
2 + 32*b^3)*(f*x + e)/a^6 + 30*(3*a^3*b + 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*a
rctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^6))/f
```

Fricas [A]
time = 3.36, size = 964, normalized size = 3.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 6
0*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a^3
*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2
*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*
b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
 e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)
^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2
+ 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4
+ 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4*b + 48*a^3*
b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3
+ 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b
^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 +
16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqr
t(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos
(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 10*a^4*b)*co
s(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)^5 + 20*(6*a^4
*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2 + 20*a^2*b^3 +
 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*co
s(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 351, normalized size = 1.12

$$\frac{\frac{15\,(a^3+18\,a^2b+48\,ab^2+32\,b^3)(fx+e)}{a^6}-\frac{30\,(3\,a^3b+19\,a^2b^2+32\,ab^3+16\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^6}-\frac{6\left(7\,a^2b^2\tan(fx+e)^3+23\,ab^3\tan(fx+e)^3+16\,b^4\tan(fx+e)^3+9\,a^3b\tan(fx+e)+34\,a^2b^2\tan(fx+e)+41\,ab^3\tan(fx+e)+16\,b^4\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a+b\right)^2a^5}-\frac{33\,a^2\tan(fx+e)^5+162\,ab\tan(fx+e)^5+144\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+288\,ab\tan(fx+e)^3+288\,b^2\tan(fx+e)^3+15\,a^2\tan(fx+e)+126\,ab\tan(fx+e)+144\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3a^5}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b +
 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^2*t
```

```
an(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3*b*t
an(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4*tan(
f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5 + 162
*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 288*
a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 126*a*b
*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

Mupad [B]
time = 7.72, size = 2117, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (5*atan(((5*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176
000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - ((
(20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (tan(
e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*
5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*
(16*a*b + a^2 + 16*b^2))/(32*a^6) + (5*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (
5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*
b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^
2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6))/((4750*a*b^10
 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*
a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(1
28*a^10) - (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)
/a^15 - (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a
^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6
))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6) + (((tan(e + f*x)*(179200
*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 1230
0*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a
^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b
 + a^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^
6)))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(16*a^6*f) - ((5*tan(e + f*x)^3*(57
*a*b^3 + 14*a^3*b + a^4 + 24*b^4 + 46*a^2*b^2))/(6*a^5) + (5*tan(e + f*x)^7
*(39*a*b^3 + 3*a^3*b + 24*b^4 + 19*a^2*b^2))/(6*a^5) + (tan(e + f*x)^5*(288
0*a*b^3 + 470*a^3*b + 33*a^4 + 1440*b^4 + 1910*a^2*b^2))/(48*a^5) + (5*tan(
e + f*x)*(44*a*b^3 + 14*a^3*b + a^4 + 16*b^4 + 41*a^2*b^2))/(16*a^5) + (5*b
*tan(e + f*x)^9*(20*a*b^2 + 5*a^2*b + 16*b^3))/(16*a^5))/(f*(2*a*b + tan(e
```

```
+ f*x)^6*(8*a*b + a^2 + 10*b^2) + a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2
) + b^2*tan(e + f*x)^10 + tan(e + f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e +
f*x)^4*(12*a*b + 3*a^2 + 10*b^2))) + (atan((((a + 4*b)*(3*a + 4*b)*(-b*(a +
 b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 1760
00*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*
(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a
^15 - (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)
*(-b*(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))
*5i)/(16*a^6) + ((a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(1
79200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 +
 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35
*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*
a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a
^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6))*5i)/(16*a^6))/((4750*a*b^10
 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*a^4*b^7)/32
+ (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4)/1024 + (187
5*a^8*b^3)/1024)/a^15 - (5*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e
 + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800
*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) - (5*(a + 4*b)*((20*a^1
2*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (5*tan(e + f
*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2
))/(2048*a^16))*(3*a + 4*b)*(-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6) + (5*(a
 + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2)*((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 92
5*a^6*b^3))/(128*a^10) + (5*(a + 4*b)*((20*a^12*b^5 + 35*a^13*b^4 + (65*a^1
4*b^3)/4 + (5*a^15*b^2)/4)/a^15 + (5*tan(e + f*x)*(2048*a^12*b^3 + 1024*a^1
3*b^2)*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))^(1/2))/(2048*a^16))*(3*a + 4*b)*(
-b*(a + b))^(1/2))/(16*a^6)))/(16*a^6)))*(a + 4*b)*(3*a + 4*b)*(-b*(a + b))
^(1/2)*5i)/(8*a^6*f)
```

3.61 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=238

$$\frac{3(a^2+12ab+16b^2)\,x}{8a^5}-\frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,f}-\frac{(5a+8b)\cos(e+fx)\sin(e+f}{8a^2f\,(a+b+b\tan^2(e+fx))^2}$$

```
[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f/(a+b)^(1/2)-1/8*(5*a+8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2-1/8*b*(7*a+12*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2-3/
2*b*(a+2*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.24, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 209, 211}

$$-\frac{3b(a+2b)\tan(e+fx)}{2a^4f\,(a+b\tan^2(e+fx)+b)}-\frac{b(7a+12b)\tan(e+fx)}{8a^3f\,(a+b\tan^2(e+fx)+b)^2}-\frac{(5a+8b)\sin(e+fx)\cos(e+fx)}{8a^2f\,(a+b\tan^2(e+fx)+b)^2}-\frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f\sqrt{a+b}}+\frac{3x(a^2+12ab+16b^2)}{8a^5}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*f) - ((5*a
 + 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) +
 (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(7
*a + 12*b)*Tan[e + f*x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*(a +
 2*b)*Tan[e + f*x])/(2*a^4*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-7b)x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}}{}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{b(}{8a^3f}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{b(}{8a^3f}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{b(}{8a^3f}$$

$$= \frac{3(a^2+12ab+16b^2)\,x}{8a^5} - \frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,f} - ($$

Mathematica [C] Result contains complex when optimal does not.
time = 26.74, size = 2838, normalized size = 11.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
 + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b
]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))
/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[2*e + 2*f
*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 64
0*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Si
n[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]
```

```
*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Se
c[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*
(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 2
56*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 51
2*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a
^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2
*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*
Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e
] + 2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1
536*b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin
[2*f*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Si
n[2*f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4
*b^2*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[
2*(e + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a
^4*b^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[
4*e + 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^
2))/(65536*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Co
s[2*e + 2*f*x])^3*Sec[e + f*x]^6*((12*(7*a^2 + 32*a*b + 32*b^2)*x)/a^5 + ((
a^7 - 14*a^6*b + 336*a^5*b^2 + 5600*a^4*b^3 + 22400*a^3*b^4 + 37632*a^2*b^5
 + 28672*a*b^6 + 8192*b^7)*((3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqr
t[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*
e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos
[2*e])/(64*a^5*b^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)
/64)*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*
Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*b^2*Sqrt[a + b
]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/(a + b)^2 + (Sec[2*e]*(-(a^6*Sin[2*e
]) - 52*a^5*b*Sin[2*e] - 500*a^4*b^2*Sin[2*e] - 1920*a^3*b^3*Sin[2*e] - 352
0*a^2*b^4*Sin[2*e] - 3072*a*b^5*Sin[2*e] - 1024*b^6*Sin[2*e] + a^6*Sin[2*f*
x] + 50*a^5*b*Sin[2*f*x] + 400*a^4*b^2*Sin[2*f*x] + 1120*a^3*b^3*Sin[2*f*x]
 + 1280*a^2*b^4*Sin[2*f*x] + 512*a*b^5*Sin[2*f*x]))/(16*a^5*b*(a + b)*f*(a
+ 2*b + a*Cos[2*e + 2*f*x])^2) + (Sec[2*e]*(-3*a^7*Sin[2*e] + 42*a^6*b*Sin[
2*e] + 2192*a^5*b^2*Sin[2*e] + 16480*a^4*b^3*Sin[2*e] + 51200*a^3*b^4*Sin[2
*e] + 77824*a^2*b^5*Sin[2*e] + 57344*a*b^6*Sin[2*e] + 16384*b^7*Sin[2*e] +
3*a^7*Sin[2*f*x] - 44*a^6*b*Sin[2*f*x] - 1900*a^5*b^2*Sin[2*f*x] - 10880*a^
4*b^3*Sin[2*f*x] - 23360*a^3*b^4*Sin[2*f*x] - 21504*a^2*b^5*Sin[2*f*x] - 71
68*a*b^6*Sin[2*f*x]))/(64*a^5*b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*e + 2*f*x]
)) + (a + 2*b)*(((-12*I)*Cos[2*e + 2*f*x])/(a^4*f) - (12*Sin[2*e + 2*f*x])/
(a^4*f)) + (a + 2*b)*(((12*I)*Cos[2*e + 2*f*x])/(a^4*f) - (12*Sin[2*e + 2*f
*x])/(a^4*f)) + (2*Sin[4*e + 4*f*x])/(a^3*f)))/(512*(a + b*Sec[e + f*x]^2)^
3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[
f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b +
48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^
```

```
2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*
Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^
4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^2*(
a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Se
c[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^
3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] -
I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt
[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*S
qrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^
3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*S...
```

Maple [A]
time = 0.18, size = 193, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}a\,b^2\right)\left(\tan^3(fx+e)\right)\cdots}{\left(a+b+b\left(\mathrm{t}\cdots\right.\right.}\cdots\right.}{}}{f}\cdots$
default	$\frac{\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}a\,b^2\right)\left(\tan^3(fx+e)\right)\cdots}{\left(a+b+b\left(\mathrm{t}\cdots\right.\right.}\cdots\right.}{}}{f}\cdots$
risch	$\frac{3x}{8a^3}+\frac{9xb}{2a^4}+\frac{6x\,b^2}{a^5}+\frac{i\mathrm{e}^{2i(fx+e)}}{8a^3f}+\frac{3i\mathrm{e}^{2i(fx+e)}b}{8a^4f}-\frac{i\mathrm{e}^{4i(fx+e)}}{64a^3f}+\frac{i\mathrm{e}^{-4i(fx+e)}}{64a^3f}-\frac{ib\left(9a^3\mathrm{e}^{6i(fx+e)}+36a^2b\,\mathrm{e}^{6i(f\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^5*(((-3/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-3/2*a*b)*tan(f*x+e))
/(tan(f*x+e)^2+1)^2+3/8*(a^2+12*a*b+16*b^2)*arctan(tan(f*x+e)))-b/a^5*(((7/
8*a^2*b+3/2*a*b^2)*tan(f*x+e)^3+3/8*a*(3*a^2+7*a*b+4*b^2)*tan(f*x+e))/(a+b+
b*tan(f*x+e)^2)^2+3/8*(5*a^2+20*a*b+16*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*
x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 309, normalized size = 1.30

$$-\frac{\frac{12\,(ab^2+2\,b^3)\tan(fx+e)^7+(19\,a^2b+72\,ab^2+72\,b^3)\tan(fx+e)^5+(5\,a^3+46\,a^2b+108\,ab^2+72\,b^3)\tan(fx+e)^3+3\,(a^3+9\,a^2b+16\,ab^2+8\,b^3)\tan(fx+e)}{a^4b^2\tan(fx+e)^8+2\,(a^5b+2\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b^2+(a^6+6\,a^5b+6\,a^4b^2)\tan(fx+e)^4+2\,(a^6+3\,a^5b+2\,a^4b^2)\tan(fx+e)^2}-\frac{3\,(a^2+12\,ab+16\,b^2)(fx+e)}{a^5}+\frac{3\,(5\,a^2b+20\,ab^2+16\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^5}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((12*(a*b^2 + 2*b^3)*tan(f*x + e)^7 + (19*a^2*b + 72*a*b^2 + 72*b^3)*t
an(f*x + e)^5 + (5*a^3 + 46*a^2*b + 108*a*b^2 + 72*b^3)*tan(f*x + e)^3 + 3*
(a^3 + 9*a^2*b + 16*a*b^2 + 8*b^3)*tan(f*x + e))/(a^4*b^2*tan(f*x + e)^8 +
2*(a^5*b + 2*a^4*b^2)*tan(f*x + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + (a^6 + 6*a
^5*b + 6*a^4*b^2)*tan(f*x + e)^4 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x +
e)^2) - 3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 + 3*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))/f
```

Fricas [A]
time = 3.39, size = 835, normalized size = 3.51

$$\left[\frac{12\left(a^{4}+12a^{3}b+16a^{2}b^{2}\right)fx\cos\left(fx+e\right)^{4}+24\left(a^{3}b+12a^{2}b^{2}+16ab^{3}\right)fx\cos\left(fx+e\right)^{2}+12\left(a^{2}b^{2}+12ab^{3}+16b^{4}\right)fx+3\left(\left(5a^{4}+20a^{3}b+16a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+5a^{2}b^{2}+20ab^{3}+16b^{4}+2\left(5a^{3}b+20a^{2}b^{2}+16ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3ab+4b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+3ab+2b^{2}\right)\cos\left(fx+e\right)^{3}-\left(ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^{2}}{a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}}\right)+4\left(2a^{4}\cos\left(fx+e\right)^{7}-\left(5a^{4}+8a^{3}b\right)\cos\left(fx+e\right)^{5}-\left(19a^{3}b+36a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}-12\left(a^{2}b^{2}+2ab^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{32\left(a^{7}f\cos\left(fx+e\right)^{4}+2a^{6}bf\cos\left(fx+e\right)^{2}+a^{5}b^{2}f\right)}, \frac{6\left(a^{4}+12a^{3}b+16a^{2}b^{2}\right)fx\cos\left(fx+e\right)^{4}+12\left(a^{3}b+12a^{2}b^{2}+16ab^{3}\right)fx\cos\left(fx+e\right)^{2}+6\left(a^{2}b^{2}+12ab^{3}+16b^{4}\right)fx+3\left(\left(5a^{4}+20a^{3}b+16a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+5a^{2}b^{2}+20ab^{3}+16b^{4}+2\left(5a^{3}b+20a^{2}b^{2}+16ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left(\left(a+2b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{b}{a+b}}}{2b\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+2\left(2a^{4}\cos\left(fx+e\right)^{7}-\left(5a^{4}+8a^{3}b\right)\cos\left(fx+e\right)^{5}-\left(19a^{3}b+36a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}-12\left(a^{2}b^{2}+2ab^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{16\left(a^{7}f\cos\left(fx+e\right)^{4}+2a^{6}bf\cos\left(fx+e\right)^{2}+a^{5}b^{2}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 12
*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4)*
f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20*a*
b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-b/
(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*cos(f*x +
e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*b^3)*cos(
f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 +
a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 12*(
a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2 + 12*a*b^3 +
 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^
2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b
)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2
*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f
*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.63, size = 306, normalized size = 1.29

$$\frac{\frac{3\,(a^2+12\,ab+16\,b^2)(fx+e)}{a^5}-\frac{3\,(5\,a^2b+20\,ab^2+16\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{12\,ab^2\tan(fx+e)^7+24\,b^3\tan(fx+e)^7+19\,a^2b\tan(fx+e)^5+72\,ab^2\tan(fx+e)^5+72\,b^3\tan(fx+e)^5+5\,a^3\tan(fx+e)^3+46\,a^2b\tan(fx+e)^3+108\,ab^2\tan(fx+e)^3+72\,b^3\tan(fx+e)^3+3\,a^3\tan(fx+e)+27\,a^2b\tan(fx+e)+48\,ab^2\tan(fx+e)+24\,b^3\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)^2a^4}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*x +
 e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(f*x
+ e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*tan(f*x
 + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f*x + e
) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

Mupad [**B**]

time = 7.19, size = 1317, normalized size = 5.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4
 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2
)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(
20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a
*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^
2 + 16*b^2)*3i)/(16*(a^5*b + a^6)) + (((tan(e + f*x)*(4608*a*b^6 + 2304*b^7
 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3*((12*a^10*b^4
+ 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*a^10*b^3 + 128*
a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a
^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b
*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6)))/((540*a*b
^7 + 216*b^8 + (999*a^2*b^6)/2 + (837*a^3*b^5)/4 + (1215*a^4*b^4)/32 + (135
*a^5*b^3)/64)/a^12 - (3*((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^
5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3
+ (3*a^12*b^2)/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*
(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a +
 b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2
)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)) + (3*((tan(e + f*x)*(4608*a
```

```
*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3
*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*
a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256
*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5
*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)
)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(8*f*(a^5*b + a^6)) -
(atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (81*b^3)/(64*a) + (27*b^4)
/(16*a^2))) + (81*b^3*tan(e + f*x))/(64*((27*a*b^2)/256 + (81*b^3)/64 + (27
*b^4)/(16*a))) + (27*b^4*tan(e + f*x))/(16*((81*a*b^3)/64 + (27*b^4)/16 + (
27*a^2*b^2)/256)))*(a*b*12i + a^2*1i + b^2*16i)*3i)/(8*a^5*f) - ((tan(e + f
*x)^5*(72*a*b^2 + 19*a^2*b + 72*b^3))/(8*a^4) + (tan(e + f*x)^3*(108*a*b^2
+ 46*a^2*b + 5*a^3 + 72*b^3))/(8*a^4) + (3*tan(e + f*x)*(16*a*b^2 + 9*a^2*b
 + a^3 + 8*b^3))/(8*a^4) + (3*b*tan(e + f*x)^7*(a*b + 2*b^2))/(2*a^4))/(f*(
2*a*b + tan(e + f*x)^4*(6*a*b + a^2 + 6*b^2) + a^2 + b^2 + tan(e + f*x)^6*(
2*a*b + 4*b^2) + b^2*tan(e + f*x)^8 + tan(e + f*x)^2*(6*a*b + 2*a^2 + 4*b^2
)))
```

3.62 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=184

$$\frac{(a+6b)x}{2a^4}-\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2}-\frac{3b\,\text{ta}}{4a^2f\,(a+b}$$

```
[Out] 1/2*(a+6*b)*x/a^4-1/8*(15*a^2+40*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b
)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2-3/4*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+12*b
)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 482, 541, 536, 209, 211}

$$\frac{x(a+6b)}{2a^4}-\frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\,(a+b\tan^2(e+fx)+b)}-\frac{3b\tan(e+fx)}{4a^2f\,(a+b\tan^2(e+fx)+b)^2}-\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*x)/(2*a^4) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*f) - (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*a^2*f*
(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 12*b)*Tan[e + f*x])/(8*a^3*(a +
b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4a^2f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(1}{8a^3(a+}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4a^2f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(1}{8a^3(a+}$$

$$= \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+f}{2af\,(a+b+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.91, size = 1915, normalized size = 10.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
+ 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]
*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e
 + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/
(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])
^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*
b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*
x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*
e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a +
 b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a
```

^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^
4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b
^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x
*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[
6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] +
2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*
b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f
*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*
f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2
Sin[2(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e
 + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b
^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e
+ 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/
(4096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e
])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 6
4*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f
*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5
 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^
2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(4096*b^2*(a + b)^2*f*(a + b*Sec[e +
f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2
*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 307
2*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a
+ 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2
624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*
b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan
[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(8192*a^4*(a + b*
Sec[e + f*x]^2)^3)

Maple [A]
time = 0.30, size = 155, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\frac{b\left(\frac{\frac{ab(7a+8b)\left(\tan^3(fx+e)\right)}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan\left(\cdots\right.}{8(a+b)\sqrt{(a+\cdots}}\right.}{a^4}}{f}$

default	$\dfrac{\dfrac{-\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)+1\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\dfrac{b\left(\dfrac{\frac{ab(7a+8b)\left(\tan^3(fx+e)\right)}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\dfrac{\left(15a^2+40ab+24b^2\right)\arctan}{8(a+b)\sqrt{(a}}\right.}{a^4}}{f}$
risch	$\frac{x}{2a^3}+\frac{3xb}{a^4}+\frac{i\mathrm{e}^{2i(fx+e)}}{8a^3f}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^3f}-\frac{ib\left(9a^3\mathrm{e}^{6i(fx+e)}+32a^2b\,\mathrm{e}^{6i(fx+e)}+24a\,b^2\mathrm{e}^{6i(fx+e)}+27a^3\mathrm{e}^{4i(fx+e)}+10\right.}{4a^4(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(-1/2*a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2*(a+6*b)*arctan(tan(f*x+e
)))-1/a^4*b*((1/8*a*b*(7*a+8*b)/(a+b)*tan(f*x+e)^3+1/8*a*(9*a+8*b)*tan(f*x+
e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+40*a*b+24*b^2)/(a+b)/((a+b)*b)^(1/2)
*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 280, normalized size = 1.52

$$-\frac{\frac{(15\,a^2b+40\,ab^2+24\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+a^4b)\sqrt{(a+b)b}}+\frac{(11\,ab^2+12\,b^3)\tan(fx+e)^5+(17\,a^2b+40\,ab^2+24\,b^3)\tan(fx+e)^3+(4\,a^3+21\,a^2b+29\,ab^2+12\,b^3)\tan(fx+e)}{(a^4b^2+a^3b^3)\tan(fx+e)^6+a^6+3\,a^5b+3\,a^4b^2+a^3b^3+(2\,a^5b+5\,a^4b^2+3\,a^3b^3)\tan(fx+e)^4+(a^6+5\,a^5b+7\,a^4b^2+3\,a^3b^3)\tan(fx+e)^2}-\frac{4\,(fx+e)(a+6\,b)}{a^4}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + a^4*b)*sqrt((a + b)*b)) + ((11*a*b^2 + 12*b^3)*tan(f*x + e)^5 + (1
7*a^2*b + 40*a*b^2 + 24*b^3)*tan(f*x + e)^3 + (4*a^3 + 21*a^2*b + 29*a*b^2
+ 12*b^3)*tan(f*x + e))/((a^4*b^2 + a^3*b^3)*tan(f*x + e)^6 + a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3 + (2*a^5*b + 5*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^4 +
 (a^6 + 5*a^5*b + 7*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^2) - 4*(f*x + e)*(a +
 6*b)/a^4)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(174) = 348$.
time = 3.05, size = 845, normalized size = 4.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a^
2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*a^2*b^2)
*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7
 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^
2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 1
6*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 7*a*b^3 +
 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^
3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*si
n(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x
 + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 208, normalized size = 1.13

$$-\frac{\frac{(15a^2b+40ab^2+24b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}+\frac{7ab^2\tan(fx+e)^3+8b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+17ab^2\tan(fx+e)+8b^3\tan(fx+e)}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)^2}-\frac{4(fx+e)(a+6b)}{a^4}+\frac{4\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) +
(7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) + 17*
a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)^2 +
 a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)^2 +
1)*a^3))/f
```

Mupad [B]
time = 7.83, size = 2628, normalized size = 14.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4
 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^
4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e +
 f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*
b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i +
b*6i)*1i)/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 +
 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5
 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^
2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^
3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a
^4))*(a*1i + b*6i)*1i)/(4*a^4))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4
+ (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan
(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^
3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^
10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a*1i +
 b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*a^
4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6i))/(4*a^4
) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 +
241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^5 + (29*a^9*b^4)/
2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*
x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2
))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1i + b*6
i))/(4*a^4)))*(a*1i + b*6i)*1i)/(2*a^4*f) - ((tan(e + f*x)*(17*a*b + 4*a^2
+ 12*b^2))/(8*a^3) + (tan(e + f*x)^5*(11*a*b^2 + 12*b^3))/(8*a^3*(a + b)) +
 (b*tan(e + f*x)^3*(40*a*b + 17*a^2 + 24*b^2))/(8*a^3*(a + b)))/(f*(2*a*b +
 tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*a*b +
 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((-b*(a + b)^3)^(1/2)*((tan(e + f*x
)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*
(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^
4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e +
 f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a
^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a
^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*
b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*
b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(326
4*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7
*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b^5 + (29*a^9*b^4)/2 +
```

```
(21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*
(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a^8*b^5 + 1280*a^9*b^4
 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b +
 a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))/(16*(3*a^6*b + a^
7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2)*1i)/(16*(3*a^6*b + a^
7 + a^4*b^3 + 3*a^5*b^2)))/(((297*a*b^6)/4 + 27*b^7 + (279*a^2*b^5)/4 + (80
5*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b^2) - ((-b*(a + b
)^3)^(1/2)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*
b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b*(a + b)^3)^(1/2)*
((6*a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^
11 + a^9*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^
2)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7
*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^
2 + 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 +
 24*b^2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2
)*((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4 + 241
*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b*(a + b)^3)^(1/2)*((6*a^8*b
^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*
b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(40*a*b + 15*a^2 + 24*b^2)*(512*a
^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*(2*a^7*b + a^8
+ a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^
2))/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(40*a*b + 15*a^2 + 24*b^2))
/(16*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b*(a + b)^3)^(1/2)*(40*a*b
+ 15*a^2 + 24*b^2)*1i)/(8*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f}-\frac{b\tan(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2}-\frac{b(7a+}{8a^2(a+b)^2f}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\,(a+b\tan^2(e+fx)+b)}-\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+fx)}{4af(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx,x,\tan(e\right.}{4a(a+b)f}$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$=-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right.}$$

$$=\frac{x}{a^3}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f}-\frac{b\tan(e+f}{4a(a+b)f\left(a+b+b\,\mathrm{t}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.40, size = 332, normalized size = 2.31

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\mathrm{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))^2(\cos(2e)-i\sin(2e))}{(a+b)^{5/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}-\frac{4b^2((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}+\frac{b(a+2b+a\cos(2(e+fx)))((9a^2+28ab+16b^2)\sin(2e)-3a(3a+2b)\sin(2fx))}{(a+b)^2f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{64a^3\left(a+b\sec^2(e+fx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.23, size = 147, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2b\,e^{4i(fx+e)}+120a\,b^2e^{4i(fx+e)}+48b^3e^{4i}\right.}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan
(f*x+e)^3+1/8*(9*a+4*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*
a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2))))
```

Maxima [A]
time = 0.48, size = 237, normalized size = 1.65

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}}+\frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)\tan(fx+e)^2}-\frac{8\,(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(135) = 270.
time = 3.24, size = 847, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Giac [A]
time = 0.46, size = 196, normalized size = 1.36

$$-\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan\left(fx+e\right)^3+4\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)+13\,ab^2\tan\left(fx+e\right)+4\,b^3\tan\left(fx+e\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}-\frac{8\,(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B]
time = 8.61, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
```

56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^

```
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 2...
```

3.64 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{15\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f}-\frac{15\cot(e+fx)}{8(a+b)^3f}+\frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{5\cot(e+}{8(a+b)^2f\,(a+b+}$$

```
[Out] -15/8*cot(f*x+e)/(a+b)^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/(a+b)^(7/2)/f+1/4*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+5/8*cot(f*x
+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 296, 331, 211}

$$-\frac{15\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{15\cot(e+fx)}{8f(a+b)^3}+\frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*f
) - (15*Cot[e + f*x])/(8*(a + b)^3*f) + Cot[e + f*x]/(4*(a + b)*f*(a + b +
b*Tan[e + f*x]^2)^2) + (5*Cot[e + f*x])/(8*(a + b)^2*f*(a + b + b*Tan[e + f
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^2}\,dx, x, \tan(e+f$$

$$= \frac{\cot(e+fx)}{4(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\,(a+b+b\tan^2(e+fx))} +$$

$$= -\frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{5\cot(e}{8(a+b)^2f\,(a+b+}$$

$$= -\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\,(a+b+b\tan^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.91, size = 987, normalized size = 7.96

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((15*b*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Cos[2*e])/(64*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin
```

```
[4*e]]) - (((15*I)/64)*b*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I
*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])
/(Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e
+ f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]
*Sec[e + f*x]^6*(-32*a^4*Sin[f*x] - 64*a^3*b*Sin[f*x] + 22*a^2*b^2*Sin[f*x]
 + 80*a*b^3*Sin[f*x] + 16*b^4*Sin[f*x] + 32*a^4*Sin[3*f*x] + 46*a^3*b*Sin[3
*f*x] - 54*a^2*b^2*Sin[3*f*x] - 8*a*b^3*Sin[3*f*x] - 48*a^4*Sin[2*e - f*x]
- 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*x] + 80*a*b^3*Sin[2*e
- f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*x] + 146*a^3*b*Sin[2*e
+ f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[2*e + f*x] + 16*b^4*Sin[
2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[4*e + f*x] - 54*a^2*b^2*S
in[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*Sin[4*e + f*x] - 8*a^4*Sin
[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a^2*b^2*Sin[2*e + 3*f*x] + 8
*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*x] + 73*a^3*b*Sin[4*e + 3*f*
x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin[4*e + 3*f*x] - 8*a^4*Sin[6*e
 + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2*b^2*Sin[6*e + 3*f*x] - 8*a*b^
3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] - 9*a^3*b*Sin[2*e + 5*f*x] - 2*
a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e + 5*f*x] + 2*a^2*b^2*Sin[4*e + 5
*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/(512*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^3)
```

Maple [A]
time = 0.25, size = 97, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{7b\left(\tan^{3}(fx+e)\right)}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^{3}}-\frac{1}{(a+b)^{3}\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{7b\left(\tan^{3}(fx+e)\right)}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^{3}}-\frac{1}{(a+b)^{3}\tan(fx+e)}}{f}$
risch	$-\frac{i\left(8a^{4}e^{8i(fx+e)}+9a^{3}b\,e^{8i(fx+e)}+24a^{2}b^{2}e^{8i(fx+e)}+8a\,b^{3}e^{8i(fx+e)}+32a^{4}e^{6i(fx+e)}+82a^{3}b\,e^{6i(fx+e)}+54a^{2}b^{2}e^{6i(fx+e)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^3*b*((7/8*b*tan(f*x+e)^3+(9/8*a+9/8*b)*tan(f*x+e))/(a+b+b*tan
(f*x+e)^2)^2+15/8/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(
a+b)^3/tan(f*x+e))
```

Maxima [A]
time = 0.48, size = 225, normalized size = 1.81

$$-\frac{\frac{15\,b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{15\,b^2\tan\left(fx+e\right)^4+25\,\left(ab+b^2\right)\tan\left(fx+e\right)^2+8\,a^2+16\,ab+8\,b^2}{\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\tan\left(fx+e\right)^5+2\,\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)\tan\left(fx+e\right)^3+\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\tan\left(fx+e\right)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sqrt((a + b)*b)) + (15*b^2*tan(f*x + e)^4 + 25*(a*b + b^2)*tan(f*x
+ e)^2 + 8*a^2 + 16*a*b + 8*b^2)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan
(f*x + e)^5 + 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x + e
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e
)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(114) = 228$.
time = 2.01, size = 643, normalized size = 5.19

$$\left[-\frac{4\,(8\,a^2-9\,ab-2\,b^2)\cos\left(fx+e\right)^5+20\,(5\,ab-b^2)\cos\left(fx+e\right)^3-15\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin\left(fx+e\right)+60\,b^2\cos\left(fx+e\right)}{32\,\left((a^5+3\,a^4b+3\,a^3b^2+a^2b^3)f\cos\left(fx+e\right)^4+2\,(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)f\cos\left(fx+e\right)^2+(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)f\right)\sin\left(fx+e\right)},-\frac{2\,(8\,a^2-9\,ab-2\,b^2)\cos\left(fx+e\right)^5+10\,(5\,ab-b^2)\cos\left(fx+e\right)^3-15\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)+30\,b^2\cos\left(fx+e\right)}{16\,\left((a^5+3\,a^4b+3\,a^3b^2+a^2b^3)f\cos\left(fx+e\right)^4+2\,(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)f\cos\left(fx+e\right)^2+(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*x
 + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x + e))
, -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/(a +
 b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
 e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a
*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x +
 e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 177, normalized size = 1.43

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{7b^2\tan(fx+e)^3+9ab\tan(fx+e)+9b^2\tan(fx+e)}{(a^3+3a^2b+3ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2*t
an(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)))/f
```

Mupad [B]
time = 5.11, size = 146, normalized size = 1.18

$$-\frac{\frac{1}{a+b}+\frac{25\,b\tan(e+f\,x)^2}{8\,(a+b)^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,(a+b)^3}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(2\,b^2+2\,a\,b\right)+\tan\left(e+f\,x\right)\,\left(a^2+2\,a\,b+b^2\right)+b^2\tan\left(e+f\,x\right)^5\right)}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{(a+b)^{7/2}}\right)}{8\,f\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(a + b) + (25*b*tan(e + f*x)^2)/(8*(a + b)^2) + (15*b^2*tan(e + f*x)^4
)/(8*(a + b)^3))/(f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b +
 a^2 + b^2) + b^2*tan(e + f*x)^5)) - (15*b^(1/2)*atan((b^(1/2)*tan(e + f*x)
*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(8*f*(a + b)^(7/2))
```

3.65 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{5(3a-4b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f}-\frac{(a-2b)\cot(e+fx)}{(a+b)^4f}-\frac{\cot^3(e+fx)}{3(a+b)^3f}-\frac{ab\tan(e+fx)}{4(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^2}$$

```
[Out] -(a-2*b)*cot(f*x+e)/(a+b)^4/f-1/3*cot(f*x+e)^3/(a+b)^3/f-5/8*(3*a-4*b)*arct
an(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/f-1/4*a*b*tan(f*x+e)
/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-1/8*(7*a-4*b)*b*tan(f*x+e)/(a+b)^4/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 467, 1273, 1275, 211}

$$-\frac{5\sqrt{b}\,(3a-4b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}-\frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\cot^3(e+fx)}{3f(a+b)^3}-\frac{(a-2b)\cot(e+fx)}{f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a +
 b)^(9/2)*f) - ((a - 2*b)*Cot[e + f*x])/((a + b)^4*f) - Cot[e + f*x]^3/(3*(
a + b)^3*f) - (a*b*Tan[e + f*x])/(4*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^
2) - ((7*a - 4*b)*b*Tan[e + f*x])/(8*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\,(a+b+b\tan^2(e+fx))^2} - \frac{b\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4(a+b+bx^2)^2}\,dx, x\right.}{4f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\,(a+b+b\tan^2(e+fx))^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\,(a+b+b\tan^2(e+fx))^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab\tan(e+fx)}{4(a+b)^3 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{5(3a-4b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2} f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+f}{3(a+b)^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.69, size = 994, normalized size = 6.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*
x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2 + 12
6*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208*a*b^3
 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e - f*x] +
 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*Sin[2*e -
 f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^2*b^2*Sin
[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] + 176*a^4*S
in[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e + f*x] + 170
4*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*e + 3*f*x] +
 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 312*a*b^3*Sin[
```

```
2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f*x] - 152*a^3*
b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^3*Sin[4*e + 3*f
*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 192*a^3*b*Sin[6*e
 + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6*e + 3*f*x] + 48*
b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*b^2*Sin[2*e + 5*f*
x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*x] + 150*a^2*b^2*Si
n[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*Sin[6*e + 5*f*x] + 27*a
^3*b*Sin[6*e + 5*f*x] - 388*a^2*b^2*Sin[6*e + 5*f*x] + 45*a^3*b*Sin[8*e + 5
*f*x] - 60*a^2*b^2*Sin[8*e + 5*f*x] + 16*a^4*Sin[4*e + 7*f*x] - 83*a^3*b*Si
n[4*e + 7*f*x] + 6*a^2*b^2*Sin[4*e + 7*f*x] + 27*a^3*b*Sin[6*e + 7*f*x] - 6
*a^2*b^2*Sin[6*e + 7*f*x] + 16*a^4*Sin[8*e + 7*f*x] - 56*a^3*b*Sin[8*e + 7*
f*x]))/a))/(6144*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.24, size = 140, normalized size = 0.85

method	result
derivativedivides	$-\frac{\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\left(\tan^3(fx+e)\right)+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\sqrt{(a+b)\,b}}\right)}{(a+b)^4}}{f}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-$
default	$-\frac{\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\left(\tan^3(fx+e)\right)+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\sqrt{(a+b)\,b}}\right)}{(a+b)^4}}{f}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-$
risch	$-\frac{i\left(144b^4e^{8i(fx+e)}-83a^3b+6a^2b^2+16a^4+60a^2b^2e^{12i(fx+e)}-192a^3b\,e^{10i(fx+e)}-558a^2b^2e^{10i(fx+e)}+168a\,b^3e^{10i(fx+e)}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^4*b*(((7/8*a*b-1/2*b^2)*tan(f*x+e)^3+(9/8*a^2+5/8*a*b-1/2*b^2
)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a-4*b)/((a+b)*b)^(1/2)*arctan(b
*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^3/tan(f*x+e)^3-(a-2*b)/(a+b)^4/tan(
f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(155) = 310$.
time = 0.50, size = 330, normalized size = 2.01

$$-\frac{\frac{15\,(3\,ab-4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}}+\frac{15\,(3\,ab^2-4\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-ab^2-4\,b^3)\tan(fx+e)^4+8\,a^3+24\,a^2b+24\,ab^2+8\,b^3+8\,(3\,a^3+2\,a^2b-5\,ab^2-4\,b^3)\tan(fx+e)^2}{(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6)\tan(fx+e)^7+2\,(a^5b+5\,a^4b^2+10\,a^3b^3+10\,a^2b^4+5\,ab^5+b^6)\tan(fx+e)^5+(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)\tan(fx+e)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a*b^2 - 4*b^3)
*tan(f*x + e)^6 + 25*(3*a^2*b - a*b^2 - 4*b^3)*tan(f*x + e)^4 + 8*a^3 + 24*
a^2*b + 24*a*b^2 + 8*b^3 + 8*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*tan(f*x +
e)^2)/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(f*x + e)^7 + 2
*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*tan(f*x + e)
^5 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
*tan(f*x + e)^3))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(155) = 310$.
time = 2.06, size = 1043, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a^
2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b^3
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^2*b
 + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*b^2 - 4*
b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*co
s(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5
)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5
 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6
)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 -
 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b
- 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6
- (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b
- 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x +
 e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a
^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3
 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3
 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 264, normalized size = 1.61

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-4\,b^2)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}}+\frac{3\left(7\,ab^2\tan(fx+e)^3-4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+5\,ab^2\tan(fx+e)-4\,b^3\tan(fx+e)\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8\left(3\,a\tan(fx+e)^2-6\,b\tan(fx+e)^2+a+b\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(fx+e)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*a^2
*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*tan(f
*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*tan(f*x + e)^3))/f
```

Mupad [**B**]
time = 6.88, size = 207, normalized size = 1.26

$$-\frac{\frac{1}{3\,(a+b)}+\frac{25\tan(e+f\,x)^4\,(3\,a\,b-4\,b^2)}{24\,(a+b)^3}+\frac{\tan(e+f\,x)^2\,(3\,a-4\,b)}{3\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(3\,a\,b^2-4\,b^3)}{8\,(a+b)^4}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^5\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^7\right)}-\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{(a+b)^{9/2}}\right)\,(3\,a-4\,b)}{8\,f\,(a+b)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(3*(a + b)) + (25*tan(e + f*x)^4*(3*a*b - 4*b^2))/(24*(a + b)^3) + (ta
n(e + f*x)^2*(3*a - 4*b))/(3*(a + b)^2) + (5*tan(e + f*x)^6*(3*a*b^2 - 4*b^
3))/(8*(a + b)^4))/(f*(tan(e + f*x)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)^5*
(2*a*b + 2*b^2) + b^2*tan(e + f*x)^7)) - (5*b^(1/2)*atan((b^(1/2)*tan(e + f
*x)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(a + b)^(9/2))*(3*a - 4*b)
)/(8*f*(a + b)^(9/2))
```

3.66 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{b}\,(15a^2-40ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f}-\frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5f}-\frac{(10a+b)\cot^3(e+fx}{15(a+b)^4f}$$

```
[Out] -1/5*(5*a^2-20*a*b+2*b^2)*cot(f*x+e)/(a+b)^5/f-1/15*(10*a+b)*cot(f*x+e)^3/(
a+b)^4/f-1/8*(15*a^2-40*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b
^(1/2)/(a+b)^(11/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/20*
b*(5*a^2+4*b^2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^2-1/40*b*(35*a^2-
40*a*b+24*b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.27, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 473, 467, 1273, 1275, 211}

$$-\frac{\sqrt{b}\,(15a^2-40ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}}-\frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40f(a+b)^5\,(a+b\tan^2(e+fx)+b)}-\frac{b(5a^2+4b^2)\tan(e+fx)}{20f(a+b)^4\,(a+b\tan^2(e+fx)+b)^2}-\frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5f(a+b)^5}-\frac{(10a+b)\cot^3(e+fx)}{15f(a+b)^4}-\frac{\cot^5(e+fx)}{5f(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/((a + b)^(11/2)*f) - ((5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(5*(a
 + b)^5*f) - ((10*a + b)*Cot[e + f*x]^3)/(15*(a + b)^4*f) - Cot[e + f*x]^5/
(5*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Tan[e + f*x
])/(20*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 40*a*b + 24
*b^2)*Tan[e + f*x])/(40*(a + b)^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{10a+b+5(a+b)x^2}{x^4(a+b+bx^2)^3}\,dx, x, \tan(e+\right.}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

$$= -\frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5 f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4 f} - \frac{\cot^5}{5(a+b)f\,(a+}$$

$$= -\frac{\sqrt{b}\,(15a^2-40ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{(5a^2-20ab+2b^2)\cot(e}{5(a+b)^5 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.66, size = 479, normalized size = 1.98

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
+ 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b + 8
*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])
^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b)*Sec[2*e]
```

```
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*Sin[2*f*x]))
)/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.30, size = 180, normalized size = 0.74

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-a\,b^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\sqrt{(a+b)\,b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-a\,b^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\sqrt{(a+b)\,b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx}}{f}$
risch	$i\left(-25120b^4e^{8i(fx+e)}+607a^3b-274a^2b^2-64a^4-10750a^2b^2e^{12i(fx+e)}-9210a^3b\,e^{10i(fx+e)}-15450a^2b^2e^{10i(fx+e)}-42680\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)^5*b*(((7/8*a^2*b-a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+a*b-8*b^2)*
tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2-40*a*b+8*b^2)/((a+b)*b)^(1/2
)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(2*a-b
)/(a+b)^4/tan(f*x+e)^3-(a^2-4*a*b+b^2)/(a+b)^5/tan(f*x+e))
```

Maxima [A]
time = 0.51, size = 442, normalized size = 1.83

$$-\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{(a+b)b}}+\frac{15\,(15\,a^2b^2-40\,ab^3+8\,b^4)\tan(fx+e)^8+25\,(15\,a^3b-25\,a^2b^2-32\,ab^3+8\,b^4)\tan(fx+e)^6+8\,(15\,a^4-10\,a^3b-57\,a^2b^2-24\,ab^3+8\,b^4)\tan(fx+e)^4+24\,a^4+96\,a^3b+144\,a^2b^2+96\,ab^3+24\,b^4+8\,(10\,a^4+31\,a^3b+33\,a^2b^2+13\,ab^3+b^4)\tan(fx+e)^2}{(a^5b^2+5\,a^4b^3+10\,a^3b^4+10\,a^2b^5+5\,ab^6+b^7)\tan(fx+e)^9+2\,(a^6b+6\,a^5b^2+15\,a^4b^3+20\,a^3b^4+15\,a^2b^5+6\,ab^6+b^7)\tan(fx+e)^7+(a^7+7\,a^6b+21\,a^5b^2+35\,a^4b^3+35\,a^3b^4+21\,a^2b^5+7\,ab^6+b^7)\tan(fx+e)^5}}{120\,f}$$

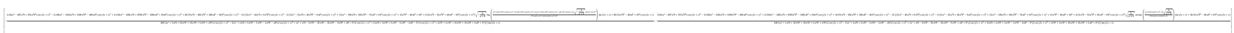
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt((a + b
)*b)) + (15*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b -
 25*a^2*b^2 - 32*a*b^3 + 8*b^4)*tan(f*x + e)^6 + 8*(15*a^4 - 10*a^3*b - 57*
a^2*b^2 - 24*a*b^3 + 8*b^4)*tan(f*x + e)^4 + 24*a^4 + 96*a^3*b + 144*a^2*b^
```

```
2 + 96*a*b^3 + 24*b^4 + 8*(10*a^4 + 31*a^3*b + 33*a^2*b^2 + 13*a*b^3 + b^4)
*tan(f*x + e)^2)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*tan(f*x + e)^9 + 2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15
*a^2*b^5 + 6*a*b^6 + b^7)*tan(f*x + e)^7 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*tan(f*x + e)^5))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(231) = 462$.
time = 2.20, size = 1463, normalized size = 6.05



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4 -
 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 1205*
a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b
- 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a^3*b
 + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a*b^3)
*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^4)*cos
(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b^2 + 48
*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 6
0*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b
+ 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*
b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*
x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*
x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)
^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x
 + e)^5 + 10*(75*a^3*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 -
 15*((15*a^4 - 40*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b
+ 48*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2
- 72*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*
a^3*b - 55*a^2*b^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f
*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(
((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b
```

```
^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^
4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 367, normalized size = 1.52

$$-\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{ab+b^2}}+\frac{15\left(7\,a^2b^2\tan(fx+e)^3-8\,ab^3\tan(fx+e)^3+9\,a^3b\tan(fx+e)+a^2b^2\tan(fx+e)-8\,ab^3\tan(fx+e)\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8\left(15\,a^2\tan(fx+e)^4-60\,ab\tan(fx+e)^4+15\,b^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2+5\,ab\tan(fx+e)^2-5\,b^2\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\tan(fx+e)^5}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x + e)^
3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e) -
8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan
(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*tan(f*x
 + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

Mupad [B]
time = 7.49, size = 267, normalized size = 1.10

$$-\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(10\,a+b)}{15\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(15\,a^2\,b-40\,a\,b^2+8\,b^3)}{24\,(a+b)^4}+\frac{\tan(e+f\,x)^4\,(15\,a^2-40\,a\,b+8\,b^2)}{15\,(a+b)^3}+\frac{\tan(e+f\,x)^8\,(15\,a^2\,b^2-40\,a\,b^3+8\,b^4)}{8\,(a+b)^5}}{f\,\left(\tan\left(e+f\,x\right)^5\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^7\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^9\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5)}{(a+b)^{11/2}}\right)\,\left(15\,a^2-40\,a\,b+8\,b^2\right)}{8\,f\,(a+b)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] - (1/(5*(a + b)) + (tan(e + f*x)^2*(10*a + b))/(15*(a + b)^2) + (5*tan(e +
f*x)^6*(15*a^2*b - 40*a*b^2 + 8*b^3))/(24*(a + b)^4) + (tan(e + f*x)^4*(15*
a^2 - 40*a*b + 8*b^2))/(15*(a + b)^3) + (tan(e + f*x)^8*(8*b^4 - 40*a*b^3 +
 15*a^2*b^2))/(8*(a + b)^5))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + tan(e
 + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) - (b^(1/2)*atan((b^(1/2)*t
an(e + f*x)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(a +
 b)^(11/2))*(15*a^2 - 40*a*b + 8*b^2))/(8*f*(a + b)^(11/2))
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^5(e+fx)\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec\right.}{15a^2f}$$

[Out] 2/15*(5*a+b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a^2/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f

Rubi [A]

time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 473, 462, 283, 223, 212}

$$\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]

[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (2*(5*a + b)*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2))/(5*a*f)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(-2(5a+b)+5ax^2}{x^4}\right.}{}$$

$$= \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\,(a}{}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\,(a}{15a^2f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\,(a}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2}}{f}$$

Mathematica [A]
time = 0.95, size = 152, normalized size = 1.09

$$-\frac{\cos(e+fx)\left(-2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+2\sqrt{b+a\cos^2(e+fx)}-\frac{2(2a+b)\left(b+a\cos^2(e+fx)\right)^{3/2}}{3a^2}+\frac{2\left(b+a\cos^2(e+fx)\right)^{5/2}}{5a^2}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}\,f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] -((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] + 2
*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/(3
*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(123) = 246.
time = 1.08, size = 1840, normalized size = 13.24

method	result	size
default	Expression too large to display	1840

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/30/f*(cos(f*x+e)-1)^2*(15*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*4^(1/2)*a^2-15*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*4^(1/2)*a^2-10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*(a+b)^(3/2)*a+15*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-15*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*4^(1/2)*a^3-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^6*(a+b)^(3/2)*a-24*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^5*(a+b)^(3/2)*a-16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^4*(a+b)^(3/2)*a+56*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^3*(a+b)^(3/2)*a+84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^2*(a+b)^(3/2)*a+15*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*cos(f*x+e)*4^(1/2)*a^2-15*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(5/2)*cos(f*x+e)*4^(1/2)*a^2+20*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)*(a+b)^(3/2)*a+15*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*cos(f*x+e)*4^(1/2)*a^3-15*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)*cos(f*x+e)*4^(1/2)*a^3+15*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*(a+b)^(3/2)*4^(1/2)*a^2*b+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)*(a+b)^(3/2)*4^(1/2)*a-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*(a+b)^(3/2)*4^(1/2)*a^2+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*(a+b)^(3/2)+15*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*a^2*b-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*a^2+4*((b+a*cos(f*x+e)^2)/

(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)^4*(a+b)^(3/2)+16*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)^3*(a+b)^(3/2)+24*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)^2*(a+b)^(3/2)+16*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)*(a+b)^(3/2))*cos(
f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)/sin(f*x+e)^4/b^(1/2)/(a+b)^(3/2)/a^2

Maxima [A]
time = 0.47, size = 183, normalized size = 1.32

$$\frac{\frac{20\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a}-30\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-15\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)-\frac{2\left(3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-5\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3\right)}{a^2}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 30*sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 2
*(3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a + b/cos(f*x + e)^2)^
(3/2)*b*cos(f*x + e)^3)/a^2)/f
```

Fricas [A]
time = 2.90, size = 313, normalized size = 2.25

$$\left[\frac{15\,a^2\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e)^3+(15\,a^2-10\,ab-2\,b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{30\,a^2f},\ -\frac{15\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e)^3+(15\,a^2-10\,ab-2\,b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\,a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*cos
(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*(15*
a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e)/b) + (3*a^2*cos(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15
*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(a^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(123) = 246.
time = 0.85, size = 1281, normalized size = 9.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2/15*(15*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) - 2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*b + 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*b - 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*(16*a^2 + 5*a*b - 27*b^2) + 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(32*a^2 - 83*a*b + 33*b^2)*sqrt(a + b) + 2*(416*a^3 + 625*a^2*b - 1230*a*b^2 - 15*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5 - 10*(256*a^3 - 391*a^2*b + 90*a*b^2 + 81*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) + 20*(16*a^4 - 161*a^3*b + 157*a^2*b^2 + 45*a*b^3 - 33*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 + 20*(160*a^4 - 159*a^3*b - 49*a^2*b^2 + 75*a*b^3 - 3*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b) - 5*(576*a^5 - 667*a^4*b - 76*a^3*b^2 + 286*a^2*b^3 - 140*a*b^4 - 27*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (768*a^5 - 1539*a^4*b + 1028*a^3*b^2 - 450*a^2*b^3 + 100*a*b^4 + 45*b^5)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1

```
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a +
b) - 3*a + b)^5)*sgn(cos(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^3(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{3af}$$

```
[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a
+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4219, 462, 283, 223, 212}

$$\frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{3af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a + b*Sec[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, \ldots\right.}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e \ldots\right.}{3af}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e \ldots\right.}{3af}$$

$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec \ldots}}{f}$$

Mathematica [A]
time = 0.40, size = 120, normalized size = 1.20

$$\frac{\sqrt{2}\,\cos(e+fx)\left(3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}\,(-3a+b+a\cos^2(e+fx))\right)\sqrt{a+b\sec^2(e+fx)}}{3af\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[
b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a + b*
Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. $2(88) = 176$.
time = 0.18, size = 1525, normalized size = 15.25

method	result	size
default	Expression too large to display	1525

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)-1)^2*(2*cos(f*x+e)^4*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(3/2)+8*cos(f*x+e)^3*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+6*cos(f*x+e)^2*(a+b)^(3/2)*b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-3*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-4*(cos
(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a+3*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-2*(cos(f*x+e)
-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*
x+e)^2/(a+b)^(1/2))*a-6*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*arctanh(
1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2
/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b-4*cos(f*x
+e)*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-3*cos(f
*x+e)*4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+3*cos(f*x+e)
*4^(1/2)*b^(3/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+3*(a+b)^(3/2)*4^(
1/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-3*4^(1/2)*b^(5/2)*
ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f
*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+3*4^(1/2)*b^(5/2)*ln(-2*(cos(f*x+e)-
1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x
+e)^2/(a+b)^(1/2))*a-3*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a+3*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos
(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b-4*(a+b)^(3/2)*b^(1/2)*((b+a*cos(
```

```
f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-3*4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)-1)*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2
/(a+b)^(1/2))*a^2+3*4^(1/2)*b^(3/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^
2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/b^(1/2)/(a+b)^(3/2)/a
```

Maxima [A]
time = 0.48, size = 124, normalized size = 1.24

$$\frac{\frac{2\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 6\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right) - 3\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 6*sqrt(a + b/cos(f*x
 + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x +
 e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Fricas [A]
time = 2.51, size = 248, normalized size = 2.48

$$\left[\frac{3\,a\sqrt{b}\,\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\left(a\cos\left(fx+e\right)^3-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{6\,af},\ -\frac{3\,a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{b}\right)-\left(a\cos\left(fx+e\right)^3-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x + e
)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/b) - (a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(88) = 176.

time = 0.63, size = 805, normalized size = 8.05

$$-\frac{2\left(\frac{3b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}-\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{2\left(3\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{5}b-3\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{4}(4a-3b)\sqrt{a+b}+2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{3}(8a^{2}-9ab+3b^{2})+6\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{2}(4a^{2}-ab-b^{2})\sqrt{a+b}-3(16a^{3}-5a^{2}b-2ab^{2}+3b^{3})\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)+(20a^{3}-19a^{2}b+6ab^{2}-3b^{3})\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{2}+2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)\sqrt{a+b}-3a+b\right)^{3}}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) - 2*(3*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^5*b - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(4*a - 3*b)*sqrt(a + b) + 2*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^3*(8*a^2 - 9*a*b + 3*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(4*a^2 - a*b - b^2)*sqrt(a +
 b) - 3*(16*a^3 - 5*a^2*b - 2*a*b^2 + 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (20*a^3 - 19*a^
2*b + 6*a*b^2 - 3*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
 + b) - 3*a + b)^3)*sgn(cos(f*x + e))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(
a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4219, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x,$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+}}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e}{f}$$

Mathematica [A]
time = 0.16, size = 98, normalized size = 1.48

$$\frac{\sqrt{2}\cos(e+fx)\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{b+a\cos^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
[Out] (Sqrt[2]*Cos[e + f*x]*(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]]
- Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]])
```

Maple [A]
time = 0.03, size = 94, normalized size = 1.42

method	result

derivativedivides	$\frac{-\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{a\sec(fx+e)}+\frac{2b\left(\frac{\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sec(fx+e)\sqrt{b}+\sqrt{a+b\left(\sec^2(fx-\right.}\right.}{2\sqrt{b}}\right.}{a}}{f}$
default	$\frac{-\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{a\sec(fx+e)}+\frac{2b\left(\frac{\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sec(fx+e)\sqrt{b}+\sqrt{a+b\left(\sec^2(fx-\right.}\right.}{2\sqrt{b}}\right.}{a}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+2*b/a*(1/2*sec(f*x+e)*(a+b*se
c(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1
/2))))
```

Maxima [A]
time = 0.49, size = 94, normalized size = 1.42

$$-\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x
+ e) + sqrt(b))))/f
```

Fricas [A]
time = 2.13, size = 196, normalized size = 2.97

$$\left[-\frac{2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b)
*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,\sin\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A]
time = 0.44, size = 55, normalized size = 0.83

$$-\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \sqrt{a\cos\left(fx+e\right)^{2}+b}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*cos(f*x +
 e)^2 + b))*sgn(cos(f*x + e))/f
```

Mupad [B]
time = 6.77, size = 87, normalized size = 1.32

$$-\frac{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{f}-\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{a}\,\cos\left(e+f\,x\right)}\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,1{}\mathrm{i}}{\sqrt{a}\,f\,\sqrt{\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] - (cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2))/f - (b^(1/2)*asin((b^(1/2)*1i
)/(a^(1/2)*cos(e + f*x)))*(a + b/cos(e + f*x)^2)^(1/2)*1i)/(a^(1/2)*f*(b/(a
*cos(e + f*x)^2) + 1)^(1/2))
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-arctanh(sec(
f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4219, 399, 223, 212, 385, 213}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{}{(-1+x}\right.}{}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(a+b)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.}{f}$$

Mathematica [A]
time = 0.15, size = 119, normalized size = 1.45

$$\frac{\sqrt{2}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
+ b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(70) = 140$.
time = 0.19, size = 688, normalized size = 8.39

method	result
default	$-\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\,\sqrt{4}\,\cos(fx+e)\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\cos(fx+e)\sqrt{a+b}+\sqrt{\frac{b+a(\cos^2(fx+e}{(1+\cos(fx+e))}}\right.}{\sin(fx+e)^2\sqrt{a+b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f*x+e)*(ln(-2*(c
os(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^
(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+
b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(3/2)-2*b^(3/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b
)^(1/2))+2*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1
/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(
1/2))*(a+b)^(1/2)*b-ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^(1/2)-ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e
)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-
1))*b^(1/2)*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(cos(f*x+e)-1))*b^(3/2))*(cos(f*x+e)-1)/sin(f*x+e)^2/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/b^(1/2)/(a+b)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)

Fricas [A]
time = 1.89, size = 528, normalized size = 6.44

$$\left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+a+2b\right)}{\cos\left(fx+e\right)^{2}-1}\right)+\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)}{2f},\ \frac{2\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a+b}\right)+\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)}{2f},\ -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{b}\right)-\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+a+2b\right)}{\cos\left(fx+e\right)^{2}-1}\right)}{2f},\ \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a+b}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{b}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/f, (sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^{2}\left(e+fx\right)}\,\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(70) = 140.
time = 0.87, size = 405, normalized size = 4.94

$$-\frac{\left(\frac{4b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}-\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}+\sqrt{a+b}\right|\right)-\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}-\sqrt{a+b}\right|\right)-\sqrt{a+b}\log\left(\left|\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

```
[Out] -1/2*(4*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + sqrt(
a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) - sqrt(a + b))) - sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*
(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}\,f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{}}{2f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+2*b)*
arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)-1/2*
cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4219, 478, 537, 223, 212, 385, 213}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*Sqrt[a + b]*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2
*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{\quad}{(-1+x}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\left(\int \frac{\quad}{\sqrt{a}}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.}{2\sqrt{a+b}}$$

Mathematica [**A**]
time = 0.47, size = 163, normalized size = 1.31

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)-(a+b)\csc^2(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\right)}{\sqrt{2}\,(a+b)f\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
+ b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b - a*S
in[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. $2(106) = 212$.
time = 0.14, size = 3015, normalized size = 24.31

method	result	size
default	Expression too large to display	3015

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(cos(f*x+e)-1)*(-2*cos(f*x+e)*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-4*cos(f*x+e)^2*(a+b)^(3/2)*4^(1/2)*arct
anh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+
e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b+cos(f
*x+e)^2*b^(1/2)*4^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3-2*cos(f*x+
e)*b^(3/2)*(a+b)^(3/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
2*(a+b)^(3/2)*4^(1/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
+4*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos
(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*4^(1/2))*a*b-4*b^(7/2)*4^(1/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2)
)-2*b^(7/2)*4^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/(cos(f*x+e)-1))+2*b^(7/2)*4^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(
a+b)^(1/2))+4*cos(f*x+e)^2*b^(7/2)*4^(1/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1
/2))+2*cos(f*x+e)^2*b^(7/2)*4^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))-2*cos(f*x+e)^2*b^(7/2)*4^(1
/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-c
os(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))-5*b^(5/2)*4^(1/2)*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a-4*
b^(3/2)*4^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/(cos(f*x+e)-1))*a^2+4*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*(cos(f*x+
e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2-b^(1/2)*4^(1/2)*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x
+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e
)-1))*a^3-b^(1/2)*4^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3-8*cos(f*
x+e)^2*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-16*c
os(f*x+e)*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)+2
*(a+b)^(3/2)*4^(1/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-8*
4^(1/2)*b^(5/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
```

```
a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+3*4^(1/2)*b^(5/2)*ln
(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-4*4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)-1)
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e
)^2/(a+b)^(1/2))*a^2-8*(a+b)^(3/2)*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(3/2)+4*cos(f*x+e)^2*b^(1/2)*(a+b)^(3/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a+8*cos(f*x+e)^2*b^(5/2)*4^(1/2)*ln(-4*(cos(f*x+e)-
1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x
+e)^2/(a+b)^(1/2))*a+5*cos(f*x+e)^2*b^(5/2)*4^(1/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a-3*cos(f*x
+e)^2*b^(5/2)*4^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+4*cos(f*x+e)^2
*b^(3/2)*4^(1/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+4*cos(f*x+e)^2*b^
(3/2)*4^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [A]
time = 2.31, size = 923, normalized size = 7.44

$$\left[\frac{2(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+((a+2b)\cos(fx+e)^2-a-2b)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)+2((a+b)\cos(fx+e)^2-a-b)\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{4((a+b)f\cos(fx+e)^2-(a+b)f)}, \frac{((a+2b)\cos(fx+e)^2-a-2b)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)+(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+((a+b)\cos(fx+e)^2-a-b)\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2((a+b)f\cos(fx+e)^2-(a+b)f)}, -\frac{4((a+b)\cos(fx+e)^2-a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-2(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-((a+2b)\cos(fx+e)^2-a-2b)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{4((a+b)f\cos(fx+e)^2-(a+b)f)}, \frac{((a+2b)\cos(fx+e)^2-a-2b)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)-2((a+b)\cos(fx+e)^2-a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{2((a+b)f\cos(fx+e)^2-(a+b)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 -
2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
+ 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*l
```

```
og((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*
f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a
+ b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a + b)*cos(f*x + e)^2 - a - b)
*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/b) - 2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) - ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x +
e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 2*((a
 + b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(106) = 212.
time = 1.00, size = 578, normalized size = 4.66

$$\frac{\left(\frac{4\,(a+2\,b)\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{16\,b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{2\,(a+2\,b)\log\left(\left|-\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{\sqrt{a+b}}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\frac{2\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)(a-b)-(a+b)^{\frac{3}{2}}\right)}{\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)^2-a-b}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(a + 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) - 16*b*arc
tan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + 2*(a + 2*b)*log(ab
```

```
s(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b) - (a + b)^(3/
2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=183

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f}-\frac{(3a+4b)\cot\ldots$$

```
[Out] -1/8*(3*a^2+12*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*
b^(1/2)/f-1/8*(3*a+4*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b
)/f-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4219, 478, 592, 537, 223, 212, 385, 213}

$$-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}-\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]])/(8*(a + b)^(3/2)*f) - ((3*a + 4*b)*Cot[e + f*x]*Csc[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqr
t[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{}{(-1}\right.}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(3a^2+12ab+8b^2)\tanh}{8(a}$$

Mathematica [A]
time = 1.49, size = 198, normalized size = 1.08

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(8\sqrt{b}(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)-(a+b)\csc^2(e+fx)(3a+4b+2(a+b)\csc^2(e+fx))\sqrt{a+b-a\sin^2(e+fx)}\right)}{4\sqrt{2}(a+b)^2f\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[
a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*A
rcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2
*(3*a + 4*b + 2*(a + b)*Csc[e + f*x]^2)*Sqrt[a + b - a*Sin[e + f*x]^2]))/(4
*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9757 vs. 2(161) = 322.
time = 0.16, size = 9758, normalized size = 53.32

method	result	size
default	Expression too large to display	9758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(171) = 342$.
time = 2.63, size = 1548, normalized size = 8.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2
)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e
)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4
- 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*
cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3
 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f
*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b +
 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/(a + b)) + 4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*cos
(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5
*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(16*((a^2 + 2*a*b + b^2)*cos(f*x
```

```
 + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*
cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^
2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^
3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b
+ 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/(a + b)) - 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 -
2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(s
qrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a
^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*
x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2(e + fx)}\,\csc^5(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(161) = 322.
time = 1.35, size = 867, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2*
e)^2 + (9*a + 11*b)/(a + b)) + 128*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))
/sqrt(-b))/sqrt(-b) - 8*(3*a^2 + 12*a*b + 8*b^2)*arctan(-(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(
```

```
-a - b))/((a + b)*sqrt(-a - b)) - 4*(3*a^2 + 12*a*b + 8*b^2)*log(abs(-(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3
*(2*a^2 - 3*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))^2*(3*a^2 + 10*a*b + 7*b^2)*sqrt(a + b) - 2*
(3*a^3 + 3*a^2*b - 2*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(a + b)))*sgn(cos(f*x +
e))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^6(e+fx)\,dx$

Optimal. Leaf size=240

$$\frac{(5a^3-15a^2b-5ab^2-b^3)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] 1/16*(5*a^3-15*a^2*b-5*a*b^2-b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+
e)^2)^(1/2))/a^(5/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*b^(1/2)/f-1/16*(a-b)*(5*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^2/f-1/24*(5*a-b)*cos(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.25, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4217, 478, 592, 537, 223, 212, 385, 209}

$$-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16a^2f}+\frac{(5a^3-15a^2b-5ab^2-b^3)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^5(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}-\frac{(5a-b)\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f - ((a - b)*(5*a + b)*Cos[e + f*x]*
Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a^2*f) - ((5*a - b)*Cos[e
+ f*x]*Sin[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) - (Cos[e + f
*x]*Sin[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} -$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}}{16a^2f}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}}{16a^2f}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}}{16a^2f}$$

$$= -\frac{(16a^2b-(5a+b)(a^2-b^2))\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{16a^{5/2}f}$$

Mathematica [F]
time = 9.66, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^6(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.47, size = 2665, normalized size = 11.10

method	result	size
default	Expression too large to display	2665

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/48/f*(-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)+10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-40*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+40*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+33*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-14*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+90*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/

```
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a^2*b*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a*b^2*sin(f*x+e)-96*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^3+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-26*cos
(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^4*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^3-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*c
os(f*x+e)^2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Fricas [A]
time = 6.79, size = 1805, normalized size = 7.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
 e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x
```

```
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 - 2*
(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), 1/384
*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
 b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f
*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^3*f), 1/192*(48*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) - 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8
*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*co
s(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos
(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))
) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 -
 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*f), 1/192*(96*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(5*a^3 - 15*a^2*b -
 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^
3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^4(e+fx)\,dx$

Optimal. Leaf size=181

$$\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(3a-}{}$$

```
[Out] 1/8*(3*a^2-6*a*b-b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
-1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cos(f
*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4217, 478, 592, 537, 223, 212, 385, 209}

$$\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}-\frac{(3a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/f - ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b +
b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \text{co}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \text{co}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \text{co}$$

$$= \frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\,\text{tanh}}{}$$

Mathematica [F]
time = 5.99, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^4(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.21, size = 1939, normalized size = 10.71

method	result	size
default	Expression too large to display	1939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-2*cos(f*x
+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a*b*sin(f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
b^2*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-12*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(
f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+16*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-5*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b-5*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+5*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*si
n(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(b+a*cos(f*
```

```
x+e)^2)/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [A]
time = 3.28, size = 1651, normalized size = 9.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)
, 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e
)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3
- (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(a^2*f), 1/32*(8*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x
+ e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
 + 8*b^2)/cos(f*x + e)^4) - (3*a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
```

```
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
4*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), 1/32*(16*a^2*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - (3*
a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)
*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
- 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*a^2*cos(f*x + e)^3 - (5*a^
2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\sin^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^4\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\,\sin^2(e+fx)\,dx$

Optimal. Leaf size=123

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}\,f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sin(e$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+a
rctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cos(f*x
+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 478, 537, 223, 212, 385, 209}

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}\,f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Sub}\cdots}{\cdots}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}\,f} + \frac{\sqrt{b}\,\tanh^{-1}\Big(\frac{\cdots}{\sqrt{a\cdots}}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.32, size = 432, normalized size = 3.51

$$\frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(i\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2afx-2bfx-i(a-b)\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+i(a-b)\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-4\sqrt{a}\sqrt{b}\log\left(\frac{\left(-\sqrt{b}\left(-1+e^{2i(e+fx)}\right)+i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{2b\left(1+e^{2i(e+fx)}\right)}\right)\right)}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{4\sqrt{2}\,f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*f*
x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E^((2
*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sqrt[b]*
(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x))))]))/(Sqrt[a]*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e +
f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 1290, normalized size = 10.49

method	result	size
default	Expression too large to display	1290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)
*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-4*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin
(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f
*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*cos(f*x+e)*sin(f*x+e)*((
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(112) = 224$.
time = 3.37, size = 1499, normalized size = 12.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a - b)*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*f), -1/8*(4*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b
)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 +
 (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*
x + e)^2)*sin(f*x + e))) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/(a*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*a*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
```

```
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\,\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+b\ldots}}\right.}{\ldots}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \ldots\right.}{\ldots}$$

$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+f\ldots}{\sqrt{a+b+b\tan^2(\ldots}}\right.}{f}$$

Mathematica [F]
time = 1.36, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.26, size = 588, normalized size = 7.44

method	result
default	$-\frac{\sqrt{2}\left(\text{EllipticF}\left(\frac{(\cos(fx+e)-1)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin(fx+e)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^2+6ab-b^2}{(a+b)^2}}\right)a+\text{EllipticF}\left(\frac{(\cos(fx+e)-\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a+EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*Ellipti
cPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)*sin(f*x+e)^2*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)/(cos(f*x
+e)-1)/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(71) = 142$.

time = 2.45, size = 1293, normalized size = 16.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^{2}\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4217, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx}}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 61, normalized size = 0.90

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b -
 a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 1004, normalized size = 14.76

method	result
default	$-\frac{\sqrt{\frac{b+a\left(\cos^{2}(fx+e)\right)}{\cos(fx+e)^{2}}}\,\cos(fx+e)\left(-\cos(fx+e)\sin(fx+e)\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(-cos(f*x+e)*sin(f*
x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b+2*cos(f*x+e)*sin(f*x+
e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*b-2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*
x+e)+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x+e)^2
)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.78

$$\frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - sqrt(b*tan(f*x + e)^2 +
a + b)/tan(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(64) = 128$.
time = 1.46, size = 330, normalized size = 4.85

$$\left[\frac{\sqrt{b}\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)\sin\left(fx+e\right)-4\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{4\,f\sin\left(fx+e\right)}, \frac{\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin\left(fx+e\right)-2\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{2\,f\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f
*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*si
n(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f
*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f
*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cot^3(e+fx)\,(a+b+b\tan^2(e+fx))^{3/2}}{3(a+b)f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/(
a+b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 462, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\,(a+b\tan^2(e+fx)+b)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b
+ b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+b+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+}}{x^2}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\,(a+b+}{3(a+}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\,(a+b+}{3(a+}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.81, size = 285, normalized size = 2.71

$$-\frac{\sqrt{2}\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{4b\,{}_2F_1\left(2,2;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\sec^2(e+fx)\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}(a+b-a\sin^2(e+fx))^2\tan^2(e+fx)}{(a+b)^2}+(a+b+2a\sin^2(e+fx))\left(\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}+\text{ArcSin}\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\right)}{3f\sqrt{a+2b+a\cos(2e+2fx)}\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}\sqrt{a+b-a\sin^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (
a*Sin[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a +
b - a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e + f*
x]^2)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e + f*x]
^2)/(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))])))/(f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e
+ f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 3847, normalized size = 36.64

method	result	size
default	Expression too large to display	3847

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a*b-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*b^2+6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f
*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+
```

e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*b^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*sin(f*x+e)*cos(f*x+e)^2*a*b-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b
+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+3*sin(f*x+e)*cos(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+3*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)*b^2-6*sin(f*x+e)*cos(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a*b-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+co
s(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((
cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)*b^2+2*cos(f*x+e)^4*((2

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^4*a*b+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^...
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.83

$$\frac{3\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-\frac{3\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)}-\frac{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 3*sqrt(b*tan(f*x +
 e)^2 + a + b)/tan(f*x + e) - (b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan
(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(99) = 198.
time = 3.06, size = 466, normalized size = 4.44

$$\left[\frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\left((2a+3b)\cos(fx+e)^3-(3a+4b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left((a+b)f\cos(fx+e)^2-(a+b)f\right)\sin(fx+e)},\ \frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)-2\left((2a+3b)\cos(fx+e)^3-(3a+4b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6\left((a+b)f\cos(fx+e)^2-(a+b)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*
cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e
)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(
f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x + e)
^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec^2(e+fx)}\,\csc^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{{\sin\left(e+f\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{2(5a+4b)\cot^3(e+fx)}{15(a}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a+4*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)
^2)^(3/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {4217, 473, 462, 283, 223, 212}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{15f(a+b)^2}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (2*(5*a + 4*b)*Cot[e +
f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(15*(a + b)^2*f) - (Cot[e + f*x]^5
*(a + b + b*Tan[e + f*x]^2)^(3/2))/(5*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2\sqrt{a+b+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \sqrt{a+b+\cdots}\right.}{\cdots}$$

$$= -\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2 f} - \frac{\cot^5(e+\cdots}{\cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx\cdots}{15(\cdots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx\cdots}{15(\cdots}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\cdots}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 9.73, size = 941, normalized size = 6.32

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Csc[e + f*x]^5*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Sin
[e + f*x]^2)/(a + b))*(-3*(a + b)*Cos[e + f*x]^2 - 4*a*Cos[e + f*x]^2*Sin[e
 + f*x]^2 - 16*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))
]*Sin[e + f*x]^2 + 8*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^2 - (8*a^2*Cos[e + f*x]^2*Sin[e + f*x]^4)/(a
 + b) - (8*a*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sin[e + f*x]^4)/(a + b) - (16*a*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -(
(b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4)/(a + b) + (24*a^2*b*Hypergeome
tric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6)/(a + b)^2
 + (8*a^2*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a
```

```
+ b))]*Sin[e + f*x]^6)/(a + b)^2 - (16*b^2*Hypergeometric2F1[2, 2, 3/2, -((
b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(a + b) + (8*b^2
*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[
e + f*x]^2*Tan[e + f*x]^2)/(a + b) - (8*a*b^2*Hypergeometric2F1[2, 2, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 - (
16*a*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b
))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 + (24*a^2*b^2*Hypergeometric2F
1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/
(a + b)^3 + (8*a^2*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/(a + b)^3 - (4*a*ArcSin[Sq
rt[-((b*Tan[e + f*x]^2)/(a + b))]]*Cos[e + f*x]^2*Sin[e + f*x]^2*Sqrt[-((b*
Tan[e + f*x]^2)/(a + b))])/Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + (3*b*ArcS
in[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^2)/Sqrt[-((b*Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + (8*a^2*b*Arc
Sin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^6)/((a + b)^2*Sqrt[-(
(b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))
/(15*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 8587, normalized size = 57.63

method	result	size
default	Expression too large to display	8587

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 152, normalized size = 1.02

$$\frac{15\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-\frac{15\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)}-\frac{10\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3}+\frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)^2\tan(fx+e)^3}-\frac{3\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 15*sqrt(b*tan(f*
x + e)^2 + a + b)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a +
b)*tan(f*x + e)^3) + 2*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)^2*tan(f*
x + e)^3) - 3*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(141) = 282$.

time = 4.47, size = 692, normalized size = 4.64

$$\left[\frac{15\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\left((8a^2+25ab+15b^2)\cos(fx+e)^5-(20a^2+59ab+35b^2)\cos(fx+e)^3+(15a^2+40ab+23b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{60\left((a^2+2ab+b^2)f\cos(fx+e)^4-2(a^2+2ab+b^2)f\cos(fx+e)^2+(a^2+2ab+b^2)f\right)\sin(fx+e)}, \frac{15\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)-2\left((8a^2+25ab+15b^2)\cos(fx+e)^5-(20a^2+59ab+35b^2)\cos(fx+e)^3+(15a^2+40ab+23b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{30\left((a^2+2ab+b^2)f\cos(fx+e)^4-2(a^2+2ab+b^2)f\cos(fx+e)^2+(a^2+2ab+b^2)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x
+ e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*
b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*
a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2
 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*ar
ctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (20*a^2 + 59*a
*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x +
e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(
f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

3.80 $\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=196

$$\frac{(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{(3a-4b)\cos(}{}$$

```
[Out] -1/3*(3*a-4*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+2/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(5/2)/a/f+1/2
*(3*a-4*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*(3*a-4*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4219, 473, 464, 283, 201, 223, 212}

$$\frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}+\frac{\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 4*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 4*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (2*C
os[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f) - (Cos[e + f*x]^5*(a +
b*Sec[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}
$$

$$
= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(-10a+5ax^2\right)(a+}{x^4}\right.}{5}
$$

$$
= \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2\right.}{5af}
$$

$$
= -\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)}{}
$$

$$
= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+}{}
$$

$$
= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+}{}
$$

$$
= \frac{(3a-4b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec}{}
$$

Mathematica [A]
time = 1.45, size = 188, normalized size = 0.96

$$
\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{6b\left(a+b-a\sin^2(e+fx)\right)^{5/2}}{a}+15\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{5/2}-5(3a-4b)\left(-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{a+b-a\sin^2(e+fx)}\left(a+4b-a\sin^2(e+fx)\right)\right)\right)}{15bf\left(a+2b+a\cos(2(e+fx))\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin[
e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2) -
 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]
+ Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2536 vs. $2(172) = 344$.
time = 0.34, size = 2537, normalized size = 12.94

method	result	size
default	Expression too large to display	2537

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/f*(cos(f*x+e)-1)^3*(60*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2
))*b^(13/2)*a-225*cos(f*x+e)^2*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(11/2
)*a^2+225*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(11/2)*a^2-31
5*cos(f*x+e)^2*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(9/2)*a^3+315*cos(f*x
+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(9/2)*a^3-195*cos(f*x+e)^2*ln(
-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^(7/2)*a^4+195*cos(f*x+e)^2*ln(-2*(cos(f
*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/s
in(f*x+e)^2/(a+b)^(1/2))*b^(7/2)*a^4-45*cos(f*x+e)^2*ln(-4*(cos(f*x+e)-1)*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^
2/(a+b)^(1/2))*b^(5/2)*a^5+45*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1
/2))*b^(5/2)*a^5+6*cos(f*x+e)^7*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(7/2)*b^(3/2)*a^2+6*cos(f*x+e)^7*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(7/2)*b^(1/2)*a^3+6*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(3/2)*a^2+12*cos(f*x+e)^5*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(5/2)*a+6*cos(f*x+e)^6*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(1/2)*a^3-8*cos(f*x+e)^5*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(3/2)*a^2+12*cos(f*x+
e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(5/2)*a-20*c
os(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(1/2)
*a^3-8*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)
*b^(3/2)*a^2-74*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
```

```
+b)^(7/2)*b^(5/2)*a-20*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(7/2)*b^(1/2)*a^3-50*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(7/2)*b^(3/2)*a^2-74*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(5/2)*a+30*cos(f*x+e)^3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(1/2)*a^3-50*cos(f*x+e)^2*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(3/2)*a^2-15*cos(f*x+e
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(5/2)*a+30*cos(
f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(1/2)*a^
3-15*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^(
3/2)*a^2-45*cos(f*x+e)^2*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*c
os(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*4^(1/2))*(a+b)^(7/2)*a^3*b+15*cos(f*x+e)^2*arctanh(1/8*(cos(f*x+
e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*(a+b)^(7/2)*a^2*b^2+60*cos
(f*x+e)^2*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/
2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1
/2))*(a+b)^(7/2)*a*b^3+6*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(7/2)*cos(f*x+e)^3+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(7/2)*b^(7/2)-15*(a+b)^(7/2)*b^(5/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a-15*(a+b)^(7/2)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^2-60*cos(f*x+e)^2*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b
^(13/2)*a)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/a/b^(1/2)/(a+b)^(9/2)
```

Maxima [A]
time = 0.48, size = 297, normalized size = 1.52

$$-\frac{\frac{12\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a}-40\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3+60\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,a\cos\left(fx+e\right)-120\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)-\frac{30\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+45\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+\sqrt{b}}\right)-60\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/a - 40*(a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e
) - 120*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) - 30*sqrt(a + b/cos(f*x +
 e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 45*a*
sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b
/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 60*b^(3/2)*log((sqrt(a + b/cos(
f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e
) + sqrt(b))))/f
```

Fricas [A]

time = 3.25, size = 373, normalized size = 1.90

$$\left[-\frac{15\,(3\,a^2-4\,ab)\sqrt{b}\,\cos(fx+e)\log\left(\frac{a\cos(fx+e)^2-2\,\sqrt{b}\,\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\,(6\,a^2\cos(fx+e)^6-4\,(5\,a^2-3\,ab)\cos(fx+e)^4+2\,(15\,a^2-40\,ab+3\,b^2)\cos(fx+e)^2-15\,ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{60\,af\cos(fx+e)},\ -\frac{15\,(3\,a^2-4\,ab)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\cos(fx+e)}{b}\right)\cos(fx+e)+(6\,a^2\cos(fx+e)^6-4\,(5\,a^2-3\,ab)\cos(fx+e)^4+2\,(15\,a^2-40\,ab+3\,b^2)\cos(fx+e)^2-15\,ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{30\,af\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(
f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4 +
2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*sqrt(-
b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
/b)*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. $2(172) = 344$.
time = 2.10, size = 1952, normalized size = 9.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/15*(15*(3*a*b - 4*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))*
sgn(cos(f*x + e))/sqrt(-b) + 30*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos(f*x +
 e)) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
```

1/2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + (3*a^
2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b - a*b^2 + 2*
b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) + a - 3*b)^2 - 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b - b^2)*sgn(cos(f*x + e)) +
15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))^8*(22*a*b - 7*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) - 20*(16*
a^3 - 6*a^2*b - 51*a*b^2 + 15*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) + 20*(3
2*a^3 - 150*a^2*b + 105*a*b^2 - 21*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(
f*x + e)) + 2*(416*a^4 - 30*a^3*b - 2685*a^2*b^2 + 840*a*b^3 - 105*b^4)*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b))^5*sgn(cos(f*x + e)) - 10*(256*a^4 - 942*a^3*b + 567*a^2*b^2 - 21*
b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(16*a^5 - 178*a^4*b
+ 531*a^3*b^2 - 63*a^2*b^3 - 63*a*b^4 + 21*b^5)*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x
+ e)) + 20*(160*a^5 - 590*a^4*b + 81*a^3*b^2 + 171*a^2*b^3 - 69*a*b^4 + 15*
b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 5*(576*a^6 - 2230*a^5*b
+ 739*a^4*b^2 + 632*a^3*b^3 - 486*a^2*b^4 + 102*a*b^5 - 21*b^6)*(sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
))*sgn(cos(f*x + e)) + (768*a^6 - 3654*a^5*b + 4103*a^4*b^2 - 2200*a^3*b^3
+ 690*a^2*b^4 - 90*a*b^5 + 15*b^6)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a
+ b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a +
b))^2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^5)/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+fx)^5 \left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.81 $\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{(3a-2b)\cos(}{}$$

```
[Out] -1/3*(3*a-2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+1/3*cos(f*x+e)^3*(a+
b*sec(f*x+e)^2)^(5/2)/a/f+1/2*(3*a-2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec
(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-2*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(
1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 464, 283, 201, 223, 212}

$$\frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 2*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 2*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a-2b)\text{Subst}\left(\int \frac{(a+bx^2}{x^2}\right.}{3a}$$

$$= -\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\,(}{}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+}{}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+}{}$$

$$= \frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec}{}$$

Mathematica [A]
time = 0.80, size = 164, normalized size = 1.01

$$\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(3\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{5/2}-(3a-2b)\left(-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{a+b-a\sin^2(e+fx)}\left(a+4b-a\sin^2(e+fx)\right)\right)\right)}{3bf\left(a+2b+a\cos(2(e+fx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b -
 a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*S
in[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(142) = 284$.
time = 0.16, size = 1913, normalized size = 11.81

method	result	size
default	Expression too large to display	1913

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)

[Out] -1/12/f*(cos(f*x+e)-1)^3*(-6*cos(f*x+e)^2*b^(13/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))+6*cos(f*x+e)^2*b^(13/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*(a+b)^(7/2)+2*cos(f*x+e)^5*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+2*cos(f*x+e)^5*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^4*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+2*cos(f*x+e)^4*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^3*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-6*cos(f*x+e)^3*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*cos(f*x+e)^2*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-6*cos(f*x+e)^2*b^(7/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+6*cos(f*x+e)^2*b^(7/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+8*cos(f*x+e)^3*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)+8*cos(f*x+e)^2*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)-6*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^3-18*cos(f*x+e)^2*b^(11/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+18*cos(f*x+e)^2*b^(11/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-18*cos(f*x+e)^2*b^(9/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+18*cos(f*x+e)^2*b^(9/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+3*cos(f*x+e)*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)+3*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a-6*cos(f*x+e)^2*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+3*cos(f*x+e)*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+9*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^2*b+3*cos(f*x+e)^2*(a+b)^(7/2)*arcta

```
nh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e
)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b^2)*cos
(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/(a+b)^(9/2)/b^(1/2)
```

Maxima [A]
time = 0.48, size = 268, normalized size = 1.65

$$\frac{4\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-12\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,a\cos\left(fx+e\right)+12\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)+\frac{6\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}-9\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+\sqrt{b}}\right)+6\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a + b/cos(f*x
 + e)^2)*a*cos(f*x + e) + 12*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) + 6*
sqrt(a + b/cos(f*x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x
 + e)^2 - b) - 9*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*b^(3/2)*lo
g((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x +
 e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Fricas [A]
time = 3.26, size = 298, normalized size = 1.84

$$\left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2-2\,\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(2\,a\cos\left(fx+e\right)^4-2\,(3\,a-4\,b)\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{12\,f\cos\left(fx+e\right)},\ -\frac{3\,(3\,a-2\,b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)}{b}\right)\cos\left(fx+e\right)-\left(2\,a\cos\left(fx+e\right)^4-2\,(3\,a-4\,b)\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{6\,f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x +
 e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a - 2*b
)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b)*cos(f*x + e) - (2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)
^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(142) = 284$.
time = 1.42, size = 1388, normalized size = 8.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(3*a*b - 2*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))*sg
n(cos(f*x + e))/sqrt(-b) + 6*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos(f*x + e)
) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + (3*a^2*b
 - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b - a*b^2 + 2*b^3
)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a +
b) + a - 3*b)^2 - 8*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*a*b*sgn(cos(f*x + e)) - 3*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^4*(2*a^2 - 3*a*b)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(4*a^3 - 9*a^2*b + 3*
a*b^2)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^3 - 3*a^2*b - a*b^2)*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(8*a^4 - 13*a^3*b - 2*a^2*b^2 +
 3*a*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*
x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (10*a^4 - 23*a^3*b + 12*a^2*b^2
- 3*a*b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e
```

```
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) - 3*a + b)^3)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+fx)^3\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.82 $\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\,(a+b\sec^2(e+}{f}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*a*arctanh(sec(f*x+e)*b^(1/2)/(a+
b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/
f
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4219, 283, 201, 223, 212}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{f}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*f) + (3*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(
a + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\text{Subst}\left(\int \sqrt{a+bx^2}\, d\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a+b\sec^2(e\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a+b\sec^2(e\right.}{f}$$

$$= \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a+b}}{2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.68, size = 73, normalized size = 0.73

$$-\frac{a\cos(e+fx)(a+2b+a\cos(2(e+fx)))^2\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1+\frac{a\cos^2(e+fx)}{b}\right)\sqrt{a+b\sec^2(e+fx)}}{20b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] -1/20*(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2,
 5/2, 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f)
```

Maple [A]
time = 0.03, size = 120, normalized size = 1.20

method	result
derivativedivides	$\frac{-\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{5}{2}}}{a\sec(fx+e)}+\frac{4b\left(\frac{\sec(fx+e)\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sec(fx\right.}{}\right.}{4}\right.}{a}}{f}$
default	$\frac{-\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{5}{2}}}{a\sec(fx+e)}+\frac{4b\left(\frac{\sec(fx+e)\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sec(fx\right.}{}\right.}{4}\right.}{a}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+4*b/a*(1/4*sec(f*x+e)*(a+b*se
c(f*x+e)^2)^(3/2)+3/4*a*(1/2*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)+1/2*a/b^(1
/2)*ln(sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)))))
```

Maxima [A]
time = 0.48, size = 152, normalized size = 1.52

$$-\frac{4\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos(fx+e)-\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+3\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e) - 2*sqrt(a + b/cos(f*x +
e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 3*a*sq
rt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/c
os(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Fricas [A]
time = 3.06, size = 249, normalized size = 2.49

$$\left[\frac{3\,a\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\,\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\,\left(2\,a\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,f\cos\left(fx+e\right)},\ -\frac{3\,a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)}{b}\right)\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\,f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(
2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos
(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + (2*a*cos(f*x + e)^2 - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

Giac [A]
time = 0.47, size = 87, normalized size = 0.87

$$-\frac{\left(\frac{3\,b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\,\sqrt{a\cos\left(fx+e\right)^{2}+b}-\frac{\sqrt{a\cos\left(fx+e\right)^{2}+b}\,b}{a\cos\left(fx+e\right)^{2}}\right)a\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*c
os(f*x + e)^2 + b) - sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2))*a*sgn
(cos(f*x + e))/f
```

Mupad [B]
time = 6.11, size = 61, normalized size = 0.61

$$-\frac{\cos\left(e+f\,x\right)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\cos(e+f\,x)^2}\right)}{f\,\left(\frac{b}{a\cos(e+f\,x)^2}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] -(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -b
/(a*cos(e + f*x)^2)))/(f*(b/(a*cos(e + f*x)^2) + 1)^(3/2))
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{}}{}$$

```
[Out] -(a+b)^(3/2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f+1/2
*(3*a+2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1
/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4219, 427, 537, 223, 212, 385, 213}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/f + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a+b)+b(3a+2b)x^2}{(-1+x^2)\sqrt{a+bx^2}}\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{}}\right.}{}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1-(-a-b}\right.}{}$$

$$= \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\tanh}{}$$

Mathematica [A]
time = 0.64, size = 171, normalized size = 1.40

$$\frac{\left(2\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx)-4(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\cos^2(e+fx)+\sqrt{2}\,b\sqrt{a+2b+a\cos(2(e+fx))}\right)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\sqrt{2}\,f\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos
[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Sec[e
 + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(104) = 208$.
time = 0.14, size = 2563, normalized size = 21.01

method	result	size
default	Expression too large to display	2563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)*cos(f*x+e)*(cos(f*x+
e)-1)^3*(cos(f*x+e)^2*b^(13/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))-2*cos(f
*x+e)^2*b^(13/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(5/2)*(a+b)^(7/2)-20*cos(f*x+e)^2*b^(7/2)*ln(-4*(c
os(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^
(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+
b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+2*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(co
s(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^3+6*cos(f*x+e)^2*b
^(11/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-12*cos(f*x+e)^2*b^(11/2)*ln(
-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+9*cos(f*x+e)^2*b^(9/2)*ln(-2*(cos(f*x+e
)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f
*x+e)^2/(a+b)^(1/2))*a^2-24*cos(f*x+e)^2*b^(9/2)*ln(-4*(cos(f*x+e)-1)*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a
+b)^(1/2))*a^2+cos(f*x+e)*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(7/2)+b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7
/2)*a-cos(f*x+e)^2*b^(13/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))-6*cos(f*x+e)^2*b^(3/2)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*
a^5-cos(f*x+e)^2*b^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6-cos(f*x+e
)^2*b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+b)/(cos(f*x+e)-1))*a^6-6*cos(f*x+e)^2*b^(11/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a-15*cos(f*
x+e)^2*b^(9/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)+b)/(cos(f*x+e)-1))*a^2-20*cos(f*x+e)^2*b^(7/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3-6*co
s(f*x+e)^2*b^(5/2)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))

```
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-9*cos(f*x+e)^2*
b^(5/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-15*cos(f*x+e)^2*b^(5/2)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos
(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f
*x+e)-1))*a^4-6*cos(f*x+e)^2*b^(3/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a
^5+cos(f*x+e)*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a+3*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1
/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*4^(1/2))*a^2*b+5*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*(co
s(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a*b^2)/sin(f*x+e)^6/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/b^(1/2)/(a+b)^(9/2)
```

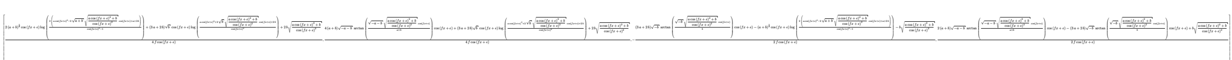
Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Fricas [A]
time = 5.06, size = 767, normalized size = 6.29



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) + (3*a + 2*b)*s
qrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt((a*co
```

```
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*((3*a + 2*b)*sqrt
(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b)*cos(f*x + e) - (a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2
*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
 2*b)/(cos(f*x + e)^2 - 1)) - b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(f*cos(f*x + e)), 1/2*(2*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) - (
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e-$$

```
[Out] -1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+1/2*(3*a+4*b)*arctanh
(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+4*b)*arctanh
(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+b*sec(f*x+e
)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4219, 478, 542, 537, 223, 212, 385, 213}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqr
t[a + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2
])/f - (Cot[e + f*x]*Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{(-1+x^2)^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}+\frac{\text{Subst}\Big(\int \ldots}{}$$

$$=\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\,(\ldots}{2f}$$

$$=\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\,(\ldots}{2f}$$

$$=\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\,(\ldots}{2f}$$

$$=\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}\,(a+\ldots}{}$$

Mathematica [A]
time = 1.58, size = 202, normalized size = 1.25

$$-\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}\,(a+(a+2b)\cos(2(e+fx)))-\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\sin^2(2(e+fx))+\sqrt{a+b}\,(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sin^2(2(e+fx))\right)}{4\sqrt{2}\,f\sqrt{a+2b+a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt[
a + 2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*(3
*a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x)]^
2 + Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b
]]*Sin[2*(e + f*x)]^2))/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5177 vs. 2(139) = 278.
time = 0.16, size = 5178, normalized size = 32.16

method	result	size
default	Expression too large to display	5178

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [A]
time = 3.70, size = 1052, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x +
e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e
)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos(f*x +
e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + ((3*a + 4*b)*cos(f
*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*a + 4*b)*co
s(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 4*b)*cos(f*x + e
)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(
a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/
(cos(f*x + e)^2 - 1)) - 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/2*(((a +
4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a -
```

```
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - ((3
*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((a + 2*b)
*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*
x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.85 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$\frac{3\sqrt{b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{3(a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8\sqrt{a+b}\,f}+\frac{3(a+$$

```
[Out] -1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a+2*b)*arctanh
(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-3/8*(a^2+8*a*b+8*b^
2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)+3
/8*(a+4*b)*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+2*b)*csc(f*x+e)^2*s
ec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.23, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4219, 478, 591, 596, 537, 223, 212, 385, 213}

$$-\frac{3(a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}+\frac{3\sqrt{b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (3*(a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/S
qrt[a + b*Sec[e + f*x]^2]])/(8*Sqrt[a + b]*f) + (3*(a + 4*b)*Sec[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*f) - (3*(a + 2*b)*Csc[e + f*x]^2*Sec[e + f*x]
*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a + b*Se
c[e + f*x]^2)^(3/2))/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\Big(\int$$

$$= -\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \text{c}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e}{}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e}{}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e}{}$$

$$= \frac{3\sqrt{b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3(a^2+8ab}{}$$

Mathematica [A]

time = 3.70, size = 262, normalized size = 1.20

$$-\frac{(b+a\cos^2(e+fx))\left(-12b^{3/2}(a^2+3ab+2b^2)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx)+3b\sqrt{a+b}\,(a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\cos^2(e+fx)+\frac{b(a+b)\sqrt{a+2b+a\cos(2(e+fx))}\,(11a+4b+8(a+3b)\cos(2(e+fx))-3(a+4b)\cos(4(e+fx)))\csc^4(e+fx)}{8\sqrt{2}}\right)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\sqrt{2}\,b(a+b)f(a+2b+a\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sqr
t[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^2
+ 8*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos[e
+ f*x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b + 8*(a
 + 3*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^4)/(8
*Sqrt[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*(a + b)*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10198 vs. $2(190) = 380$.
time = 0.19, size = 10199, normalized size = 46.78

method	result	size
default	Expression too large to display	10199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [**A**]
time = 3.13, size = 1595, normalized size = 7.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos
(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)
*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
```

```
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 +
 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 +
 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos
(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*
((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + 6*((a^2
 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 +
 (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b
^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*co
s(f*x + e)), -1/16*(24*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
- 3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x
+ e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 5*a*b + 4*b^2)*cos(f*x
+ e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*
f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b
^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 12*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*
cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e)/b) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2
+ 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3
 + (a + b)*f*cos(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.86 $\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=298

$$\frac{(5a^3-45a^2b+15ab^2+b^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f}+\frac{(3a-5b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+}{\sqrt{a+b+b\tan^2}}\right.}{2f}$$

```
[Out] 1/16*(5*a^3-45*a^2*b+15*a*b^2+b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/a^(3/2)/f+1/2*(3*a-5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan
(f*x+e)^2)^(1/2))*b^(1/2)/f-1/16*(5*a^2-26*a*b+b^2)*(a+b+b*tan(f*x+e)^2)^(1
/2)*tan(f*x+e)/a/f+1/48*(5*a^2-40*a*b+3*b^2)*sin(f*x+e)^2*(a+b+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)/a/f+1/24*(5*a-3*b)*sin(f*x+e)^4*(a+b+b*tan(f*x+e)^2)^(
1/2)*tan(f*x+e)/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4217, 478, 591, 592, 596, 537, 223, 212, 385, 209}

$$-\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}+\frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48af}+\frac{(5a^3-45a^2b+15ab^2+b^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24f}+\frac{\sqrt{b}\,(3a-5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\sin^5(e+fx)\cos(e+fx)\,(a+b\tan^2(e+fx)+b)^{3/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] ((5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - ((5*a^2 - 26*a*
b + b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a^2 -
40*a*b + 3*b^2)*Sin[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(48*a*f) + ((5*a - 3*b)*Sin[e + f*x]^4*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(24*f) - (Cos[e + f*x]*Sin[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
```

```
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)^{3/2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} -$$

$$= \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e-}}{48af}$$

$$= -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a}{}$$

$$= -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a}{}$$

$$= -\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a}{}$$

$$= \frac{(5a^3-45a^2b+15ab^2+b^3)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f}$$

Mathematica [F]
time = 11.22, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.50, size = 3067, normalized size = 10.29

method	result	size
default	Expression too large to display	3067

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)

[Out] 1/48/f*(-15*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3-270*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+94*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-94*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-9*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+68*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+9*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-68*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+24*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+90*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+144*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b

```
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a^2*b-240*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+63*sin(f*x+e)*cos(
f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+75*sin(f*x+e)
*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+22*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b+17*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*b^3-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*b^3-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^3-24*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-22*cos(f*x+e)^7
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-17*cos(f*x+e)^5*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-8*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3+8*cos(f*x+e)^8*((2*I*a^(1/2)*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Fricas [**A**]
time = 34.12, size = 1957, normalized size = 6.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")

[Out] [-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a^3 - 5*
a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*
b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4
 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), 1/384*(96*
(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a^2*b + 15*a*b^2
+ b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x
 + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), -1/
192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
 e) + 24*(3*a^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*co
s(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a
^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f
*x + e)), -1/192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*cos(f*x + e) - 48*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(8*a^3*
cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 -
```

```
68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.87 $\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=217

$$\frac{3(a^2-6ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{3(a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] 3/8*(a^2-6*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
/a^(1/2)+3/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b
^(1/2)/f-3/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-b)*sin(
f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]

time = 0.23, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4217, 478, 591, 596, 537, 223, 212, 385, 209}

$$\frac{3(a^2-6ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}\,f}-\frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3\sqrt{b}\,(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\sin^3(e+fx)\cos(e+fx)\,(a+b\tan^2(e+fx)+b)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 3*b)*Tan[e + f*x]*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(8*f) + (3*(a - b)*Sin[e + f*x]^2*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b +
 b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subs}\cdots}{\cdots}$$

$$= \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \cdots$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\,\text{s}\cdots}{\cdots}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\,\text{s}\cdots}{\cdots}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\,\text{s}\cdots}{\cdots}$$

$$= \frac{3(a^2-6ab+b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}\,f} + \frac{3(a-\cdots}{\cdots}$$

Mathematica [A]

time = 5.92, size = 211, normalized size = 0.97

$$\frac{3\left(\frac{(a^2-6ab+b^2)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} + 4(a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\right)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2\sqrt{2}\,f(a+2b+a\cos(2e+2fx))^{3/2}} + \frac{(-7a+26b+(-6a+10b)\cos(2(e+fx))+a\cos(4(e+fx)))\sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]])/Sqrt[a] + 4*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/
(2*Sqrt[2]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((-7*a + 26*b + (-6*a
+ 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)])*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(32*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.29, size = 2309, normalized size = 10.64

method	result	size
default	Expression too large to display	2309

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(2*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-2*cos(f*x
+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^2*sin(f*x+e)
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+9*sin(f*x+e)*cos(f*x+
e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*cos(f*x+e)^2*sin(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*a^2-36*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
```

```
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6
*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+24*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*c
os(f*x+e)^2*a*b-24*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-5*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+7*cos(f*x+e)^5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2-7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b-co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^3*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2
*b^2+4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Fricas [A]
time = 17.81, size = 1765, normalized size = 8.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")

[Out] [-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*(48*(a^2 - a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a^2 - 6*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + 12*(a^2 - a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a^2 - 6*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 24*(a^2 - a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e))]

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.88 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{a}\,(a-3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{(3a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\,\text{ta}}{}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f
+1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)
/f+b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4217, 478, 542, 537, 223, 212, 385, 209}

$$\frac{\sqrt{a}\,(a-3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+f}{}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+f}{}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+f}{}$$

$$= \frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-b)\sqrt{b}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.48, size = 493, normalized size = 3.06

$$\frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(-4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2e^{2i(e+fx)}\left(2\sqrt{a}(a-3b)fx-i\sqrt{a}(a-3b)\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+i\sqrt{a}(a-3b)\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+2\sqrt{b}(-3a+b)\log\left(\frac{\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{b(-3a+b)\left(1+e^{2i(e+fx)}\right)}\right)\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2\sqrt{2}f\left(a+2b+a\cos(2e+2fx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x))*(
2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
+ f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(b*(-3*a +
 b)*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
```

```
(2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*
x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 1583, normalized size = 9.83

method	result	size
default	Expression too large to display	1583

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+6*cos(f*x+e)^2*sin(f
*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a*b+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+2*si
n(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)
```

```
+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^
(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(148) = 296$.
time = 13.54, size = 1629, normalized size = 10.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b
)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7
*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x + e)^2 - b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e))
, 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3*b)*cos(f*x +
e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(a*
cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
```

```
))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) +
(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) + 4*(a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e) - 2*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(a*cos(f*x +
e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.89 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+f\ldots}{}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,\cdots\right.}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,d\cdots\right.}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\cdots}{\sqrt{a+b+\cdots}}\right.}{f}$$

$$=\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{a+b+\cdots}}\right.}{2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.90, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+ia^{3/2}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-3a\sqrt{b}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)-b^{3/2}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.13, size = 1557, normalized size = 13.19

method	result	size
default	Expression too large to display	1557

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*
x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f
*x+e)^2*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((co
s(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+2*sin(f*x+e)*cos(f
*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*b^2+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((
```

```
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*
x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(106) = 212$.
time = 7.41, size = 1547, normalized size = 13.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
```

```
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2(e + fx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=105

$$\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\,(}{}$$

[Out] 3/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 283, 201, 223, 212}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) + (3*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f) - (Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/f

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b+bx^2\right)^{3/2}}{x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}+\frac{(3b)\text{Subst}\left(\int\sqrt{a\cdots}\right.}{}$$

$$=\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b\cdots\right.}{}$$

$$=\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b\cdots\right.}{}$$

$$=\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 64, normalized size = 0.61

$$-\frac{(a+b)\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)/
(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 2032, normalized size = 19.35

method	result	size
default	Expression too large to display	2032

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(-3*sin(f*x+e)*co
s(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*sin(f*x+e)*
cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*sin(f*x+e
)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos
(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
b^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^
2*a*b-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
```

```
/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+6*sin(f*x+e)*cos(f*x+e
)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*b^2+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b+cos(f*x+e
)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^2*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2
)/sin(f*x+e)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [**A**]
time = 0.27, size = 104, normalized size = 0.99

$$\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b\tan\left(fx+e\right)-\frac{2\left(b\tan(fx+e)^{2}+a+b\right)^{\frac{3}{2}}}{\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*b^(3/2)*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(
f*x + e) - 2*(b*tan(f*x + e)^2 + a + b)^(3/2)/tan(f*x + e))/f
```

Fricas [**A**]
time = 3.99, size = 398, normalized size = 3.79

$$\left[\frac{3\,(a+b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos(fx+e)^{4}+8\,\left(ab-b^{2}\right)\cos(fx+e)^{2}+4\,\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\sin(fx+e)+8\,b^{2}}{\cos(fx+e)^{4}}\right)\sin\left(fx+e\right)-4\left(\left(2\,a+3\,b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{8\,f\cos\left(fx+e\right)\sin\left(fx+e\right)},\ \frac{3\,(a+b)\sqrt{-b}\,\arctan\left(-\frac{\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{-b}\,\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\,\left(ab\cos(fx+e)^{2}+b^{2}\right)\sin(fx+e)}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\left(\left(2\,a+3\,b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*
b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*sqrt
```

```
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e)), 1/4
*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin
(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.91 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3\cdots}{}$$

```
[Out] 1/2*(3*a+5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
5*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 464, 283, 201, 223, 212}

$$\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3f(a+b)}-\frac{(3a+5b)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 5*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a
 + b)*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^{3/2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} + \frac{(3a+5b)\text{Subst}\Big(}{}$$

$$= -\frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+}{}$$

$$= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)}{}$$

$$= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)}{}$$

$$= \frac{\sqrt{b}\,(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.87, size = 369, normalized size = 2.15

$$\frac{\sqrt{2}\,e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(4a\left(1+e^{2i(e+fx)}\right)^2\left(1-4e^{2i(e+fx)}+e^{4i(e+fx)}\right)+b\left(15-20e^{2i(e+fx)}-22e^{4i(e+fx)}-20e^{6i(e+fx)}+15e^{8i(e+fx)}\right)\right)}{\left(-1+e^{2i(e+fx)}\right)^3\left(1+e^{2i(e+fx)}\right)^2}-\frac{3\sqrt{b}\,(3a+5b)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(4*a*(1 + E^((2*I)*(e + f*x)))^2*(1 - 4
*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x))) + b*(15 - 20*E^((2*I)*(e + f*x)
) - 22*E^((4*I)*(e + f*x)) - 20*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/((-1 + E^((2*I)*(e + f*x)))^3*(1 + E^((2*I)*(e + f*x)))^2) - (3*Sqrt[b
]*(3*a + 5*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e +
 f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a
 + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 3925, normalized size = 22.82

method	result	size
default	Expression too large to display	3925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-9*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a*b-15*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2+18*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+30*cos(f*x+e)^5*
sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*b^2-9*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-15*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+18*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)
*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b
```

```
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a*b+30*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+9*sin(f*x+e)*cos(f*x+e)
^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+15*sin(f*x+e)*cos(f*x
+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-18*sin(f*x+e)*cos(
f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a*b-30*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+4
*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+15*cos(f*x+e)^6*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c...
```

Maxima [A]

time = 0.28, size = 257, normalized size = 1.49

$$\frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}+9\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}+9\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b\tan\left(fx+e\right)+\frac{6\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b^2\tan(fx+e)}{a+b}-\frac{6\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{\tan(fx+e)}-\frac{4\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)\tan(fx+e)}-\frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}}{(a+b)\tan(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 6*a*b^(3/2)*arcs
inh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 9*b^(3/2)*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b)) + 6*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a
 + b) + 9*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 6*sqrt(b*tan(f*x
```

```
+ e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 6*(b*tan(f*x + e)^2 + a + b)^(3/
2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*tan(f*x + e
)) - 2*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3))/f
```

Fricas [A]
time = 6.96, size = 506, normalized size = 2.94

$$\left[\frac{3\left((3a+5b)\cos(fx+e)^3-(3a+5b)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\left((4a+15b)\cos(fx+e)^4-2(3a+10b)\cos(fx+e)^2+3b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}, \frac{3\left((3a+5b)\cos(fx+e)^3-(3a+5b)\cos(fx+e)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)-2\left((4a+15b)\cos(fx+e)^4-2(3a+10b)\cos(fx+e)^2+3b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*(
(4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*s
in(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e
))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*s
in(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b
)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a}{}$$

```
[Out] 1/2*(3*a+7*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2
)/f+1/2*b*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*a+
7*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-2/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(5/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(5/2)/(a+
b)/f
```

Rubi [A]
time = 0.14, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 473, 464, 283, 201, 223, 212}

$$\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5f(a+b)}-\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3f(a+b)}-\frac{(3a+7b)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 7*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 7*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*
(a + b)*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)*f
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{(1+x^2)^2(a+b+bx^2)^{3/2}}{x^6}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f}+\frac{\text{Subst}\left(\int\frac{(a+b+bx^2)}{}\right.}{}$$

$$=-\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}-\frac{\cot^5(e+fx)\,(a}{5}$$

$$=-\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}-\frac{2\cot^3(e}{}$$

$$=\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a+7b)\,\mathrm{c}}{}$$

$$=\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a+7b)\,\mathrm{c}}{}$$

$$=\frac{\sqrt{b}\,(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+7}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.77, size = 512, normalized size = 2.45

$$\frac{\sqrt{2}\,e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(1-6e^{2i(e+fx)}+16e^{4i(e+fx)}-6e^{6i(e+fx)}+e^{8i(e+fx)}\right)+b^2\left(105-350e^{2i(e+fx)}+231e^{4i(e+fx)}+412e^{6i(e+fx)}+231e^{8i(e+fx)}-350e^{10i(e+fx)}+105e^{12i(e+fx)}\right)+ab\left(115-402e^{2i(e+fx)}+317e^{4i(e+fx)}+708e^{6i(e+fx)}+317e^{8i(e+fx)}-402e^{10i(e+fx)}+115e^{12i(e+fx)}\right)\right)}{(a+b)\left(-1+e^{2i(e+fx)}\right)^5\left(1+e^{2i(e+fx)}\right)^2}-\frac{15\sqrt{b}\,(3a+7b)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15f\left(a+2b+a\cos(2(e+fx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(1
- 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) +
E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I)*(e
 + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^((10*I
)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e + f*x
)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I)*(e +
```

```
f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a + b)*(-1
 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[b]*(3*a +
 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec
[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.27, size = 8726, normalized size = 41.75

method	result	size
default	Expression too large to display	8726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 289, normalized size = 1.38

$$\frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{60\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+45\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+\frac{60\,b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{a+b}+45\,\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b\tan\left(fx+e\right)+\frac{60\,\sqrt{b\tan\left(fx+e\right)^2+a+b}\,b^2\tan\left(fx+e\right)}{a+b}-\frac{30\,\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}}{\tan\left(fx+e\right)}-\frac{40\,\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)\tan\left(fx+e\right)}-\frac{20\,\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{5}{2}}}{(a+b)\tan\left(fx+e\right)^3}-\frac{6\,\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{5}{2}}}{(a+b)\tan\left(fx+e\right)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 60*a*b^(3/2)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 45*b^(3/2)*arcsinh(b*tan(f
*x + e)/sqrt((a + b)*b)) + 60*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
))/(a + b) + 45*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 60*sqrt(b*t
an(f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 30*(b*tan(f*x + e)^2 + a
+ b)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*ta
n(f*x + e)) - 20*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3)
- 6*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^5))/f
```

Fricas [A]
time = 17.02, size = 722, normalized size = 3.45

$$\left[\frac{15\left(\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^5-2\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^3+\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-6ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin\left(fx+e\right)-4\left(\left(16a^2+115ab+105b^2\right)\cos\left(fx+e\right)^6-\left(40a^2+273ab+245b^2\right)\cos\left(fx+e\right)^4+\left(30a^2+185ab+161b^2\right)\cos\left(fx+e\right)^2-15ab-15b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{120\left((a+b)f\cos\left(fx+e\right)^5-2(a+b)f\cos\left(fx+e\right)^3+(a+b)f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}, \frac{15\left(\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^5-2\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)^3+\left(3a^2+10ab+7b^2\right)\cos\left(fx+e\right)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin\left(fx+e\right)-2\left(\left(16a^2+115ab+105b^2\right)\cos\left(fx+e\right)^6-\left(40a^2+273ab+245b^2\right)\cos\left(fx+e\right)^4+\left(30a^2+185ab+161b^2\right)\cos\left(fx+e\right)^2-15ab-15b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{60\left((a+b)f\cos\left(fx+e\right)^5-2(a+b)f\cos\left(fx+e\right)^3+(a+b)f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7
*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log((
```

```
(a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16
*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^2)*cos
(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b - 15*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^5 -
 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)), 1/60*(
15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7*b^2)*co
s(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 2
45*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a
*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f
*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x +
e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}+\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\text{co}$$

```
[Out] -1/15*(15*a^2+20*a*b+8*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+2/15*
(5*a+2*b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/5*cos(f*x+e)^5*(a+b
*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4219, 473, 464, 270}

$$\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 + 20*a*b + 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(
a^3*f) + (2*(5*a + 2*b)*Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^2*
f) - (Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/(5*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-2(5a+2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+
}{5af}$$

$$= \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+
}{5af}$$

$$= -\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+
}{}$$

Mathematica [A]
time = 1.03, size = 93, normalized size = 0.76

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(89a^2+144ab+64b^2-4a(7a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\sec(e+fx)}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/240*((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7*
a + 4*b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(a^3*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Maple [A]
time = 0.21, size = 105, normalized size = 0.85

method	result	size
default	$-\frac{\left(b+a\left(\cos^{2}\left(fx+e\right)\right)\right)\left(3\left(\cos^{4}\left(fx+e\right)\right)a^{2}-10\left(\cos^{2}\left(fx+e\right)\right)a^{2}-4\left(\cos^{2}\left(fx+e\right)\right)ab+15a^{2}+20ab+8b^{2}\right)}{15f\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)a^{3}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-10*cos(f*x+e)^2*a^2-4*cos(f*
x+e)^2*a*b+15*a^2+20*a*b+8*b^2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos
(f*x+e)/a^3
```

Maxima [A]
time = 0.28, size = 174, normalized size = 1.41

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}-3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b\cos\left(fx+e\right)\right)}{a^{2}}+\frac{3\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{5}-10\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}b\cos\left(fx+e\right)^{3}+15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b^{2}\cos\left(fx+e\right)}{a^{3}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a
^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x + e
)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e
))/a^3)/f
```

Fricas [A]
time = 1.13, size = 92, normalized size = 0.75

$$-\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{5}-2\left(5\,a^{2}+2\,ab\right)\cos\left(fx+e\right)^{3}+\left(15\,a^{2}+20\,ab+8\,b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
a^3*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(111) = 222.
time = 0.84, size = 928, normalized size = 7.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -256/15*(5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))^7*(a + b) - 5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(2*a - b)*sqrt(a + b)
- (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*
e)^2 + a + b))^5*(13*a^2 + 40*a*b + 15*b^2) + 5*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(8*a^2 + 5*a
*b - 3*b^2)*sqrt(a + b) - 5*(a^3 - 9*a^2*b - 13*a*b^2 - 3*b^3)*(sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))
^3 - 5*(10*a^3 + 17*a^2*b + 4*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b) + 5
*(9*a^4 + 14*a^3*b - 6*a*b^3 - b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) - (12*a^4 + 9*a^3*b - 13*a
^2*b^2 - 5*a*b^3 + 5*b^4)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sq
rt(a + b) - 3*a + b)^5*f*sgn(cos(f*x + e)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}+\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}$$

```
[Out] -1/3*(3*a+2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f+1/3*cos(f*x+e)^3*(
a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4219, 464, 270}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a^2*f) + (Cos[e
 + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
&& IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af} + \frac{(3a+2b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \text{s}\right.}{3af}$$

$$= -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e}}{3af}$$

Mathematica [A]
time = 0.29, size = 64, normalized size = 0.86

$$\frac{(-5a-4b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{12a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] ((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A]
time = 0.14, size = 69, normalized size = 0.93

method	result	size
default	$\frac{(b+a(\cos^2(fx+e)))(a(\cos^2(fx+e))-3a-2b)}{3f\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(fx+e)a^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)/cos(f*x+e)/a^2
```

Maxima [A]
time = 0.28, size = 89, normalized size = 1.20

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a} - \frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2)
^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)/f
```

Fricas [A]
time = 1.52, size = 61, normalized size = 0.82

$$\frac{\left(a\cos\left(fx+e\right)^{3}-\left(3\,a+2\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(a^2*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(66) = 132.
time = 0.75, size = 559, normalized size = 7.55

$$-\frac{16\left(3\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{4}\sqrt{a+b}-4\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{3}a-6\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{2}(a+b)^{\frac{3}{2}}+12\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)(a^{2}+ab)-(5a^{2}+2ab-3b^{2})\sqrt{a+b}\right)}{3\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{2}+2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)\sqrt{a+b}-3a+b\right)^{3}f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/3*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e)^2 + a + b))^4*sqrt(a + b) - 4*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*a - 6*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2
```

```
*(a + b)^(3/2) + 12*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))*(a^2 + a*b) - (5*a^2 + 2*a*b - 3*b^2)*sqrt
(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^3*f*sgn(c
os(f*x + e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+fx)^3}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

```
[Out] -cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4219, 270}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 1.60

$$-\frac{(a+2b+a\cos(2e+2fx))\sec(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(a*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [A]
time = 0.08, size = 31, normalized size = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{fa\sec(fx+e)}$	31
default	$-\frac{\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{fa\sec(fx+e)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 30, normalized size = 1.00

$$-\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Fricas [A]
time = 1.69, size = 40, normalized size = 1.33

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.49, size = 31, normalized size = 1.03

$$-\frac{\sqrt{a\cos(fx+e)^2+b}}{af\,\mathrm{sgn}(\cos(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a*cos(f*x + e)^2 + b)/(a*f*sgn(cos(f*x + e)))
```

Mupad [B]
time = 5.38, size = 46, normalized size = 1.53

$$-\frac{\cos(e+f\,x)\,\sqrt{\frac{a+2\,b+a\,\cos(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] -(cos(e + f*x)*((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2
))/(a*f)
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}\,f}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4219, 385, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
b]*f))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}\,f}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 2.00

$$
-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\,\sec(e+fx)}{\sqrt{2}\,\sqrt{a+b}\,f\,\sqrt{a+b\sec^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Cos
[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*x]
^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(37) = 74$.
time = 0.15, size = 280, normalized size = 6.51

method	result
default	$\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\cos(fx+e)\sqrt{a+b}+\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\sqrt{a+b}-\cos(f\right.}{\sin(fx+e)^2\sqrt{a+b}}\right.\right.}{2f\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(cos(f*x+e)-1)*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/
(a+b)^(1/2))+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/(cos(f*x+e)-1)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1
/2)/cos(f*x+e)/(cos(f*x+e)-1)/(a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 1.69, size = 148, normalized size = 3.44

$$\left[\frac{\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a+b}\,f}, \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*f)
, sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b))/((a + b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

[Out] Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(37) = 74.
time = 0.79, size = 305, normalized size = 7.09

$$-\frac{\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right|\right)}{\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\sqrt{a+b}\right|\right)}{\sqrt{a+b}}-\frac{\log\left(\left|-\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)(a+b)+\sqrt{a+b}(a-b)\right|\right)}{\sqrt{a+b}}}{2f\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*(log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))/sqrt(a + b) - log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b)))/sqrt(a + b) - log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b))/(f*sgn(cos(f*x + e)))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)

[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

```
[Out] -1/2*a*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]

time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4219, 482, 12, 385, 213}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/((a
 + b)^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*
(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{a}{(-1+x^2)\sqrt{a+bx^2}}\right.}{2(a+b)}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\right.}{2(a+b)}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{a\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x\right.}{2(a+}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e}{2(a+b)f}$$

Mathematica [A]

time = 1.09, size = 140, normalized size = 1.61

$$-\frac{a\sqrt{a+2b+a\cos(2e+2fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)}{2\sqrt{2}\,(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin[
e + f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x]^
2)/(a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(Sqrt[2]*(a + b)^2*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2198 vs. $2(75) = 150$.
time = 0.20, size = 2199, normalized size = 25.28

method	result	size
default	Expression too large to display	2199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^
2/(a+b)^(1/2))*cos(f*x+e)^3*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(
f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a*b-((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a^2-((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a*b+2*(a+b)^(3/2)*cos(f*x+e)
^2*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a
+b)^(1/2))*cos(f*x+e)^2*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
```

```
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^2*a*b-((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a^2-((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^2+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*c
os(f*x+e)*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f
*x+e)*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e
)*a*b+2*(a+b)^(3/2)*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(co
s(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b
)/sin(f*x+e)^2/(a+b)^(1/2))*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(
f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/(cos(f*x+e)-1))*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f
*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x
+e)-1))*a*b)*sin(f*x+e)^2/(cos(f*x+e)-1)^2/cos(f*x+e)/((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2/(a+b)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]

time = 2.16, size = 323, normalized size = 3.71

$$\left[\frac{2\,(a+b)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{4\left((a^2+2ab+b^2)f\cos\left(fx+e\right)^2-(a^2+2ab+b^2)f\right)}, \frac{\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a+b}\right)+(a+b)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{2\left((a^2+2ab+b^2)f\cos\left(fx+e\right)^2-(a^2+2ab+b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
(a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)
*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a + b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(e+f\,x\right)}^3\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f}-\frac{\cot^3(e+$$

```
[Out] -3/8*a^2*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/
2)/f-1/8*(5*a+2*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f
-1/4*cot(f*x+e)^3*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 481, 541, 12, 385, 213}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(8*
(a + b)^(5/2)*f) - ((5*a + 2*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(8*(a + b)^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(4*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{-a-2(2a+b)x^2}{(-1+x^2)^2\sqrt{a+b}}\right.}{4(a+}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc}{}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc}{}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc}{}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 78, normalized size = 0.57

$$-\frac{a^2(a+2b+a\cos(2(e+fx)))\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\sec(e+fx)}{2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1 -
 (a*Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/((a + b)^3*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4982 vs. 2(122) = 244.
time = 0.16, size = 4983, normalized size = 36.11

method	result	size
default	Expression too large to display	4983

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/16/f*(-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2-6*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3*b-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^2*b^2-6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2-6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3*b-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^2*b^2+12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^2+12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3*b+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^2*b^2-10*(a+b)^(5/2)*a*b-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co

```
s(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^4+6*(a+b)^(5/2)*cos(f*x+e)^4*a^2-10*(a+
b)^(5/2)*cos(f*x+e)^2*a^2+2*(a+b)^(5/2)*cos(f*x+e)^2*a*b-6*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3*b-3*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(cos(f*x+e)-1))*a^2*b^2-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4-3*cos(f*x+e)^
5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^4-3*cos(f*x+e)^
4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)
^(1/2))*a^4-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f
*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x
+e)-1))*a^4+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e
)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.15, size = 515, normalized size = 3.73

$$\left[\frac{3\,{\left(a^{2} \cos\left(f x + e\right)^{4} - 2\,a^{2} \cos\left(f x + e\right)^{2} + a^{2}\right)} \sqrt{a + b} \log\left(\frac{2\,{\left(a \cos\left(f x + e\right)^{2} - 2\,\sqrt{a + b} \sqrt{\frac{a \cos\left(f x + e\right)^{2} + b}{\cos\left(f x + e\right)^{2}}} \cos\left(f x + e\right) + a + 2\,b\right)}}{\cos\left(f x + e\right)^{2} - 1}\right) + 2\,{\left(3\,{\left(a^{2} + a b\right)} \cos\left(f x + e\right)^{3} - {\left(5\,a^{2} + 7\,a b + 2\,b^{2}\right)} \cos\left(f x + e\right)\right)} \sqrt{\frac{a \cos\left(f x + e\right)^{2} + b}{\cos\left(f x + e\right)^{2}}}}{16\,{\left({\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f \cos\left(f x + e\right)^{4} - 2\,{\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f \cos\left(f x + e\right)^{2} + {\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f\right)}}, \frac{3\,{\left(a^{2} \cos\left(f x + e\right)^{4} - 2\,a^{2} \cos\left(f x + e\right)^{2} + a^{2}\right)} \sqrt{-a - b} \arctan\left(\frac{\sqrt{-a - b} \sqrt{\frac{a \cos\left(f x + e\right)^{2} + b}{\cos\left(f x + e\right)^{2}}} \cos\left(f x + e\right)}{a + b}\right) + {\left(3\,{\left(a^{2} + a b\right)} \cos\left(f x + e\right)^{3} - {\left(5\,a^{2} + 7\,a b + 2\,b^{2}\right)} \cos\left(f x + e\right)\right)} \sqrt{\frac{a \cos\left(f x + e\right)^{2} + b}{\cos\left(f x + e\right)^{2}}}}{8\,{\left({\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f \cos\left(f x + e\right)^{4} - 2\,{\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f \cos\left(f x + e\right)^{2} + {\left(a^{3} + 3\,a^{2} b + 3\,a b^{2} + b^{3}\right)} f\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*cos(f
*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos
(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(122) = 244.
time = 1.01, size = 785, normalized size = 5.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*((a + b)*tan(1/2*f*
x + 1/2*e)^2/(a^2 + 2*a*b + b^2) + 3*(3*a + b)/(a^2 + 2*a*b + b^2)) - 24*a^
2*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*
x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b)) - 1
2*a^2*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - a + b))/((a^2 + 2*a*b + b^2)*sqrt(
a + b)) + 4*(2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
```

```
/2*f*x + 1/2*e)^2 + a + b))^3*(2*a^2 - 2*a*b - b^2) - 3*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2
 + 2*a*b + b^2)*sqrt(a + b) - 2*(3*a^3 + a^2*b - 2*a*b^2)*(sqrt(a + b)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (5
*a^3 + 11*a^2*b + 7*a*b^2 + b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(
a^2 + 2*a*b + b^2)))/(f*sgn(cos(f*x + e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=193

$$\frac{5(a+b)^3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f}-\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f}$$

```
[Out] 5/16*(a+b)^3*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f-1/48*(33*a^2+40*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1
/2)/a^3/f+1/24*(9*a+5*b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a^2/f+1/6*cos(f*x+e)^3*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 481, 592, 541, 12, 385, 209}

$$\frac{5(a+b)^3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{(9a+5b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f}-\frac{(33a^2+40ab+15b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}+\frac{\sin^3(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (5*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(16*a^(7/2)*f) - ((33*a^2 + 40*a*b + 15*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqr
t[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + ((9*a + 5*b)*Cos[e + f*x]^3*Sin[e
 + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos[e + f*x]^3*Sin[e
+ f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-2(3a\ldots}{(1+x^2)^3\sqrt{a+b\ldots}}\right.}{\ldots}$$

$$= \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\,\text{s}\ldots}{\ldots}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9\ldots}{\ldots}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9\ldots}{\ldots}$$

$$= -\frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9\ldots}{\ldots}$$

$$= \frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{(33a^2+40ab+15b^2)\cos(e+\ldots}{\ldots}$$

Mathematica [A]
time = 1.53, size = 163, normalized size = 0.84

$$\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(15(a+b)^3\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,(15(a+b)^2+10a(a+b)\sin^2(e+fx)+8a^2\sin^4(e+fx))\right)}{48\sqrt{2}\,a^{7/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sq
rt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2 +
8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.34, size = 2425, normalized size = 12.56

method	result	size
default	Expression too large to display	2425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*sin(f*x+e)*(-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-2*cos(f*x+e)^5*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2*b+14*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-
14*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-5*cos(f*x+e)^
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+33*cos(f*x+e)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+40*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b^2-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2*b-40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-90*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
^2*b*sin(f*x+e)-90*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
```

```
i((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+45*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+45*2^(1/2)*((I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^3+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*
x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-26*cos(f*x+e)^5*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*cos(f*
x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/((b+a*cos(
f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.06, size = 670, normalized size = 3.47

$$\left[-\frac{15\,(a^3+3a^2b+3ab^2+b^3)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b^2)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32(a^4-7a^3b+7a^2b^2-ab^3)\cos(fx+e)^2+8\left(16a^3\cos(fx+e)^7-24(a^3-a^2b)\cos(fx+e)^5+2(5a^3-14a^2b+5ab^2)\cos(fx+e)^3-(a^3-7a^2b+7ab^2-b^3)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)+8\left(8a^3\cos(fx+e)^5-2(13a^3+5a^2b)\cos(fx+e)^3+(33a^3+40a^2b+15ab^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{384a^4f},\ -\frac{15\,(a^3+3a^2b+3ab^2+b^3)\sqrt{a}\arctan\left(\frac{\left(8a^3\cos(fx+e)^5-8(a^3-ab)\cos(fx+e)^3+(a^3-6a^2b+ab^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\left(8a^3\cos(fx+e)^5-2(13a^3+5a^2b)\cos(fx+e)^3+(33a^3+40a^2b+15ab^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{192a^4f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
```

```
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 - 2*(1
3*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/1
92*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^3*co
s(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 1
5*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f}$$

```
[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/4*c
os(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 481, 541, 12, 385, 209}

$$\frac{3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*a^(5/2)*f) - ((5*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+b}{(1+x^2)^2\sqrt{a+b}}\right.}{4}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)}{}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)}{}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)}{}$$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e-}{8}$$

Mathematica [A]
time = 0.59, size = 145, normalized size = 1.07

$$\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,(3(a+b)+2a\sin^2(e+fx))\right)}{8\sqrt{2}\,a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt[
a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2]*a
^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1701, normalized size = 12.60

method	result	size

default	Expression too large to display	1701

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*(2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
2-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+6*2^(1/2)*((I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))
/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e
)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^2*sin(f*x+e)+12*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*
sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-
6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-5*cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*b^2+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2
```

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 2.15, size = 594, normalized size = 4.40

$$\left[-\frac{3\left(a^{2}+2ab+b^{2}\right)\sqrt{-a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5a^{4}-14a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28a^{3}b+70a^{2}b^{2}-28ab^{3}+b^{4}-32\left(a^{4}-7a^{3}b+7a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5a^{3}-14a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7a^{2}b+7ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)-8\left(2a^{2}\cos\left(fx+e\right)^{3}-\left(5a^{2}+3ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{64a^{3}f},\ -\frac{3\left(a^{2}+2ab+b^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)-4\left(2a^{2}\cos\left(fx+e\right)^{3}-\left(5a^{2}+3ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{32a^{3}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
 a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x
+ e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=85

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f}-\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 482, 12, 385, 209}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a+b}{(1+x^2)\sqrt{a+\cdots}}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\text{Subst}\left(\int \frac{\cdots}{(1+x^2\cdots}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+ax^2\cdots}\right.}{\cdots}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+\cdots}}{2af}$$

Mathematica [A]

time = 0.29, size = 125, normalized size = 1.47

$$\frac{\sqrt{a+2b+a\cos(2(e+fx))}\,\sec(e+fx)\left((a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\right)}{2\sqrt{2}\,a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 1055, normalized size = 12.41

method	result
default	$-\frac{\sin(fx+e)\left(\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}-\cos(f\right.}{(1+\cos(fx+e))(a+b)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x
+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
```

```
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(cos(f*x+e)-1)/((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(1/2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(78) = 156.
time = 3.72, size = 524, normalized size = 6.16

$$\left[-\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-a}\left(a+b\right)\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{16\,a^{2}f},\ -\frac{4\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(a+b\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{8\,a^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4213, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 0.07, size = 87, normalized size = 2.23

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\,\sec(e+fx)}{\sqrt{2}\,\sqrt{a}\,f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 380, normalized size = 9.74

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}-\cos(fx+e)a-b\right)}{(1+\cos(fx+e))(a+b)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(35) = 70$.
time = 0.59, size = 1046, normalized size = 26.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e

```
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(35) = 70$.
time = 3.46, size = 427, normalized size = 10.95

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\,\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\,\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{8\,af},\ -\frac{\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\,\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{4\,\sqrt{a}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec^{2}\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

```
[Out] -cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]

time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4217, 270}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 1.67

$$-\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/((a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A]
time = 0.12, size = 48, normalized size = 1.45

method	result	size
default	$-\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(fx+e)}{f\sin(fx+e)(a+b)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)/(a+b)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.06

$$-\frac{\sqrt{b\tan\left(fx+e\right)^2+a+b}}{(a+b)f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*f*tan(f*x + e))
```

Fricas [A]
time = 3.42, size = 51, normalized size = 1.55

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{(a+b)f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f*
x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.65, size = 74, normalized size = 2.24

$$-\frac{(2\sin(2e+2fx)+\sin(4e+4fx))\sqrt{\frac{a+2b+a\cos(2e+2fx)}{\cos(2e+2fx)+1}}}{2f\sin(2e+2fx)^2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] -((2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((a + 2*b + a*cos(2*e + 2*f*x))/(
cos(2*e + 2*f*x) + 1))^(1/2))/(2*f*sin(2*e + 2*f*x)^2*(a + b))
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f}-\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

```
[Out] -1/3*(3*a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)
^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4217, 464, 270}

$$-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
```

```
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{(3a+b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\right.}{3(a+b)f}$$

$$= -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\,\mathrm{ta}}}{3(a+b)f}$$

Mathematica [A]
time = 0.22, size = 74, normalized size = 0.95

$$\frac{(-2a-b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x
]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A]
time = 0.14, size = 66, normalized size = 0.85

method	result	size
default	$\frac{(2a(\cos^2(fx+e))-3a-b)\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(fx+e)}{3f\sin(fx+e)^3(a+b)^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(2*a*cos(f*x+e)^2-3*a-b)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(
f*x+e)/sin(f*x+e)^3/(a+b)^2
```

Maxima [A]
time = 0.27, size = 102, normalized size = 1.31

$$-\frac{\frac{3\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan(fx+e)}-\frac{2\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan(fx+e)}+\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan(fx+e)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 2*sqrt(b*ta
n(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 +
a + b)/((a + b)*tan(f*x + e)^3))/f
```

Fricas [A]
time = 5.38, size = 102, normalized size = 1.31

$$-\frac{\left(2\,a\cos\left(fx+e\right)^{3}-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{2}+2\,ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2\,ab+b^{2}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b +
 b^2)*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{4}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [B]
time = 9.92, size = 123, normalized size = 1.58

$$-\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i-b\,e^{e\,2i+f\,x\,2i}\,2i\right)}{3\,f\,\left(a+b\right)^2\,\left(e^{e\,2i+f\,x\,2i}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(2*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2)^2)^(1/2)*(a*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i
 - b*exp(e*2i + f*x*2i)*2i))/(3*f*(a + b)^2*(exp(e*2i + f*x*2i) - 1)^3)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=132

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f}-\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+}{15(a+b)^2f}$$

```
[Out] -1/15*(15*a^2+10*a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f
-2/15*(5*a+3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f
*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4217, 473, 464, 270}

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}-\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/((a + b)^3*f) - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{2(5a+3b)+5(a+b)x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \text{ta}\right.}{5(a+b)f}$$

$$= -\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+}}{5(a+b)}$$

$$= -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} - \frac{2(5a+3b)\cot^3}{}$$

Mathematica [A]
time = 0.37, size = 100, normalized size = 0.76

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(8a^2+8ab+3b^2-2a(3a+b)\cos(2(e+fx))+a^2\cos(4(e+fx))\right)\csc^5(e+fx)\sec(e+fx)}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a + b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/((a
 + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A]
time = 0.17, size = 101, normalized size = 0.77

method	result	size
default	$-\frac{\left(8\left(\cos^{4}\left(fx+e\right)\right)a^{2}-20\left(\cos^{2}\left(fx+e\right)\right)a^{2}-4\left(\cos^{2}\left(fx+e\right)\right)ab+15a^{2}+10ab+3b^{2}\right)\sqrt{\frac{b+a\left(\cos^{2}\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)}{15f\sin\left(fx+e\right)^{5}\left(a+b\right)^{3}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(8*cos(f*x+e)^4*a^2-20*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b+15*a^2+1
0*a*b+3*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^
5/(a+b)^3
```

Maxima [A]
time = 0.28, size = 203, normalized size = 1.54

$$-\frac{\frac{15\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan(fx+e)}-\frac{20\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan(fx+e)}+\frac{8\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b^{2}}{(a+b)^{3}\tan(fx+e)}+\frac{10\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan(fx+e)^{3}}-\frac{4\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}{(a+b)^{2}\tan(fx+e)^{3}}+\frac{3\sqrt{b\tan\left(fx+e\right)^{2}+a+b}}{(a+b)\tan(fx+e)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 20*sqrt(b
*tan(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)
^2 + a + b)*b^2/((a + b)^3*tan(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a + b
)/((a + b)*tan(f*x + e)^3) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*b/((a + b)^2*
tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)^5)
)/f
```

Fricas [A]
time = 6.61, size = 179, normalized size = 1.36

$$-\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-4\left(5\,a^{2}+ab\right)\cos\left(fx+e\right)^{3}+\left(15\,a^{2}+10\,ab+3\,b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\left(\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 10
*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{6}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [B]
time = 15.15, size = 723, normalized size = 5.48

$$\frac{\left(\frac{32\,a+16\,b}{5\,f\,(6\,a+6\,b)\,(a\,1\mathrm{i}+b\,1\mathrm{i})}+\frac{32\,a+80\,b}{5\,f\,(6\,a+6\,b)\,(a\,1\mathrm{i}+b\,1\mathrm{i})}\right)\sqrt{a+\frac{b}{{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a+\frac{b}{{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(\frac{a\,(2\,a+b)\,32\mathrm{i}}{15\,f\,{(a\,1\mathrm{i}+b\,1\mathrm{i})}^2\,(8\,a+8\,b)}+\frac{a\,(2\,a+3\,b)\,32\mathrm{i}}{15\,f\,{(a\,1\mathrm{i}+b\,1\mathrm{i})}^2\,(8\,a+8\,b)}\right)\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{a+\frac{b}{{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)\,\left(\frac{96\,a+32\,b}{5\,f\,(a\,1\mathrm{i}+b\,1\mathrm{i})\,(16\,a+16\,b)}+\frac{160\,a+160\,b}{5\,f\,(a\,1\mathrm{i}+b\,1\mathrm{i})\,(16\,a+16\,b)}+\frac{256\,a+320\,b}{5\,f\,(a\,1\mathrm{i}+b\,1\mathrm{i})\,(16\,a+16\,b)}\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^4\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a+\frac{b}{{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)\,32\mathrm{i}}{f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^5\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,(10\,a+10\,b)}-\frac{8\,a^2\,\sqrt{a+\frac{b}{{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{15\,f\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,{(a\,1\mathrm{i}+b\,1\mathrm{i})}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] (((32*a + 16*b)/(5*f*(6*a + 6*b)*(a*1i + b*1i)) + (32*a + 80*b)/(5*f*(6*a +
 6*b)*(a*1i + b*1i)))*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2
)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*
x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2)^2)^(1/2)*((a*(2*a + b)*32i)/(15*f*(a*1i + b*1i)^2*(8*
a + 8*b)) + (a*(2*a + 3*b)*32i)/(15*f*(a*1i + b*1i)^2*(8*a + 8*b)))*(2*exp(
e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(
e*2i + f*x*2i) + 1)) + ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)
/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*((96*a + 32*b)
/(5*f*(a*1i + b*1i)*(16*a + 16*b)) + (160*a + 160*b)/(5*f*(a*1i + b*1i)*(16
*a + 16*b)) + (256*a + 320*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b))))/((exp(e*2
i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x
*4i) + 1)*32i)/(f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(10*a
 + 10*b)) - (8*a^2*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2
)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(15*f*(exp(e*2i +
f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a*1i + b*1i)^3)
```

3.106 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(15a^2+40ab+24b^2)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}}-\frac{2b(15a^2+40}{15a^4f\sqrt{}}$$

```
[Out] -1/15*(15*a^2+40*a*b+24*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)+2/15
*(5*a+3*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x+e)^5/a/f
/(a+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+40*a*b+24*b^2)*sec(f*x+e)/a^4/f/(a
+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4219, 473, 464, 277, 197}

$$\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(\frac{8b(5a+3b)}{a^2}+15\right)\cos(e+fx)}{15af\sqrt{a+b\sec^2(e+fx)}}-\frac{2b(15a^2+40ab+24b^2)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15 + (8*b*(5*a + 3*b))/a^2)*Cos[e + f*x])/(a*f*Sqrt[a + b*Sec[e + f
*x]^2]) + (2*(5*a + 3*b)*Cos[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cos[e + f*x]^5/(5*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(15*a^2 + 40
*a*b + 24*b^2)*Sec[e + f*x])/(15*a^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a+3b)+5ax^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \ldots$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \ldots$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \ldots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 432 vs. $2(171) = 342$.
time = 7.64, size = 432, normalized size = 2.53

$$\frac{3(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{64af\sqrt{a+2b+a\cos(2(e+fx))}\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(2a+4b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{32a^2f\sqrt{a+2b+a\cos(2(e+fx))}\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\left(27a^2+128ab+128b^2+16a(a+2b)\cos(2(e+fx))-2a^2\cos(4(e+fx))\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{192a^3f\sqrt{a+2b+a\cos(2(e+fx))}\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\left(40a^3+336a^2b+768ab^2+512b^3+a\left(25a^2+128ab+128b^2\right)\cos(2(e+fx))-4a^2(a+2b)\cos(4(e+fx))+a^3\cos(6(e+fx))\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{320a^4f\sqrt{a+2b+a\cos(2(e+fx))}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(64*a*f*Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) + ((2*a + 4*b + a*Cos
[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(32*a^2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((27*
a^2 + 128*a*b + 128*b^2 + 16*a*(a + 2*b)*Cos[2*(e + f*x)] - 2*a^2*Cos[4*(e
+ f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(192*a^3*f*Sq
rt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((40*a^3 +
 336*a^2*b + 768*a*b^2 + 512*b^3 + a*(25*a^2 + 128*a*b + 128*b^2)*Cos[2*(e
+ f*x)] - 4*a^2*(a + 2*b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(320*a^4*f*Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35189 vs. $2(155) = 310$.
time = 1.99, size = 35190, normalized size = 205.79

method	result	size
default	Expression too large to display	35190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.27, size = 268, normalized size = 1.57

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3}+\frac{15b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}+\frac{30b^2}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^3\cos(fx+e)}+\frac{15b^3}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^4\cos(fx+e)}+\frac{3\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-5\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b^2\cos(fx+e)\right)}{a^4}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt(a
+ b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)^2)*
a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a +
 b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b
^2*cos(f*x + e))/a^4)/f
```

Fricas [A]
time = 3.58, size = 143, normalized size = 0.84

$$-\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{7}-2\,\left(5\,a^{3}+3\,a^{2}b\right)\cos\left(fx+e\right)^{5}+\left(15\,a^{3}+40\,a^{2}b+24\,ab^{2}\right)\cos\left(fx+e\right)^{3}+2\,\left(15\,a^{2}b+40\,ab^{2}+24\,b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\,\left(a^{5}f\cos\left(fx+e\right)^{2}+a^{4}bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
+ 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)
^2 + a^4*b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1439 vs. 2(155) = 310.
time = 1.25, size = 1439, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*((a^5*b^2*sgn(cos(f*x + e)) + 2*a^4*b^3*sgn(cos(f*x + e)) + a^3*b
^4*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^7*b) + (a^5*b^2*sgn(cos(f*x
 + e)) + 2*a^4*b^3*sgn(cos(f*x + e)) + a^3*b^4*sgn(cos(f*x + e)))/(a^7*b))/
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + 4*(15*(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(a*
b + b^2) + 15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b))^8*(11*a*b + 7*b^2)*sqrt(a + b) + 20*(16*a^3 + 27
*a^2*b + 30*a*b^2 + 15*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
```

```
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7 - 20*(32*a^3 + 51*a^2*b + 6*a*b^
2 - 21*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b) - 2*(416*a^4 + 1935*a^3*b + 2445*a^2
*b^2 + 885*a*b^3 - 105*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5 + 10*(256*a^4 + 711*a^3*b + 9*a^
2*b^2 - 243*a*b^3 - 21*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) - 20*(16*a^5 - 127*a
^4*b - 564*a^3*b^2 - 342*a^2*b^3 + 36*a*b^4 + 21*b^5)*(sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 - 20*(
160*a^5 + 703*a^4*b + 420*a^3*b^2 - 198*a^2*b^3 - 60*a*b^4 + 15*b^5)*(sqrt(
a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b))^2*sqrt(a + b) + 5*(576*a^6 + 2459*a^5*b + 967*a^4*b^2 - 1378*a^3*b^3
 - 258*a^2*b^4 + 183*a*b^5 - 21*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) - (768*a^6 + 2691*a^5*b -
 1477*a^4*b^2 - 1810*a^3*b^3 + 1350*a^2*b^4 - 225*a*b^5 + 15*b^6)*sqrt(a +
b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^5*a^3*sgn(cos(
f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.107 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/3*(3*a+4*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)^3/a
/f/(a+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+4*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e
)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4219, 464, 277, 197}

$$-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a + 4*b)*Cos[e + f*x])/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Cos[e
+ f*x]^3/(3*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(3*a + 4*b)*Sec[e + f*x]
)/(3*a^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a+4b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx\right.}{3af}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b(3a+4b))\text{S}}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\,\text{se}}{3a^3f\sqrt{a+b\,\text{se}}}$$

Mathematica [A]
time = 3.70, size = 93, normalized size = 0.82

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(9a^2+64ab+64b^2+8a(a+2b)\cos(2(e+fx))-a^2\cos(4(e+fx))\right)\sec^3(e+fx)}{48a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a + 2
*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(a^3*f*(a + b*
Sec[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 12781 vs. $2(102) = 204$.
time = 0.42, size = 12782, normalized size = 112.12

method	result	size

default	Expression too large to display	12782

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 151, normalized size = 1.32

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2}-\frac{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-6\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)}{a^3}+\frac{3b}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,a^2\cos\left(fx+e\right)}+\frac{3b^2}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,a^3\cos\left(fx+e\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^3
+ 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/cos
(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

Fricas [A]
time = 1.72, size = 104, normalized size = 0.91

$$\frac{\left(a^2\cos\left(fx+e\right)^5-\left(3\,a^2+4\,ab\right)\cos\left(fx+e\right)^3-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*x
+ e)^2 + a^3*b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(102) = 204.
time = 1.05, size = 852, normalized size = 7.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*((a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(cos(f*x + e)))*tan(1/2*f*
x + 1/2*e)^2/(a^5*b) + (a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(cos(f*x + e
)))/(a^5*b))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + 4*(3*(sqrt(
a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b))^5*b + 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))^4*(4*a + 3*b)*sqrt(a + b) - 2*(sqrt(a + b)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(8
*a^2 + 9*a*b - 3*b^2) - 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(4*a^2 + 9*a*b + b^2)*sqrt(a + b)
+ 3*(16*a^3 + 37*a^2*b + 2*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) - (20*a^3 + 35*a^2*b
 - 30*a*b^2 + 3*b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) - 3*a + b)^3*a^2*sgn(cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.108 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/a^2/f/(a+b*sec(f*x+
e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4219, 277, 197}

$$-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{af}$$
$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A]
time = 1.42, size = 64, normalized size = 1.03

$$-\frac{(a+2b+a\cos(2(e+fx)))(a+4b+a\cos(2(e+fx)))\sec^3(e+fx)}{4a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e +
 f*x]^3)/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.03, size = 59, normalized size = 0.95

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\left(\sec^2(fx+e)\right)}}}{f}$	59
default	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\left(\sec^2(fx+e)\right)}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\left(\sec^2(fx+e)\right)}}}{f}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(f
*x+e)^2)^(1/2))
```

Maxima [A]

time = 0.27, size = 61, normalized size = 0.98

$$-\frac{\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,\cos(fx+e)}{a^{2}}+\frac{b}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,a^{2}\cos(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)^
2)*a^2*cos(f*x + e)))/f
```

Fricas [A]
time = 3.36, size = 72, normalized size = 1.16

$$-\frac{\left(a\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{a^{3}f\cos\left(fx+e\right)^{2}+a^{2}bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [A]
time = 0.51, size = 48, normalized size = 0.77

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}+b}+\frac{b}{\sqrt{a\cos\left(fx+e\right)^{2}+b}}}{a^{2}f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*cos(f*x + e)^2 + b) + b/sqrt(a*cos(f*x + e)^2 + b))/(a^2*f*sgn(cos
(f*x + e)))
```

Mupad [B]
time = 10.87, size = 155, normalized size = 2.50

$$-\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,\left(e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+\frac{b}{{\left(\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(a+2\,a\,e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,e^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+8\,b\,e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}{2\,a^2\,f\,\left(a+2\,a\,e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,e^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(- e*1i - f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i
)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*
4i + f*x*4i) + 8*b*exp(e*2i + f*x*2i)))/(2*a^2*f*(a + 2*a*exp(e*2i + f*x*2i
) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i)))
```

3.109 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f}-\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f-b*s
ec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4219, 390, 385, 213}

$$-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
3/2)*f)) - (b*Sec[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e$$

$$= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e-}{\sqrt{a+b\sec}}\right.}{(a+b)f}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f} - \frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A]
time = 0.84, size = 113, normalized size = 1.41

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(b\sqrt{a+b}+a\tanh^{-1}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+b-a\sin^2(e+fx)}\right)}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*ArcT
anh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*x]
^2]))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(72) = 144.
time = 0.17, size = 1094, normalized size = 13.68

method	result	size
default	Expression too large to display	1094

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e
)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^2+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a*b+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a^2+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a*b+2*(a+b)^(3/2)*
b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)
^(1/2))*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1
)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+
e)^2/(a+b)^(1/2))*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)
*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1
))*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a*b)/cos(f*x
+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(a+b)^(5/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(76) = 152$.

time = 2.29, size = 362, normalized size = 4.52

$$\left[-\frac{2\,(ab+b^2)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-(a^2\cos\left(fx+e\right)^2+ab)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\left((a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f\right)},\ \frac{(a^2\cos\left(fx+e\right)^2+ab)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a+b}\right)-(ab+b^2)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{(a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqr
t(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b
)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3
*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.110 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}-\frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -1/2*(a-2*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(5/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-3/2*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 482, 541, 12, 385, 213}

$$-\frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/((a + b)^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2]) - (3*b*Sec[e + f*x])/(2*(a + b)^2*f*Sqrt[a + b*Sec[e + f
*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+\right.}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a}{}$$

$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+f}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.36, size = 97, normalized size = 0.77

$$-\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\csc^2(e+fx)-(a-2b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sec^3(e+fx)}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]
^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3288 vs. 2(110) = 220.
time = 0.14, size = 3289, normalized size = 26.10

method	result	size
default	Expression too large to display	3289

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/f/(b+a*cos(f*x+e)^2)^2*(cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3-2*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^3+cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3-2*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^3+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^3-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos

```
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a*b^2+3*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1
/2))*a*b^2-2*cos(f*x+e)^2*(a+b)^(5/2)*a+4*cos(f*x+e)^2*(a+b)^(5/2)*b-((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*
a^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(
a+b)^(1/2))*b^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^3
+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^3-6*(a+b)^(5/2
)*b-3*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*
a*b^2-3*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos
(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)
/sin(f*x+e)^2/(a+b)^(1/2))*a*b^2-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(cos(f*x+e)-1))*a*b^2-3*cos(f*x+e)...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(117) = 234.
time = 4.94, size = 571, normalized size = 4.53

$$\left[-\frac{\left((a^2-2ab)\cos(fx+e)^4-(a^2-3ab+2b^2)\cos(fx+e)^2-ab+2b^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2+2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)-2\left((a^2-ab-2b^2)\cos(fx+e)^3+3(ab+b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left((a^4+3a^3b+3a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2a^3b-2ab^3-b^4)f\cos(fx+e)^2-(a^3b+3a^2b^2+3ab^3+b^4)f\right)}, -\frac{\left((a^2-2ab)\cos(fx+e)^4-(a^2-3ab+2b^2)\cos(fx+e)^2-ab+2b^2\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)+\left((a^2-ab-2b^2)\cos(fx+e)^3+3(ab+b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((a^4+3a^3b+3a^2b^2+ab^3)f\cos(fx+e)^4-(a^4+2a^3b-2ab^3-b^4)f\cos(fx+e)^2-(a^3b+3a^2b^2+3ab^3+b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2
 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^
2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)
^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos(f*x +
e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a - b)*arc
tan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
a + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2
 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

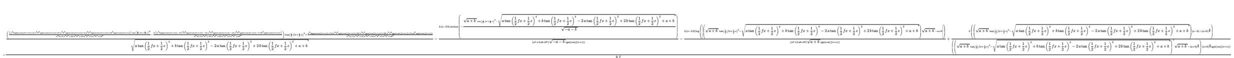
$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(110) = 220.
time = 1.32, size = 903, normalized size = 7.17



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*((((a^5*b*sgn(cos(f*x + e)) + 4*a^4*b^2*sgn(cos(f*x + e)) + 6*a^3*b^3*
sgn(cos(f*x + e)) + 4*a^2*b^4*sgn(cos(f*x + e)) + a*b^5*sgn(cos(f*x + e)))*
tan(1/2*f*x + 1/2*e)^2/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2
*b^5 + a*b^6) - 2*(a^5*b*sgn(cos(f*x + e)) - 2*a^4*b^2*sgn(cos(f*x + e)) -
12*a^3*b^3*sgn(cos(f*x + e)) - 14*a^2*b^4*sgn(cos(f*x + e)) - 5*a*b^5*sgn(c
os(f*x + e)))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*
b^6))*tan(1/2*f*x + 1/2*e)^2 + (a^5*b*sgn(cos(f*x + e)) + 12*a^4*b^2*sgn(co
s(f*x + e)) + 30*a^3*b^3*sgn(cos(f*x + e)) + 28*a^2*b^4*sgn(cos(f*x + e)) +
 9*a*b^5*sgn(cos(f*x + e)))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 +
5*a^2*b^5 + a*b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
```

```
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 4*(a
 - 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b
)*sgn(cos(f*x + e))) - 2*(a - 2*b)*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - a + b
))/((a^2 + 2*a*b + b^2)*sqrt(a + b)*sgn(cos(f*x + e))) + 2*((sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a
 - b) - (a + b)^(3/2))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b) - (a + b)^(3/2))*(a + b
)^(3/2)*sgn(cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+fx)^3\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.111 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f}-\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^3(e+fx)\csc(e+f}{4(a+b)f\sqrt{a+b\sec^2(e+}$$

```
[Out] -3/8*a*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+
b)^(7/2)/f-5/8*a*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1
/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-2*b)*
b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 481, 541, 12, 385, 213}

$$-\frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{5a\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/(8*(a + b)^(7/2)*f) - (5*a*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*S
qrt[a + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*Sqr
t[a + b*Sec[e + f*x]^2]) - ((13*a - 2*b)*b*Sec[e + f*x])/(8*(a + b)^3*f*Sqr
t[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2}{(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+\right.}{4(a+b)f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(}{8(a-}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(}{8(a-}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(}{8(a-}$$

$$= -\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+f}{8(a+b)^2f\sqrt{a+b\sec^2(e+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.52, size = 100, normalized size = 0.56

$$-\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)^2\csc^4(e+fx)-a(a-4b)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sec^3(e+fx)}{8(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*b
)*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e +
f*x]^3)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8267 vs. 2(157) = 314.
time = 0.18, size = 8268, normalized size = 46.71

method	result	size
default	Expression too large to display	8268

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(167) = 334$.
time = 2.53, size = 903, normalized size = 5.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*cos
(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*s
qrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a
^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b
^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*
b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*
b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 +
b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^
2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e
)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a + b)) + (3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5
- (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b
^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*
```

```
a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*
a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b
+ 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1625 vs. $2(157) = 314$.
time = 1.78, size = 1625, normalized size = 9.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(((((a^8*b + 7*a^7*b^2 + 21*a^6*b^3 + 35*a^5*b^4 + 35*a^4*b^5 + 21*a^
3*b^6 + 7*a^2*b^7 + a*b^8)*tan(1/2*f*x + 1/2*e)^2/(a^9*b*sgn(cos(f*x + e))
+ 8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 56*a^6*b^4*s
gn(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn(cos(f*x +
e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sgn(cos(f*x + e)) + a*b^9*sg
n(cos(f*x + e))) + (7*a^8*b + 39*a^7*b^2 + 87*a^6*b^3 + 95*a^5*b^4 + 45*a^4
*b^5 - 3*a^3*b^6 - 11*a^2*b^7 - 3*a*b^8)/(a^9*b*sgn(cos(f*x + e)) + 8*a^8*b
^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 56*a^6*b^4*sgn(cos(f*
x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn(cos(f*x + e)) + 28*
a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sgn(cos(f*x + e)) + a*b^9*sgn(cos(f*x
 + e))))*tan(1/2*f*x + 1/2*e)^2 - (17*a^8*b - 9*a^7*b^2 - 291*a^6*b^3 - 725
*a^5*b^4 - 765*a^4*b^5 - 363*a^3*b^6 - 49*a^2*b^7 + 9*a*b^8)/(a^9*b*sgn(cos
(f*x + e)) + 8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 5
6*a^6*b^4*sgn(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn
(cos(f*x + e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sgn(cos(f*x + e))
 + a*b^9*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (9*a^8*b + 113*a^7*b^
2 + 425*a^6*b^3 + 745*a^5*b^4 + 675*a^4*b^5 + 299*a^3*b^6 + 43*a^2*b^7 - 5*
a*b^8)/(a^9*b*sgn(cos(f*x + e)) + 8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*
sgn(cos(f*x + e)) + 56*a^6*b^4*sgn(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x +
 e)) + 56*a^4*b^6*sgn(cos(f*x + e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*
b^8*sgn(cos(f*x + e)) + a*b^9*sgn(cos(f*x + e))))/sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
```

```
f*x + 1/2*e)^2 + a + b) - 12*(a^2 - 4*a*b)*sqrt(a + b)*log(abs(-(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)*(a + b) + sqrt(a + b)*(a - b)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*sgn(cos(f*x + e))) - 24*(a^2 - 4*a*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b)
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a - b)*sgn(cos(f*x + e))) + 4*(2*(
sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^3*(2*a^2 - 4*a*b + b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(3*a^2 + 2*a*b - b^2)*
sqrt(a + b) - 2*(3*a^3 - a^2*b - 2*a*b^2 + 2*b^3)*(sqrt(a + b)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (5*a^3 + 7
*a^2*b - a*b^2 - 3*b^3)*sqrt(a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
 a + b))^2 - a - b)^2*sgn(cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+fx)^5 \left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.112 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{5(a+b)^2(a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f}-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(9a+}{24a^2}$$

```
[Out] 5/16*(a+b)^2*(a+7*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(9/2)/f-1/48*(a+b)*(33*a+35*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x
+e)^2)^(1/2)+1/24*(9*a+7*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)
^2)^(1/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/48
*b*(81*a^2+190*a*b+105*b^2)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 481, 592, 541, 12, 385, 209}

$$\frac{5(a+b)^2(a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}-\frac{(a+b)(33a+35b)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(9a+7b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (5*(a + b)^2*(a + 7*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*a^(9/2)*f) - ((a + b)*(33*a + 35*b)*Cos[e + f*x]*Sin[e + f*x
])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((9*a + 7*b)*Cos[e + f*x]^3*
Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*S
in[e + f*x]^3)/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(81*a^2 + 190*a*
b + 105*b^2)*Tan[e + f*x])/(48*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+2(b-3(a+b))x^2)}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e \right.}{6af}$$

$$= \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}}{}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e}{24a^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e}{24a^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e}{24a^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e}{24a^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= \frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)}{48a^3f\sqrt{a+b}}$$

Mathematica [A]

time = 8.81, size = 256, normalized size = 1.06

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(120(a+b)^2(a+7b)\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\left(37a^3+439a^2b+830ab^2+420b^3+a(29a^2+108ab+70b^2)\cos(2(e+fx))-7a^2(a+b)\cos(4(e+fx))+a^3\cos(6(e+fx))\right)\sin(e+fx)\right)}{1536a^{9/2}\sqrt{a+b}\,f\left(a+b\sec^2(e+fx)\right)^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*
```

```
Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(3
7*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)*Cos
[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*Sin[
e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a
 + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.44, size = 2437, normalized size = 10.07

method	result	size
default	Expression too large to display	2437

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*(b+a*cos(f*x+e)^2)*(-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-14*cos(f*x+e
)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*cos(f*x+e)^4*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+68*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^2*b+35*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2-68*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-35
*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+81*cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+190*cos(f*x+e)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-210*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+15*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+105*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-81*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^2*b-190*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-270*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
```

```
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-450*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+135*2
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+225*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-105*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-26*
cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^3+105*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*sin
(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 20.24, size = 850, normalized size = 3.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2
*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 7*a^
3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e)^3 + (81
*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), -1/192*(15
*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2*b^2 + 7*a*
b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*
b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^
3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(f*x + e)^7 - 2*(
13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x
 + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.113 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{3(a+b)(a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f}-\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\cos^3(e+fx)\,\text{s}}{4af\sqrt{a+b+b}}$$

```
[Out] 3/8*(a+b)*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(
7/2)/f-5/8*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+15*b)*t
an(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 481, 541, 12, 385, 209}

$$\frac{3(a+b)(a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*a^(7/2)*f) - (5*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*Sqr
t[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*Sqrt[a
+ b + b*Tan[e + f*x]^2]) - (b*(13*a + 15*b)*Tan[e + f*x])/(8*a^3*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b-4(a+b)x^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{4af}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(}{8a^3f}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(}{8a^3f}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(}{8a^3f}$$

$$= \frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+f}{8a^2f\sqrt{a+b+b\,t}}$$

Mathematica [A]
time = 4.00, size = 229, normalized size = 1.31

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(24(a^2+6ab+5b^2)\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,(7a^2+62ab+60b^2+2a(3a+5b)\cos(2(e+fx))-a^2\cos(4(e+fx)))\sin(e+fx)\right)}{256a^{7/2}\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(
7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
```

```
f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3
/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1714, normalized size = 9.79

method	result	size
default	Expression too large to display	1714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^2-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-18*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+36*2
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*a*b*sin(f*x+e)+30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-5*cos(f*x+e
```

```
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-
13*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-15*cos(f*x+e)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*sin(f*x+e)/(co
s(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 10.82, size = 738, normalized size = 4.22

$$\left[-\frac{3\left(a^{2} b+6 a b^{2}+5 b^{3}+\left(a^{3}+6 a^{2} b+5 a b^{2}\right) \cos\left(f x+e\right)^{2}\right) \sqrt{-a} \log\left(128 a^{4} \cos\left(f x+e\right)^{8}-256\left(a^{4}-a^{3} b\right) \cos\left(f x+e\right)^{6}+32\left(5 a^{4}-14 a^{3} b+5 a^{2} b^{2}\right) \cos\left(f x+e\right)^{4}+a^{4}-28 a^{3} b+70 a^{2} b^{2}-28 a b^{3}+b^{4}-32\left(a^{4}-7 a^{3} b+7 a^{2} b^{2}-a b^{3}\right) \cos\left(f x+e\right)^{2}+8\left(16 a^{3} \cos\left(f x+e\right)^{7}-24\left(a^{3}-a^{2} b\right) \cos\left(f x+e\right)^{5}+2\left(5 a^{3}-14 a^{2} b+5 a b^{2}\right) \cos\left(f x+e\right)^{3}-\left(a^{3}-7 a^{2} b+7 a b^{2}-b^{3}\right) \cos\left(f x+e\right)\right) \sqrt{-a} \sqrt{\frac{a \cos\left(f x+e\right)^{2}+b}{\cos\left(f x+e\right)^{2}}} \sin\left(f x+e\right)\right)-8\left(2 a^{3} \cos\left(f x+e\right)^{5}-5\left(a^{3}+a^{2} b\right) \cos\left(f x+e\right)^{3}-\left(13 a^{2} b+15 a b^{2}\right) \cos\left(f x+e\right)\right) \sqrt{\frac{a \cos\left(f x+e\right)^{2}+b}{\cos\left(f x+e\right)^{2}}} \sin\left(f x+e\right)}{64\left(a^{5} f \cos\left(f x+e\right)^{2}+a^{4} b f\right)}, -\frac{3\left(a^{2} b+6 a b^{2}+5 b^{3}+\left(a^{3}+6 a^{2} b+5 a b^{2}\right) \cos\left(f x+e\right)^{2}\right) \sqrt{a} \arctan\left(\frac{\left(8 a^{2} \cos\left(f x+e\right)^{5}-8\left(a^{2}-a b\right) \cos\left(f x+e\right)^{3}+\left(a^{2}-6 a b+b^{2}\right) \cos\left(f x+e\right)\right) \sqrt{a} \sqrt{\frac{a \cos\left(f x+e\right)^{2}+b}{\cos\left(f x+e\right)^{2}}}}{4\left(2 a^{3} \cos\left(f x+e\right)^{4}-a^{2} b+a b^{2}-\left(a^{3}-3 a^{2} b\right) \cos\left(f x+e\right)^{2}\right) \sin\left(f x+e\right)}\right)-4\left(2 a^{3} \cos\left(f x+e\right)^{5}-5\left(a^{3}+a^{2} b\right) \cos\left(f x+e\right)^{3}-\left(13 a^{2} b+15 a b^{2}\right) \cos\left(f x+e\right)\right) \sqrt{\frac{a \cos\left(f x+e\right)^{2}+b}{\cos\left(f x+e\right)^{2}}} \sin\left(f x+e\right)}{32\left(a^{5} f \cos\left(f x+e\right)^{2}+a^{4} b f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)
^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b +
 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b^2 + 5*b^3
 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b + 15*a*b^2)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.114 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{(a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan(f*x+e)/
a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 482, 541, 12, 385, 209}

$$\frac{(a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - (3*b*Tan[e + f*x])/(2*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3}{}$$

$$= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+f}}$$

Mathematica [A]
time = 1.35, size = 190, normalized size = 1.57

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(4(a+3b)\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}(a+6b+a\cos(2(e+fx)))\sin(e+fx)\right)}{32a^{5/2}\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sqrt
[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b + a*C
os[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*
x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 1069, normalized size = 8.83

method	result	size
default	Expression too large to display	1069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(cos(f*x+e)-1)/cos
(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(112) = 224.

time = 5.13, size = 640, normalized size = 5.29

$$\left[-\frac{\left(\left(a^{2}+3\,ab\right)\cos\left(fx+e\right)^{2}+ab+3\,b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\,\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\,\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)+8\,\left(a^{2}\cos\left(fx+e\right)^{3}+3\,ab\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{16\,\left(a^{4}f\cos\left(fx+e\right)^{2}+a^{3}bf\right)},\ -\frac{\left(\left(a^{2}+3\,ab\right)\cos\left(fx+e\right)^{2}+ab+3\,b^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\,\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)+4\,\left(a^{2}\cos\left(fx+e\right)^{3}+3\,ab\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{8\,\left(a^{4}f\cos\left(fx+e\right)^{2}+a^{3}bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)

[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.115 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4213, 390, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x,\right.}{af}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b+b\,\text{ta}}}\right.}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.
time = 1.46, size = 168, normalized size = 2.18

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\,\sqrt{a}\,b\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\sin(e+fx)\right)}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 1007, normalized size = 13.08

method	result
default	$-\frac{\left(b+a\left(\cos^2(fx+e)\right)\right)\left(\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}\,-i\sqrt{a}\,\sqrt{b}\,+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}\,-i\sqrt{a}\right.}{(1+\cos(fx+e))(}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*s
in(f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b)*sin(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(3/2)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. $2(73) = 146$.
time = 0.70, size = 2169, normalized size = 28.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
```

```
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(73) = 146$.
time = 3.02, size = 632, normalized size = 8.21

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{8\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)},-\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{4\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.116 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/(a+b)^2/f/(a+
b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4217, 277, 197}

$$-\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A]
time = 1.79, size = 76, normalized size = 1.12

$$-\frac{(a+2b+a\cos(2(e+fx)))(a+3b+(a-b)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]*Sec[e + f*x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.14, size = 89, normalized size = 1.31

method	result	size
default	$-\frac{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+2b\right)\left(\cos^3(fx+e)\right)\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{3}{2}}}{f\left(b+a\left(\cos^2(fx+e)\right)\right)^2\sin(fx+e)(a+b)^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/f/(b+a*cos(f*x+e)^2)^2*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)*cos(f*x+e)^3*
((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/(a+b)^2
```

Maxima [A]
time = 0.27, size = 68, normalized size = 1.00

$$-\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^2}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + 1/(sqrt(b*t
an(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)))/f
```

Fricas [A]
time = 2.73, size = 108, normalized size = 1.59

$$-\frac{\left((a-b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{\left((a^{3}+2\,a^{2}b+ab^{2})f\cos\left(fx+e\right)^{2}+(a^{2}b+2\,ab^{2}+b^{3})f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B]
time = 12.12, size = 2151, normalized size = 31.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] -((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*
2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*4i + f*x*4i)*(((a + 4*b)*(((a
 + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a +
3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(
a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a
+ 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3
- ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b +
a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (
a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b
 + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b
 + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a
*b^2 + a^2*b))))/a + ((a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2)
)/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i
)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*
b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*
b))))/a + (((a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b)
 - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*
(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3
*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3
*(a + 3*b)*1i)/(4*f*(a*b + a^2)*(a*b^2 + a^2*b))) + exp(e*2i + f*x*2i)*((((
a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b
)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - ((a + 4*b)*(((a + 4*
b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^
2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3
*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b
)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3*b)^3 - ((a
 + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))
*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a - b) - (a + 3
*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^
2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^
2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2
+ a^2*b))))/a + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (
a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(4*f*(a*b^2 + a
^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (
a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))) - ((a + 4*b)
*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*
(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b
)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*
(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a + (((a + 3*b)^3 - ((a +
 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1
i)/(4*f*(a*b^2 + a^2*b)*(a + 3*b)) + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b
```

```
)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2)
)*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)
*(a*b^2 + a^2*b)) - (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 +
a^2*b))))/((exp(e*2i + f*x*2i) + 1)*(a - a*exp(e*6i + f*x*6i) + exp(e*2i +
f*x*2i)*(a + 4*b) - exp(e*4i + f*x*4i)*(a + 4*b)))
```

3.117 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{2(3a-b)b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -1/3*(3*a-b)*cot(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)
^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-b)*b*tan(f*x+e)/(a+b)^3/f/(a
+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4217, 464, 277, 197}

$$-\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a - b)*Cot[e + f*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
- Cot[e + f*x]^3/(3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (2*(3*a - b
)*b*Tan[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{3/2}}\,dx,\right.}{3(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx}}$$

Mathematica [A]
time = 0.72, size = 102, normalized size = 0.83

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-2a(a-3b)+(a^2-2ab-3b^2)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)\right)\sec^2(e+fx)\tan(e+fx)}{6(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^2
)*Csc[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])/(
(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.16, size = 137, normalized size = 1.11

method	result

default	$\frac{\left(2\left(\cos^4(fx+e)\right)a^2-6\left(\cos^4(fx+e)\right)ab-3\left(\cos^2(fx+e)\right)a^2+10\left(\cos^2(fx+e)\right)ab-3\left(\cos^2(fx+e)\right)b^2-6ab+2b^2\right)\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{3}{2}}(\mathrm{co}}{3f\left(b+a\left(\cos^2(fx+e)\right)\right)^2\sin(fx+e)^3(a+b)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(b+a*cos(f*x+e)^2)^2*(2*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4*a*b-3*cos(f*x
+e)^2*a^2+10*cos(f*x+e)^2*a*b-3*cos(f*x+e)^2*b^2-6*a*b+2*b^2)*((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3/sin(f*x+e)^3/(a+b)^3
```

Maxima [A]
time = 0.27, size = 166, normalized size = 1.35

$$-\frac{\frac{6\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}-\frac{8\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{3}}+\frac{3}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)\tan\left(fx+e\right)}-\frac{4\,b}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}\tan\left(fx+e\right)}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 8*b^2*t
an(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 3/(sqrt(b*tan(f*x
+ e)^2 + a + b)*(a + b)*tan(f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a + b)
*(a + b)^2*tan(f*x + e)) + 1/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)^3))/f
```

Fricas [A]
time = 4.84, size = 197, normalized size = 1.60

$$-\frac{\left(2\left(a^{2}-3\,ab\right)\cos\left(fx+e\right)^{5}-\left(3\,a^{2}-10\,ab+3\,b^{2}\right)\cos\left(fx+e\right)^{3}-2\left(3\,ab-b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{4}-\left(a^{4}+2\,a^{3}b-2\,ab^{3}-b^{4}\right)f\cos\left(fx+e\right)^{2}-\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x + e
)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f
)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B]
time = 27.53, size = 2500, normalized size = 20.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (a^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((
cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128i)
/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 32*a
^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(cos(
4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x) + s
in(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2*f*x)*
1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4
*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(
6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i
) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 128
*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) - 128*a^3*b*
f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a*b^3*f*(cos(
6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f*(cos(6*f*x)
+ sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*f*x) + sin(4*f*
x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x) + sin(6*f*x)*1i
)*(cos(6*e) + sin(6*e)*1i))) - (5*a*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(
cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(
2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*3i + a^2*b*f*3
i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - b^3*f*
(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*b^2*f*(cos(4*f
*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(cos(4*f*x) + si
```

```
n(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (5*b*(a + b/(((cos(2*f*x) - si
n(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(c
os(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b^2*f*
3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1
i)*1i - b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*1i - a*
b^2*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i - a^2*b*f*(c
os(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i)*3i)) - (a^8*(a + b/(((c
os(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin
(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*1i)/(4*(a^3*b^7*f + 6
*a^4*b^6*f + 15*a^5*b^5*f + 20*a^6*b^4*f + 15*a^7*b^3*f + 6*a^8*b^2*f + a^9
*b*f + 2*a^9*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 2*
a^9*b*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^9*b*f*(co
s(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) + 4*a^2*b^8*f*(cos(2*f*x
) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 26*a^3*b^7*f*(cos(2*f*x) + si
n(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 72*a^4*b^6*f*(cos(2*f*x) + sin(2*f*
x)*1i)*(cos(2*e) + sin(2*e)*1i) + 110*a^5*b^5*f*(cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i) + 100*a^6*b^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(co
s(2*e) + sin(2*e)*1i) + 54*a^7*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e)
 + sin(2*e)*1i) + 16*a^8*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin
(2*e)*1i) - 4*a^2*b^8*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1
i) - 26*a^3*b^7*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 7
2*a^4*b^6*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 110*a^5
*b^5*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 100*a^6*b^4*
f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 54*a^7*b^3*f*(cos
(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 16*a^8*b^2*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - a^3*b^7*f*(cos(8*f*x) + sin(8
*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 6*a^4*b^6*f*(cos(8*f*x) + sin(8*f*x)*1
i)*(cos(8*e) + sin(8*e)*1i) - 15*a^5*b^5*f*(cos(8*f*x) + sin(8*f*x)*1i)*(co
s(8*e) + sin(8*e)*1i) - 20*a^6*b^4*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e)
 + sin(8*e)*1i) - 15*a^7*b^3*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin
(8*e)*1i) - 6*a^8*b^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1
i))) - ((a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 +
 ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*32
i)/(24*a^2*f + 24*b^2*f + 48*a*b*f - 48*a^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*
(cos(2*e) + sin(2*e)*1i) + 48*a^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e)
+ sin(6*e)*1i) - 24*a^2*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)
*1i) - 48*b^2*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 48*
b^2*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 24*b^2*f*(cos
(8*f*x) + sin(8*f*x)*1i)*(cos(8*e) + sin(8*e)*1i) - 96*a*b*f*(cos(2*f*x) +
sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) + 96*a*b*f*(cos(6*f*x) + sin(6*f*x)
*1i)*(cos(6*e) + sin(6*e)*1i) - 48*a*b*f*(cos(8*f*x) + sin(8*f*x)*1i)*(cos(
8*e) + sin(8*e)*1i)) + (b^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e)
 - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i)
)/4 + 1/2))^(1/2)*128i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3
*f + 128*a^3*b*f - 32*a^4*f*(cos(2*f*x) + sin(2...
```

3.118 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -1/15*(15*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2
/15*(5*a+2*b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f*x
+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-10*a*b-b^2)*tan(f*x
+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 473, 464, 277, 197}

$$-\frac{2b(15a^2-10ab-b^2)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{2(5a+2b)\cot^3(e+fx)}{15f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15*a^2 - 10*a*b - b^2)*Cot[e + f*x])/((a + b)^3*f*Sqrt[a + b + b*Ta
n[e + f*x]^2]) - (2*(5*a + 2*b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2]) - Cot[e + f*x]^5/(5*(a + b)*f*Sqrt[a + b + b*Tan[e + f
*x]^2]) - (2*b*(15*a^2 - 10*a*b - b^2)*Tan[e + f*x])/(15*(a + b)^4*f*Sqrt[a
 + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a+2b)+5(a+b)x^2}{x^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(\right.}{5(a+b)f}$$

$$= -\frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx}}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

Mathematica [A]

time = 1.01, size = 126, normalized size = 0.69

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-8a^2(a-5b)+4a(a^2-4ab-5b^2)\csc^2(e+fx)+(a-5b)(a+b)^2\csc^4(e+fx)+3(a+b)^3\csc^6(e+fx)\right)\sec^2(e+fx)\tan(e+fx)}{30(a+b)^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b
- 5*b^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^3*
Csc[e + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/((a + b)^4*f*(a + b*Sec[e + f*
x]^2)^(3/2))
```

Maple [A]
time = 0.24, size = 204, normalized size = 1.11

method	result
default	$-\frac{\left(8\left(\cos^6(fx+e)\right)a^3-40\left(\cos^6(fx+e)\right)a^2b-20\left(\cos^4(fx+e)\right)a^3+104\left(\cos^4(fx+e)\right)a^2b-20\left(\cos^4(fx+e)\right)a\,b^2+15\left(\cos^2(fx+e)\right)a^3-85\right.}{15f\left(b+a\left(\cos^2(fx+e)\right)\right)^2\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(b+a*cos(f*x+e)^2)^2*(8*cos(f*x+e)^6*a^3-40*cos(f*x+e)^6*a^2*b-20*c
os(f*x+e)^4*a^3+104*cos(f*x+e)^4*a^2*b-20*cos(f*x+e)^4*a*b^2+15*cos(f*x+e)^
2*a^3-85*cos(f*x+e)^2*a^2*b+49*cos(f*x+e)^2*a*b^2+5*cos(f*x+e)^2*b^3+30*a^2
*b-20*a*b^2-2*b^3)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin
(f*x+e)^5/(a+b)^4
```

Maxima [A]
time = 0.28, size = 300, normalized size = 1.64

$$-\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2}-\frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^3}+\frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^4}+\frac{15}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)}-\frac{40\,b}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2\tan(fx+e)}+\frac{24\,b^2}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^3\tan(fx+e)}+\frac{10}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)^3}-\frac{6\,b}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2\tan(fx+e)^3}+\frac{3}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 80*b^
2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 48*b^3*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) + 15/(sqrt(b*tan(f*x + e)^2
 + a + b)*(a + b)*tan(f*x + e)) - 40*b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
 b)^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(
f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^3) - 6*
b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)^3) + 3/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^5))/f
```

Fricas [A]
time = 8.83, size = 324, normalized size = 1.77

$$-\frac{\left(8\left(a^{3}-5a^{2}b\right)\cos\left(fx+e\right)^{7}-4\left(5a^{3}-26a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{5}+\left(15a^{3}-85a^{2}b+49ab^{2}+5b^{3}\right)\cos\left(fx+e\right)^{3}+2\left(15a^{2}b-10ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\left(\left(a^{5}+4a^{4}b+6a^{3}b^{2}+4a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{6}-\left(2a^{5}+7a^{4}b+8a^{3}b^{2}+2a^{2}b^{3}-2ab^{4}-b^{5}\right)f\cos\left(fx+e\right)^{4}+\left(a^{5}+2a^{4}b-2a^{3}b^{2}-8a^{2}b^{3}-7ab^{4}-2b^{5}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b+4a^{3}b^{2}+6a^{2}b^{3}+4ab^{4}+b^{5}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*co
s(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*(1
5*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)
^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x +
e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{6}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.119 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}}+\frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}}-\frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}-\frac{4b(5a^2+2\cdots}{15a^4f\,(\cdots}$$

```
[Out] -1/5*(5*a^2+20*a*b+16*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/2)+2/15*(
5*a+4*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^5/a/f/(
a+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^4/f/(a+b*
sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^5/f/(a+b*sec(
f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 473, 464, 277, 198, 197}

$$\frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}}-\frac{\left(\frac{4b(5a+4b)}{a^2}+5\right)\cos(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}-\frac{8b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\,(a+b\sec^2(e+fx))^{3/2}}-\frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5 + (4*b*(5*a + 4*b))/a^2)*Cos[e + f*x])/(a*f*(a + b*Sec[e + f*x]^2)
^(3/2)) + (2*(5*a + 4*b)*Cos[e + f*x]^3)/(15*a^2*f*(a + b*Sec[e + f*x]^2)^(
3/2)) - Cos[e + f*x]^5/(5*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 +
 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^4*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (
8*b*(5*a^2 + 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^5*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a+4b)+5ax^2}{x^4(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(5a^2+20ab}{}$$

$$= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{}{5af\,(}$$

$$= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{}{5af\,(}$$

$$= -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{}{5af\,(}$$

Mathematica [A]
time = 3.44, size = 182, normalized size = 0.83

$$-\frac{(a+2b+a\cos(2(e+fx)))\,(425a^4+6400a^3b+22784a^2b^2+32768ab^3+16384b^4+48a(11a^3+150a^2b+384ab^2+256b^3)\cos(2(e+fx))+12a^2(7a^2+64ab+64b^2)\cos(4(e+fx))-16a^4\cos(6(e+fx))-32a^3b\cos(6(e+fx))+3a^4\cos(8(e+fx)))\sec^5(e+fx)}{3840a^5f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3840*((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*b
^2 + 32768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256*b
^3)*Cos[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)] -
16*a^4*Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e + f*x)
])*Sec[e + f*x]^5)/(a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 1.22, size = 229, normalized size = 1.05

method	result
default	$\frac{(a+b)^7\left(b+a\left(\cos^2(fx+e)\right)\right)\left(3\left(\cos^8(fx+e)\right)a^4-10\left(\cos^6(fx+e)\right)a^4-8\left(\cos^6(fx+e)\right)a^3b+15\left(\cos^4(fx+e)\right)a^4+60\left(\cos^4(fx+e)\right)a^3b+4\right.}{30f\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(a+b)^7*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^8*a^4-10*cos(f*x+e)^6*a^4-8
*cos(f*x+e)^6*a^3*b+15*cos(f*x+e)^4*a^4+60*cos(f*x+e)^4*a^3*b+48*cos(f*x+e)
^4*a^2*b^2+60*cos(f*x+e)^2*a^3*b+240*cos(f*x+e)^2*a^2*b^2+192*cos(f*x+e)^2*
a*b^3+40*a^2*b^2+160*a*b^3+128*b^4)*4^(1/2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^
2)^(5/2)/cos(f*x+e)^5*a^2/((-a*b)^(1/2)+a)^7/((-a*b)^(1/2)-a)^7
```

Maxima [A]
time = 0.29, size = 358, normalized size = 1.63

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^4}+\frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-20\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+90\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b^2\cos(fx+e)}{a^5}+\frac{5\left(6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2\right)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}+\frac{10\left(9\left(a+\frac{b}{\cos(fx+e)^2}\right)b^2\cos(fx+e)^2-b^3\right)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^4\cos(fx+e)^3}+\frac{5\left(12\left(a+\frac{b}{\cos(fx+e)^2}\right)b^3\cos(fx+e)^2-b^4\right)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^5\cos(fx+e)^3}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*x +
 e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x +
 e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(
f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*b^2*co
s(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3) + 5*(
12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x + e)^2)
^(3/2)*a^5*cos(f*x + e)^3))/f
```

Fricas [A]
time = 4.34, size = 198, normalized size = 0.90

$$-\frac{\left(3\,a^4\cos(fx+e)^9-2\left(5\,a^4+4\,a^3b\right)\cos(fx+e)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos(fx+e)^5+12\left(5\,a^3b+20\,a^2b^2+16\,ab^3\right)\cos(fx+e)^3+8\left(5\,a^2b^2+20\,ab^3+16\,b^4\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\left(a^7f\cos(fx+e)^4+2\,a^6bf\cos(fx+e)^2+a^5b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^4
 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*a*
b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*co
s(f*x + e)^2 + a^5*b^2*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. $2(199) = 398$.
time = 2.03, size = 1638, normalized size = 7.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*((((6*a^16*b^3*sgn(cos(f*x + e)) + 23*a^15*b^4*sgn(cos(f*x + e)) +
 28*a^14*b^5*sgn(cos(f*x + e)) + 11*a^13*b^6*sgn(cos(f*x + e)))*tan(1/2*f*x
 + 1/2*e)^2/(a^18*b^2) - 3*(2*a^16*b^3*sgn(cos(f*x + e)) + a^15*b^4*sgn(cos
(f*x + e)) - 12*a^14*b^5*sgn(cos(f*x + e)) - 11*a^13*b^6*sgn(cos(f*x + e)))
/(a^18*b^2))*tan(1/2*f*x + 1/2*e)^2 - 3*(2*a^16*b^3*sgn(cos(f*x + e)) + a^1
5*b^4*sgn(cos(f*x + e)) - 12*a^14*b^5*sgn(cos(f*x + e)) - 11*a^13*b^6*sgn(c
os(f*x + e)))/(a^18*b^2))*tan(1/2*f*x + 1/2*e)^2 + (6*a^16*b^3*sgn(cos(f*x
+ e)) + 23*a^15*b^4*sgn(cos(f*x + e)) + 28*a^14*b^5*sgn(cos(f*x + e)) + 11*
a^13*b^6*sgn(cos(f*x + e)))/(a^18*b^2))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b)^(3/2) + 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b + 3*b^2) + 15*(sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*
(22*a*b + 21*b^2)*sqrt(a + b) + 20*(16*a^3 + 38*a^2*b + 63*a*b^2 + 45*b^3)*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^7 - 20*(32*a^3 + 118*a^2*b + 51*a*b^2 - 63*b^3)*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6
*sqrt(a + b) - 2*(416*a^4 + 2590*a^3*b + 4665*a^2*b^2 + 2640*a*b^3 - 315*b^
4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))^5 + 10*(256*a^4 + 1262*a^3*b + 405*a^2*b^2 - 648*a*b^3 - 63
*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b))^4*sqrt(a + b) - 20*(16*a^5 - 110*a^4*b - 911*a^3*b^2 - 8
37*a^2*b^3 + 75*a*b^4 + 63*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 - 20*(160*a^5 + 1134*a^4*b +
 1019*a^3*b^2 - 375*a^2*b^3 - 183*a*b^4 + 45*b^5)*(sqrt(a + b)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
```

```
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b
) + 5*(576*a^6 + 4022*a^5*b + 2825*a^4*b^2 - 2696*a^3*b^3 - 722*a^2*b^4 + 5
22*a*b^5 - 63*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b)) - (768*a^6 + 4806*a^5*b - 907*a^4*b^2 - 492
0*a^3*b^3 + 3190*a^2*b^4 - 630*a*b^5 + 45*b^6)*sqrt(a + b))/(((sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^
2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^5*a^4*sgn(cos(f*x + e))))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.120 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{8b(a+2b)\sec(}{3a^4f\sqrt{a+b\sec^2}}$$

```
[Out] -(a+2*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/a/f/(a+
b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+2*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/
2)-8/3*b*(a+2*b)*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4219, 464, 277, 198, 197}

$$-\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + 2*b)*Cos[e + f*x])/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))) + Cos[e +
f*x]^3/(3*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + 2*b)*Sec[e + f*x])/
(3*a^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a + 2*b)*Sec[e + f*x])/(3*a^
4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)}{3af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(4b(a+2b))\text{Su}}{}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)}{3af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b(a+2b)\,\text{s}}{3a^3f\,(a+b\sec}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)}{3af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b(a+2b)\,\text{s}}{3a^3f\,(a+b\sec}$$

Mathematica [A]
time = 2.52, size = 129, normalized size = 0.88

$$-\frac{(a+2b+a\cos(2(e+fx)))\,(26a^3+264a^2b+640ab^2+512b^3+3a(11a^2+96ab+128b^2)\cos(2(e+fx))+6a^2(a+4b)\cos(4(e+fx))-a^3\cos(6(e+fx)))\sec^5(e+fx)}{192a^4f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 51
2*b^3 + 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b)*
Cos[4*(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(a^4*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.52, size = 159, normalized size = 1.09

method	result
default	$-\frac{(a+b)^5\left(b+a\left(\cos^2(fx+e)\right)\right)\left(\left(\cos^6(fx+e)\right)a^3-3\left(\cos^4(fx+e)\right)a^3-6\left(\cos^4(fx+e)\right)a^2b-12\left(\cos^2(fx+e)\right)a^2b-24\left(\cos^2(fx+e)\right)a\,b^2-\right.}{6f\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5\left(\sqrt{-ab}-a\right)^5\left(\sqrt{-ab}+a\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(a+b)^5*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^6*a^3-3*cos(f*x+e)^4*a^3-6*co
s(f*x+e)^4*a^2*b-12*cos(f*x+e)^2*a^2*b-24*cos(f*x+e)^2*a*b^2-8*a*b^2-16*b^3
)*4^(1/2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5*a/((-a*b)^(1
/2)-a)^5/((-a*b)^(1/2)+a)^5
```

Maxima [A]
time = 0.27, size = 209, normalized size = 1.43

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^4}+\frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}+\frac{9\left(a+\frac{b}{\cos(fx+e)^2}\right)b^2\cos(fx+e)^2-b^3}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^4\cos(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^4
+ (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)
^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 -
 b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

Fricas [A]
time = 5.98, size = 146, normalized size = 1.00

$$\frac{\left(a^3\cos(fx+e)^7-3\left(a^3+2\,a^2b\right)\cos(fx+e)^5-12\left(a^2b+2\,ab^2\right)\cos(fx+e)^3-8\left(ab^2+2\,b^3\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^6f\cos(fx+e)^4+2\,a^5bf\cos(fx+e)^2+a^4b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2*
a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 +
 a^4*b^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(132) = 264$.
time = 1.46, size = 1017, normalized size = 6.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(((((3*a^12*b^3*sgn(cos(f*x + e)) + 7*a^11*b^4*sgn(cos(f*x + e)) + 4*a
^10*b^5*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^14*b^2) - 3*(a^12*b^3*
sgn(cos(f*x + e)) - a^11*b^4*sgn(cos(f*x + e)) - 4*a^10*b^5*sgn(cos(f*x + e
)))/(a^14*b^2))*tan(1/2*f*x + 1/2*e)^2 - 3*(a^12*b^3*sgn(cos(f*x + e)) - a^
11*b^4*sgn(cos(f*x + e)) - 4*a^10*b^5*sgn(cos(f*x + e)))/(a^14*b^2))*tan(1/
2*f*x + 1/2*e)^2 + (3*a^12*b^3*sgn(cos(f*x + e)) + 7*a^11*b^4*sgn(cos(f*x +
 e)) + 4*a^10*b^5*sgn(cos(f*x + e)))/(a^14*b^2))/(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b)^(3/2) + 4*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*b + 3*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(2*a +
3*b)*sqrt(a + b) - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(4*a^2 + 9*a*b - 3*b^2) - 6*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^2*(2*a^2 + 7*a*b + b^2)*sqrt(a + b) + 3*(8*a^3 + 29*a^2*b + 2*a*b^2 - 3*b
^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b)) - (10*a^3 + 31*a^2*b - 24*a*b^2 + 3*b^3)*sqrt(a + b))/(((s
```

```
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^2 + 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - 3*a + b)^3*a^3*sgn(cos(f*x + e)
)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.121 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/a^2/f/(a+b*sec(f*
x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4219, 277, 198, 197}

$$-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e + f*x])/(3*
a^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x])/(3*a^3*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af} \\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(8b)\text{Subst}\Big(\int \cdots}{\cdots} \\
&= -\frac{\cos(e+fx)}{af\,(a+b\sec^2(e+fx))^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{8b\sec(\cdots}{3a^3f\sqrt{a+b\cdots}}
\end{aligned}$$

Mathematica [A]
time = 1.42, size = 88, normalized size = 0.91

$$-\frac{(a+2b+a\cos(2(e+fx)))\left((3a+8b)^2+12a(a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\sec^5(e+fx)}{48a^3f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[2
*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(a^3*f*(a + b*Sec[e +
 f*x]^2)^(5/2))
```

Maple [A]
time = 0.03, size = 90, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\left(\sec^2(fx+e)\right)}}\right)}{a}}{f}$	90

default	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\left(\sec^2(fx+e)\right)}}\right)}{a}}{f}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b*
sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 92, normalized size = 0.95

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3}+\frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x + e
)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)
^3))/f
```

Fricas [A]
time = 4.20, size = 108, normalized size = 1.11

$$-\frac{\left(3\,a^2\cos(fx+e)^5+12\,ab\cos(fx+e)^3+8\,b^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos(fx+e)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f
*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [A]
time = 0.61, size = 70, normalized size = 0.72

$$-\frac{3\sqrt{a\cos\left(fx+e\right)^2+b}+\frac{6\left(a\cos(fx+e)^2+b\right)b-b^2}{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}}}{3\,a^3f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(a*cos(f*x + e)^2 + b) + (6*(a*cos(f*x + e)^2 + b)*b - b^2)/(a*
cos(f*x + e)^2 + b)^(3/2))/(a^3*f*sgn(cos(f*x + e)))
```

Mupad [B]
time = 17.23, size = 2500, normalized size = 25.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
+ f*x*3i)*(((2*a + 4*b)*(((2*a + 4*b)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(48*
a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a +
 b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) - (16
*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a - (a + 6*b)/(24*a*b*f*(a + b)
) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a + 4*b)*(8*a*
b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))/a - (((32*a*b^2 + 30*a^2*b + 3
*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^
4*b*f*(a + b)))*(2*a + 4*b))/a - (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a +
 b)) + (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b)) + (32*a*b^2 + 40*a^2*b
+ 3*a^3)/(48*a^3*b*f*(a + b))) + exp(e*1i + f*x*1i)*(((2*a + 4*b)*((((32*a*
b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^
2*b + a^3))/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3
)/(48*a^3*b*f*(a + b)) - (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a +
 (48*a*b^2 + 18*a^2*b + a^3 + 32*b^3)/(48*a^3*b*f*(a + b)) - (a + 6*b)/(24*
a*b*f*(a + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a
 + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))*(2*exp(e*2i + f*x
*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)*(a + exp(e*2i + f
*x*2i)*(2*a + 4*b) + a*exp(e*4i + f*x*4i))) - ((a + b/(exp(- e*1i - f*x*1i)
```

/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*1i + f*x*1i)*(((a*2i + b*4i)*(((
a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a
b + a^2)(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^
2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))
/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b
^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((
8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*2i)/(3*f*(2*a*b + a^2)) + (
(a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^
2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*
b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a*2i + b*
4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*
a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*
a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a
*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)
/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2
*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b
^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(4
8*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(1
2*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)
^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (3*a*(a + 2*b)*(8*a*b
 + a^2 + 8*b^2))/(16*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a + 2*b)*(
a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b))
) + exp(e*3i + f*x*3i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a
^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a
^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2
b)(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a + ((
a*2i + b*4i)*(((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) - ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i +
b*4i)*(8*a*b + a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*
1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a
^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2
*b)))*1i)/a - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i
)/(3*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b +
a^2)*(a*b^2 + a^2*b)))*1i)/a + ((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^
2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) + (a*(a + 2*b)*(8*a*b + a^2 + 8
*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*2i + b*4i)*(((a*2i
 + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b + a^2 + 8*b^2)*1i
)/(48*f*(2*a*b + a^2)*(a*b^2 + a^2*b)) - ((a*1i + ((8*a*b + a^2 + 8*b^2)^2*
1i)/(4*(a*b^2 + a^2*b)))*1i)/(12*f*(2*a*b + a^2)) + (a*(a + 2*b)*(8*a*b + a
^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a - ((a*1i + ((8*a*b
 + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*1i)/(3*f*(2*a*b + a^2)) + (a*(a
+ 2*b)*(8*a*b + a^2 + 8*b^2))/(12*f*(2*a*b + a^2)*(a*b^2 + a^2*b)))*1i)/a -

```
((a*1i + ((8*a*b + a^2 + 8*b^2)^2*1i)/(4*(a*b^2 + a^2*b)))*3i)/(4*f*(2*a*b
 + a^2)) + ((a*2i + b*4i)*(((a*2i + b*4i)*(((a + 2*b)*(a*2i + b*4i)*(8*a*b
+ a^2 + 8*b^2)*1i)/(48*f*(2*a*b + a^2)*(a*b^2 +...
```

3.122 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f}-\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f-1/3
*b*sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+2*b)*sec(f*x+e)
/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4219, 425, 541, 12, 385, 213}

$$-\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\sec(e+fx)}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
5/2)*f)) - (b*Sec[e + f*x])/(3*a*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) -
(b*(5*a + 2*b)*Sec[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+2b-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e-\right.}{3a(a+b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} +$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} +$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} +$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.35, size = 108, normalized size = 0.85

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(a^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)\left(-2(2a+b)+3a\sin^2(e+fx)\right)\right)}{6a^2(a+b)f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2,
 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*Sin
[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5055 vs. 2(113) = 226.
time = 0.33, size = 5056, normalized size = 39.81

method	result	size
default	Expression too large to display	5056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(119) = 238$.
time = 3.34, size = 616, normalized size = 4.85

$$\left[\frac{3\left(a^{4}\cos\left(fx+e\right)^{4}+2a^{3}b\cos\left(fx+e\right)^{2}+a^{2}b^{2}\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^{2}-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^{2}-1}\right)-2\left(3\left(2a^{3}b+3a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+7ab^{3}+2b^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{6\left(\left(a^{7}+3a^{6}b+3a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+3a^{5}b^{2}+3a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+3a^{4}b^{3}+3a^{3}b^{4}+a^{2}b^{5}\right)f\right)},\frac{3\left(a^{4}\cos\left(fx+e\right)^{4}+2a^{3}b\cos\left(fx+e\right)^{2}+a^{2}b^{2}\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos(fx+e)}{a+b}\right)-\left(3\left(2a^{3}b+3a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+7ab^{3}+2b^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{7}+3a^{6}b+3a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+3a^{5}b^{2}+3a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+3a^{4}b^{3}+3a^{3}b^{4}+a^{2}b^{5}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b)
*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b + 3
*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^
2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)
*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f), 1/3*(3*
(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b
^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
+ a^2*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.123 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+\right.}$$

```
[Out] -1/2*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)
^(7/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-5/6*b*s
ec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-2*b)*b*sec(f*x+e)/a/
(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 482, 541, 12, 385, 213}

$$-\frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{5b\sec(e+fx)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/((a + b)^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*(a + b*Se
c[e + f*x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x
]^2)^(3/2)) - ((13*a - 2*b)*b*Sec[e + f*x])/(6*a*(a + b)^3*f*Sqrt[a + b*Sec
[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e\right.}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} +$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.52, size = 151, normalized size = 0.88

$$-\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\left(3a^2+6ab-2b^2+(3a^2+2b^2)\cos(2(e+fx))\right)\csc^2(e+fx)-3a(a-4b)(a+2b+a\cos(2(e+fx)))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sec^5(e+fx)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3*
a^2 + 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b + a*
Cos[2*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a
 + b)])*Sec[e + f*x]^5)/(a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11109 vs. 2(151) = 302.
time = 0.53, size = 11110, normalized size = 64.97

method	result	size
default	Expression too large to display	11110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(160) = 320$.
time = 3.55, size = 971, normalized size = 5.68

$$\left[-\frac{3\left(\left(a^{4}-4a^{3}b\right)\cos\left(fx+e\right)^{6}-\left(a^{4}-6a^{3}b+8a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}-a^{2}b^{2}+4ab^{3}-\left(2a^{3}b-9a^{2}b^{2}+4ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}+2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+a+2b\right)}{\cos\left(fx+e\right)^{2}-1}\right)-2\left(3\left(a^{4}-3a^{3}b-4a^{2}b^{2}\right)\cos\left(fx+e\right)^{5}+2\left(9a^{3}b+4a^{2}b^{2}-4ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{3}+\left(13a^{2}b^{2}+11ab^{3}-2b^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{12\left(\left(a^{7}+4a^{6}b+6a^{5}b^{2}+4a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{6}-\left(a^{7}+2a^{6}b-2a^{5}b^{2}-8a^{4}b^{3}-7a^{3}b^{4}-2a^{2}b^{5}\right)f\cos\left(fx+e\right)^{4}-\left(2a^{6}b+7a^{5}b^{2}+8a^{4}b^{3}+2a^{3}b^{4}-2a^{2}b^{5}-ab^{6}\right)f\cos\left(fx+e\right)^{2}-\left(a^{5}b^{2}+4a^{4}b^{3}+6a^{3}b^{4}+4a^{2}b^{5}+ab^{6}\right)f\right)},\ \frac{3\left(\left(a^{4}-4a^{3}b\right)\cos\left(fx+e\right)^{6}-\left(a^{4}-6a^{3}b+8a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}-a^{2}b^{2}+4ab^{3}-\left(2a^{3}b-9a^{2}b^{2}+4ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a+b}\right)+\left(3\left(a^{4}-3a^{3}b-4a^{2}b^{2}\right)\cos\left(fx+e\right)^{5}+2\left(9a^{3}b+4a^{2}b^{2}-4ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{3}+\left(13a^{2}b^{2}+11ab^{3}-2b^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{6\left(\left(a^{7}+4a^{6}b+6a^{5}b^{2}+4a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{6}-\left(a^{7}+2a^{6}b-2a^{5}b^{2}-8a^{4}b^{3}-7a^{3}b^{4}-2a^{2}b^{5}\right)f\cos\left(fx+e\right)^{4}-\left(2a^{6}b+7a^{5}b^{2}+8a^{4}b^{3}+2a^{3}b^{4}-2a^{2}b^{5}-ab^{6}\right)f\cos\left(fx+e\right)^{2}-\left(a^{5}b^{2}+4a^{4}b^{3}+6a^{3}b^{4}+4a^{2}b^{5}+ab^{6}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x +
 e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) -
 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 -
 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^
2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*
a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 +
8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*
a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^3*b)*cos(f*
x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 4*a*b^3 -
 (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
```

```
4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4
*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4
*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1980 vs. $2(151) = 302$.
time = 1.79, size = 1980, normalized size = 11.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(((((3*(a^12*b^2*sgn(cos(f*x + e)) + 10*a^11*b^3*sgn(cos(f*x + e)) +
45*a^10*b^4*sgn(cos(f*x + e)) + 120*a^9*b^5*sgn(cos(f*x + e)) + 210*a^8*b^6
*sgn(cos(f*x + e)) + 252*a^7*b^7*sgn(cos(f*x + e)) + 210*a^6*b^8*sgn(cos(f*
x + e)) + 120*a^5*b^9*sgn(cos(f*x + e)) + 45*a^4*b^10*sgn(cos(f*x + e)) + 1
0*a^3*b^11*sgn(cos(f*x + e)) + a^2*b^12*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/
2*e)^2/(a^13*b^2 + 11*a^12*b^3 + 55*a^11*b^4 + 165*a^10*b^5 + 330*a^9*b^6 +
 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6*b^9 + 165*a^5*b^10 + 55*a^4*b^11 + 11*
a^3*b^12 + a^2*b^13) - 4*(3*a^12*b^2*sgn(cos(f*x + e)) + 12*a^11*b^3*sgn(co
s(f*x + e)) - 25*a^10*b^4*sgn(cos(f*x + e)) - 270*a^9*b^5*sgn(cos(f*x + e))
 - 810*a^8*b^6*sgn(cos(f*x + e)) - 1344*a^7*b^7*sgn(cos(f*x + e)) - 1386*a^
6*b^8*sgn(cos(f*x + e)) - 900*a^5*b^9*sgn(cos(f*x + e)) - 345*a^4*b^10*sgn(
cos(f*x + e)) - 60*a^3*b^11*sgn(cos(f*x + e)) + 3*a^2*b^12*sgn(cos(f*x + e)
) + 2*a*b^13*sgn(cos(f*x + e)))/(a^13*b^2 + 11*a^12*b^3 + 55*a^11*b^4 + 165
*a^10*b^5 + 330*a^9*b^6 + 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6*b^9 + 165*a^5
*b^10 + 55*a^4*b^11 + 11*a^3*b^12 + a^2*b^13))*tan(1/2*f*x + 1/2*e)^2 + 6*(
3*a^12*b^2*sgn(cos(f*x + e)) + 14*a^11*b^3*sgn(cos(f*x + e)) + 27*a^10*b^4*
sgn(cos(f*x + e)) + 68*a^9*b^5*sgn(cos(f*x + e)) + 262*a^8*b^6*sgn(cos(f*x
+ e)) + 644*a^7*b^7*sgn(cos(f*x + e)) + 910*a^6*b^8*sgn(cos(f*x + e)) + 752
*a^5*b^9*sgn(cos(f*x + e)) + 343*a^4*b^10*sgn(cos(f*x + e)) + 62*a^3*b^11*s
gn(cos(f*x + e)) - 9*a^2*b^12*sgn(cos(f*x + e)) - 4*a*b^13*sgn(cos(f*x + e)
))/(a^13*b^2 + 11*a^12*b^3 + 55*a^11*b^4 + 165*a^10*b^5 + 330*a^9*b^6 + 462
```

```
*a^8*b^7 + 462*a^7*b^8 + 330*a^6*b^9 + 165*a^5*b^10 + 55*a^4*b^11 + 11*a^3*
b^12 + a^2*b^13))*tan(1/2*f*x + 1/2*e)^2 - 12*(a^12*b^2*sgn(cos(f*x + e)) +
 12*a^11*b^3*sgn(cos(f*x + e)) + 49*a^10*b^4*sgn(cos(f*x + e)) + 82*a^9*b^5
*sgn(cos(f*x + e)) + 2*a^8*b^6*sgn(cos(f*x + e)) - 224*a^7*b^7*sgn(cos(f*x
+ e)) - 406*a^6*b^8*sgn(cos(f*x + e)) - 356*a^5*b^9*sgn(cos(f*x + e)) - 163
*a^4*b^10*sgn(cos(f*x + e)) - 28*a^3*b^11*sgn(cos(f*x + e)) + 5*a^2*b^12*sg
n(cos(f*x + e)) + 2*a*b^13*sgn(cos(f*x + e)))/(a^13*b^2 + 11*a^12*b^3 + 55*
a^11*b^4 + 165*a^10*b^5 + 330*a^9*b^6 + 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6
*b^9 + 165*a^5*b^10 + 55*a^4*b^11 + 11*a^3*b^12 + a^2*b^13))*tan(1/2*f*x +
1/2*e)^2 + (3*a^12*b^2*sgn(cos(f*x + e)) + 78*a^11*b^3*sgn(cos(f*x + e)) +
559*a^10*b^4*sgn(cos(f*x + e)) + 2016*a^9*b^5*sgn(cos(f*x + e)) + 4374*a^8*
b^6*sgn(cos(f*x + e)) + 6132*a^7*b^7*sgn(cos(f*x + e)) + 5670*a^6*b^8*sgn(c
os(f*x + e)) + 3384*a^5*b^9*sgn(cos(f*x + e)) + 1191*a^4*b^10*sgn(cos(f*x +
 e)) + 174*a^3*b^11*sgn(cos(f*x + e)) - 21*a^2*b^12*sgn(cos(f*x + e)) - 8*a
*b^13*sgn(cos(f*x + e)))/(a^13*b^2 + 11*a^12*b^3 + 55*a^11*b^4 + 165*a^10*b
^5 + 330*a^9*b^6 + 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6*b^9 + 165*a^5*b^10 +
 55*a^4*b^11 + 11*a^3*b^12 + a^2*b^13))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b)^(3/2) - 12*(a - 4*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*sqrt(-a - b)*sgn(cos(f*x + e))) - 6*(a - 4*b)*log(a
bs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))*sqrt(a + b) - a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt
(a + b)*sgn(cos(f*x + e))) + 6*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b) - (a + b)^(3/2))/(((sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b))^2*sqrt(a + b) - (a + b)^(3/2))*(a^2 + 2*a*b + b^2)*sqrt(a + b)*sg
n(cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+fx)^3 \left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.124 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{(3a^2-24ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f}-\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+f}{4(a+b)f\,(a+}$$

```
[Out] -1/8*(3*a^2-24*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))/(a+b)^(9/2)/f-1/8*(5*a-2*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*se
c(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3
/2)-1/24*(23*a-12*b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)-5/24*(
11*a-10*b)*b*sec(f*x+e)/(a+b)^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4219, 481, 541, 12, 385, 213}

$$-\frac{(3a^2-24ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}}-\frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{b(23a-12b)\sec(e+fx)}{24f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a +
b*Sec[e + f*x]^2]])/((a + b)^(9/2)*f) - ((5*a - 2*b)*Cot[e + f*x]*Csc[e + f
*x])/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e +
 f*x])/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 12*b)*b*Sec[e
+ f*x])/(24*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 10*b)*b*
Sec[e + f*x])/(24*(a + b)^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-2(2a-b)x^2}{(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e\right.}{4(a+b)f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(3a^2-24ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+}{8(a+b)^2f\,(a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.95, size = 129, normalized size = 0.55

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(3(a+b)(3a-4b+(a+8b)\cos(2(e+fx)))\csc^4(e+fx)-2(3a^2-24ab+8b^2)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sec^5(e+fx)}{96(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/96*((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric2F
1[-3/2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/((a + b)^
3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15550 vs. $2(210) = 420$.
time = 0.92, size = 15551, normalized size = 66.46

method	result	size
default	Expression too large to display	15551

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(222) = 444$.
time = 5.06, size = 1343, normalized size = 5.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3
*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2
 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a
^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(a*
cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*b - 16
*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2 + 104*
a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3 + 88*b^4
)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2
- 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(
a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2
```

```
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/24
*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3*b +
32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2 - 56
*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a^3*b
- 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(3*a^4 - 21*a^3*b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117
*a^3*b + 4*a^2*b^2 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^
2*b^2 - 61*a*b^3 + 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 +
(a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^
6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5
+ 5*a*b^6 + b^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3235 vs. $2(210) = 420$.
time = 2.31, size = 3235, normalized size = 13.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(12*(3*a^2 - 24*a*b + 8*b^2)*sqrt(a + b)*log(abs(-(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b
) + sqrt(a + b)*(a - b)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*sgn(cos(f*x + e))) - ((((3*((a^17*b^2 + 15*a^16*b^3 + 105*a^15*b^
4 + 455*a^14*b^5 + 1365*a^13*b^6 + 3003*a^12*b^7 + 5005*a^11*b^8 + 6435*a^1
0*b^9 + 6435*a^9*b^10 + 5005*a^8*b^11 + 3003*a^7*b^12 + 1365*a^6*b^13 + 455
*a^5*b^14 + 105*a^4*b^15 + 15*a^3*b^16 + a^2*b^17)*tan(1/2*f*x + 1/2*e)^2/(
a^18*b^2*sgn(cos(f*x + e)) + 16*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*s
```

gn(cos(f*x + e)) + 560*a^15*b^5*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f
*x + e)) + 4368*a^13*b^7*sgn(cos(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)
) + 11440*a^11*b^9*sgn(cos(f*x + e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) +
11440*a^9*b^11*sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a
^7*b^13*sgn(cos(f*x + e)) + 1820*a^6*b^14*sgn(cos(f*x + e)) + 560*a^5*b^15*
sgn(cos(f*x + e)) + 120*a^4*b^16*sgn(cos(f*x + e)) + 16*a^3*b^17*sgn(cos(f*
x + e)) + a^2*b^18*sgn(cos(f*x + e))) + (5*a^17*b^2 + 61*a^16*b^3 + 329*a^1
5*b^4 + 1001*a^14*b^5 + 1729*a^13*b^6 + 1001*a^12*b^7 - 3003*a^11*b^8 - 986
7*a^10*b^9 - 15873*a^9*b^10 - 17017*a^8*b^11 - 13013*a^7*b^12 - 7189*a^6*b^
13 - 2821*a^5*b^14 - 749*a^4*b^15 - 121*a^3*b^16 - 9*a^2*b^17)/(a^18*b^2*sg
n(cos(f*x + e)) + 16*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*sgn(cos(f*x
+ e)) + 560*a^15*b^5*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f*x + e)) +
4368*a^13*b^7*sgn(cos(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)) + 11440*a
^11*b^9*sgn(cos(f*x + e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) + 11440*a^9*b
^11*sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a^7*b^13*sgn
(cos(f*x + e)) + 1820*a^6*b^14*sgn(cos(f*x + e)) + 560*a^5*b^15*sgn(cos(f*x
 + e)) + 120*a^4*b^16*sgn(cos(f*x + e)) + 16*a^3*b^17*sgn(cos(f*x + e)) + a
^2*b^18*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - 2*(45*a^17*b^2 + 267*a
^16*b^3 - 427*a^15*b^4 - 9373*a^14*b^5 - 43407*a^13*b^6 - 113113*a^12*b^7 -
 191191*a^11*b^8 - 214929*a^10*b^9 - 149721*a^9*b^10 - 39039*a^8*b^11 + 390
39*a^7*b^12 + 53417*a^6*b^13 + 32123*a^5*b^14 + 11277*a^4*b^15 + 2243*a^3*b
^16 + 197*a^2*b^17)/(a^18*b^2*sgn(cos(f*x + e)) + 16*a^17*b^3*sgn(cos(f*x +
 e)) + 120*a^16*b^4*sgn(cos(f*x + e)) + 560*a^15*b^5*sgn(cos(f*x + e)) + 18
20*a^14*b^6*sgn(cos(f*x + e)) + 4368*a^13*b^7*sgn(cos(f*x + e)) + 8008*a^12
*b^8*sgn(cos(f*x + e)) + 11440*a^11*b^9*sgn(cos(f*x + e)) + 12870*a^10*b^10
*sgn(cos(f*x + e)) + 11440*a^9*b^11*sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(c
os(f*x + e)) + 4368*a^7*b^13*sgn(cos(f*x + e)) + 1820*a^6*b^14*sgn(cos(f*x
+ e)) + 560*a^5*b^15*sgn(cos(f*x + e)) + 120*a^4*b^16*sgn(cos(f*x + e)) + 1
6*a^3*b^17*sgn(cos(f*x + e)) + a^2*b^18*sgn(cos(f*x + e))))*tan(1/2*f*x + 1
/2*e)^2 + 6*(25*a^17*b^2 + 177*a^16*b^3 + 421*a^15*b^4 + 181*a^14*b^5 - 363
*a^13*b^6 + 2365*a^12*b^7 + 11649*a^11*b^8 + 20097*a^10*b^9 + 12507*a^9*b^1
0 - 12221*a^8*b^11 - 33825*a^7*b^12 - 35153*a^6*b^13 - 21449*a^5*b^14 - 804
9*a^4*b^15 - 1733*a^3*b^16 - 165*a^2*b^17)/(a^18*b^2*sgn(cos(f*x + e)) + 16
*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*sgn(cos(f*x + e)) + 560*a^15*b^5
*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f*x + e)) + 4368*a^13*b^7*sgn(co
s(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)) + 11440*a^11*b^9*sgn(cos(f*x
+ e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) + 11440*a^9*b^11*sgn(cos(f*x + e)
) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a^7*b^13*sgn(cos(f*x + e)) + 182
0*a^6*b^14*sgn(cos(f*x + e)) + 560*a^5*b^15*sgn(cos(f*x + e)) + 120*a^4*b^1
6*sgn(cos(f*x + e)) + 16*a^3*b^17*sgn(cos(f*x + e)) + a^2*b^18*sgn(cos(f*x
+ e))))*tan(1/2*f*x + 1/2*e)^2 - 3*(35*a^17*b^2 + 493*a^16*b^3 + 2763*a^15*
b^4 + 7749*a^14*b^5 + 9151*a^13*b^6 - 8943*a^12*b^7 - 51689*a^11*b^8 - 8787
9*a^10*b^9 - 72567*a^9*b^10 - 7513*a^8*b^11 + 52129*a^7*b^12 + 64143*a^6*b^
13 + 40533*a^5*b^14 + 15259*a^4*b^15 + 3261*a^3*b^16 + 307*a^2*b^17)/(a^18*
b^2*sgn(cos(f*x + e)) + 16*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*sgn(co

```
s(f*x + e)) + 560*a^15*b^5*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f*x +
e)) + 4368*a^13*b^7*sgn(cos(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)) + 1
1440*a^11*b^9*sgn(cos(f*x + e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) + 11440
*a^9*b^11*sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a^7*b^
13*sgn(cos(f*x + e)) + 1820*a^6*b^14*sgn(cos(f*x + e)) + 560*a^5*b^15*sgn(c
os(f*x + e)) + 120*a^4*b^16*sgn(cos(f*x + e)) + 16*a^3*b^17*sgn(cos(f*x + e
)) + a^2*b^18*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (27*a^17*b^2 + 7
23*a^16*b^3 + 6647*a^15*b^4 + 32903*a^14*b^5 + 102687*a^13*b^6 + 216359*a^1
2*b^7 + 314171*a^11*b^8 + 305019*a^10*b^9 + 166881*a^9*b^10 - 8151*a^8*b^11
 - 105963*a^7*b^12 - 102427*a^6*b^13 - 55003*a^5*b^14 - 18147*a^4*b^15 - 34
63*a^3*b^16 - 295*a^2*b^17)/(a^18*b^2*sgn(cos(f*x + e)) + 16*a^17*b^3*sgn(c
os(f*x + e)) + 120*a^16*b^4*sgn(cos(f*x + e)) + 560*a^15*b^5*sgn(cos(f*x +
e)) + 1820*a^14*b^6*sgn(cos(f*x + e)) + 4368*a^...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.125 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$\frac{5(a+b)\left(a^2+14ab+21b^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f}-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+f}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] 5/16*(a+b)*(a^2+14*a*b+21*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(11/2)/f-1/48*b*(113*a^2+420*a*b+315*b^2)*tan(f*x+e)/a^5/f/(a+b
+b*tan(f*x+e)^2)^(1/2)-1/16*(a+b)*(11*a+21*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(
a+b+b*tan(f*x+e)^2)^(3/2)+3/8*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*ta
n(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(3
/2)-7/48*b*(a+b)*(7*a+15*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4217, 481, 592, 541, 12, 385, 209}

$$-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(a+b)(11a+21b)\sin(e+fx)\cos(e+fx)}{16a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{3(a+b)\sin(e+fx)\cos^3(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{5(a+b)\left(a^2+14ab+21b^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}-\frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(16*a^(11/2)*f) - ((a + b)*(11*a + 21*b)*Cos[e + f*x
]*Sin[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (3*(a + b)*Co
s[e + f*x]^3*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Co
s[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (7*
b*(a + b)*(7*a + 15*b)*Tan[e + f*x])/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^(
3/2)) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(48*a^5*f*Sqrt[a + b
 + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)-6(a+b)x^2)}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{6af}$$

$$= \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}}{}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+}{8a^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= \frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} - \frac{(a+b)(1}{16a^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1705 vs. $2(288) = 576$.
time = 19.77, size = 1705, normalized size = 5.92

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] -1/3072*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 1120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Sin[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[a + b]*(a^4 + 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b + 25200*a^4*b^2 + 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) - 2*a*(1151*a^5 + 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4 + 13440*b^5)*Sin[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin[e + f*x]^4 - 576*a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)^2*Sin[e + f*x]^8)))/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)/(a + b))^2))/a^4))/(12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5*(b + (3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])

```
^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.99, size = 4477, normalized size = 15.55

method	result	size
default	Expression too large to display	4477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(630*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^4+45
0*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3*b+1050*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*
x+e)^2*sin(f*x+e)*a^2*b^2+630*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a*b^3
+18*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-18*cos(f*x+e
)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-96*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-63*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^2*b^2+96*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^3*b+63*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^
2-162*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-574*cos(f*
x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-420*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+162*cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+574*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2*b^2+420*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b^3-113*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-420
```

```
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-225*2^(1/2)*((I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)
*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*EllipticF((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b-525*2^(1/2)
*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*EllipticF((
cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2-315
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b
^3+315*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-26*cos(f*x+e)^6*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-8*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^4+8*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^4+33*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-33*cos(f*x+
e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-315*cos(f*x+e)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+26*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^4+113*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+420*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-315*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4+30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [A]

time = 89.90, size = 1046, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b + 3
5*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b^3
 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b
)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e)^5 + 2*(8
1*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b^2 + 420*a^
2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*
f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b
+ 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*
b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^5*cos(f*x +
 e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3
*b^2)*cos(f*x + e)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e
)^3 + (113*a^3*b^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*
f*cos(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.126 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=227

$$\frac{(3a^2+30ab+35b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f}-\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+}{4af\,(a+b+}$$

```
[Out] 1/8*(3*a^2+30*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f-5/24*b*(11*a+21*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/
2)-1/8*(5*a+7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/24*b*(23*a+35*b)*
tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]

time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 481, 541, 12, 385, 209}

$$-\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(5a+7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{(3a^2+30ab+35b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a + 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 35*b)*Tan[e + f*x])/(24*a
^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 21*b)*Tan[e + f*x])/(
24*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+3b)x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+f\right.}{4af}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Sub}}{}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= \frac{(3a^2+30ab+35b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e-}{8a^2f\,(a+b+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1315 vs. $2(227) = 454$.
time = 13.01, size = 1315, normalized size = 5.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/768*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b
```

```
^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*
(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b
)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 11
20*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)
*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Si
n[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a
+ b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e +
f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)
])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a
*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*
x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
- a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3
))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(
3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
 + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b
- a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e +
 f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a +
 b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6
*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a
 + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(
a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e +
f*x]^2)/(a + b))^2))/a^4))/(3072*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a
+ b - a*Sin[e + f*x]^2)^(3/2)) + ((2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(256*(a + b)^2*f*
(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b +
(3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e +
f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*
(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.45, size = 3223, normalized size = 14.20

method	result	size
default	Expression too large to display	3223

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(9*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
```

```
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-18*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)
*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a^3-180*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+21*cos(f*x+e
)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-21*cos(f*x+e)^4*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+78*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^2*b+140*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a*b^2-78*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-1
40*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+55*cos(f*x+e)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-210*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(
f*x+e)+105*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-21
0*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+90*sin(f*x+e)*cos(f*x+e)^2*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+105*sin(f*x+e)*cos(f*x
+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
```

```
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-55*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-18*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-180*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a*b^2*sin(f*x+e)+9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*b*sin(f*x+e)+90*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

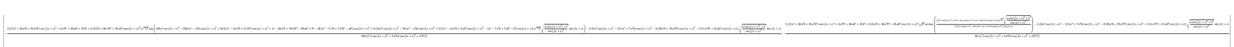
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [A]
time = 26.51, size = 914, normalized size = 4.03



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30
*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
```

```
*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70
*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2
*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3*b + 35*a^2*
b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b
^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6*a^4*cos(f*x +
e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70*a^2*b^2)*cos(f
*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*
x + e)^2 + a^5*b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.127 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+\right.}$$

```
[Out] 1/2*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
-1/6*b*(13*a+15*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/2*co
s(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/a^2/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 482, 541, 12, 385, 209}

$$\frac{(a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) - (5*b*Tan[e + f*x])/(6*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) -
(b*(13*a + 15*b)*Tan[e + f*x])/(6*a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e\ -\right.}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \text{Sul}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= \frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx}{2af\,(a+b+b\tan^2(e+fx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 983 vs. 2(167) = 334.
time = 7.69, size = 983, normalized size = 5.89

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(S
in[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a
+ b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 -
(a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos
```

[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)/(a + b))^2))/a^4))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b + (3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2))

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 3158, normalized size = 18.91

method	result	size
default	Expression too large to display	3158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+18*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+15*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
```

cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos(f*x+e)^2*sin(f*x+e)
2^(1/2)((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a^3-36*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-30*co
s(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*
cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-36*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co

s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*a*b^2*sin(f*x+e)-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(156) = 312$.
time = 9.32, size = 918, normalized size = 5.50

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 +
 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a
^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b
 + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4 + a^3*b
)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 +
15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)
^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b

```
)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(f*x + e)^5 + 2*(9
*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*b^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f
*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3
)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.128 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 425, 541, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \text{t}\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 17.46, size = 1927, normalized size = 15.42

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.23, size = 3024, normalized size = 24.19

method	result	size
default	Expression too large to display	3024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a^3-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*cos(f*x+e)^2*s
in(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
```

```
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e
)-12*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+3*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+6*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-
4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(117) = 234.

time = 5.67, size = 918, normalized size = 7.34

$$\left[-\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5a^{4}-14a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28a^{3}b+70a^{2}b^{2}-28ab^{3}+b^{4}-32\left(a^{4}-7a^{3}b+7a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5a^{3}-14a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7a^{2}b+7ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)+8\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{24\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)},\ -\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)+4\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{12\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.129 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{8b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -8/3*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/(a+b)/f/(
a+b+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(
3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4217, 277, 198, 197}

$$-\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e +
 f*x])/(3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x]
)/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan\right.}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\,(a+b+b\tan^2(e+fx)}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\,(a+b+b\tan^2(e+fx)}$$

Mathematica [A]
time = 2.13, size = 108, normalized size = 1.02

$$-\frac{(a+2b+a\cos(2(e+fx)))\,(3a^2-6ab-b^2-6(a^2-b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx))\sec^2(e+fx)\tan^3(e+fx)}{6(a+b)^3f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*C
sc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3)/
((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.23, size = 133, normalized size = 1.25

method	result
default	$-\frac{(3(\cos^4(fx+e))a^2-6(\cos^4(fx+e))ab-(\cos^4(fx+e))b^2+12(\cos^2(fx+e))ab-4(\cos^2(fx+e))b^2+8b^2)(\cos^5(fx+e))\left(\frac{b+a(\cos^2(fx}{\cos(fx+e)}\right.}{3f(b+a(\cos^2(fx+e)))^4\sin(fx+e)(a+b)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/f/(b+a*cos(f*x+e)^2)^4*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4*a*b-cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-4*cos(f*x+e)^2*b^2+8*b^2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)/(a+b)^3

Maxima [A]
time = 0.28, size = 100, normalized size = 0.94

$$-\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}+\frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) + 3/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)))/f

Fricas [A]
time = 5.26, size = 200, normalized size = 1.89

$$-\frac{\left((3\,a^2-6\,ab-b^2)\cos\left(fx+e\right)^5+4\,(3\,ab-b^2)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left((a^5+3\,a^4b+3\,a^3b^2+a^2b^3)f\cos\left(fx+e\right)^4+2\,(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)f\cos\left(fx+e\right)^2+(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] -1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [B]
time = 16.30, size = 336, normalized size = 3.17

$$-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+\frac{b}{{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(-a\,b\,6\mathrm{i}+a^2\,3\mathrm{i}-b^2\,1\mathrm{i}+a^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,12\mathrm{i}+a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,18\mathrm{i}+a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,12\mathrm{i}+a^2\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,3\mathrm{i}-b^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,20\mathrm{i}+b^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,90\mathrm{i}-b^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,20\mathrm{i}-b^2\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,1\mathrm{i}+a\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,24\mathrm{i}+a\,b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,60\mathrm{i}+a\,b\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,24\mathrm{i}-a\,b\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,6\mathrm{i}\right)}{3\,f\,{\left(a+b\right)}^3\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,{\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2)^2)^(1/2)*(a^2*3i - a*b*6i - b^2*1i + a^2*exp(e*2i + f*x*2i)*12i + a^
2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(e*8i + f*x*
8i)*3i - b^2*exp(e*2i + f*x*2i)*20i + b^2*exp(e*4i + f*x*4i)*90i - b^2*exp(
e*6i + f*x*6i)*20i - b^2*exp(e*8i + f*x*8i)*1i + a*b*exp(e*2i + f*x*2i)*24i
 + a*b*exp(e*4i + f*x*4i)*60i + a*b*exp(e*6i + f*x*6i)*24i - a*b*exp(e*8i +
 f*x*8i)*6i))/(3*f*(a + b)^3*(exp(e*2i + f*x*2i) - 1)*(a + 2*a*exp(e*2i + f
*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

3.130 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{4(a-b)b\tan(e+fx)}{3(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -8/3*(a-b)*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+
e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/(a+b)/f/(a+b+b*tan
(f*x+e)^2)^(3/2)-4/3*(a-b)*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4217, 464, 277, 198, 197}

$$-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(a-b)\cot(e+fx)}{f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)*Cot[e + f*x])/((a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) -
Cot[e + f*x]^3/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*(a -
b)*b*Tan[e + f*x])/(3*(a + b)^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{x^2(a+b+bx^2)^{5/2}}\,dx,\right.}{(a+b)f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right.}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right.}$$

Mathematica [A]
time = 4.96, size = 138, normalized size = 0.87

$$\frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{4b^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{4b(-3a+b)}{a+2b+a\cos(2(e+fx))}-2(a-3b)\csc^2(e+fx)-(a+b)\csc^4(e+fx)\right)\sec^4(e+fx)\tan(e+fx)}{24(a+b)^4f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*Cs
c[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*(a
+ b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.18, size = 212, normalized size = 1.34

method	result
default	$\frac{(2(\cos^6(fx+e))a^3-12(\cos^6(fx+e))a^2b+2(\cos^6(fx+e))a\,b^2-3(\cos^4(fx+e))a^3+21(\cos^4(fx+e))a^2b-21(\cos^4(fx+e))a\,b^2+3(\cos^4(f}{3f(b+a(\cos^2(fx+e)))^4\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(b+a*cos(f*x+e)^2)^4*(2*cos(f*x+e)^6*a^3-12*cos(f*x+e)^6*a^2*b+2*cos(
f*x+e)^6*a*b^2-3*cos(f*x+e)^4*a^3+21*cos(f*x+e)^4*a^2*b-21*cos(f*x+e)^4*a*b
^2+3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*a^2*b+24*cos(f*x+e)^2*a*b^2-12*cos(f*
x+e)^2*b^3-8*a*b^2+8*b^3)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5
/2)/sin(f*x+e)^3/(a+b)^4
```

Maxima [A]
time = 0.28, size = 230, normalized size = 1.46

$$-\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{16\,b^2\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^4}-\frac{8\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}-\frac{6\,b}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2\tan(fx+e)}+\frac{1}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*tan
(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 16*b^2*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 8*b^2*tan(f*x + e)/((b*tan(f
*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/2)*(
a + b)*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(
f*x + e)) + 1/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(152) = 304$.
time = 11.84, size = 330, normalized size = 2.09

$$-\frac{\left(2\,(a^3-6\,a^2b+ab^2)\cos\left(fx+e\right)^7-3\,(a^3-7\,a^2b+7\,ab^2-b^3)\cos\left(fx+e\right)^5-12\,(a^2b-2\,ab^2+b^3)\cos\left(fx+e\right)^3-8\,(ab^2-b^3)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left((a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)f\cos\left(fx+e\right)^6-(a^6+2\,a^5b-2\,a^4b^2-8\,a^3b^3-7\,a^2b^4-2\,ab^5)f\cos\left(fx+e\right)^4-(2\,a^5b+7\,a^4b^2+8\,a^3b^3+2\,a^2b^4-2\,ab^5-b^6)f\cos\left(fx+e\right)^2-(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a*b
^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^6
+ 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^
5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^
5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(\right.}$$

```
[Out] -8/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
1/5*(5*a^2-10*a*b+b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-2/15
*(5*a+b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e)^5
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a
+b)^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4217, 473, 464, 277, 198, 197}

$$-\frac{8b(5a^2-10ab+b^2)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^5(e+fx)}{5f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{2(5a+b)\cot^3(e+fx)}{15f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5*a^2 - 10*a*b + b^2)*Cot[e + f*x])/((a + b)^3*f*(a + b + b*Tan[e +
f*x]^2)^(3/2)) - (2*(5*a + b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*(a + b + b*Ta
n[e + f*x]^2)^(3/2)) - Cot[e + f*x]^5/(5*(a + b)*f*(a + b + b*Tan[e + f*x]^
2)^(3/2)) - (4*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*(a + b)^4*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(1
5*(a + b)^5*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a+b)+5(a+b)x^2}{x^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(\right.}{5(a+b)f}$$

$$= -\frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\,(a+b+b\tan^2(e+fx}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\,(a+b+b\tan^2(e+f}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\,(a+b+b\tan^2(e+f}$$

$$= -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\,(a+b+b\tan^2(e+f}$$

Mathematica [A]
time = 7.61, size = 173, normalized size = 0.77

$$\frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{20ab^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{10ab(-6a+5b)}{a+2b+a\cos(2(e+fx))}+(-8a^2+50ab-15b^2)\csc^2(e+fx)+2(a+b)(-2a+5b)\csc^4(e+fx)-3(a+b)^2\csc^6(e+fx)\right)\sec^4(e+fx)\tan(e+fx)}{120(a+b)^5f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(e
 + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*a^
2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x]^4
 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*
f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.19, size = 324, normalized size = 1.43

method	result
default	$-\frac{(8(\cos^8(fx+e))a^4-80(\cos^8(fx+e))a^3b+40(\cos^8(fx+e))a^2b^2-20(\cos^6(fx+e))a^4+212(\cos^6(fx+e))a^3b-220(\cos^6(fx+e))a^2b^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/15/f/(b+a*cos(f*x+e)^2)^4*(8*cos(f*x+e)^8*a^4-80*cos(f*x+e)^8*a^3*b+40*cos(f*x+e)^8*a^2*b^2-20*cos(f*x+e)^6*a^4+212*cos(f*x+e)^6*a^3*b-220*cos(f*x+e)^6*a^2*b^2+60*cos(f*x+e)^6*a*b^3+15*cos(f*x+e)^4*a^4-180*cos(f*x+e)^4*a^3*b+378*cos(f*x+e)^4*a^2*b^2-180*cos(f*x+e)^4*a*b^3+15*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b-220*cos(f*x+e)^2*a^2*b^2+212*cos(f*x+e)^2*a*b^3-20*cos(f*x+e)^2*b^4+40*a^2*b^2-80*a*b^3+8*b^4)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)*cos(f*x+e)^5/sin(f*x+e)^5/(a+b)^3/(a^2+2*a*b+b^2)

Maxima [A]
time = 0.29, size = 397, normalized size = 1.76

$$-\frac{\frac{40\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^3}+\frac{20\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{160\,b^2\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^4}-\frac{80\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{128\,b^3\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}\,(a+b)^5}+\frac{64\,b^3\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^4}+\frac{15}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}-\frac{60\,b}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2\tan(fx+e)}+\frac{48\,b^2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3\tan(fx+e)}+\frac{10}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)^3}-\frac{8\,b}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2\tan(fx+e)^3}+\frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] -1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 20*b*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 160*b^2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 80*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 128*b^3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^5) + 64*b^3*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^4) + 15/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)) + 48*b^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3*tan(f*x + e)) + 10/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3) - 8*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^5))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(216) = 432$.
time = 29.62, size = 472, normalized size = 2.09

$$-\frac{\left(8\left(a^4-10\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^9-4\left(5\,a^4-53\,a^3b+55\,a^2b^2-15\,ab^3\right)\cos\left(fx+e\right)^7+3\left(5\,a^4-60\,a^3b+126\,a^2b^2-60\,ab^3+5\,b^4\right)\cos\left(fx+e\right)^5+4\left(15\,a^3b-55\,a^2b^2+53\,ab^3-5\,b^4\right)\cos\left(fx+e\right)^3+8\left(5\,a^2b^2-10\,ab^3+b^4\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{15\left(\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^8-2\left(a^7+4\,a^6b+5\,a^5b^2-5\,a^3b^4-4\,a^2b^5-ab^6\right)f\cos\left(fx+e\right)^6+\left(a^7+a^6b-9\,a^5b^2-25\,a^4b^3-25\,a^3b^4-9\,a^2b^5+ab^6+b^7\right)f\cos\left(fx+e\right)^4+2\left(a^6b+4\,a^5b^2+5\,a^4b^3-5\,a^2b^5-4\,ab^6-b^7\right)f\cos\left(fx+e\right)^2+\left(a^5b^2+5\,a^4b^3+10\,a^3b^4+10\,a^2b^5+5\,ab^6+b^7\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] -1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b + 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b^2 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^3 - 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5

```
*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4
+ 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x +
 e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*
sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.132 $\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m \, dx$

Optimal. Leaf size=123

$$\frac{F_1\left(\frac{1+m}{2};\frac{1}{2}+p,-p;\frac{3+m}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^{\frac{1}{2}+p}\left(a+b\sec^2(e+fx)\right)^p(d\sin(e+fx))^m\left(\right.}{f(1+m)}$$

[Out] AppellF1(1/2+1/2*m,1/2+p,-p,3/2+1/2*m,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^(1/2+p)*(a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m*tan(f*x+e)/f/(1+m)/(((a+b-a*sin(f*x+e)^2)/(a+b))^p)

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m \, dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]

[Out] Defer[Int][(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m, x]

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m \, dx = \int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m \, dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(123) = 246.
time = 4.30, size = 286, normalized size = 2.33

$$\frac{F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin(e+fx)(d\sin(e+fx))^m}{f(1+m)\left(F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)-\frac{\left(-2bpF_1\left(\frac{3+m}{2};\frac{2+m}{2},1-p;\frac{5+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)+(a+b)(2+m)F_1\left(\frac{3+m}{2};\frac{4+m}{2},-p;\frac{5+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\right)\tan^2(e+fx)}{(a+b)(3+m)}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]

[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Sin[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 + m)/

```
2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*(3 + m)
)))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p} \left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
[Out] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\sin(e+fx))^m \left(a+\frac{b}{\cos(e+fx)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.133 $\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$

Optimal. Leaf size=182

$$\frac{(10a+b(3-2p))\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{15a^2f}-\frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{5af}-\frac{(15a^2+10ab(\ldots}{\ldots}$$

```
[Out] 1/15*(10*a+b*(3-2*p))*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1+p)/a^2/f-1/5*cos(f
*x+e)^5*(a+b*sec(f*x+e)^2)^(1+p)/a/f-1/15*(15*a^2+10*a*b*(1-2*p)+b^2*(4*p^2
-8*p+3))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(
f*x+e)^2)^p/a^2/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4219, 473, 464, 372, 371}

$$-\frac{(15a^2+10ab(1-2p)+b^2(4p^2-8p+3))\cos(e+fx)\,(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{15a^2f}+\frac{(10a+b(3-2p))\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{p+1}}{15a^2f}-\frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{p+1}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] ((10*a + b*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(15*a^
2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2 +
 10*a*b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Cos[e + f*x]*Hypergeometric2F1[-
1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(15*a^2*f*
(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^p}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} + \frac{\text{Subst}\left(\int \frac{(-10a-b(3-2p)+}{x^4}\right.}{}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e}{}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e}{}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e}{}$$

Mathematica [A]

time = 8.57, size = 253, normalized size = 1.39

$$\frac{2\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin^4(e+fx)\left(4\left(15a^2+10ab(1-2p)+b^2\left(3-8p+4p^2\right)\right){}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)+(a+2b+a\cos(2(e+fx)))(-17a-6b+4bp+3a\cos(2(e+fx)))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p\right)}{15a^2f\left(4\cos(2(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p-2^{-p}\left(3\left(\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)}{a}\right)^p+2^p\cos(4(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] (2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*b
*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*S
ec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p +
3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*Cos[2
*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e + f*x
]^2)/a)^p)/2^p))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\sin^5\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.134 $\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx$

Optimal. Leaf size=117

$$\frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{3af}-\frac{(3a+b-2bp)\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e-$$

```
[Out] 1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1+p)/a/f-1/3*(-2*b*p+3*a+b)*cos(f*x+e)
*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^p/a/f/((1
+b*sec(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4219, 464, 372, 371}

$$\frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{p+1}}{3af}-\frac{(3a-2bp+b)\cos(e+fx)\,(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a
 + b*Sec[e + f*x]^2)^p)/(3*a*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{(3a+b-2bp)\text{Subst}\left(\int \right.}{3}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{\left((3a+b-2bp)\left(a+b\,\text{se}\right.\right.}{}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+f}{}$$

Mathematica [A]
time = 4.28, size = 178, normalized size = 1.52

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin^2(e+fx)\left(-2(3a+b-2bp)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)+(a+2b+a\cos(2(e+fx)))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p\right)}{3af\left(-2\left(1+\frac{b\sec^2(e+fx)}{a}\right)^p+\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p+\cos(2(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b - 2
*b*p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*b
+ a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(a*f*(-2*(1 + (b*S
ec[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x)]*(
(a + b + b*Tan[e + f*x]^2)/a)^p))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2(fx+e)\right)\right)^p \left(\sin^3(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin{(e+f\,x)}^3\,{\left(a+\frac{b}{{\cos{(e+f\,x)}}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.135 $\int (a + b\sec^2(e + fx))^p \sin(e + fx)\, dx$

Optimal. Leaf size=68

$$-\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2
)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4219, 372, 371}

$$-\frac{\cos(e+fx)(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\sec^2(e+fx))^p \left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx\right.}{f}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p}{f}$$

Mathematica [**A**]
time = 1.90, size = 68, normalized size = 1.00

$$-\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [B]
time = 4.92, size = 79, normalized size = 1.16

$$\frac{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a\,{\cos\left(e+f\,x\right)}^2}{b}\right)}{f\,\left(2\,p-1\right)\,{\left(\frac{a\,{\cos\left(e+f\,x\right)}^2}{b}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] (cos(e + f*x)*(a + b/cos(e + f*x)^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -(
a*cos(e + f*x)^2)/b))/(f*(2*p - 1)*((a*cos(e + f*x)^2)/b + 1)^p)
```

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=77

$$-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)*(a+b*sec(
f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4219, 441, 440}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
 + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{-1+x^2}\,dx,\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1532 vs. $2(77) = 154$.
time = 17.46, size = 1532, normalized size = 19.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*Se
c[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
-(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a +
b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*T
an[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1
+ p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2, -p, 1
/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2
])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[
e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*b + a*Cos[2
*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/2 - p, -1/2,
 -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e +
f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]
) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) + ((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2
- p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[
e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x
]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
 -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*(1 + ((a + b)*Cot[e + f*x]^2)/
```

```
b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*(
(-2*(a + b)*(-1/2 - p)*p*AppellF1[1/2 - p, -1/2, 1 - p, 3/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(b*(1/2
- p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(
((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(1/2 - p))*Sqrt[S
ec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e +
f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a
+ b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 + 2*p)*(1 +
 ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[1, 1
/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*T
an[e + f*x]^3*((a + b + b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (2*A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/((a + b + b*Tan[e + f*x]^2)/(a + b))^p - (Tan[e +
f*x]^2*((b*p*AppellF1[2, 1/2, 1 - p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(a + b) - (AppellF1[2, 3/2, -p, 3
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/2))/((a + b + b*Tan[e + f*x]^2)/(a + b))^p))/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \csc \left(fx+e\right) \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x), x)
```

3.137 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)^3*(a+b
*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4219, 525, 524}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e +
 f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4219

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/
```

```
x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{\left(-1+x^2\right)^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec\right.}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(81) = 162.
time = 4.98, size = 266, normalized size = 3.28

$$\frac{F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)\csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p}{f(-1+2p)\left(-\frac{\left(2(a+b)pF_1\left(\frac{3}{2}-p;-\frac{1}{2},1-p;\frac{5}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)+bF_1\left(\frac{3}{2}-p;\frac{1}{2},-p;\frac{5}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)\right)\cot(e+fx)\csc(e+fx)}{b(-3+2p)}+F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)\sec(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f
*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*(a
 + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f*x]^
2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 + 2*p)
)) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e
 + f*x]^2)/b)]*Sec[e + f*x]))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^3(fx+e)\right)\left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.138 $\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{5f}$$

```
[Out] 1/5*AppellF1(5/2,3,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 525, 524}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
```

```
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\right.}{(1+}\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+}{5f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5878 vs. $2(88) = 176$.
time = 25.96, size = 5878, normalized size = 66.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
[Out] Result too large to show
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\left(\sin^4\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.139 $\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 525, 524}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
```

```
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx}{a+}\right.}{(1+x^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)(a+b}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(88) = 176.
time = 21.60, size = 3781, normalized size = 42.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*Appel
lF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) +
2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2
, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*
Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f
*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[
```

e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 6*a*(a + b)*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 6*(a + b)*(-2 + p)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e + f*x]^2*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e + f*x]*(((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3)/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -(

```
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[
1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(4*(-(b*p*
AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e...
```

Maple [**F**]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(\sin^2\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.140 $\int (a+b\sec^2(e+fx))^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan\right.}{f}$$
$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2137 vs. $2(83) = 166$.
time = 15.39, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
```

```
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec^{2}{\left(e + f x \right)}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=73

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*
x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 372, 371}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2} \ldots\right.}{f}$$
$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+\ldots\right.}{f}$$

Mathematica [**A**]
time = 1.22, size = 72, normalized size = 0.99

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=128

$$-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f}-\frac{(3a+2b(1+p))\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)(a+$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)/f-1/3*(3*a+2*b*(1+p))*co
t(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*x+e
)^2)^p/(a+b)/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 464, 372, 371}

$$-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f(a+b)}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/((a + b)*f) - ((3*
a + 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
 f*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*f*(1 + (b*Tan[e
 + f*x]^2)/(a + b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{(3a+2b(1+p))\text{Su}\ldots}{\ldots}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{\Big((3a+2b(1+p))\,(\ldots}{\ldots}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} - \frac{(3a+2b(1+p))\,\text{co}\ldots}{\ldots}$$

Mathematica [A]
time = 2.54, size = 132, normalized size = 1.03

$$-\frac{\cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((3a+2b(1+p))\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+\cot^2(e+fx)\left(a+b+b\tan^2(e+fx)\right)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^p\right)}{3(a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeome
tric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a +
 b + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/((a + b)*f*(1 +
 (b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^4(fx+e)\right)\left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=192

$$-\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f}-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f}-\frac{(1$$

```
[Out] -1/15*(10*a+b*(7+2*p))*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)^2/f-1/
5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1+p)/(a+b)/f-1/15*(15*a^2+20*a*b*(1+p)
+4*b^2*(p^2+3*p+2))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(
a+b))*(a+b+b*tan(f*x+e)^2)^p/(a+b)^2/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 473, 464, 372, 371}

$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{15f(a+b)^2}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{5f(a+b)}-\frac{(10a+b(2p+7))\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/15*((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 +
p))/((a + b)^2*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(5*
(a + b)*f) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x
]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b +
b*Tan[e + f*x]^2)^p)/(15*(a + b)^2*f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)^p}{x^6}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f}+\frac{\operatorname{Subst}\left(\int\frac{(a+b+bx^2)^p(\ldots}{\ldots}\right.}{\ldots}$$

$$=-\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f}-\frac{\mathrm{c}\ldots}{\ldots}$$

$$=-\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f}-\frac{\mathrm{c}\ldots}{\ldots}$$

$$=-\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f}-\frac{\mathrm{c}\ldots}{\ldots}$$

Mathematica [A]
time = 2.31, size = 149, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,{}_2F_1\left(-\frac{5}{2},-p;-\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+10\cot^2(e+fx)\,{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+15\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\right)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((
b*Tan[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p,
 -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1/2,
 -((b*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e
 + f*x]^2)/(a + b))^p)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^6\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

3.144 $\int (a - a\sec^2(c+dx))^4 \, dx$

Optimal. Leaf size=74

$$a^4x - \frac{a^4\tan(c+dx)}{d} + \frac{a^4\tan^3(c+dx)}{3d} - \frac{a^4\tan^5(c+dx)}{5d} + \frac{a^4\tan^7(c+dx)}{7d}$$

```
[Out] a^4*x-a^4*tan(d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d-1/5*a^4*tan(d*x+c)^5/d+1/7*a^
4*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 3554, 8}

$$\frac{a^4\tan^7(c+dx)}{7d} - \frac{a^4\tan^5(c+dx)}{5d} + \frac{a^4\tan^3(c+dx)}{3d} - \frac{a^4\tan(c+dx)}{d} + a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x - (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d) - (a^4*Tan[c + d*
x]^5)/(5*d) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-a\sec^2(c+dx)\right)^4\,dx &= a^4\int \tan^8(c+dx)\,dx\\
&=\frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^6(c+dx)\,dx\\
&=-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int \tan^4(c+dx)\,dx\\
&=\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^2(c+dx)\,dx\\
&=-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\\
&=a^4x-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.97

$$
a^4\left(\frac{\text{ArcTan}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c
 + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

Maple [A]
time = 0.10, size = 125, normalized size = 1.69

method	result
risch	$a^4x-\frac{8ia^4\left(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44\right)}{105d\left(e^{2i(dx+c)}+1\right)^7}$
derivativedivides	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)-}{d}$
default	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)-}{d}$
norman	$\frac{a^4x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^4x+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{44a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{706a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{3048a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{706a^4(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)-4*a^4*tan(d*x+c)-6*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+
4*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-a^4*(-16/35-1/7
*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [**A**]
time = 0.27, size = 129, normalized size = 1.74

$$a^4x+\frac{\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^4}{35\,d}-\frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^4}{15\,d}+\frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4}{d}-\frac{4\,a^4\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35
*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(
d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(d*x
 + c)/d
```

Fricas [**A**]
time = 2.71, size = 82, normalized size = 1.11

$$\frac{105\,a^4dx\cos\left(dx+c\right)^7-\left(176\,a^4\cos\left(dx+c\right)^6-122\,a^4\cos\left(dx+c\right)^4+66\,a^4\cos\left(dx+c\right)^2-15\,a^4\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(d
*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)^7
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 1\,dx+\int\left(-4\sec^2\left(c+dx\right)\right)\,dx+\int 6\sec^4\left(c+dx\right)dx+\int\left(-4\sec^6\left(c+dx\right)\right)\,dx+\int\sec^8\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c +
 d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8,
x))
```

Giac [**A**]
time = 0.45, size = 66, normalized size = 0.89

$$\frac{15\,a^4\tan\left(dx+c\right)^7-21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+105\,(dx+c)a^4-105\,a^4\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^
3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

Mupad [B]
time = 4.62, size = 61, normalized size = 0.82

$$\frac{\frac{a^4\tan(c+d\,x)^7}{7}-\frac{a^4\tan(c+d\,x)^5}{5}+\frac{a^4\tan(c+d\,x)^3}{3}-a^4\tan(c+d\,x)+d\,x\,a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] ((a^4*tan(c + d*x)^3)/3 - a^4*tan(c + d*x) - (a^4*tan(c + d*x)^5)/5 + (a^4*
tan(c + d*x)^7)/7 + a^4*d*x)/d
```

3.145 $\int \left(a-a\sec^2(c+dx)\right)^3 \, dx$

Optimal. Leaf size=56

$$a^3x-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x-a^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d-1/5*a^3*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - (a^3*Tan[c + d*x])/d + (a^3*Tan[c + d*x]^3)/(3*d) - (a^3*Tan[c + d*
x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(a - a\sec^2(c + dx)\right)^3 \, dx = -\left(a^3 \int \tan^6(c + dx) \, dx\right)$$
$$= -\frac{a^3 \tan^5(c + dx)}{5d} + a^3 \int \tan^4(c + dx) \, dx$$
$$= \frac{a^3 \tan^3(c + dx)}{3d} - \frac{a^3 \tan^5(c + dx)}{5d} - a^3 \int \tan^2(c + dx) \, dx$$
$$= -\frac{a^3 \tan(c + dx)}{d} + \frac{a^3 \tan^3(c + dx)}{3d} - \frac{a^3 \tan^5(c + dx)}{5d} + a^3 \int 1 \, dx$$
$$= a^3 x - \frac{a^3 \tan(c + dx)}{d} + \frac{a^3 \tan^3(c + dx)}{3d} - \frac{a^3 \tan^5(c + dx)}{5d}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 1.04

$$-a^3\left(-\frac{\text{ArcTan}(\tan(c + dx))}{d} + \frac{\tan(c + dx)}{d} - \frac{\tan^3(c + dx)}{3d} + \frac{\tan^5(c + dx)}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
Tan[c + d*x]^5/(5*d)))
```

Maple [A]
time = 0.05, size = 81, normalized size = 1.45

method	result
risch	$a^3 x - \frac{2 i a^3 \left(45\,e^{8 i (dx+c)}+90\,e^{6 i (dx+c)}+140\,e^{4 i (dx+c)}+70\,e^{2 i (dx+c)}+23\right)}{15 d \left(e^{2 i (dx+c)}+1\right)^5}$
derivativedivides	$\frac{a^3 (dx+c) - 3a^3 \tan(dx+c) - 3a^3 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c) + a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^3 (dx+c) - 3a^3 \tan(dx+c) - 3a^3 \left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c) + a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
norman	$\frac{a^3 x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - a^3 x + \frac{2a^3 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} - \frac{32 a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{356 a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d} - \frac{32 a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{2a^3\left(\tan^9 \right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c)-3*a^3*tan(d*x+c)-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+
a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 1.45

$$a^3x - \frac{\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)a^3}{15\,d} + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^3}{d} - \frac{3\,a^3\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/d
 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

Fricas [A]
time = 2.53, size = 69, normalized size = 1.23

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5 - \left(23\,a^3\cos\left(dx+c\right)^4 - 11\,a^3\cos\left(dx+c\right)^2 + 3\,a^3\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x +
 c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3\left(\int (-1)\,dx + \int 3\sec^2\left(c+dx\right)dx + \int \left(-3\sec^4\left(c+dx\right)\right)\,dx + \int \sec^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(c
 + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

Giac [A]
time = 0.46, size = 53, normalized size = 0.95

$$-\frac{3\,a^3\tan\left(dx+c\right)^5 - 5\,a^3\tan\left(dx+c\right)^3 - 15\,(dx+c)a^3 + 15\,a^3\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15*
a^3*tan(d*x + c))/d
```

Mupad [B]
time = 4.47, size = 49, normalized size = 0.88

$$-\frac{\frac{a^3\tan(c+d\,x)^5}{5}-\frac{a^3\tan(c+d\,x)^3}{3}+a^3\tan(c+d\,x)-d\,x\,a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^3,x)
```

```
[Out] -(a^3*tan(c + d*x) - (a^3*tan(c + d*x)^3)/3 + (a^3*tan(c + d*x)^5)/5 - a^3*
d*x)/d
```

3.146 $\int (a - a\sec^2(c+dx))^2\, dx$

Optimal. Leaf size=38

$$a^2x - \frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

```
[Out] a^2*x-a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 3554, 8}

$$\frac{a^2\tan^3(c+dx)}{3d} - \frac{a^2\tan(c+dx)}{d} + a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = a^2 \int \tan^4(c+dx)\, dx$$
$$= \frac{a^2\tan^3(c+dx)}{3d} - a^2 \int \tan^2(c+dx)\, dx$$
$$= -\frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d} + a^2 \int 1\, dx$$
$$= a^2x - \frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.11

$$a^2\left(\frac{\text{ArcTan}(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

Maple [A]
time = 0.04, size = 49, normalized size = 1.29

method	result
derivativedivides	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$a^2x-\frac{4ia^2\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{a^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^2x\left(\tan^4 \ldots\right.}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c)-2*a^2*tan(d*x+c)-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 45, normalized size = 1.18

$$a^2x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^2}{3\,d} - \frac{2\,a^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

Fricas [A]
time = 2.98, size = 56, normalized size = 1.47

$$\frac{3\,a^2dx\cos\left(dx+c\right)^3-\left(4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 1\,dx+\int\left(-2\sec^2\left(c+dx\right)\right)\,dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c + d
*x)**4, x))
```

Giac [A]
time = 0.43, size = 39, normalized size = 1.03

$$\frac{a^2\tan\left(dx+c\right)^3+3\left(dx+c\right)a^2-3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 4.40, size = 33, normalized size = 0.87

$$a^2\,x-\frac{a^2\,\left(3\tan(c+d\,x)-\tan(c+d\,x)^3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - (a^2*(3*tan(c + d*x) - tan(c + d*x)^3))/(3*d)
```

3.147 $\int \left(a-a\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$ax-\frac{a\tan(c+dx)}{d}$$

```
[Out] a*x-a*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3852, 8}

$$ax-\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sec[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)\,dx &= ax-a\int \sec^2(c+dx)\,dx\\ &= ax+\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= ax-\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.62

$$-a\left(-\frac{\text{ArcTan}(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sec[c + d*x]^2,x]
[Out] -(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

Maple [A]
time = 0.03, size = 17, normalized size = 1.06

method	result	size
default	$ax-\frac{a\tan(dx+c)}{d}$	17
derivativedivides	$\frac{(dx+c)a-a\tan(dx+c)}{d}$	22
risch	$ax-\frac{2ia}{d(e^{2i(dx+c)}+1)}$	25
norman	$\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-ax+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] a*x-a*tan(d*x+c)/d
```

Maxima [A]
time = 0.26, size = 16, normalized size = 1.00

$$ax-\frac{a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - a*tan(d*x + c)/d
```

Fricas [A]
time = 2.44, size = 32, normalized size = 2.00

$$\frac{adx\cos(dx+c)-a\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
[Out] (a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\int(-1)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)**2,x)
[Out] -a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

Giac [A]
time = 0.40, size = 16, normalized size = 1.00

$$ax - \frac{a\tan\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
[Out] a*x - a*tan(d*x + c)/d
```

Mupad [B]
time = 4.44, size = 16, normalized size = 1.00

$$a\,x - \frac{a\tan(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a - a/cos(c + d*x)^2,x)
[Out] a*x - (a*tan(c + d*x))/d
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{x}{a}+\frac{\cot(c+dx)}{ad}$$

```
[Out] x/a+cot(d*x+c)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 3554, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] x/a + Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\sec^2(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 31, normalized size = 1.63

$$\frac{\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

Maple [A]
time = 0.05, size = 24, normalized size = 1.26

method	result	size
derivativedivides	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
default	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
risch	$\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}$	29
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{1}{2ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/tan(d*x+c)+arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.48, size = 26, normalized size = 1.37

$$\frac{\frac{dx+c}{a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] ((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

Fricas [A]
time = 2.44, size = 31, normalized size = 1.63

$$\frac{dx\sin(dx+c)+\cos(dx+c)}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2),x)
```

```
[Out] -Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.
time = 0.43, size = 45, normalized size = 2.37

$$\frac{\frac{2\,(dx+c)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 4.70, size = 19, normalized size = 1.00

$$\frac{x}{a}+\frac{\cot(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2),x)
```

```
[Out] x/a + cot(c + d*x)/(a*d)
```

3.149 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}$$

[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4205, 3554, 8}

$$-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sec[c + d*x]^2)^(-2),x]

[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx = \frac{\int \cot^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\int \cot^2(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\int 1\,dx}{a^2}$$
$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 36, normalized size = 0.97

$$-\frac{\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] -1/3*(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(a^
2*d)
```

Maple [**A**]
time = 0.05, size = 34, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
risch	$\frac{x}{a^2}+\frac{4i\left(3\,e^{4i(dx+c)}-3\,e^{2i(dx+c)}+2\right)}{3d\,a^2\left(e^{2i(dx+c)}-1\right)^3}$	53
norman	$\frac{\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{24ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/tan(d*x+c))
```

Maxima [A]
time = 0.49, size = 40, normalized size = 1.08

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,\tan(dx+c)^2-1}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.
time = 3.52, size = 75, normalized size = 2.03

$$\frac{4\,\cos\left(dx+c\right)^3+3\,\left(dx\cos\left(dx+c\right)^2-dx\right)\sin\left(dx+c\right)-3\,\cos\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(d
*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^4\left(c+dx\right)-2\sec^2\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.
time = 0.44, size = 80, normalized size = 2.16

$$\frac{\frac{24\,(dx+c)}{a^2}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

Mupad [B]
time = 4.41, size = 31, normalized size = 0.84

$$\frac{x}{a^2}+\frac{\tan(c+d\,x)^2-\frac{1}{3}}{a^2\,d\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^2,x)
```

```
[Out] x/a^2 + (tan(c + d*x)^2 - 1/3)/(a^2*d*tan(c + d*x)^3)
```

3.150 $\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx$

Optimal. Leaf size=55

$$\frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 3554, 8}

$$\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a - a\sec^2(c+dx))^3}\,dx = -\frac{\int \cot^6(c+dx)\,dx}{a^3}$$
$$= \frac{\cot^5(c+dx)}{5a^3d} + \frac{\int \cot^4(c+dx)\,dx}{a^3}$$
$$= -\frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\int \cot^2(c+dx)\,dx}{a^3}$$
$$= \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} + \frac{\int 1\,dx}{a^3}$$
$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 36, normalized size = 0.65

$$\frac{\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] (Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*d
)
```

Maple [A]
time = 0.06, size = 44, normalized size = 0.80

method	result
derivativedivides	$\frac{\arctan(\tan(dx+c)) - \frac{1}{3\tan(dx+c)^3} + \frac{1}{5\tan(dx+c)^5} + \frac{1}{\tan(dx+c)}}{d\,a^3}$
default	$\frac{\arctan(\tan(dx+c)) - \frac{1}{3\tan(dx+c)^3} + \frac{1}{5\tan(dx+c)^5} + \frac{1}{\tan(dx+c)}}{d\,a^3}$
risch	$\frac{x}{a^3} + \frac{2i\left(45\,e^{8i(dx+c)} - 90\,e^{6i(dx+c)} + 140\,e^{4i(dx+c)} - 70\,e^{2i(dx+c)} + 23\right)}{15d\,a^3\left(e^{2i(dx+c)} - 1\right)^5}$
norman	$\frac{\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} + \frac{1}{160ad} - \frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad} + \frac{11\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad} - \frac{11\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad} + \frac{7\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad} - \frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c))
```

Maxima [A]
time = 0.48, size = 50, normalized size = 0.91

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*ta
n(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.
time = 3.47, size = 109, normalized size = 1.98

$$\frac{23\,\cos\left(dx+c\right)^5-35\,\cos\left(dx+c\right)^3+15\,\left(dx\cos\left(dx+c\right)^4-2\,dx\cos\left(dx+c\right)^2+dx\right)\sin\left(dx+c\right)+15\,\cos\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^4-2\,a^3d\cos\left(dx+c\right)^2+a^3d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d*
x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x + c
)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sec^6\left(c+dx\right)-3\sec^4\left(c+dx\right)+3\sec^2\left(c+dx\right)-1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.
time = 0.43, size = 111, normalized size = 2.02

$$\frac{\frac{480\,(dx+c)}{a^3}+\frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1
/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
- 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 4.54, size = 41, normalized size = 0.75

$$\frac{x}{a^3}+\frac{\tan(c+d\,x)^4-\frac{\tan(c+d\,x)^2}{3}+\frac{1}{5}}{a^3\,d\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^3,x)
```

```
[Out] x/a^3 + (tan(c + d*x)^4 - tan(c + d*x)^2/3 + 1/5)/(a^3*d*tan(c + d*x)^5)
```

3.151 $\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx$

Optimal. Leaf size=73

$$\frac{x}{a^4}+\frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}$$

```
[Out] x/a^4+cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^4/d+1/5*cot(d*x+c)^5/a^4/d-1/7*co
t(d*x+c)^7/a^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 3554, 8}

$$-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 + Cot[c + d*x]/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + Cot[c + d*x]^5/(5
*a^4*d) - Cot[c + d*x]^7/(7*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx &= \frac{\int \cot^8(c+dx)\,dx}{a^4}\\
&= -\frac{\cot^7(c+dx)}{7a^4d}-\frac{\int \cot^6(c+dx)\,dx}{a^4}\\
&= \frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\int \cot^4(c+dx)\,dx}{a^4}\\
&= -\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}-\frac{\int \cot^2(c+dx)\,dx}{a^4}\\
&= \frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\int 1\,dx}{a^4}\\
&= \frac{x}{a^4}+\frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 36, normalized size = 0.49

$$
-\frac{\cot^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\tan^2(c+dx)\right)}{7a^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] -1/7*(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(a^
4*d)
```

Maple [A]
time = 0.07, size = 54, normalized size = 0.74

method	result
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^4}$
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^4}$
risch	$\frac{x}{a^4}+\frac{8i\left(105\,e^{12i(dx+c)}-315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}-770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}-203\,e^{2i(dx+c)}+44\right)}{105d\,a^4\left(e^{2i(dx+c)}-1\right)^7}$
norman	$\frac{\frac{x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{896ad}+\frac{9\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}-\frac{37\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}+\frac{93\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{93\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{37\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{384ad}}{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(arctan(tan(d*x+c))-1/7/tan(d*x+c)^7-1/3/tan(d*x+c)^3+1/5/tan(d*x+c
)^5+1/tan(d*x+c))
```

Maxima [A]
time = 0.49, size = 60, normalized size = 0.82

$$\frac{\frac{105\,(dx+c)}{a^4}+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(67) = 134.
time = 3.57, size = 147, normalized size = 2.01

$$\frac{176\,\cos(dx+c)^7-406\,\cos(dx+c)^5+350\,\cos(dx+c)^3+105\,\left(dx\cos(dx+c)^6-3\,dx\cos(dx+c)^4+3\,dx\cos(dx+c)^2-dx\right)\sin(dx+c)-105\,\cos(dx+c)}{105\,\left(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-a^4d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*(
d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*sin
(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^8(c+dx)-4\sec^6(c+dx)+6\sec^4(c+dx)-4\sec^2(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*sec
(c + d*x)**2 + 1), x)/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(67) = 134.
time = 0.47, size = 139, normalized size = 1.90

$$\frac{\frac{13440\,(dx+c)}{a^4}+\frac{9765\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1295\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1295\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9765\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2*
d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*c)
^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5 + 1
295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]

time = 4.93, size = 51, normalized size = 0.70

$$\frac{x}{a^4}+\frac{\tan(c+d\,x)^6-\frac{\tan(c+d\,x)^4}{3}+\frac{\tan(c+d\,x)^2}{5}-\frac{1}{7}}{a^4\,d\tan\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^4,x)
```

```
[Out] x/a^4 + (tan(c + d*x)^2/5 - tan(c + d*x)^4/3 + tan(c + d*x)^6 - 1/7)/(a^4*d
*tan(c + d*x)^7)
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=98

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\,\mathrm{se}}{}$$

```
[Out] 1/16*(6*a+5*b)*arctanh(sin(f*x+e))/f+1/16*(6*a+5*b)*sec(f*x+e)*tan(f*x+e)/f
+1/24*(6*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f
```

Rubi [A]

time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4131, 3853, 3855}

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*a + 5*b)*ArcTanh[Sin[e + f*x]])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]*Tan[
e + f*x])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Se
c[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^5(e+fx)\tan(e+fx)}{6f} + \frac{1}{6}(6a+5b)\int \sec^5(e+fx)\,dx$$
$$= \frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$
$$= \frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f} + \frac{(6a+5b)\sec^3(e+fx)\tan(}{24f}$$
$$= \frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f} + \frac{(6a+5b)\sec(e+fx)\tan(e+f}{16f}$$

Mathematica [A]
time = 0.37, size = 75, normalized size = 0.77

$$\frac{3(6a+5b)\tanh^{-1}(\sin(e+fx)) + \sec(e+fx)\left(3(6a+5b)+2(6a+5b)\sec^2(e+fx)+8b\sec^4(e+fx)\right)\tan(e+fx)}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
[Out] (3*(6*a + 5*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(6*a + 5*b) + 2*(6*a
 + 5*b)*Sec[e + f*x]^2 + 8*b*Sec[e + f*x]^4)*Tan[e + f*x])/(48*f)
```

Maple [A]
time = 0.13, size = 108, normalized size = 1.10

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.}{f}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.}{f}$
norman	$\frac{\frac{(2a+15b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{(2a+15b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{(10a+11b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{(42a-5b)\left(\tan^3\left(\right.\right.}{24f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}$
risch	$-\frac{i\left(18a\,e^{11i(fx+e)}+15b\,e^{11i(fx+e)}+102a\,e^{9i(fx+e)}+85b\,e^{9i(fx+e)}+84a\,e^{7i(fx+e)}+198b\,e^{7i(fx+e)}-84a\,e^{5i(fx+e)}-198b\right.}{24f\left(e^{2i(fx+e)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+ta
n(f*x+e)))+b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*
x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 134, normalized size = 1.37

$$\frac{3\left(6\,a+5\,b\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(6\,a+5\,b\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(6\,a+5\,b\right)\sin\left(fx+e\right)^{5}-8\left(6\,a+5\,b\right)\sin\left(fx+e\right)^{3}+3\left(10\,a+11\,b\right)\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{6}-3\,\sin\left(fx+e\right)^{4}+3\,\sin\left(fx+e\right)^{2}-1}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
 - 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3*(
10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x +
 e)^2 - 1))/f
```

Fricas [A]
time = 3.72, size = 122, normalized size = 1.24

$$\frac{3\left(6\,a+5\,b\right)\cos\left(fx+e\right)^{6}\log\left(\sin\left(fx+e\right)+1\right)-3\left(6\,a+5\,b\right)\cos\left(fx+e\right)^{6}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\left(6\,a+5\,b\right)\cos\left(fx+e\right)^{4}+2\left(6\,a+5\,b\right)\cos\left(fx+e\right)^{2}+8\,b\right)\sin\left(fx+e\right)}{96\,f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*co
s(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2*(
6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Giac [A]
time = 0.45, size = 121, normalized size = 1.23

$$\frac{3\left(6\,a+5\,b\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\left(6\,a+5\,b\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(18\,a\sin\left(fx+e\right)^{5}+15\,b\sin\left(fx+e\right)^{5}-48\,a\sin\left(fx+e\right)^{3}-40\,b\sin\left(fx+e\right)^{3}+30\,a\sin\left(fx+e\right)+33\,b\sin\left(fx+e\right)\right)}{\left(\sin\left(fx+e\right)^{2}-1\right)^{3}}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(abs(sin(f*x + e) + 1)) - 3*(6*a + 5*b)*log(abs(sin(
f*x + e) - 1)) - 2*(18*a*sin(f*x + e)^5 + 15*b*sin(f*x + e)^5 - 48*a*sin(f*
x + e)^3 - 40*b*sin(f*x + e)^3 + 30*a*sin(f*x + e) + 33*b*sin(f*x + e))/(si
n(f*x + e)^2 - 1)^3)/f
```

Mupad [B]
time = 4.65, size = 102, normalized size = 1.04

$$\frac{\operatorname{atanh}(\sin(e+f\,x))\,\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)}{f}-\frac{\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,\sin(e+f\,x)^5+\left(-a-\frac{5\,b}{6}\right)\,\sin(e+f\,x)^3+\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\sin(e+f\,x)}{f\,\left(\sin(e+f\,x)^6-3\sin(e+f\,x)^4+3\sin(e+f\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^5,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a)/8 + (5*b)/16))/f - (sin(e + f*x)^5*((3*a)/8 + (
5*b)/16) + sin(e + f*x)*((5*a)/8 + (11*b)/16) - sin(e + f*x)^3*(a + (5*b)/6
))/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

```
[Out] 1/8*(4*a+3*b)*arctanh(sin(f*x+e))/f+1/8*(4*a+3*b)*sec(f*x+e)*tan(f*x+e)/f+1
/4*b*sec(f*x+e)^3*tan(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4131, 3853, 3855}

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]])/(8*f) + ((4*a + 3*b)*Sec[e + f*x]*Tan[e
 + f*x])/(8*f) + (b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{1}{4}(4a+3b)\int \sec^3(e+fx)\,dx$$
$$= \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$
$$= \frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{(4a+3b)\sec(e+fx)\tan(e+f}{8f}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.77

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx)) + \sec(e+fx)\left(4a+3b+2b\sec^2(e+fx)\right)\tan(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(4*a + 3*b + 2*b*Sec[e + f*x]^2)*Tan[e + f*x])/(8*f)
```

Maple [A]
time = 0.08, size = 85, normalized size = 1.21

method	result
derivativedivides	$\frac{a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{f}$
default	$\frac{a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{f}$
norman	$\frac{-\frac{(4a-3b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{(4a-3b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{(4a+5b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{(4a+3b)\ln\left(\tan\left(\frac{f}{2}\right.\right.}{8f}$
risch	$-\frac{i\left(4a\,e^{7i(fx+e)}+3b\,e^{7i(fx+e)}+4a\,e^{5i(fx+e)}+11b\,e^{5i(fx+e)}-4a\,e^{3i(fx+e)}-11b\,e^{3i(fx+e)}-4a\,e^{i(fx+e)}-3b\,e^{i(fx+e)}\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+b*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.47

$$\frac{(4a+3b)\log(\sin(fx+e)+1)-(4a+3b)\log(\sin(fx+e)-1)-\frac{2\left((4a+3b)\sin(fx+e)^3-(4a+5b)\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1)
 - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.61, size = 102, normalized size = 1.46

$$\frac{(4a+3b)\cos(fx+e)^4\log(\sin(fx+e)+1)-(4a+3b)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left((4a+3b)\cos(fx+e)^2+2b\right)\sin(fx+e)}{16f\cos(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f*
x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)\sec^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Giac [A]
time = 0.46, size = 98, normalized size = 1.40

$$\frac{(4a+3b)\log(|\sin(fx+e)+1|)-(4a+3b)\log(|\sin(fx+e)-1|)-\frac{2\left(4a\sin(fx+e)^3+3b\sin(fx+e)^3-4a\sin(fx+e)-5b\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/16*((4*a + 3*b)*log(abs(sin(f*x + e) + 1)) - (4*a + 3*b)*log(abs(sin(f*x
+ e) - 1)) - 2*(4*a*sin(f*x + e)^3 + 3*b*sin(f*x + e)^3 - 4*a*sin(f*x + e)
- 5*b*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

Mupad [B]
time = 0.14, size = 78, normalized size = 1.11

$$\frac{\operatorname{atanh}(\sin(e+fx))\left(\frac{a}{2}+\frac{3b}{8}\right)}{f}-\frac{\sin(e+fx)^3\left(\frac{a}{2}+\frac{3b}{8}\right)-\sin(e+fx)\left(\frac{a}{2}+\frac{5b}{8}\right)}{f\left(\sin(e+fx)^4-2\sin(e+fx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^3,x)
```

```
[Out] (atanh(sin(e + f*x))*(a/2 + (3*b)/8))/f - (sin(e + f*x)^3*(a/2 + (3*b)/8) -
 sin(e + f*x)*(a/2 + (5*b)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1))
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e))/f+1/2*b*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4131, 3855}

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2*
f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(2a+b)\int \sec(e+fx)\,dx\\ &= \frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.20

$$\frac{a\tanh^{-1}(\sin(e+fx))}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
f*x]*Tan[e + f*x])/(2*f)
```

Maple [A]
time = 0.04, size = 55, normalized size = 1.38

method	result	size
derivativedivides	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
default	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
norman	$\frac{\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	93
risch	$-\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{\ln\left(e^{i(fx+e)}-i\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b}{2f}+\frac{\ln\left(e^{i(fx+e)}+i\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b}{2f}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*ln(sec(f*x+e)+tan(f*x+e))+b*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*
x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 62, normalized size = 1.55

$$\frac{(2a+b)\log(\sin(fx+e)+1)-(2a+b)\log(\sin(fx+e)-1)-\frac{2b\sin(fx+e)}{\sin(fx+e)^2-1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2*
b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 2.26, size = 78, normalized size = 1.95

$$\frac{(2a+b)\cos(fx+e)^2\log(\sin(fx+e)+1)-(2a+b)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2b\sin(fx+e)}{4f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x + e
)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)\sec\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Giac [A]
time = 0.44, size = 60, normalized size = 1.50

$$\frac{(2\,a+b)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-(2\,a+b)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\,b\sin(fx+e)}{\sin(fx+e)^{2}-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*((2*a + b)*log(abs(sin(f*x + e) + 1)) - (2*a + b)*log(abs(sin(f*x + e)
- 1)) - 2*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B]
time = 4.41, size = 41, normalized size = 1.02

$$\frac{\mathrm{atanh}(\sin\left(e+f\,x\right))\,\left(a+\frac{b}{2}\right)}{f}-\frac{b\,\sin\left(e+f\,x\right)}{2\,f\,\left({\sin\left(e+f\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x),x)
```

```
[Out] (atanh(sin(e + f*x))*(a + b/2))/f - (b*sin(e + f*x))/(2*f*(sin(e + f*x)^2 -
 1))
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\tanh^{-1}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}$$

```
[Out] b*arctanh(sin(f*x+e))/f+a*sin(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4130, 3855}

$$\frac{a\sin(e+fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx&=\frac{a\sin(e+fx)}{f}+b\int \sec(e+fx)\,dx\\&=\frac{b\tanh^{-1}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.46

$$\frac{b\tanh^{-1}(\sin(e+fx))}{f}+\frac{a\cos(fx)\sin(e)}{f}+\frac{a\cos(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/f
```

Maple [A]
time = 0.06, size = 30, normalized size = 1.25

method	result	size
derivativedivides	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
default	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
risch	$-\frac{ia\,\mathrm{e}^{i(fx+e)}}{2f}+\frac{ia\,\mathrm{e}^{-i(fx+e)}}{2f}+\frac{\ln(\mathrm{e}^{i(fx+e)}+i)b}{f}-\frac{\ln(\mathrm{e}^{i(fx+e)}-i)b}{f}$	71
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{f}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{f}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(sin(f*x+e)*a+b*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 1.71

$$\frac{b(\log(\sin(fx+e)+1)-\log(\sin(fx+e)-1))+2\,a\sin(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))/
f
```

Fricas [A]
time = 3.05, size = 43, normalized size = 1.79

$$\frac{b\log(\sin(fx+e)+1)-b\log(-\sin(fx+e)+1)+2\,a\sin(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e))
/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\cos\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Giac [A]
time = 0.44, size = 40, normalized size = 1.67

$$\frac{b\log\left(\left|\sin\left(fx + e\right) + 1\right|\right) - b\log\left(\left|\sin\left(fx + e\right) - 1\right|\right) + 2\,a\sin\left(fx + e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(sin(f*x + e) + 1)) - b*log(abs(sin(f*x + e) - 1)) + 2*a*sin(
f*x + e))/f
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.92

$$\frac{a\,\sin\left(e + f\,x\right) + b\,\mathrm{atanh}\left(\sin\left(e + f\,x\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*sin(e + f*x) + b*atanh(sin(e + f*x)))/f
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*a*sin(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4129, 3092}

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - (a*Sin[e + f*x]^3)/(3*f)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \int \cos(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx\\ &= -\frac{\operatorname{Subst}\left(\int \left(a+b-ax^2\right)\,dx,x,-\sin(e+fx)\right)}{f}\\ &= \frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.67

$$\frac{b\cos(fx)\sin(e)}{f}+\frac{b\cos(e)\sin(fx)}{f}+\frac{a\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
[Out] (b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Sin
[e + f*x]^3)/(3*f)
```

Maple [A]
time = 0.08, size = 33, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\frac{a\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
default	$\frac{\frac{a\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
risch	$\frac{3a\sin(fx+e)}{4f}+\frac{\sin(fx+e)b}{f}+\frac{a\sin(3fx+3e)}{12f}$	40
norman	$\frac{\frac{2(a-3b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2(a-3b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*a*(2+cos(f*x+e)^2)*sin(f*x+e)+sin(f*x+e)*b)
```

Maxima [A]
time = 0.27, size = 29, normalized size = 0.97

$$-\frac{a\sin\left(fx+e\right)^3-3\left(a+b\right)\sin\left(fx+e\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

Fricas [A]
time = 2.71, size = 30, normalized size = 1.00

$$\frac{\left(a\cos\left(fx+e\right)^2+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)\cos^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**3, x)
```

Giac [A]
time = 0.43, size = 34, normalized size = 1.13

$$-\frac{a\sin\left(fx+e\right)^{3}-3\,a\sin\left(fx+e\right)-3\,b\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.93

$$-\frac{\frac{a\sin\left(e+f\,x\right)^{3}}{3}-\sin\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] -((a*sin(e + f*x)^3)/3 - sin(e + f*x)*(a + b))/f
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[Out] (a+b)*sin(f*x+e)/f-1/3*(2*a+b)*sin(f*x+e)^3/f+1/5*a*sin(f*x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4129, 3092, 380}

$$-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - ((2*a + b)*Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*
x]^5)/(5*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \int \cos^3(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx \\
&= -\frac{\text{Subst}\left(\int (1-x^2)\left(a+b-ax^2\right)\,dx, x, -\sin(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(2a+b)x^2+ax^4\right)\,dx, x, -\sin(e+fx)\right)}{f} \\
&= \frac{(a+b)\sin(e+fx)}{f} - \frac{(2a+b)\sin^3(e+fx)}{3f} + \frac{a\sin^5(e+fx)}{5f}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 1.48

$$\frac{5a\sin(e+fx)}{8f} + \frac{b\sin(e+fx)}{f} - \frac{b\sin^3(e+fx)}{3f} + \frac{5a\sin(3(e+fx))}{48f} + \frac{a\sin(5(e+fx))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (5*a*Sin[e + f*x])/(8*f) + (b*Sin[e + f*x])/f - (b*Sin[e + f*x]^3)/(3*f) +
(5*a*Sin[3*(e + f*x)])/(48*f) + (a*Sin[5*(e + f*x)])/(80*f)
```

Maple [A]
time = 0.11, size = 54, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(fx+e)+\frac{4\left(\cos^2(fx+e)\right)}{3}\right)\sin(fx+e)}{5}+\frac{b\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}}{f}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(fx+e)+\frac{4\left(\cos^2(fx+e)\right)}{3}\right)\sin(fx+e)}{5}+\frac{b\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}}{f}$
risch	$\frac{5a\sin(fx+e)}{8f}+\frac{3\sin(fx+e)b}{4f}+\frac{a\sin(5fx+5e)}{80f}+\frac{5a\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)b}{12f}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(a+5b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{2(a+5b)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(19a+5b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*a*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+1/3*b*(2+cos(f*x+
e)^2)*sin(f*x+e))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.92

$$\frac{3\,a\sin\left(fx+e\right)^{5}-5\left(2\,a+b\right)\sin\left(fx+e\right)^{3}+15\left(a+b\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
+ e))/f
```

Fricas [A]
time = 3.06, size = 48, normalized size = 0.96

$$\frac{\left(3\,a\cos\left(fx+e\right)^{4}+\left(4\,a+5\,b\right)\cos\left(fx+e\right)^{2}+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*x
 + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cos^{5}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**5, x)
```

Giac [A]
time = 0.42, size = 57, normalized size = 1.14

$$\frac{3\,a\sin\left(fx+e\right)^{5}-10\,a\sin\left(fx+e\right)^{3}-5\,b\sin\left(fx+e\right)^{3}+15\,a\sin\left(fx+e\right)+15\,b\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a*
sin(f*x + e) + 15*b*sin(f*x + e))/f
```

Mupad [B]
time = 4.32, size = 43, normalized size = 0.86

$$\frac{\frac{a\sin(e+f\,x)^5}{5}+\left(-\frac{2\,a}{3}-\frac{b}{3}\right)\,\sin\left(e+f\,x\right)^3+\left(a+b\right)\,\sin\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*sin(e + f*x)^5)/5 - sin(e + f*x)^3*((2*a)/3 + b/3) + sin(e + f*x)*(a +
b))/f
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan^5(e+fx)}{35f}$$

```
[Out] 1/7*(7*a+6*b)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^6*tan(f*x+e)/f+2/21*(7*a+6*b)*t
an(f*x+e)^3/f+1/35*(7*a+6*b)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4131, 3852}

$$\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((7*a + 6*b)*Tan[e + f*x])/(7*f) + (b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) +
(2*(7*a + 6*b)*Tan[e + f*x]^3)/(21*f) + ((7*a + 6*b)*Tan[e + f*x]^5)/(35*f)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{1}{7}(7a+6b)\int \sec^6(e+fx)\,dx\\ &= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}-\frac{(7a+6b)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,d\right.}{7f}\\ &= \frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b}{}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 81, normalized size = 0.93

$$\frac{a\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}+\frac{b\left(\tan(e+fx)+\tan^3(e+fx)+\frac{3}{5}\tan^5(e+fx)+\frac{1}{7}\tan^7(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
+ f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.90

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+}{f}$
risch	$\frac{16i\left(70a\,e^{8i(fx+e)}+175a\,e^{6i(fx+e)}+210b\,e^{6i(fx+e)}+147a\,e^{4i(fx+e)}+126b\,e^{4i(fx+e)}+49a\,e^{2i(fx+e)}+42b\,e^{2i(fx+e)}+7a+6b\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4(5a+3b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{4(5a+3b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{8(91a+53b)\left(\tan^7\left(\frac{fx}{2}+\right.\right.}{35f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/7
*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.74

$$\frac{15\,b\tan\left(fx+e\right)^7+21\left(a+3\,b\right)\tan\left(fx+e\right)^5+35\left(2\,a+3\,b\right)\tan\left(fx+e\right)^3+105\left(a+b\right)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*t
an(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

Fricas [A]
time = 2.74, size = 79, normalized size = 0.91

$$\frac{\left(8\left(7a+6b\right)\cos\left(fx+e\right)^{6}+4\left(7a+6b\right)\cos\left(fx+e\right)^{4}+3\left(7a+6b\right)\cos\left(fx+e\right)^{2}+15b\right)\sin\left(fx+e\right)}{105f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*a
 + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Giac [A]
time = 0.44, size = 79, normalized size = 0.91

$$\frac{15b\tan\left(fx+e\right)^{7}+21a\tan\left(fx+e\right)^{5}+63b\tan\left(fx+e\right)^{5}+70a\tan\left(fx+e\right)^{3}+105b\tan\left(fx+e\right)^{3}+105a\tan\left(fx+e\right)+105b\tan\left(fx+e\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 70
*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan(f
*x + e))/f
```

Mupad [B]
time = 4.31, size = 56, normalized size = 0.64

$$\frac{\frac{b\tan(e+f\,x)^7}{7}+\left(\frac{a}{5}+\frac{3\,b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)^5*(a/5 + (3*b)/5) + (b*tan(e + f*x)^7)/7 + tan(e + f*x)^3*((2
*a)/3 + b) + tan(e + f*x)*(a + b))/f
```

3.159 $\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$

Optimal. Leaf size=65

$$\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\tan^3(e+fx)}{15f}$$

```
[Out] 1/5*(5*a+4*b)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^4*tan(f*x+e)/f+1/15*(5*a+4*b)*t
an(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4131, 3852}

$$\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tan[e + f*x])/(5*f) + (b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) +
((5*a + 4*b)*Tan[e + f*x]^3)/(15*f)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{1}{5}(5a+4b)\int \sec^4(e+fx)\,dx\\
&= \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}-\frac{(5a+4b)\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\text{t}\right.}{5f}\\
&= \frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\,\text{ta}}{15}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 61, normalized size = 0.94

$$\frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}+\frac{b\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*x
]^3)/3 + Tan[e + f*x]^5/5))/f
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.89

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)}{f}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)}{f}$
risch	$\frac{4i\left(15a\,e^{6i(fx+e)}+35a\,e^{4i(fx+e)}+40b\,e^{4i(fx+e)}+25a\,e^{2i(fx+e)}+20b\,e^{2i(fx+e)}+5a+4b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{8(2a+b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{8(2a+b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(25a+29b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{15f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*s
ec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.71

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,(a+2\,b)\tan\left(fx+e\right)^3+15\,(a+b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
+ e))/f
```

Fricas [A]
time = 3.46, size = 60, normalized size = 0.92

$$\frac{\left(2\left(5a+4b\right)\cos\left(fx+e\right)^{4}+\left(5a+4b\right)\cos\left(fx+e\right)^{2}+3b\right)\sin\left(fx+e\right)}{15f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin(
f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [A]
time = 0.42, size = 57, normalized size = 0.88

$$\frac{3b\tan\left(fx+e\right)^{5}+5a\tan\left(fx+e\right)^{3}+10b\tan\left(fx+e\right)^{3}+15a\tan\left(fx+e\right)+15b\tan\left(fx+e\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a*
tan(f*x + e) + 15*b*tan(f*x + e))/f
```

Mupad [B]
time = 4.50, size = 42, normalized size = 0.65

$$\frac{\frac{b\tan(e+f\,x)^{5}}{5}+\left(\frac{a}{3}+\frac{2\,b}{3}\right)\,\tan(e+f\,x)^{3}+(a+b)\,\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^4,x)
```

```
[Out] (tan(e + f*x)^3*(a/3 + (2*b)/3) + (b*tan(e + f*x)^5)/5 + tan(e + f*x)*(a +
b))/f
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=43

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}$$

```
[Out] 1/3*(3*a+2*b)*tan(f*x+e)/f+1/3*b*sec(f*x+e)^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4131, 3852, 8}

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}+\frac{1}{3}(3a+2b)\int \sec^2(e+fx)\,dx\\ &= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}-\frac{(3a+2b)\text{Subst}(\int 1\,dx,x,-\tan(e-}{3f}\\ &= \frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 36, normalized size = 0.84

$$\frac{a\tan(e+fx)}{f}+\frac{b\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] (a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A]
time = 0.04, size = 35, normalized size = 0.81

method	result	size
derivativedivides	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$	35
default	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$	35
risch	$\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+3a+2b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	63
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4(3a+b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.86

$$\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

Fricas [A]
time = 3.26, size = 40, normalized size = 0.93

$$\frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec ^{2}\left(e+fx\right)\right)\sec ^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [**A**]
time = 0.43, size = 34, normalized size = 0.79

$$\frac{b\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)+3\,b\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

Mupad [**B**]
time = 4.46, size = 28, normalized size = 0.65

$$\frac{b\tan(e+f\,x)^{3}}{3\,f}+\frac{\tan(e+f\,x)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x)^3)/(3*f) + (tan(e + f*x)*(a + b))/f
```

3.161 $\int (a+b\sec^2(e+fx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3852, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^2(e+fx))\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.00, size = 16, normalized size = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
norman	$\frac{ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.07

$$ax+\frac{b\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16)=32$.
time = 2.79, size = 34, normalized size = 2.27

$$\frac{afx\cos(fx+e)+b\sin(fx+e)}{f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.43, size = 15, normalized size = 1.00

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [B]
time = 4.42, size = 17, normalized size = 1.13

$$\frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*(a+2*b)*x+1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4130, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}(a+2b)\int 1\,dx\\ &= \frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 1.06

$$bx+\frac{a(e+fx)}{2f}+\frac{a\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
[Out] b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

Maple [A]
time = 0.06, size = 37, normalized size = 1.19

method	result
risch	$\frac{ax}{2}+xb+\frac{a\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
default	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
norman	$\frac{(-\frac{a}{2}-b)x+(-\frac{a}{2}-b)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{a}{2}+b)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{a}{2}+b)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\left(\tan^3\left(\frac{fx}{2}\cdots\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(f*x+e))
```

Maxima [A]
time = 0.48, size = 40, normalized size = 1.29

$$\frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 1.95, size = 30, normalized size = 0.97

$$\frac{(a+2\,b)fx+a\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A]
time = 3.45, size = 51, normalized size = 1.65

$$a\left(\begin{cases} \frac{x\sin^2(e+fx)}{2} + \frac{x\cos^2(e+fx)}{2} + \frac{\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f \neq 0 \\ x\cos^2(e) & \text{otherwise} \end{cases}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(e
 + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

Giac [A]
time = 0.42, size = 37, normalized size = 1.19

$$\frac{(fx+e)(a+2b) + \frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B]
time = 4.31, size = 25, normalized size = 0.81

$$\frac{\frac{a\sin(2e+2fx)}{4} + fx\left(\frac{a}{2}+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*sin(2*e + 2*f*x))/4 + f*x*(a/2 + b))/f
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*si
n(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4130, 2715, 8}

$$\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{1}{4}(3a+4b)\int \cos^2(e+fx)\,dx$$
$$= \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$
$$= \frac{1}{8}(3a+4b)x + \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+f}{4}$$

Mathematica [A]
time = 0.11, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)])
/(32*f)
```

Maple [A]
time = 0.10, size = 65, normalized size = 1.07

method	result
risch	$\frac{3ax}{8}+\frac{xb}{2}+\frac{\sin(4fx+4e)a}{32f}+\frac{a\sin(2fx+2e)}{4f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
norman	$\frac{\left(-\frac{3a}{8}-\frac{b}{2}\right)x+\left(-\frac{9a}{8}-\frac{3b}{2}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{3a}{4}-b\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{3a}{4}+b\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{3a}{8}+\frac{b}{2}\right)x\left(\tan^1\right.}{\left(1+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+b*(1/2*
sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.49, size = 78, normalized size = 1.28

$$\frac{(fx+e)(3a+4b)+\frac{(3a+4b)\tan(fx+e)^3+(5a+4b)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan(
f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.46, size = 52, normalized size = 0.85

$$\frac{(3a+4b)fx+\left(2a\cos\left(fx+e\right)^3+(3a+4b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin(
f*x + e))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cos^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

Giac [A]
time = 0.41, size = 73, normalized size = 1.20

$$\frac{(fx+e)(3a+4b)+\frac{3a\tan(fx+e)^3+4b\tan(fx+e)^3+5a\tan(fx+e)+4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*a
*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f
```

Mupad [B]
time = 4.48, size = 67, normalized size = 1.10

$$x\left(\frac{3a}{8}+\frac{b}{2}\right)+\frac{\left(\frac{3a}{8}+\frac{b}{2}\right)\tan(e+fx)^3+\left(\frac{5a}{8}+\frac{b}{2}\right)\tan(e+fx)}{f\left(\tan\left(e+fx\right)^4+2\tan\left(e+fx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 + b/2) + (tan(e + f*x)^3*((3*a)/8 + b/2) + tan(e + f*x)*((5*a)/8
 + b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{16}(5a+6b)x+\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+$$

```
[Out] 1/16*(5*a+6*b)*x+1/16*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(5*a+6*b)*cos(
f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4130, 2715, 8}

$$\frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 6*b)*x)/16 + ((5*a + 6*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((5*a
 + 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{6}(5a+6b)\int \cos^4(e+fx)\,dx$$
$$= \frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$
$$= \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(5a+6b)\cos^3(e+fx)\sin(e}{24f}$$
$$= \frac{1}{16}(5a+6b)x + \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(5a+6b)\text{c}}{}$$

Mathematica [A]
time = 0.13, size = 68, normalized size = 0.76

$$\frac{60ae+72be+60afx+72bfx+(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+fx))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] + (
9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.97

method	result
risch	$\frac{5ax}{16}+\frac{3xb}{8}+\frac{a\sin(6fx+6e)}{192f}+\frac{3\sin(4fx+4e)a}{64f}+\frac{\sin(4fx+4e)b}{32f}+\frac{15a\sin(2fx+2e)}{64f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3}{8}\right.}{f}$
default	$\frac{a\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3}{8}\right.}{f}$
norman	$\frac{\left(-\frac{5a}{16}-\frac{3b}{8}\right)x+\left(-\frac{45a}{16}-\frac{27b}{8}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5}{1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16
*f*x+5/16*e)+b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)
)
```

Maxima [A]
time = 0.48, size = 110, normalized size = 1.24

$$\frac{3\,(f x+e)(5\,a+6\,b)+\frac{3\,(5\,a+6\,b)\tan(fx+e)^5+8\,(5\,a+6\,b)\tan(fx+e)^3+3\,(11\,a+10\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6*
b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f
*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.87, size = 72, normalized size = 0.81

$$\frac{3\,(5\,a+6\,b)fx+\left(8\,a\cos\left(fx+e\right)^5+2\,(5\,a+6\,b)\cos\left(fx+e\right)^3+3\,(5\,a+6\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)^
3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 96, normalized size = 1.08

$$\frac{3\,(fx+e)(5\,a+6\,b)+\frac{15\,a\tan(fx+e)^5+18\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3+48\,b\tan(fx+e)^3+33\,a\tan(fx+e)+30\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
+ 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*tan(
f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B]
time = 4.94, size = 91, normalized size = 1.02

$$x\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)+\frac{\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}+b\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a}{16}+\frac{5\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left({\tan\left(e+f\,x\right)}^6+3\,{\tan\left(e+f\,x\right)}^4+3\,{\tan\left(e+f\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 + (3*b)/8) + (tan(e + f*x)^5*((5*a)/16 + (3*b)/8) + tan(e + f*x
)*((11*a)/16 + (5*b)/8) + tan(e + f*x)^3*((5*a)/6 + b))/(f*(3*tan(e + f*x)^
2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=165

$$\frac{(48a^2+80ab+35b^2)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f}+\frac{(48a^2+80ab+3}{}$$

```
[Out] 1/128*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e))/f+1/128*(48*a^2+80*a*b+35*
b^2)*sec(f*x+e)*tan(f*x+e)/f+1/192*(48*a^2+80*a*b+35*b^2)*sec(f*x+e)^3*tan(
f*x+e)/f+1/48*b*(10*a+7*b)*sec(f*x+e)^5*tan(f*x+e)/f+1/8*b*sec(f*x+e)^7*(a+
b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 424, 393, 205, 212}

$$\frac{(48a^2+80ab+35b^2)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{(48a^2+80ab+35b^2)\tan(e+fx)\sec^3(e+fx)}{192f}+\frac{(48a^2+80ab+35b^2)\tan(e+fx)\sec(e+fx)}{128f}+\frac{b(10a+7b)\tan(e+fx)\sec^5(e+fx)}{48f}+\frac{b\tan(e+fx)\sec^7(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]])/(128*f) + ((48*a^2 + 80*
a*b + 35*b^2)*Sec[e + f*x]*Tan[e + f*x])/(128*f) + ((48*a^2 + 80*a*b + 35*b
^2)*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (b*(10*a + 7*b)*Sec[e + f*x]^5*T
an[e + f*x])/(48*f) + (b*Sec[e + f*x]^7*(a + b - a*Sin[e + f*x]^2)*Tan[e +
f*x])/(8*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^5}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} - \frac{\text{Subst}\Big(\int$$

$$= \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b\sec^7(e+fx)\,(a+b-$$

$$= \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{b(10a+7b)\,\text{se}$$

$$= \frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} + \frac{(48a^2+80ab+$$

$$= \frac{(48a^2+80ab+35b^2)\tanh^{-1}(\sin(e+fx))}{128f} + \frac{(48a^2+80ab+35$$

Mathematica [A]
time = 0.60, size = 119, normalized size = 0.72

$$\frac{3(48a^2+80ab+35b^2)\tanh^{-1}(\sin(e+fx))+\sec(e+fx)\left(3(48a^2+80ab+35b^2)+2(48a^2+80ab+35b^2)\sec^2(e+fx)+8b(16a+7b)\sec^4(e+fx)+48b^2\sec^6(e+fx)\right)\tan(e+fx)}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(48*a
^2 + 80*a*b + 35*b^2) + 2*(48*a^2 + 80*a*b + 35*b^2)*Sec[e + f*x]^2 + 8*b*(
16*a + 7*b)*Sec[e + f*x]^4 + 48*b^2*Sec[e + f*x]^6)*Tan[e + f*x])/(384*f)
```

Maple [A]
time = 0.16, size = 180, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.}{}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.}{}$
norman	$\frac{\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{\left(80a^2+176ab+93b^2\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}-\frac{\left(432a^2+1360ab-1085b^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192f}-\frac{\left(432a^2+13\right.}{}}{}$
risch	$-\frac{i\left(144a^2e^{15i(fx+e)}+240ab\,e^{15i(fx+e)}+105b^2e^{15i(fx+e)}+1104a^2e^{13i(fx+e)}+1840ab\,e^{13i(fx+e)}+805b^2e^{13i(fx+e)}+2448\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+
tan(f*x+e)))+2*a*b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*
tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/8*sec(f*x+e)^7-7/48*se
c(f*x+e)^5-35/192*sec(f*x+e)^3-35/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(sec(
f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 210, normalized size = 1.27

$$\frac{3\,(48\,a^2+80\,ab+35\,b^2)\log\left(\sin\left(fx+e\right)+1\right)-3\,(48\,a^2+80\,ab+35\,b^2)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(48\,a^2+80\,ab+35\,b^2)\sin(fx+e)^7-11\,(48\,a^2+80\,ab+35\,b^2)\sin(fx+e)^5+(624\,a^2+1168\,ab+511\,b^2)\sin(fx+e)^3-3\,(80\,a^2+176\,ab+93\,b^2)\sin(fx+e)\right)}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80*
a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin(f
*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 1168*
a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x + e))
/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 +
 1))/f
```

Fricas [A]
time = 2.33, size = 177, normalized size = 1.07

$$\frac{3\,(48\,a^2+80\,ab+35\,b^2)\cos(fx+e)^8\log(\sin(fx+e)+1)-3\,(48\,a^2+80\,ab+35\,b^2)\cos(fx+e)^8\log(-\sin(fx+e)+1)+2\,(3\,(48\,a^2+80\,ab+35\,b^2)\cos(fx+e)^6+2\,(48\,a^2+80\,ab+35\,b^2)\cos(fx+e)^4+8\,(16\,ab+7\,b^2)\cos(fx+e)^2+48\,b^2)\sin(fx+e)}{768\,f\cos(fx+e)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3*(
48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*cos
(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))/(f*
cos(f*x + e)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^2\sec^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

Giac [A]
time = 0.48, size = 221, normalized size = 1.34

$$\frac{3\,(48\,a^2+80\,ab+35\,b^2)\log(|\sin(fx+e)+1|)-3\,(48\,a^2+80\,ab+35\,b^2)\log(|\sin(fx+e)-1|)-\frac{2\left(144\,a^2\sin(fx+e)^7+240\,ab\sin(fx+e)^7+105\,b^2\sin(fx+e)^7-528\,a^2\sin(fx+e)^5-880\,ab\sin(fx+e)^5-385\,b^2\sin(fx+e)^5+624\,a^2\sin(fx+e)^3+1168\,ab\sin(fx+e)^3+511\,b^2\sin(fx+e)^3-240\,a^2\sin(fx+e)-528\,ab\sin(fx+e)-279\,b^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^4}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(48*a^2
+ 80*a*b + 35*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(144*a^2*sin(f*x + e)^7 +
 240*a*b*sin(f*x + e)^7 + 105*b^2*sin(f*x + e)^7 - 528*a^2*sin(f*x + e)^5 -
 880*a*b*sin(f*x + e)^5 - 385*b^2*sin(f*x + e)^5 + 624*a^2*sin(f*x + e)^3 +
 1168*a*b*sin(f*x + e)^3 + 511*b^2*sin(f*x + e)^3 - 240*a^2*sin(f*x + e) -
528*a*b*sin(f*x + e) - 279*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^4)/f
```

Mupad [B]
time = 4.68, size = 170, normalized size = 1.03

$$\frac{\left(-\frac{3\,a^2}{8}-\frac{5\,a\,b}{8}-\frac{35\,b^2}{128}\right)\sin(e+f\,x)^7+\left(\frac{11\,a^2}{8}+\frac{55\,a\,b}{24}+\frac{385\,b^2}{384}\right)\sin(e+f\,x)^5+\left(-\frac{13\,a^2}{8}-\frac{73\,a\,b}{24}-\frac{511\,b^2}{384}\right)\sin(e+f\,x)^3+\left(\frac{5\,a^2}{8}+\frac{11\,a\,b}{8}+\frac{93\,b^2}{128}\right)\sin(e+f\,x)}{f\,\left(\sin(e+f\,x)^8-4\sin(e+f\,x)^6+6\sin(e+f\,x)^4-4\sin(e+f\,x)^2+1\right)}+\frac{\mathrm{atanh}(\sin(e+f\,x))\left(\frac{3\,a^2}{8}+\frac{5\,a\,b}{8}+\frac{35\,b^2}{128}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^5,x)
```

```
[Out] (sin(e + f*x)*((11*a*b)/8 + (5*a^2)/8 + (93*b^2)/128) - sin(e + f*x)^7*((5*
a*b)/8 + (3*a^2)/8 + (35*b^2)/128) + sin(e + f*x)^5*((55*a*b)/24 + (11*a^2)
/8 + (385*b^2)/384) - sin(e + f*x)^3*((73*a*b)/24 + (13*a^2)/8 + (511*b^2)/
384))/(f*(6*sin(e + f*x)^4 - 4*sin(e + f*x)^2 - 4*sin(e + f*x)^6 + sin(e +
f*x)^8 + 1)) + (atanh(sin(e + f*x))*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/128))
/f
```

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=129

$$\frac{(8a^2+12ab+5b^2)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(8a^2+12ab+5b^2)\sec(e+fx)\tan(e+fx)}{16f}+\frac{b(8a+5b)\sec^3(e+}{24f}$$

[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctanh(sin(f*x+e))/f+1/16*(8*a^2+12*a*b+5*b^2)*sec(f*x+e)*tan(f*x+e)/f+1/24*b*(8*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4232, 424, 393, 205, 212}

$$\frac{(8a^2+12ab+5b^2)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(8a^2+12ab+5b^2)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b(8a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{b\tan(e+fx)\sec^5(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6f}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]

[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]])/(16*f) + ((8*a^2 + 12*a*b + 5*b^2)*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (b*(8*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Sec[e + f*x]^5*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x])/(6*f)

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^4}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^5(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\,(a+b-a\,\text{s}}{6}$$

$$= \frac{(8a^2+12ab+5b^2)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(8a+5b)\sec^3(e+}{24}$$

$$= \frac{(8a^2+12ab+5b^2)\tanh^{-1}(\sin(e+fx))}{16f} + \frac{(8a^2+12ab+5b^2)\sec}{16}$$

Mathematica [A]
time = 0.45, size = 94, normalized size = 0.73

$$\frac{3(8a^2+12ab+5b^2)\tanh^{-1}(\sin(e+fx)) + \sec(e+fx)\left(3(8a^2+12ab+5b^2)+2b(12a+5b)\sec^2(e+fx)+8b^2\sec^4(e+fx)\right)\tan(e+fx)}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(8*a^2 + 12*a*b + 5*b^2) + 2*b*(12*a + 5*b)*Sec[e + f*x]^2 + 8*b^2*Sec[e + f*x]^4)*Tan[e + f*x])/(48*f)

Maple [A]
time = 0.10, size = 147, normalized size = 1.14

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{f}$
default	$\frac{a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{f}$
norman	$\frac{\frac{\left(8a^2+4ab+15b^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{\left(8a^2+4ab+15b^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{\left(8a^2+20ab+11b^2\right)\left(\tan\right.}{8f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}$
risch	$-\frac{i\left(24a^2e^{11i(fx+e)}+36ab\,e^{11i(fx+e)}+15b^2e^{11i(fx+e)}+72a^2e^{9i(fx+e)}+204ab\,e^{9i(fx+e)}+85b^2e^{9i(fx+e)}+48a^2e^{7i(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+2*a*b*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e))))

Maxima [A]
time = 0.27, size = 174, normalized size = 1.35

$$\frac{3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)+1\right)-3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(8\,a^2+12\,ab+5\,b^2)\sin(fx+e)^5-8\,(6\,a^2+12\,ab+5\,b^2)\sin(fx+e)^3+3\,(8\,a^2+20\,ab+11\,b^2)\sin(fx+e)\right)}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*b^2)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f

Fricas [A]
time = 2.89, size = 151, normalized size = 1.17

$$\frac{3\,(8\,a^2+12\,ab+5\,b^2)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\,(8\,a^2+12\,ab+5\,b^2)\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\,(8\,a^2+12\,ab+5\,b^2)\cos\left(fx+e\right)^4+2\,(12\,ab+5\,b^2)\cos\left(fx+e\right)^2+8\,b^2\right)\sin\left(fx+e\right)}{96\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(
8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a^2
 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 + 8*b
^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \sec^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

Giac [A]
time = 0.47, size = 183, normalized size = 1.42

$$\frac{3\left(8a^2 + 12ab + 5b^2\right)\log\left(\left|\sin\left(fx + e\right) + 1\right|\right) - 3\left(8a^2 + 12ab + 5b^2\right)\log\left(\left|\sin\left(fx + e\right) - 1\right|\right) - \frac{2\left(24a^2\sin(fx+e)^5 + 36ab\sin(fx+e)^5 + 15b^2\sin(fx+e)^5 - 48a^2\sin(fx+e)^3 - 96ab\sin(fx+e)^3 - 40b^2\sin(fx+e)^3 + 24a^2\sin(fx+e) + 60ab\sin(fx+e) + 33b^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2 - 1\right)^3}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(8*a^2 + 12
*a*b + 5*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(24*a^2*sin(f*x + e)^5 + 36*a*
b*sin(f*x + e)^5 + 15*b^2*sin(f*x + e)^5 - 48*a^2*sin(f*x + e)^3 - 96*a*b*s
in(f*x + e)^3 - 40*b^2*sin(f*x + e)^3 + 24*a^2*sin(f*x + e) + 60*a*b*sin(f*
x + e) + 33*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B]
time = 4.55, size = 134, normalized size = 1.04

$$\frac{\mathrm{atanh}(\sin\left(e + f\,x\right))\,\left(\frac{a^2}{2} + \frac{3\,a\,b}{4} + \frac{5\,b^2}{16}\right)}{f} - \frac{\left(\frac{a^2}{2} + \frac{3\,a\,b}{4} + \frac{5\,b^2}{16}\right)\,\sin\left(e + f\,x\right)^5 + \left(-a^2 - 2\,a\,b - \frac{5\,b^2}{6}\right)\,\sin\left(e + f\,x\right)^3 + \left(\frac{a^2}{2} + \frac{5\,a\,b}{4} + \frac{11\,b^2}{16}\right)\,\sin\left(e + f\,x\right)}{f\,\left(\sin\left(e + f\,x\right)^6 - 3\sin\left(e + f\,x\right)^4 + 3\sin\left(e + f\,x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^3,x)
```

```
[Out] (atanh(sin(e + f*x))*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/f - (sin(e + f*x)*((
5*a*b)/4 + a^2/2 + (11*b^2)/16) - sin(e + f*x)^3*(2*a*b + a^2 + (5*b^2)/6)
+ sin(e + f*x)^5*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/(f*(3*sin(e + f*x)^2 - 3
*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.167 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\,(a+b-a\sin^2(}{4f}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e))/f+3/8*b*(2*a+b)*sec(f*x+e)*tan(
f*x+e)/f+1/4*b*sec(f*x+e)^3*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 424, 393, 212}

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)\,(-a\sin^2(e+fx)+a+b)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (3*b*(2*a + b)*Sec[
e + f*x]*Tan[e + f*x])/(8*f) + (b*Sec[e + f*x]^3*(a + b - a*Sin[e + f*x]^2)
*Tan[e + f*x])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^3}\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f}-\frac{\text{Subst}\left(\int\frac{-(\cdots}{}\right.}{}$$
$$=\frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\left(a+b-a\sin\cdots\right.}{4f}$$
$$=\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\sec(e+fx)\,\text{ta}\cdots}{8f}$$

Mathematica [A]
time = 0.17, size = 63, normalized size = 0.69

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))+b\sec(e+fx)\left(8a+3b+2b\sec^2(e+fx)\right)\tan(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]] + b*Sec[e + f*x]*(8*a + 3*b
+ 2*b*Sec[e + f*x]^2)*Tan[e + f*x])/(8*f)
```

Maple [A]
time = 0.07, size = 107, normalized size = 1.18

method	result
derivativedivides	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\cdots\right.\right.}{f}$
default	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\cdots\right.\right.}{f}$

norman	$\frac{-\frac{b(8a-3b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{b(8a-3b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{b(8a+5b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{(8a^2+8ab+3b^2}{}$
risch	$-\frac{ib\left(8a\,e^{7i(fx+e)}+3b\,e^{7i(fx+e)}+8a\,e^{5i(fx+e)}+11b\,e^{5i(fx+e)}-8a\,e^{3i(fx+e)}-11b\,e^{3i(fx+e)}-8a\,e^{i(fx+e)}-3b\,e^{i(fx+e)}\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*ln(sec(f*x+e)+tan(f*x+e))+2*a*b*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(
sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)
+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.37

$$\frac{(8a^2+8ab+3b^2)\log(\sin(fx+e)+1)-(8a^2+8ab+3b^2)\log(\sin(fx+e)-1)-\frac{2\left((8ab+3b^2)\sin(fx+e)^3-(8ab+5b^2)\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b^
2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5*b
^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.32, size = 123, normalized size = 1.35

$$\frac{(8a^2+8ab+3b^2)\cos(fx+e)^4\log(\sin(fx+e)+1)-(8a^2+8ab+3b^2)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left((8ab+3b^2)\cos(fx+e)^2+2b^2\right)\sin(fx+e)}{16f\cos(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^2
 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b^2
)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)^2\sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)

Giac [A]
time = 0.46, size = 120, normalized size = 1.32

$$\frac{(8a^2+8ab+3b^2)\log(|\sin(fx+e)+1|)-(8a^2+8ab+3b^2)\log(|\sin(fx+e)-1|)-\frac{2\left(8ab\sin(fx+e)^3+3b^2\sin(fx+e)^3-8ab\sin(fx+e)-5b^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(abs(sin(f*x + e) + 1)) - (8*a^2 + 8*a*b + 3*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(8*a*b*sin(f*x + e)^3 + 3*b^2*sin(f*x + e)^3 - 8*a*b*sin(f*x + e) - 5*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f

Mupad [B]
time = 4.46, size = 86, normalized size = 0.95

$$\frac{\operatorname{atanh}(\sin(e+f\,x))\,\left(a^2+a\,b+\frac{3\,b^2}{8}\right)}{f}+\frac{\sin(e+f\,x)\,\left(\frac{5\,b^2}{8}+a\,b\right)-\sin(e+f\,x)^3\,\left(\frac{3\,b^2}{8}+a\,b\right)}{f\,\left(\sin(e+f\,x)^4-2\sin(e+f\,x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x),x)

[Out] (atanh(sin(e + f*x))*(a*b + a^2 + (3*b^2)/8))/f + (sin(e + f*x)*(a*b + (5*b^2)/8) - sin(e + f*x)^3*(a*b + (3*b^2)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1))

3.168 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*b*(4*a+b)*arctanh(sin(f*x+e))/f+a^2*sin(f*x+e)/f+1/2*b^2*sec(f*x+e)*tan
(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {4232, 398, 393, 212}

$$\frac{a^2\sin(e+fx)}{f}+\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*Sin[e + f*x])/f + (b^2*Sec
[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b-ax^2)^2}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)-2abx^2}{(1-x^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{a^2\sin(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{(b(4a+b))\text{Subst}\left(\int\right.}{}\\ &= \frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\,\mathrm{t}}{2f}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 1.43

$$\frac{2ab\tanh^{-1}(\sin(e+fx))}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2\cos(fx)\sin(e)}{f}+\frac{a^2\cos(e)\sin(fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*
Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e + f*
x])/(2*f)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.23

method	result
derivativedivides	$\frac{a^2\sin(fx+e)+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{a^2\sin(fx+e)+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$

risch	$-\frac{i a^2 e^{i(fx+e)}}{2f}+\frac{i a^2 e^{-i(fx+e)}}{2f}-\frac{i b^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{2\ln\left(e^{i(fx+e)}+i\right)ab}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b^2}{2f}-\frac{2\ln(e^{i(}}{}$
norman	$\frac{\frac{(2a^2+b^2)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{(6a^2-b^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{(2a^2+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(6a^2-b^2)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{b(4a+b)\ln(t}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*sin(f*x+e)+2*a*b*ln(sec(f*x+e)+tan(f*x+e))+b^2*(1/2*sec(f*x+e)*tan
(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 1.68

$$-\frac{b^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-4ab(\log(\sin(fx+e)+1)-\log(\sin(fx+e)-1))-4a^2\sin(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)
) - 4*a^2*sin(f*x + e))/f
```

Fricas [A]
time = 3.22, size = 101, normalized size = 1.80

$$\frac{(4ab+b^2)\cos(fx+e)^2\log(\sin(fx+e)+1)-(4ab+b^2)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2(2a^2\cos(fx+e)^2+b^2)\sin(fx+e)}{4f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f*x
 + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)^2\cos(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x), x)
```

Giac [A]
time = 0.45, size = 79, normalized size = 1.41

$$\frac{4\,a^2\sin\left(fx+e\right)+\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\,b^2\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(f*x + e) + (4*a*b + b^2)*log(abs(sin(f*x + e) + 1)) - (4*a*b
 + b^2)*log(abs(sin(f*x + e) - 1)) - 2*b^2*sin(f*x + e)/(sin(f*x + e)^2 - 1
))/f
```

Mupad [B]
time = 0.11, size = 55, normalized size = 0.98

$$\frac{a^2\,\sin\left(e+f\,x\right)+\frac{b\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)\,\left(4\,a+b\right)}{2}-\frac{b^2\,\sin\left(e+f\,x\right)}{2\,\left({\sin\left(e+f\,x\right)}^2-1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*sin(e + f*x) + (b*atanh(sin(e + f*x))*(4*a + b))/2 - (b^2*sin(e + f*x)
)/(2*(sin(e + f*x)^2 - 1)))/f
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}$$

```
[Out] b^2*arctanh(sin(f*x+e))/f+a*(a+2*b)*sin(f*x+e)/f-1/3*a^2*sin(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 212}

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (a*(a + 2*b)*Sin[e + f*x])/f - (a^2*Sin[e +
 f*x]^3)/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{(a+b-ax^2)^2}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left(a(a+2b)-a^2x^2+\frac{b^2}{1-x^2}\right)\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}+\frac{b^2\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\right.}{f}$$
$$=\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx}{3f}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.47

$$\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}+\frac{2ab\cos(fx)\sin(e)}{f}+\frac{2ab\cos(e)\sin(fx)}{f}+\frac{a^2\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*S
in[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

Maple [A]
time = 0.09, size = 55, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{a^2\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+2ab\sin(fx+e)+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
default	$\frac{\frac{a^2\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+2ab\sin(fx+e)+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
risch	$-\frac{3ia^2e^{i(fx+e)}}{8f}-\frac{ie^{i(fx+e)}ab}{f}+\frac{3ia^2e^{-i(fx+e)}}{8f}+\frac{ie^{-i(fx+e)}ab}{f}+\frac{\ln(e^{i(fx+e)}+i)b^2}{f}-\frac{\ln(e^{i(fx+e)}-i)b^2}{f}+a^2s$
norman	$\frac{-\frac{4a(a-2b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4a(a-2b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a(a+2b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2a(7a+6b)\left(\tan^3\left(\frac{f}{2}\right.\right.}{3f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*a^2*(2+cos(f*x+e)^2)*sin(f*x+e)+2*a*b*sin(f*x+e)+b^2*ln(sec(f*x+e)
+tan(f*x+e)))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 1.37

$$-\frac{2\,a^2\sin\left(fx+e\right)^3-3\,b^2\log\left(\sin\left(fx+e\right)+1\right)+3\,b^2\log\left(\sin\left(fx+e\right)-1\right)-6\,\left(a^2+2\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f*
x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

Fricas [A]
time = 3.62, size = 70, normalized size = 1.43

$$\frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*co
s(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^3\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**3, x)
```

Giac [A]
time = 0.44, size = 70, normalized size = 1.43

$$-\frac{2\,a^2\sin\left(fx+e\right)^3-3\,b^2\log\left(\left|\sin\left(fx+e\right)+1\right|\right)+3\,b^2\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-6\,a^2\sin\left(fx+e\right)-12\,ab\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(abs(sin(f*x + e) + 1)) + 3*b^2*log(a
bs(sin(f*x + e) - 1)) - 6*a^2*sin(f*x + e) - 12*a*b*sin(f*x + e))/f
```

Mupad [B]
time = 4.53, size = 48, normalized size = 0.98

$$-\frac{\sin\left(e+f\,x\right)\,\left(a^2-2\,a\,\left(a+b\right)\right)+\frac{a^2\,{\sin\left(e+f\,x\right)}^3}{3}-b^2\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
[Out] -(sin(e + f*x)*(a^2 - 2*a*(a + b)) + (a^2*sin(e + f*x)^3)/3 - b^2*atanh(sin
(e + f*x)))/f
```

3.170 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\sin(e+fx)}{f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx)}{5f}$$

```
[Out] (a+b)^2*sin(f*x+e)/f-2/3*a*(a+b)*sin(f*x+e)^3/f+1/5*a^2*sin(f*x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {4232, 200}

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)^2\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Sin[e + f*x])/f - (2*a*(a + b)*Sin[e + f*x]^3)/(3*f) + (a^2*Sin[
e + f*x]^5)/(5*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a+b-ax^2\right)^2\,dx,x,\sin(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int\left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2a^2\left(1+\frac{b}{a}\right)x^2+a^2x^4\right)\,dx,x,\sin(e\right.}{f}\\&=\frac{(a+b)^2\sin(e+fx)}{f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx}{5f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(53) = 106.

time = 0.04, size = 109, normalized size = 2.06

$$\frac{b^2\cos(fx)\sin(e)}{f}+\frac{b^2\cos(e)\sin(fx)}{f}+\frac{5a^2\sin(e+fx)}{8f}+\frac{2ab\sin(e+fx)}{f}-\frac{2ab\sin^3(e+fx)}{3f}+\frac{5a^2\sin(3(e+fx))}{48f}+\frac{a^2\sin(5(e+fx))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (5*a^2*Sin[e + f*x])/(8
*f) + (2*a*b*Sin[e + f*x])/f - (2*a*b*Sin[e + f*x]^3)/(3*f) + (5*a^2*Sin[3*
(e + f*x)])/(48*f) + (a^2*Sin[5*(e + f*x)])/(80*f)
```

Maple [A]

time = 0.12, size = 67, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(fx+e)+\frac{4\left(\cos^2(fx+e)\right)}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+b^2\sin(fx+e)}{f}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(fx+e)+\frac{4\left(\cos^2(fx+e)\right)}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}+b^2\sin(fx+e)}{f}$
risch	$\frac{5a^2\sin(fx+e)}{8f}+\frac{3\sin(fx+e)ab}{2f}+\frac{\sin(fx+e)b^2}{f}+\frac{a^2\sin(5fx+5e)}{80f}+\frac{5a^2\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)ab}{6f}$
norman	$-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2\left(a^2+2ab+b^2\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{2\left(5a^2+2ab-3b^2\right)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2\left(5a^2+2ab-3b^2\right)\left(\tan^{13}\left(\frac{f}{2}\right.\right.}{3f}$ … / $(1-$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(2+cos(
f*x+e)^2)*sin(f*x+e)+b^2*sin(f*x+e))
```

Maxima [A]

time = 0.27, size = 58, normalized size = 1.09

$$\frac{3\,a^2\sin\left(fx+e\right)^5-10\left(a^2+ab\right)\sin\left(fx+e\right)^3+15\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a*
b + b^2)*sin(f*x + e))/f
```

Fricas [A]
time = 2.45, size = 62, normalized size = 1.17

$$\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{4}+2\left(2\,a^{2}+5\,ab\right)\cos\left(fx+e\right)^{2}+8\,a^{2}+20\,ab+15\,b^{2}\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20*
a*b + 15*b^2)*sin(f*x + e)/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.45, size = 76, normalized size = 1.43

$$\frac{3\,a^{2}\sin\left(fx+e\right)^{5}-10\,a^{2}\sin\left(fx+e\right)^{3}-10\,ab\sin\left(fx+e\right)^{3}+15\,a^{2}\sin\left(fx+e\right)+30\,ab\sin\left(fx+e\right)+15\,b^{2}\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
+ 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

Mupad [B]
time = 4.44, size = 44, normalized size = 0.83

$$\frac{\sin\left(e+f\,x\right)\,{\left(a+b\right)}^{2}+\frac{a^{2}\,{\sin\left(e+f\,x\right)}^{5}}{5}-\frac{2\,a\,{\sin\left(e+f\,x\right)}^{3}\,\left(a+b\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (sin(e + f*x)*(a + b)^2 + (a^2*sin(e + f*x)^5)/5 - (2*a*sin(e + f*x)^3*(a +
 b))/3)/f
```

3.171 $\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal. Leaf size=106

$$\frac{(a+b)^2\tan(e+fx)}{f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*(a+b)*(a+2*b)*tan(f*x+e)^3/f+1/5*(a^2+6*a*b+6*b^2)
*tan(f*x+e)^5/f+2/7*b*(a+2*b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 380}

$$\frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/(3*f) + (
(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x]^7
)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int\left((a+b)^2+2(a+b)(a+2b)x^2+\left(a^2+6ab+6b^2\right)x^4+\right.\right.}{f}$$
$$= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a^2+6}{}$$

Mathematica [A]

time = 0.48, size = 96, normalized size = 0.91

$$\frac{315(a+b)^2\tan(e+fx)+210(a^2+3ab+2b^2)\tan^3(e+fx)+63(a^2+6ab+6b^2)\tan^5(e+fx)+90b(a+2b)\tan^7(e+fx)+35b^2\tan^9(e+fx)}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 63
*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 35*
b^2*Tan[e + f*x]^9)/(315*f)
```

Maple [A]

time = 0.08, size = 134, normalized size = 1.26

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan}{f}$
risch	$16i\left(210a^2e^{12i(fx+e)}+945a^2e^{10i(fx+e)}+1260ab\,e^{10i(fx+e)}+1701a^2e^{8i(fx+e)}+3276ab\,e^{8i(fx+e)}+2016b^2e^{8i(fx+e)}+155\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16/
35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-1
28/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec(f*
x+e)^2)*tan(f*x+e))
```

Maxima [A]

time = 0.28, size = 108, normalized size = 1.02

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\,\left(ab+2\,b^2\right)\tan\left(fx+e\right)^7+63\,\left(a^2+6\,ab+6\,b^2\right)\tan\left(fx+e\right)^5+210\,\left(a^2+3\,ab+2\,b^2\right)\tan\left(fx+e\right)^3+315\,\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3 +
315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [**A**]
time = 2.41, size = 126, normalized size = 1.19

$$\frac{\left(8\left(21a^{2}+36ab+16b^{2}\right)\cos\left(fx+e\right)^{8}+4\left(21a^{2}+36ab+16b^{2}\right)\cos\left(fx+e\right)^{6}+3\left(21a^{2}+36ab+16b^{2}\right)\cos\left(fx+e\right)^{4}+10\left(9ab+4b^{2}\right)\cos\left(fx+e\right)^{2}+35b^{2}\right)\sin\left(fx+e\right)}{315f\cos\left(fx+e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b + 1
6*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*(9
*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\sec^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

Giac [**A**]
time = 0.46, size = 152, normalized size = 1.43

$$\frac{35b^{2}\tan\left(fx+e\right)^{9}+90ab\tan\left(fx+e\right)^{7}+180b^{2}\tan\left(fx+e\right)^{7}+63a^{2}\tan\left(fx+e\right)^{5}+378ab\tan\left(fx+e\right)^{5}+378b^{2}\tan\left(fx+e\right)^{5}+210a^{2}\tan\left(fx+e\right)^{3}+630ab\tan\left(fx+e\right)^{3}+420b^{2}\tan\left(fx+e\right)^{3}+315a^{2}\tan\left(fx+e\right)+630ab\tan\left(fx+e\right)+315b^{2}\tan\left(fx+e\right)}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e)^
5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x + e)^
3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))/f
```

Mupad [**B**]
time = 4.54, size = 94, normalized size = 0.89

$$\frac{\tan(e+f\,x)\,{(a+b)}^{2}+\frac{b^{2}\,{\tan(e+f\,x)}^{9}}{9}+{\tan(e+f\,x)}^{3}\,\left(\frac{2\,a^{2}}{3}+2\,a\,b+\frac{4\,b^{2}}{3}\right)+{\tan(e+f\,x)}^{5}\,\left(\frac{a^{2}}{5}+\frac{6\,a\,b}{5}+\frac{6\,b^{2}}{5}\right)+\frac{2\,b\,{\tan(e+f\,x)}^{7}\,(a+2\,b)}{7}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^9)/9 + tan(e + f*x)^3*(2*a*b +
(2*a^2)/3 + (4*b^2)/3) + tan(e + f*x)^5*((6*a*b)/5 + a^2/5 + (6*b^2)/5) + (
2*b*tan(e + f*x)^7*(a + 2*b))/7)/f
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+1/3*(a+b)*(a+3*b)*tan(f*x+e)^3/f+1/5*b*(2*a+3*b)*tan(f
*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 380}

$$\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/(3*f) + (b*
(2*a + 3*b)*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((a+b)^2+(a+b)(a+3b)x^2+b(2a+3b)x^4+b^2x^6\right)\,dx\right.}{f}\\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b}{}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 75, normalized size = 0.94

$$\frac{105(a+b)^2\tan(e+fx)+35(a^2+4ab+3b^2)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+15b^2\tan^7(e+fx)}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21*
b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)
```

Maple [A]
time = 0.09, size = 104, normalized size = 1.30

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}\right.}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}\right.}{f}$
risch	$\frac{4i\left(105a^2e^{10i(fx+e)}+455a^2e^{8i(fx+e)}+560ab\,e^{8i(fx+e)}+770a^2e^{6i(fx+e)}+1400ab\,e^{6i(fx+e)}+840b^2e^{6i(fx+e)}+630a^2e^{4i(f\right.}{105f\left(e^{2i(fx+\right.}}$
norman	$\frac{-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\left(a^2+2ab+b^2\right)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4\left(7a^2+10ab+3b^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{4\left(7a^2+10ab+3b^2\right)\left(\tan^1\right.}{3f}}{\left(\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4-
4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^
4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.27, size = 85, normalized size = 1.06

$$\frac{15\,b^2\tan(fx+e)^7+21\,(2\,ab+3\,b^2)\tan(fx+e)^5+35\,(a^2+4\,ab+3\,b^2)\tan(fx+e)^3+105\,(a^2+2\,ab+b^2)\tan(fx+e)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2
+ 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A]
time = 2.64, size = 99, normalized size = 1.24

$$\frac{\left(2\,\left(35\,a^{2}+56\,ab+24\,b^{2}\right)\cos\left(fx+e\right)^{6}+\left(35\,a^{2}+56\,ab+24\,b^{2}\right)\cos\left(fx+e\right)^{4}+6\,\left(7\,ab+3\,b^{2}\right)\cos\left(fx+e\right)^{2}+15\,b^{2}\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24*
b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x +
e)/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)
```

Giac [A]
time = 0.47, size = 114, normalized size = 1.42

$$\frac{15\,b^{2}\tan\left(fx+e\right)^{7}+42\,ab\tan\left(fx+e\right)^{5}+63\,b^{2}\tan\left(fx+e\right)^{5}+35\,a^{2}\tan\left(fx+e\right)^{3}+140\,ab\tan\left(fx+e\right)^{3}+105\,b^{2}\tan\left(fx+e\right)^{3}+105\,a^{2}\tan\left(fx+e\right)+210\,ab\tan\left(fx+e\right)+105\,b^{2}\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)^3
 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f
```

Mupad [B]
time = 4.67, size = 70, normalized size = 0.88

$$\frac{\tan(e+f\,x)\,\left(a+b\right)^{2}+\tan(e+f\,x)^{3}\,\left(\frac{a^{2}}{3}+\frac{4\,a\,b}{3}+b^{2}\right)+\frac{b^{2}\tan(e+f\,x)^{7}}{7}+\frac{b\tan(e+f\,x)^{5}\,\left(2\,a+3\,b\right)}{5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + tan(e + f*x)^3*((4*a*b)/3 + a^2/3 + b^2) + (b^2*t
an(e + f*x)^7)/7 + (b*tan(e + f*x)^5*(2*a + 3*b))/5)/f
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] (a+b)^2*tan(f*x+e)/f+2/3*b*(a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4231, 200}

$$\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*b*(a + b)*Tan[e + f*x]^3)/(3*f) + (b^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx &= \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^2 dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)+2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right) dx, x, \tan(e\right.}{f}\\ &= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx}{5f}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 48, normalized size = 0.91

$$\frac{15(a+b)^2\tan(e+fx)+10b(a+b)\tan^3(e+fx)+3b^2\tan^5(e+fx)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f*
x]^5)/(15*f)
```

Maple [A]
time = 0.07, size = 71, normalized size = 1.34

method	result
derivativedivides	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)}{f}$
default	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)}{f}$
risch	$\frac{2i(15a^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+90a^2e^{4i(fx+e)}+140ab\,e^{4i(fx+e)}+80b^2e^{4i(fx+e)}+60a^2e^{2i(fx+e)}+100a}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\left(a^2+2ab+b^2\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{8\left(3a^2+4ab+b^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{8\left(3a^2+4ab+b^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/5
*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.45

$$\frac{10\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ab+\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)b^2+15a^2\tan\left(fx+e\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*tan
(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 3.28, size = 73, normalized size = 1.38

$$\frac{\left(\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+2\,\left(5\,ab+2\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
+ e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.44, size = 76, normalized size = 1.43

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+10\,b^2\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)+30\,ab\tan\left(fx+e\right)+15\,b^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

Mupad [B]
time = 4.54, size = 44, normalized size = 0.83

$$\frac{\tan(e+f\,x)\,\left(a+b\right)^2+\frac{b^2\tan(e+f\,x)^5}{5}+\frac{2\,b\tan(e+f\,x)^3\,(a+b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^5)/5 + (2*b*tan(e + f*x)^3*(a +
 b))/3)/f
```

3.174 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx\right.}{f}$$
$$= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. $2(40) = 80$.
time = 0.45, size = 106, normalized size = 2.65

$$\frac{4\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(3a^2fx\cos^3(e+fx)+b^2\sec(e)\sin(fx)+2b(3a+b)\cos^2(e+fx)\sec(e)\sin(fx)+b^2\cos(e+fx)\tan(e)\right)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.00, size = 48, normalized size = 1.20

method	result
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}+3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^2x+3a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 1.18

$$a^2x + \frac{(\tan(fx+e)^3 + 3\tan(fx+e))b^2}{3f} + \frac{2ab\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Fricas [A]
time = 2.67, size = 62, normalized size = 1.55

$$\frac{3a^2fx\cos(fx+e)^3 + (2(3ab+b^2)\cos(fx+e)^2 + b^2)\sin(fx+e)}{3f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec^2(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.42, size = 49, normalized size = 1.22

$$\frac{b^2\tan(fx+e)^3 + 3(fx+e)a^2 + 6ab\tan(fx+e) + 3b^2\tan(fx+e)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

Mupad [B]
time = 4.55, size = 42, normalized size = 1.05

$$\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}a(a+4b)x+\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 398, 393, 209}

$$\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*Tan[e + f*
x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{b^2\tan(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}+\frac{(a(a+4b))\text{Subs}}{}\\ &= \frac{1}{2}a(a+4b)x+\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 52, normalized size = 1.11

$$2abx+\frac{a^2(e+fx)}{2f}+\frac{a^2\sin(2(e+fx))}{4f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[e
 + f*x])/f
```

Maple [A]
time = 0.08, size = 51, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
risch	$\frac{a^2x}{2}+2xab-\frac{ia^2\mathrm{e}^{2i(fx+e)}}{8f}+\frac{ia^2\mathrm{e}^{-2i(fx+e)}}{8f}+\frac{2ib^2}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$

norman	$\frac{(-\frac{1}{2}a^2-2ab)x+(-\frac{1}{2}a^2-2ab)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{1}{2}a^2+2ab)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{1}{2}a^2+2ab)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-a^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(f*x+e)+b^2*tan(f*
x+e))
```

Maxima [A]
time = 0.49, size = 57, normalized size = 1.21

$$\frac{2\,b^2\tan\left(fx+e\right)+\left(a^2+4\,ab\right)\left(fx+e\right)+\frac{a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.59, size = 60, normalized size = 1.28

$$\frac{\left(a^2+4\,ab\right)fx\cos\left(fx+e\right)+\left(a^2\cos\left(fx+e\right)^2+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
+ e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**2, x)
```

Giac [A]
time = 0.46, size = 53, normalized size = 1.13

$$\frac{2\,b^2\tan\left(fx+e\right)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Mupad [B]
time = 4.52, size = 66, normalized size = 1.40

$$\frac{b^2\tan(e+f\,x)}{f}+\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a+4\,b)}{2\,\left(\frac{a^2}{2}+2\,b\,a\right)}\right)\,(a+4\,b)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x))/f + (a^2*sin(2*e + 2*f*x))/(4*f) + (a*atan((a*tan(e + f*
x)*(a + 4*b))/(2*(2*a*b + a^2/2)))*(a + 4*b))/(2*f)
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal. Leaf size=81

$$\frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f
*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 424, 393, 209}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) + (a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx}{4}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a}{}$$

Mathematica [A]
time = 0.14, size = 58, normalized size = 0.72

$$\frac{4(3a^2+8ab+8b^2)(e+fx)+8a(a+2b)\sin(2(e+fx))+a^2\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)])/(32*f)
```

Maple [A]
time = 0.09, size = 78, normalized size = 0.96

method	result
risch	$\frac{3a^2x}{8}+xab+x\,b^2+\frac{a^2\sin(4fx+4e)}{32f}+\frac{\sin(2fx+2e)a^2}{4f}+\frac{\sin(2fx+2e)ab}{2f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$

default	$\frac{a^2\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
norman	$\frac{(-\frac{3}{8}a^2-ab-b^2)x+(-\frac{9}{8}a^2-3ab-3b^2)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-\frac{9}{8}a^2-3ab-3b^2)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-\frac{3}{8}a^2-ab-b^2)x\left(\tan^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+2*a*b
*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b^2*(f*x+e))
```

Maxima [A]
time = 0.48, size = 92, normalized size = 1.14

$$\frac{(3a^2+8ab+8b^2)(fx+e)+\frac{(3a^2+8ab)\tan(fx+e)^3+(5a^2+8ab)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
(5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.70, size = 65, normalized size = 0.80

$$\frac{(3a^2+8ab+8b^2)fx+\left(2a^2\cos(fx+e)^3+(3a^2+8ab)\cos(fx+e)\right)\sin(fx+e)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)*
cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)^2\cos^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**4, x)
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.07

$$\frac{(3\,a^2+8\,ab+8\,b^2)(fx+e)+\frac{3\,a^2\tan(fx+e)^3+8\,ab\tan(fx+e)^3+5\,a^2\tan(fx+e)+8\,ab\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan(
f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^
2)/f
```

Mupad [B]
time = 4.57, size = 76, normalized size = 0.94

$$x\left(\frac{3\,a^2}{8}+a\,b+b^2\right)+\frac{\left(\frac{3\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*(a*b + (3*a^2)/8 + b^2) + (tan(e + f*x)*(a*b + (5*a^2)/8) + tan(e + f*x)^
3*(a*b + (3*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{1}{16}\left(5a^2+12ab+8b^2\right)x+\frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f}+\frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f}+$$

```
[Out] 1/16*(5*a^2+12*a*b+8*b^2)*x+1/16*(5*a^2+12*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)
/f+1/24*a*(5*a+8*b)*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)
*(a+b+b*tan(f*x+e)^2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4231, 424, 393, 205, 209}

$$\frac{\left(5a^2+12ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 12*a*b + 8*b^2)*x)/16 + ((5*a^2 + 12*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(5*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (
a*Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+f}{}$$

$$= \frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a+8b)\cos^3(e}{2}$$

$$= \frac{1}{16}\left(5a^2+12ab+8b^2\right)x + \frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e}{16f}$$

Mathematica [A]
time = 0.24, size = 99, normalized size = 0.83

$$\frac{60a^2e+144abe+96b^2e+60a^2fx+144abfx+96b^2fx+(45a^2+96ab+48b^2)\sin(2(e+fx))+3a(3a+4b)\sin(4(e+fx))+a^2\sin(6(e+fx))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
(45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e + f*
x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

Maple [A]
time = 0.06, size = 116, normalized size = 0.97

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
default	$\frac{a^2\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
risch	$\frac{5a^2x}{16}+\frac{3xab}{4}+\frac{x\,b^2}{2}+\frac{a^2\sin(6fx+6e)}{192f}+\frac{3a^2\sin(4fx+4e)}{64f}+\frac{\sin(4fx+4e)ab}{16f}+\frac{15\sin(2fx+2e)a^2}{64f}+\frac{\sin(2fx+}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(a^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+2*a*b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+b^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))

Maxima [A]
time = 0.48, size = 142, normalized size = 1.19

$$\frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{3\,(5\,a^2+12\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\tan(fx+e)^3+3\,(11\,a^2+20\,ab+8\,b^2)\tan(fx+e)}{\tan(fx+e)^6+3\tan(fx+e)^4+3\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20*a*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f

Fricas [A]
time = 3.51, size = 93, normalized size = 0.78

$$\frac{3\,(5\,a^2+12\,ab+8\,b^2)fx+\left(8\,a^2\cos(fx+e)^5+2\,(5\,a^2+12\,ab)\cos(fx+e)^3+3\,(5\,a^2+12\,ab+8\,b^2)\cos(fx+e)\right)\sin(fx+e)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 + 1
2*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x +
e))/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.45, size = 150, normalized size = 1.26

$$\frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{15\,a^2\tan(fx+e)^5+36\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+96\,ab\tan(fx+e)^3+48\,b^2\tan(fx+e)^3+33\,a^2\tan(fx+e)+60\,ab\tan(fx+e)+24\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a*
b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b*t
an(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan(f*
x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B]
time = 5.32, size = 123, normalized size = 1.03

$$x\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)+\frac{\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}+2\,a\,b+b^2\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a^2}{16}+\frac{5\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a*b)/4 + (5*a^2)/16 + b^2/2) + (tan(e + f*x)*((5*a*b)/4 + (11*a^2)/16
 + b^2/2) + tan(e + f*x)^3*(2*a*b + (5*a^2)/6 + b^2) + tan(e + f*x)^5*((3*a
*b)/4 + (5*a^2)/16 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(
e + f*x)^6 + 1))
```

3.178 $\int (a+b\sec^2(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x+b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(3*a+2*b)*tan(d*x+c)^3/d+1/5
*b^3*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b^2*(3*a + 2*b)*Tan[c +
 d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(3a^2+3ab+b^2)+b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1+x^2}\right)dx, x, \tan(c+d$$
$$= \frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$
$$= a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c}{5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(73) = 146.
time = 1.13, size = 268, normalized size = 3.67

$$\frac{\sec(c)\sec^5(c+dx)\left(150a^3dx\cos(dx)+150a^3dx\cos(2c+dx)+75a^3dx\cos(2c+3dx)+75a^3dx\cos(4c+3dx)+15a^3dx\cos(4c+5dx)+15a^3dx\cos(6c+5dx)+540a^2b\sin(dx)+420ab^2\sin(dx)+160b^3\sin(dx)-360a^2b\sin(2c+dx)-180ab^2\sin(2c+dx)+360a^2b\sin(2c+3dx)+300ab^2\sin(2c+3dx)+80b^3\sin(2c+3dx)-90a^2b\sin(4c+3dx)+90a^2b\sin(4c+5dx)+60ab^2\sin(4c+5dx)+16b^3\sin(4c+5dx)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*(150*a^3*d*x*Cos[d*x] + 150*a^3*d*x*Cos[2*c + d*x] +
 75*a^3*d*x*Cos[2*c + 3*d*x] + 75*a^3*d*x*Cos[4*c + 3*d*x] + 15*a^3*d*x*Cos
[4*c + 5*d*x] + 15*a^3*d*x*Cos[6*c + 5*d*x] + 540*a^2*b*Sin[d*x] + 420*a*b^
2*Sin[d*x] + 160*b^3*Sin[d*x] - 360*a^2*b*Sin[2*c + d*x] - 180*a*b^2*Sin[2*
c + d*x] + 360*a^2*b*Sin[2*c + 3*d*x] + 300*a*b^2*Sin[2*c + 3*d*x] + 80*b^3
*Sin[2*c + 3*d*x] - 90*a^2*b*Sin[4*c + 3*d*x] + 90*a^2*b*Sin[4*c + 5*d*x] +
 60*a*b^2*Sin[4*c + 5*d*x] + 16*b^3*Sin[4*c + 5*d*x]))/(480*d)
```

Maple [A]
time = 0.07, size = 84, normalized size = 1.15

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3a\,b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
default	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3a\,b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
risch	$a^3x+\frac{2ib\left(45a^2e^{8i(dx+c)}+180a^2e^{6i(dx+c)}+90ab\,e^{6i(dx+c)}+270a^2e^{4i(dx+c)}+210ab\,e^{4i(dx+c)}+80b^2e^{4i(dx+c)}+180a^2e\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{a^3x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^3x+5a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))

Maxima [A]
time = 0.27, size = 83, normalized size = 1.14

$$a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)ab^2}{d}+\frac{\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)b^3}{15\,d}+\frac{3\,a^2b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")

[Out] a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d

Fricas [A]
time = 3.54, size = 90, normalized size = 1.23

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5+\left(\left(45\,a^2b+30\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4+3\,b^3+\left(15\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x + c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c)**2)**3,x)

[Out] Integral((a + b*sec(c + d*x)**2)**3, x)

Giac [A]
time = 0.46, size = 91, normalized size = 1.25

$$\frac{3\,b^3\tan\left(dx+c\right)^5+15\,ab^2\tan\left(dx+c\right)^3+10\,b^3\tan\left(dx+c\right)^3+15\,(dx+c)a^3+45\,a^2b\tan\left(dx+c\right)+45\,ab^2\tan\left(dx+c\right)+15\,b^3\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)^
3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15*b
^3*tan(d*x + c))/d
```

Mupad [B]
time = 4.51, size = 73, normalized size = 1.00

$$\frac{\tan(c+d\,x)\,\left(3\,b\,{(a+b)}^2-3\,b^2\,(a+b)+b^3\right)+\frac{b^3\,{\tan(c+d\,x)}^5}{5}+{\tan(c+d\,x)}^3\,\left(b^2\,(a+b)-\frac{b^3}{3}\right)+a^3\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^3,x)
```

```
[Out] (tan(c + d*x)*(3*b*(a + b)^2 - 3*b^2*(a + b) + b^3) + (b^3*tan(c + d*x)^5)/
5 + tan(c + d*x)^3*(b^2*(a + b) - b^3/3) + a^3*d*x)/d
```

3.179 $\int \left(a+b\sec^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=111

$$a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+$$

```
[Out] a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2+8*a*b+3*b^2)*
tan(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$a^4x+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*a^2 +
8*a*b + 3*b^2)*Tan[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tan[c + d*x]^5)/(5*
d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)+b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x\right.\right.}{d}$$

$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d} + \frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d} +$$

$$= a^4x + \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d} + \frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 455 vs. $2(111) = 222$.
time = 1.70, size = 455, normalized size = 4.10

$$\frac{\sec(c)\sec^7(c+dx)\left(3675a^4dx\cos(dx)+3675a^4dx\cos(2c+dx)+2205a^4dx\cos(2c+3dx)+2205a^4dx\cos(4c+3dx)+735a^4dx\cos(4c+5dx)+735a^4dx\cos(6c+5dx)+105a^4dx\cos(6c+7dx)+105a^4dx\cos(8c+7dx)+16800a^3b\sin(dx)+18480a^2b^2\sin(dx)+11200ab^3\sin(dx)+3360b^4\sin(dx)-12600a^3b\sin(2c+dx)-10920a^2b^2\sin(2c+dx)-4480ab^3\sin(2c+dx)+12600a^3b\sin(2c+3dx)+15120a^2b^2\sin(2c+3dx)+9408ab^3\sin(2c+3dx)+2016b^4\sin(2c+3dx)-5040a^3b\sin(4c+3dx)-2520a^2b^2\sin(4c+3dx)+5040a^3b\sin(4c+5dx)+5880a^2b^2\sin(4c+5dx)+3136ab^3\sin(4c+5dx)+672b^4\sin(4c+5dx)-840a^3b\sin(6c+5dx)+840a^3b\sin(6c+7dx)+840a^2b^2\sin(6c+7dx)+448ab^3\sin(6c+7dx)+96b^4\sin(6c+7dx)\right)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*(3675*a^4*d*x*Cos[d*x] + 3675*a^4*d*x*Cos[2*c + d*x]
 + 2205*a^4*d*x*Cos[2*c + 3*d*x] + 2205*a^4*d*x*Cos[4*c + 3*d*x] + 735*a^4*
d*x*Cos[4*c + 5*d*x] + 735*a^4*d*x*Cos[6*c + 5*d*x] + 105*a^4*d*x*Cos[6*c +
 7*d*x] + 105*a^4*d*x*Cos[8*c + 7*d*x] + 16800*a^3*b*Sin[d*x] + 18480*a^2*b
^2*Sin[d*x] + 11200*a*b^3*Sin[d*x] + 3360*b^4*Sin[d*x] - 12600*a^3*b*Sin[2*
c + d*x] - 10920*a^2*b^2*Sin[2*c + d*x] - 4480*a*b^3*Sin[2*c + d*x] + 12600
*a^3*b*Sin[2*c + 3*d*x] + 15120*a^2*b^2*Sin[2*c + 3*d*x] + 9408*a*b^3*Sin[2
*c + 3*d*x] + 2016*b^4*Sin[2*c + 3*d*x] - 5040*a^3*b*Sin[4*c + 3*d*x] - 252
0*a^2*b^2*Sin[4*c + 3*d*x] + 5040*a^3*b*Sin[4*c + 5*d*x] + 5880*a^2*b^2*Sin
[4*c + 5*d*x] + 3136*a*b^3*Sin[4*c + 5*d*x] + 672*b^4*Sin[4*c + 5*d*x] - 84
0*a^3*b*Sin[6*c + 5*d*x] + 840*a^3*b*Sin[6*c + 7*d*x] + 840*a^2*b^2*Sin[6*c
 + 7*d*x] + 448*a*b^3*Sin[6*c + 7*d*x] + 96*b^4*Sin[6*c + 7*d*x]))/(13440*d
)
```

Maple [A]
time = 0.10, size = 130, normalized size = 1.17

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(}{d}$

default	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx}{d}$
risch	$a^4x+\frac{8ib\left(105a^3e^{12i(dx+c)}+630a^3e^{10i(dx+c)}+315a^2b\,e^{10i(dx+c)}+1575a^3e^{8i(dx+c)}+1365a^2b\,e^{8i(dx+c)}+560a\,b^2e^{8i(dx+}\right.}{}$
norman	$\frac{a^4x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^4x+7a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21a^4x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35a^4x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-35a^4x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d
*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(-1
6/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 134, normalized size = 1.21

$$a^4x+\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2b^2}{d}+\frac{4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)ab^3}{15\,d}+\frac{\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)b^4}{35\,d}+\frac{4\,a^3b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c
)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)^7
 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^3*b
*tan(d*x + c)/d
```

Fricas [A]
time = 3.03, size = 130, normalized size = 1.17

$$\frac{105\,a^4dx\cos(dx+c)^7+\left(4\left(105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos(dx+c)^6+2\left(105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos(dx+c)^4+15\,b^4+6\left(14\,ab^3+3\,b^4\right)\cos(dx+c)^2\right)\sin(dx+c)}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
+ 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x + c)
^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(c+dx)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**4, x)
```

Giac [A]
time = 0.46, size = 148, normalized size = 1.33

$$\frac{15\,b^4\tan\left(dx+c\right)^7+84\,ab^3\tan\left(dx+c\right)^5+63\,b^4\tan\left(dx+c\right)^5+210\,a^2b^2\tan\left(dx+c\right)^3+280\,ab^3\tan\left(dx+c\right)^3+105\,b^4\tan\left(dx+c\right)^3+105\,(dx+c)a^4+420\,a^3b\tan\left(dx+c\right)+630\,a^2b^2\tan\left(dx+c\right)+420\,ab^3\tan\left(dx+c\right)+105\,b^4\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x + c
)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan(d
*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan(d*
x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

Mupad [B]
time = 4.58, size = 119, normalized size = 1.07

$$\frac{\tan(c+d\,x)\,\left(4\,b\,{(a+b)}^3+4\,b^3\,(a+b)-6\,b^2\,{(a+b)}^2-b^4\right)+{\tan(c+d\,x)}^3\,\left(2\,b^2\,{(a+b)}^2-\frac{4\,b^3\,(a+b)}{3}+\frac{b^4}{3}\right)+\frac{b^4\,{\tan(c+d\,x)}^7}{7}+{\tan(c+d\,x)}^5\,\left(\frac{4\,b^3\,(a+b)}{5}-\frac{b^4}{5}\right)+a^4\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^4,x)
```

```
[Out] (tan(c + d*x)*(4*b*(a + b)^3 + 4*b^3*(a + b) - 6*b^2*(a + b)^2 - b^4) + tan
(c + d*x)^3*(2*b^2*(a + b)^2 - (4*b^3*(a + b))/3 + b^4/3) + (b^4*tan(c + d*
x)^7)/7 + tan(c + d*x)^5*((4*b^3*(a + b))/5 - b^4/5) + a^4*d*x)/d
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,f}+\frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

```
[Out] -1/2*(2*a-b)*arctanh(sin(f*x+e))/b^2/f+a^(3/2)*arctanh(sin(f*x+e)*a^(1/2)/(
a+b)^(1/2))/b^2/f/(a+b)^(1/2)+1/2*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 425, 536, 212, 214}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}}-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (a^(3/2)*ArcTanh[(Sqrt[a]*
Sin[e + f*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*f) + (Sec[e + f*x]*Tan[e + f*x
])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{\text{Subst}\left(\int \frac{-a+b-ax^2}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{2bf}\\ &= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{b^2f} - \frac{(2a-b)\text{Su}}{}\\ &= -\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,f} + \frac{\sec(e+fx)\tan}{2bf}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.95, size = 2519, normalized size = 29.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a - b)*(a + 2*b + a*Cos[2*e + 2*f*x])*Log[Cos[e/2 + (f*x)/2] - Sin[e/2
+ (f*x)/2]]*Sec[e + f*x]^2)/(4*b^2*f*(a + b*Sec[e + f*x]^2)) + ((-2*a + b)*
(a + 2*b + a*Cos[2*e + 2*f*x])*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]
```

```
*Sec[e + f*x]^2)/(4*b^2*f*(a + b*Sec[e + f*x]^2)) + ((I/4)*a^(3/2)*ArcTan[(
(-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin
[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a
]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*S
in[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] -
(2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b
*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] -
 (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*
f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e]*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e +
 f*x]^2)/(b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e] - I*Sin[2*
e]]) - (a^(3/2)*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e
] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*
Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a +
 b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[C
os[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e]*(a + 2*b + a*Cos[2*e + 2*f*x]
)*Sec[e + f*x]^2)/(4*b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e]
 - I*Sin[2*e]]) + (a^(3/2)*Cos[e]*(a + 2*b + a*Cos[2*e + 2*f*x])*Log[a + 2*
a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b
*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sec[e + f*x
]^2)/(8*b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e] - I*Sin[2*e]
]) - (a^(3/2)*Cos[e]*(a + 2*b + a*Cos[2*e + 2*f*x])*Log[-a - 2*a*Cos[2*e] -
 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] +
2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt
[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sec[e + f*x]^2)/(8*b^2*
Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (a^(3/2
)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[
e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]
] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[
3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x
] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Co
s[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e
+ 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*
Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[
e + f*x]^2*Sin[e])/(4*b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e
] - I*Sin[2*e]]) + ((I/4)*a^(3/2)*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[
e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Si
n[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] -
I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*
Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*(a + 2*b + a*Cos[2
*e + 2*f*x])*Sec[e + f*x]^2*Sin[e])/(b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^
2)*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/8)*a^(3/2)*(a + 2*b + a*Cos[2*e + 2*f
*x])*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin
```

```
[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]
]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*
x]]*Sec[e + f*x]^2*Sin[e])/(b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[C
os[2*e] - I*Sin[2*e]]) + ((I/8)*a^(3/2)*(a + 2*b + a*Cos[2*e + 2*f*x])*Log[
-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] +
(2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*
x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sec[
e + f*x]^2*Sin[e])/(b^2*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[Cos[2*e]
- I*Sin[2*e]]) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^2)/(8*b*f*(a
+ b*Sec[e + f*x]^2)*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) - ((a + 2*
b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^2)/(8*b*f*(a + b*Sec[e + f*x]^2)*(Cos[
e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2)
```

Maple [A]
time = 0.21, size = 106, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{(a+b)a}}\right)}{b^2\sqrt{(a+b)a}}-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}}{f}$
default	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{(a+b)a}}\right)}{b^2\sqrt{(a+b)a}}-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}}{f}$
risch	$-\frac{i(e^{3i(fx+e)}-e^{i(fx+e)})}{fb(e^{2i(fx+e)}+1)^2}-\frac{\ln(e^{i(fx+e)}+i)a}{b^2f}+\frac{\ln(e^{i(fx+e)}+i)}{2bf}+\frac{\ln(e^{i(fx+e)}-i)a}{b^2f}-\frac{\ln(e^{i(fx+e)}-i)}{2bf}+\frac{\sqrt{(a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2/b^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/4/b/(s
in(f*x+e)-1)+1/4*(2*a-b)/b^2*ln(sin(f*x+e)-1)-1/4/b/(sin(f*x+e)+1)+1/4/b^2*
(-2*a+b)*ln(sin(f*x+e)+1))
```

Maxima [A]
time = 0.48, size = 130, normalized size = 1.51

$$-\frac{\frac{2\,a^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}+\frac{(2\,a-b)\log(\sin(fx+e)+1)}{b^2}-\frac{(2\,a-b)\log(\sin(fx+e)-1)}{b^2}+\frac{2\,\sin(fx+e)}{b\sin(fx+e)^2-b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*b^2) + (2*a - b)*log(sin(f*x + e) + 1)/b^2 - (
2*a - b)*log(sin(f*x + e) - 1)/b^2 + 2*sin(f*x + e)/(b*sin(f*x + e)^2 - b))
/f
```

Fricas [A]
time = 5.29, size = 290, normalized size = 3.37

$$\left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos\left(fx+e\right)^{2}\log\left(-\frac{a\cos(fx+e)^{2}-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^{2}+b}\right)-(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)+(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)+2\,b\sin\left(fx+e\right)}{4\,b^{2}f\cos\left(fx+e\right)^{2}},-\frac{4\,a\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin\left(fx+e\right)\right)\cos\left(fx+e\right)^{2}+(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)-(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)-2\,b\sin\left(fx+e\right)}{4\,b^{2}f\cos\left(fx+e\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b)
*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a - b
)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-sin(
f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqrt(-a
/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a - b)*
cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-sin(f*
x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 113, normalized size = 1.31

$$-\frac{\frac{4\,a^{2}\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^{2}-ab}}\right)}{\sqrt{-a^{2}-ab}\,b^{2}}+\frac{(2\,a-b)\log(|\sin(fx+e)+1|)}{b^{2}}-\frac{(2\,a-b)\log(|\sin(fx+e)-1|)}{b^{2}}+\frac{2\,\sin(fx+e)}{\left(\sin(fx+e)^{2}-1\right)b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
+ (2*a - b)*log(abs(sin(f*x + e) + 1))/b^2 - (2*a - b)*log(abs(sin(f*x + e)
 - 1))/b^2 + 2*sin(f*x + e)/((sin(f*x + e)^2 - 1)*b))/f
```

Mupad [B]
time = 4.95, size = 591, normalized size = 6.87

$$\frac{b\left(a\sin\left(e+f\,x\right)-a\,\mathrm{atanh}(\sin\left(e+f\,x\right))+a\sin\left(e+f\,x\right)^2\mathrm{atanh}(\sin\left(e+f\,x\right))\right)+b^2\left(\sin\left(e+f\,x\right)+\mathrm{atanh}(\sin\left(e+f\,x\right))-\sin\left(e+f\,x\right)^2\mathrm{atanh}(\sin\left(e+f\,x\right))\right)-2\,a^2\mathrm{atanh}(\sin\left(e+f\,x\right))+2\,a^2\sin\left(e+f\,x\right)^2\mathrm{atanh}(\sin\left(e+f\,x\right))+\mathrm{atan}\left(\frac{-a\sin(e+f\,x)\,{\left(a^4+b\,a^3\right)}^{3/2}\,8\mathrm{i}-b\sin(e+f\,x)\,{\left(a^4+b\,a^3\right)}^{3/2}\,4\mathrm{i}+a^5\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,8\mathrm{i}-a^2\,b^3\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,2\mathrm{i}+a^3\,b^2\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,1\mathrm{i}+a\,b^4\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,1\mathrm{i}+a^4\,b\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,12\mathrm{i}}{3\,a^5\,b^2+5\,a^4\,b^3+a^3\,b^4-a^2\,b^5}\right)\sqrt{a^4+b\,a^3}\,2\mathrm{i}-\mathrm{atan}\left(\frac{-a\sin(e+f\,x)\,{\left(a^4+b\,a^3\right)}^{3/2}\,8\mathrm{i}-b\sin(e+f\,x)\,{\left(a^4+b\,a^3\right)}^{3/2}\,4\mathrm{i}+a^5\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,8\mathrm{i}-a^2\,b^3\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,2\mathrm{i}+a^3\,b^2\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,1\mathrm{i}+a\,b^4\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,1\mathrm{i}+a^4\,b\sin(e+f\,x)\,\sqrt{a^4+b\,a^3}\,12\mathrm{i}}{3\,a^5\,b^2+5\,a^4\,b^3+a^3\,b^4-a^2\,b^5}\right)\sin\left(e+f\,x\right)^2\sqrt{a^4+b\,a^3}\,2\mathrm{i}}{f\,\left(-2\,b^3\sin\left(e+f\,x\right)^2+2\,b^3-2\,a\,b^2\sin\left(e+f\,x\right)^2+2\,a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] (b*(a*sin(e + f*x) - a*atanh(sin(e + f*x)) + a*sin(e + f*x)^2*atanh(sin(e +
 f*x))) + atan((a^5*sin(e + f*x)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*(a
^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(
e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*
1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b
+ a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*(a^3*b + a^4
)^(1/2)*2i + b^2*(sin(e + f*x) + atanh(sin(e + f*x)) - sin(e + f*x)^2*atanh
(sin(e + f*x))) - 2*a^2*atanh(sin(e + f*x)) - atan((a^5*sin(e + f*x)*(a^3*b
 + a^4)^(1/2)*8i - b*sin(e + f*x)*(a^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(
a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b
^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(
1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b + a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 +
5*a^4*b^3 + 3*a^5*b^2))*sin(e + f*x)^2*(a^3*b + a^4)^(1/2)*2i + 2*a^2*sin(e
 + f*x)^2*atanh(sin(e + f*x)))/(f*(2*a*b^2 + 2*b^3 - 2*b^3*sin(e + f*x)^2 -
 2*a*b^2*sin(e + f*x)^2))
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,f}$$

```
[Out] arctanh(sin(f*x+e))/b/f-arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/f
/(a+b)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 400, 212, 214}

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b*f) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{bf} - \frac{a\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{bf}\\ &= \frac{\tanh^{-1}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.18, size = 1022, normalized size = 18.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-(Sqrt[a]*Cos[e]*Log[a + 2*
(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]) + Sqrt[a]*Cos[e]*Lo
g[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b
*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]] - (2*I)*Sqr
t[a]*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]
*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*
Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a +
b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e]
+ I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin
[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(Cos[e] - I*Si
n[e]) - 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sqrt[(Cos[e]
 - I*Sin[e])^2] + 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sq
rt[(Cos[e] - I*Sin[e])^2] + I*Sqrt[a]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*
(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sq
```

```
rt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] - I*Sqrt[a]*Log[-a - 2*(a + b)*Cos[2*e]
+ a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] + 2*Sqrt[a]*ArcTan[((a + b)*Si
n[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Co
s[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(I*Cos[e] + Sin[e])))/(8*b*Sqrt[a + b]*
f*(a + b*Sec[e + f*x]^2)*Sqrt[(Cos[e] - I*Sin[e])^2])
```

Maple [A]
time = 0.12, size = 63, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{a \,\mathrm{arctanh}\left(\frac{a \sin\left(f x +e \right)}{\sqrt{\left(a +b \right) a}}\right)}{b \sqrt{\left(a +b \right) a}}-\frac{\ln\left(\sin\left(f x +e \right)-1\right)}{2 b}+\frac{\ln\left(\sin\left(f x +e \right)+1\right)}{2 b}}{f}$
default	$\frac{-\frac{a \,\mathrm{arctanh}\left(\frac{a \sin\left(f x +e \right)}{\sqrt{\left(a +b \right) a}}\right)}{b \sqrt{\left(a +b \right) a}}-\frac{\ln\left(\sin\left(f x +e \right)-1\right)}{2 b}+\frac{\ln\left(\sin\left(f x +e \right)+1\right)}{2 b}}{f}$
risch	$\frac{\ln\left({\mathrm e}^{i \left(f x +e \right)}+i\right)}{b f}-\frac{\ln\left({\mathrm e}^{i \left(f x +e \right)}-i\right)}{b f}+\frac{\sqrt{\left(a +b \right) a}\, \ln\left({\mathrm e}^{2 i \left(f x +e \right)}-\frac{2 i \sqrt{\left(a +b \right) a}\, {\mathrm e}^{i \left(f x +e \right)}}{a}-1\right)}{2 \left(a +b \right) f b}-\frac{\sqrt{(a+b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a/b/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/2/b*ln(si
n(f*x+e)-1)+1/2/b*ln(sin(f*x+e)+1))
```

Maxima [A]
time = 0.49, size = 87, normalized size = 1.58

$$\frac{\frac{a \log\left(\frac{a \sin\left(fx+e\right) - \sqrt{(a+b)a}}{a \sin\left(fx+e\right) + \sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\, b} + \frac{\log\left(\sin\left(fx+e\right)+1\right)}{b} - \frac{\log\left(\sin\left(fx+e\right)-1\right)}{b}}{2\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*b) + log(sin(f*x + e) + 1)/b - log(sin(f*x + e) - 1
)/b)/f
```

Fricas [A]

time = 3.51, size = 165, normalized size = 3.00

$$\left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\,\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf},\ \frac{2\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\,\sin\left(fx+e\right)\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - log
(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))
*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [A]

time = 0.44, size = 74, normalized size = 1.35

$$\frac{\frac{2\,a\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b}+\frac{\log\left(\left|\sin\left(fx+e\right)+1\right|\right)}{b}-\frac{\log\left(\left|\sin\left(fx+e\right)-1\right|\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b) + log
(abs(sin(f*x + e) + 1))/b - log(abs(sin(f*x + e) - 1))/b)/f
```

Mupad [B]

time = 4.58, size = 456, normalized size = 8.29

$$\frac{\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{b\,f}+\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2-\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2+\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}}{\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)+\frac{\left(2\,a^2\,b^2-\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\sin\left(e+f\,x\right)-\frac{\left(2\,a^2\,b^2+\frac{\sin\left(e+f\,x\right)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,\left(a+b\right)}}{b^2+a\,b}}\right)\,\sqrt{a\,\left(a+b\right)}\,1{}\mathrm{i}}{f\,\left(b^2+a\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atanh(sin(e + f*x))/(b*f) + (atan((((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (si
n(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*
(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^
3*sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a
 + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a +
b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (sin(e + f
*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b)
)^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))/(a*b + b^2) - ((2*a^3*sin(e +
f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/
2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))
/(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(f*(a*b + b^2))
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}\,f}$$

```
[Out] arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/f/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4232, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{f\sqrt{(a+b)a}}$	28
default	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{f\sqrt{(a+b)a}}$	28
risch	$\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.49, size = 52, normalized size = 1.44

$$-\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*
a)))/(sqrt((a + b)*a)*f)
```

Fricas [A]
time = 2.35, size = 121, normalized size = 3.36

$$\left[\frac{\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)}{2\sqrt{a^2+ab}\,f}, -\frac{\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)}{(a^2+ab)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*
cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.46, size = 38, normalized size = 1.06

$$-\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

Mupad [B]
time = 0.11, size = 28, normalized size = 0.78

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))/(a^(1/2)*f*(a + b)^(1/2))
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,f}+\frac{\sin(e+fx)}{af}$$

```
[Out] sin(f*x+e)/a/f-b*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/f/(a+b)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4232, 396, 214}

$$\frac{\sin(e+fx)}{af}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*f))
+ Sin[e + f*x]/(a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sin(e+fx)}{af} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{af}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,f} + \frac{\sin(e+fx)}{af}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 52, normalized size = 1.00

$$
\frac{-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \sqrt{a}\,\sin(e+fx)}{a^{3/2}f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] (-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
in[e + f*x])/(a^(3/2)*f)
```

Maple [A]
time = 0.13, size = 45, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{a} - \frac{b\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a\sqrt{(a+b)a}}}{f}$	45
default	$\frac{\frac{\sin(fx+e)}{a} - \frac{b\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a\sqrt{(a+b)a}}}{f}$	45
risch	$-\frac{i\,e^{i(fx+e)}}{2af} + \frac{i\,e^{-i(fx+e)}}{2af} + \frac{b\ln\left(e^{2i(fx+e)} - \frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}} - 1\right)}{2\sqrt{a^2+ab}\,fa} - \frac{b\ln\left(e^{2i(fx+e)} + \frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}} - 1\right)}{2\sqrt{a^2+ab}\,fa}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*sin(f*x+e)-1/a*b/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1
/2)))
```

Maxima [A]
time = 0.48, size = 70, normalized size = 1.35

$$\frac{\frac{b\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{2\sin(fx+e)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b
)*a)))/(sqrt((a + b)*a)*a) + 2*sin(f*x + e)/a)/f
```

Fricas [A]
time = 2.15, size = 170, normalized size = 3.27

$$\left[\frac{\sqrt{a^2+ab}\,b\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(a^2+ab)\sin(fx+e)}{2\,(a^3+a^2b)f},\ \frac{\sqrt{-a^2-ab}\,b\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)+(a^2+ab)\sin(fx+e)}{(a^3+a^2b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
+ e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a^3
 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a +
 b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 53, normalized size = 1.02

$$\frac{\frac{b\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a}+\frac{\sin(fx+e)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x +
 e)/a)/f
```

Mupad [B]
time = 4.40, size = 44, normalized size = 0.85

$$\frac{\sin\left(e+f\,x\right)}{a\,f}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{a^{3/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] sin(e + f*x)/(a*f) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(3/
2)*f*(a + b)^(1/2))
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,f}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

```
[Out] (a-b)*sin(f*x+e)/a^2/f-1/3*sin(f*x+e)^3/a/f+b^2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/a^(5/2)/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4232, 398, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*f) +
 ((a - b)*Sin[e + f*x])/(a^2*f) - Sin[e + f*x]^3/(3*a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-b}{a^2} - \frac{x^2}{a} + \frac{b^2}{a^2(a+b-ax^2)}\right)\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{(a-b)\sin(e+fx)}{a^2 f} - \frac{\sin^3(e+fx)}{3af} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{a^2 f}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,f} + \frac{(a-b)\sin(e+fx)}{a^2 f} - \frac{\sin^3(e+fx)}{3af}$$

Mathematica [A]

time = 0.32, size = 105, normalized size = 1.38

$$\frac{\frac{6b^2\left(-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)+\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}} + 3\sqrt{a}(3a-4b)\sin(e+fx) + a^{3/2}\sin(3(e+fx))}{12a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt[
a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(3/
2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

Maple [A]

time = 0.16, size = 70, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\sin^3(fx+e)\right)}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a^2\sqrt{(a+b)a}}}{f}$
default	$\frac{-\frac{\frac{a\left(\sin^3(fx+e)\right)}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a^2\sqrt{(a+b)a}}}{f}$

risch	$-\frac{3i e^{i(fx+e)}}{8af}+\frac{i e^{i(fx+e)} b}{2a^2 f}+\frac{3i e^{-i(fx+e)}}{8af}-\frac{i e^{-i(fx+e)} b}{2a^2 f}+\frac{b^2 \ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\, f a^2}-\frac{b^2 \ln\left(e^{2i(fx}\right.}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+sin(f*x+e)*b)+b^2/a^2/((a+b)*a
)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.49, size = 92, normalized size = 1.21

$$-\frac{\frac{3\,b^2 \log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2}+\frac{2\left(a\sin(fx+e)^3-3\,(a-b)\sin(fx+e)\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((
a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2*(a*sin(f*x + e)^3 - 3*(a - b)*sin(f*x
 + e))/a^2)/f
```

Fricas [A]
time = 3.24, size = 238, normalized size = 3.13

$$\left[\frac{3\sqrt{a^2+ab}\,b^2\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos(fx+e)^2\right)\sin(fx+e)}{6\,(a^4+a^3b)f},\ -\frac{3\sqrt{-a^2-ab}\,b^2\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos(fx+e)^2\right)\sin(fx+e)}{3\,(a^4+a^3b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2 +
(a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*sqrt
(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3 - a
^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)
*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 85, normalized size = 1.12

$$-\frac{\frac{3\,b^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a^2}+\frac{a^2\sin(fx+e)^3-3\,a^2\sin(fx+e)+3\,ab\sin(fx+e)}{a^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
+ (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

Mupad [B]
time = 4.45, size = 72, normalized size = 0.95

$$\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{5/2}\,f\,\sqrt{a+b}}-\frac{\sin\left(e+f\,x\right)^3}{3\,a\,f}-\frac{\sin\left(e+f\,x\right)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(5/2)*f*(a + b)^(1/2))
 - sin(e + f*x)^3/(3*a*f) - (sin(e + f*x)*((a + b)/a^2 - 2/a))/f
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}\,f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}$$

```
[Out] (a^2-a*b+b^2)*sin(f*x+e)/a^3/f-1/3*(2*a-b)*sin(f*x+e)^3/a^2/f+1/5*sin(f*x+e
)^5/a/f-b^3*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4232, 398, 214}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f}+\frac{\sin^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b]*f)
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/(a^3*f) - ((2*a - b)*Sin[e + f*x]^3)/(
3*a^2*f) + Sin[e + f*x]^5/(5*a*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
```

```
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{a^3}-\frac{(2a-b)x^2}{a^2}+\frac{x^4}{a}-\frac{b^3}{a^3(a+b-ax^2)}\right)dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3 f}-\frac{(2a-b)\sin^3(e+fx)}{3a^2 f}+\frac{\sin^5(e+fx)}{5af}-\frac{b^3\text{Subst}(\int$$
$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}\,f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3 f}-\frac{(2a-b)\sin^3(e+f}{3a^2 f}$$

Mathematica [A]
time = 0.73, size = 136, normalized size = 1.26

$$\frac{\frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\,\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\,\sin(e+fx)\right)\right)}{\sqrt{a+b}}+30\sqrt{a}\,(5a^2-6ab+8b^2)\sin(e+fx)+5a^{3/2}(5a-4b)\sin(3(e+fx))+3a^{5/2}\sin(5(e+fx))}{240a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[e
+ f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)
])/(240*a^(7/2)*f)
```

Maple [A]
time = 0.21, size = 110, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+\frac{ab\left(\sin^3(fx+e)\right)}{3}+a^2\sin(fx+e)-ab\sin(fx+e)+b^2\sin(fx+e)}{a^3}-\frac{b^3\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a^3\sqrt{(a+b)a}}}{f}$
default	$\frac{\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+\frac{ab\left(\sin^3(fx+e)\right)}{3}+a^2\sin(fx+e)-ab\sin(fx+e)+b^2\sin(fx+e)}{a^3}-\frac{b^3\,\text{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{a^3\sqrt{(a+b)a}}}{f}$

risch	$-\frac{5i e^{i(fx+e)}}{16af}+\frac{3i e^{i(fx+e)} b}{8a^2 f}-\frac{i e^{i(fx+e)} b^2}{2a^3 f}+\frac{5i e^{-i(fx+e)}}{16af}-\frac{3i e^{-i(fx+e)} b}{8a^2 f}+\frac{i e^{-i(fx+e)} b^2}{2a^3 f}+\frac{b^3 \ln\left(e^{2i(fx+e)}-2\right.}{2\sqrt{a^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*(1/5*sin(f*x+e)^5*a^2-2/3*a^2*sin(f*x+e)^3+1/3*a*b*sin(f*x+e)^3+
a^2*sin(f*x+e)-a*b*sin(f*x+e)+b^2*sin(f*x+e))-b^3/a^3/((a+b)*a)^(1/2)*arcta
nh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.47, size = 122, normalized size = 1.13

$$\frac{\frac{15\, b^3 \log\left(\frac{a \sin(fx+e)-\sqrt{(a+b)a}}{a \sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\, a^3}+\frac{2\left(3\, a^2 \sin(fx+e)^5-5\left(2\, a^2-ab\right) \sin(fx+e)^3+15\left(a^2-ab+b^2\right) \sin(fx+e)\right)}{a^3}}{30\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/30*(15*b^3*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*a^3) + 2*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - a
*b)*sin(f*x + e)^3 + 15*(a^2 - a*b + b^2)*sin(f*x + e))/a^3)/f
```

Fricas [A]
time = 3.28, size = 315, normalized size = 2.92

$$\left[\frac{15\sqrt{a^2+ab}\, b^3 \log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\,\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(3\left(a^4+a^3b\right)\cos(fx+e)^4+8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3+\left(4\,a^4-a^3b-5\,a^2b^2\right)\cos(fx+e)^2\right)\sin(fx+e)}{30\left(a^5+a^4b\right)f},\ \frac{15\sqrt{-a^2-ab}\, b^3 \arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)+\left(3\left(a^4+a^3b\right)\cos(fx+e)^4+8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3+\left(4\,a^4-a^3b-5\,a^2b^2\right)\cos(fx+e)^2\right)\sin(fx+e)}{15\left(a^5+a^4b\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*x
+ e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2*b^
2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2 - a*
b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b)*cos
(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 129, normalized size = 1.19

$$\frac{\frac{15\,b^3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,a^3}+\frac{3\,a^4\sin(fx+e)^5-10\,a^4\sin(fx+e)^3+5\,a^3b\sin(fx+e)^3+15\,a^4\sin(fx+e)-15\,a^3b\sin(fx+e)+15\,a^2b^2\sin(fx+e)}{a^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3)
 + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3 +
 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/a^5
)/f
```

Mupad [B]
time = 0.14, size = 111, normalized size = 1.03

$$\frac{\sin\left(e+f\,x\right)\,\left(\frac{3}{a}+\frac{\left(a+b\right)\,\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)}{f}+\frac{{\sin\left(e+f\,x\right)}^5}{5\,a\,f}+\frac{{\sin\left(e+f\,x\right)}^3\,\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right)}{f}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin\left(e+f\,x\right)}{\sqrt{a+b}}\right)}{a^{7/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (sin(e + f*x)*(3/a + ((a + b)*((a + b)/a^2 - 3/a))/a))/f + sin(e + f*x)^5/(
5*a*f) + (sin(e + f*x)^3*((a + b)/(3*a^2) - 1/a))/f - (b^3*atanh((a^(1/2)*s
in(e + f*x))/(a + b)^(1/2)))/(a^(7/2)*f*(a + b)^(1/2))
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/f/(a+b)^(1/2)-(a-b)*tan(
f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 398, 211}

$$\frac{a^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*f) -
((a - b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{a-b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{b^2f}\\
&= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.51, size = 224, normalized size = 2.91

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-3a^2\operatorname{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\sec(e+fx)\sqrt{b(i\cos(e)+\sin(e))^4}\left(\sec(e)\left(-3a+2b+b\sec^2(e+fx)\right)\sin(fx)+b\sec(e+fx)\tan(e)\right)\right)}{6b^2\sqrt{a+b}\,f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(e))^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*
Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[e +
f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a + b*Sec
[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.21, size = 70, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$

risch	$-\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-2b\right)}{3f\,b^{2}\left(e^{2i(fx+e)}+1\right)^{3}}-\frac{a^{2}\ln\Big(e^{2i(fx+e)}+\frac{2iba+2ib^{2}+a\sqrt{-ab-b^{2}}+2b\sqrt{-ab-}}{a\sqrt{-ab-b^{2}}}}{2\sqrt{-ab-b^{2}}\,f\,b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e))+a^2/b^2/((a+b)*
b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 68, normalized size = 0.88

$$\frac{\frac{3\,a^{2}\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^{2}}+\frac{b\tan(fx+e)^{3}-3\,(a-b)\tan(fx+e)}{b^{2}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b^2) + (
b*tan(f*x + e)^3 - 3*(a - b)*tan(f*x + e))/b^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 3.27, size = 372, normalized size = 4.83

$$\left[-\frac{3\sqrt{-ab-b^{2}}\,a^{2}\cos(fx+e)^{3}\log\left(\frac{(a^{2}+8\,ab+8\,b^{2})\cos(fx+e)^{4}-2\,(3\,ab+4\,b^{2})\cos(fx+e)^{2}+4\left((a+2\,b)\cos(fx+e)^{3}-b\cos(fx+e)\right)\sqrt{-ab-b^{2}}\sin(fx+e)+b^{2}}{a^{2}\cos(fx+e)^{4}+2\,ab\cos(fx+e)^{2}+b^{2}}\right)-4\left(ab^{2}+b^{3}-(3\,a^{2}b+ab^{2}-2\,b^{3})\cos(fx+e)^{2}\right)\sin(fx+e)}{12\,(ab^{3}+b^{4})f\cos(fx+e)^{3}},\ -\frac{3\sqrt{ab+b^{2}}\,a^{2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^{2}-b}{2\sqrt{ab+b^{2}}\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^{3}-2\left(ab^{2}+b^{3}-(3\,a^{2}b+ab^{2}-2\,b^{3})\cos(fx+e)^{2}\right)\sin(fx+e)}{6\,(ab^{3}+b^{4})f\cos(fx+e)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)
^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)
^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 - 2*b
^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1/6*(3
*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b +
b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (3*a^2*b
 + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x +
e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{6}(e+fx)}{a+b\sec^{2}(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.43, size = 96, normalized size = 1.25

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}\,b^2}+\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)+3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x
+ e) + 3*b^2*tan(f*x + e))/b^3)/f
```

Mupad [B]
time = 4.37, size = 72, normalized size = 0.94

$$\frac{\tan(e+f\,x)^3}{3\,b\,f}-\frac{\tan(e+f\,x)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{f}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{5/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - (tan(e + f*x)*((a + b)/b^2 - 2/b))/f + (a^2*atan((
b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(5/2)*f*(a + b)^(1/2))
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$-\frac{a\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,f}+\frac{\tan(e+fx)}{bf}$$

```
[Out] -a*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/f/(a+b)^(1/2)+tan(f*x+e)/
b/f
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 396, 211}

$$\frac{\tan(e+fx)}{bf}-\frac{a\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*f)) +
 Tan[e + f*x]/(b*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)}{bf} - \frac{a\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{bf}\\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,f} + \frac{\tan(e+fx)}{bf}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.85, size = 192, normalized size = 3.69

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(a\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\sec(e)\sec(e+fx)\sqrt{b(i\cos(e)+\sin(e))^4}\sin(fx)\right)}{2b\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Sec[e
]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b]*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.13, size = 45, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i}{fb(e^{2i(fx+e)}+1)}-\frac{a\ln\left(e^{2i(fx+e)}-\frac{2iba+2ib^2-a\sqrt{-ab-b^2}-2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,fb}+\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b*tan(f*x+e)-a/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)
))
```

Maxima [A]
time = 0.48, size = 47, normalized size = 0.90

$$-\frac{\frac{a \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\, b} - \frac{\tan(fx+e)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b) - tan(f*x +
e)/b)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.
time = 3.42, size = 302, normalized size = 5.81

$$\left[-\frac{\sqrt{-ab-b^2}\, a \cos(fx+e) \log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 2(3ab+4b^2)\cos(fx+e)^2 - 4\left((a+2b)\cos(fx+e)^3 - b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e) + b^2}{a^2\cos(fx+e)^4 + 2ab\cos(fx+e)^2 + b^2}\right) - 4(ab+b^2)\sin(fx+e)}{4(ab^2+b^3) f \cos(fx+e)}, \frac{\sqrt{ab+b^2}\, a \arctan\left(\frac{(a+2b)\cos(fx+e)^2 - b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e) + 2(ab+b^2)\sin(fx+e)}{2(ab^2+b^3) f \cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*c
os(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*b + b^
2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(e+fx)}{a + b\sec^2(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 66, normalized size = 1.27

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}\,b}-\frac{\tan(fx+e)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

Mupad [B]
time = 4.42, size = 44, normalized size = 0.85

$$\frac{\tan(e+f\,x)}{b\,f}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,f\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

```
[Out] tan(e + f*x)/(b*f) - (a*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(3/2
)*f*(a + b)^(1/2))
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,f}$$

```
[Out] arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/f/b^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.78

method	result
derivativedivides	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{f\sqrt{(a+b)\,b}}$
default	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{f\sqrt{(a+b)\,b}}$
risch	$-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2iba+2ib^2-a\sqrt{-ab-b^2}-2}{a\sqrt{-ab-b}}\right.}{2\sqrt{-ab-b^2}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.
time = 2.71, size = 219, normalized size = 6.08

$$\left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)}{4(ab+b^2)f}, -\frac{\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)}{2\sqrt{ab+b^2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sq
rt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.46, size = 48, normalized size = 1.33

$$\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2
)))/(sqrt(a*b + b^2)*f)
```

Mupad [B]
time = 4.51, size = 31, normalized size = 0.86

$$\frac{\mathrm{atan}\left(\frac{b\tan(e+fx)}{\sqrt{b^2+ab}}\right)}{f\sqrt{b^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
[Out] atan((b*tan(e + f*x))/(a*b + b^2)^(1/2))/(f*(a*b + b^2)^(1/2))
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{a}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

```
[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4212, 3260, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}$$
$$= \frac{x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af}$$
$$= \frac{x}{a} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 182, normalized size = 4.04

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}\,fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))\right)}{2a\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.00, size = 46, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-}}{}\right)}{2(a+b)fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*arctan(tan(f*x+e))-1/a*b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 46, normalized size = 1.02

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Fricas [A]
time = 2.79, size = 241, normalized size = 5.36

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\,\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\,\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af},\ \frac{2\,fx+\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left(\left(a+2\,b\right)\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 65, normalized size = 1.44

$$-\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B]
time = 4.71, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+ba}+\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+ba}}{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{a^2+ba}-\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\,(8a^3b^2+16a^2b^3)\,\sqrt{-b\,(a+b)}}{4\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{2\,(a^2+ba)}\right)\sqrt{-b\,(a+b)}}{a^2+ba}}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{f\,(a^2+ba)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$\frac{(a-2b)x}{2a^2}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,f}+\frac{\cos(e+fx)\sin(e+fx)}{2af}$$

```
[Out] 1/2*(a-2*b)*x/a^2+1/2*cos(f*x+e)*sin(f*x+e)/a/f+b^(3/2)*arctan(b^(1/2)*tan(
f*x+e)/(a+b)^(1/2))/a^2/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4231, 425, 536, 209, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a^2*Sqrt[a + b]*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{-a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}\\ &= \frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^2f} + \frac{b^2\text{Subst}}{}\\ &= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,f} + \frac{\cos(e+fx)\sin(e+fx)}{2af}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 67, normalized size = 0.89

$$\frac{2(a-2b)(e+fx) + \frac{4b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + a\sin(2(e+fx))}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

Maple [A]
time = 0.13, size = 76, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a^2 \sqrt{(a+b)\,b}}+\frac{\frac{a \tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a^2 \sqrt{(a+b)\,b}}+\frac{\frac{a \tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}-\frac{xb}{a^2}-\frac{i e^{2i(fx+e)}}{8af}+\frac{i e^{-2i(fx+e)}}{8af}+\frac{\sqrt{-(a+b)\,b}\;b \ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)\,b}-a-2b}{a}\right)}{2(a+b)f\,a^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^2*(1/
2*a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2*(a-2*b)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.48, size = 76, normalized size = 1.01

$$\frac{\frac{2\,b^2 \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\;a^2}+\frac{(fx+e)(a-2\,b)}{a^2}+\frac{\tan(fx+e)}{a\tan(fx+e)^2+a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) + (
f*x + e)*(a - 2*b)/a^2 + tan(f*x + e)/(a*tan(f*x + e)^2 + a))/f
```

Fricas [A]
time = 3.07, size = 286, normalized size = 3.81

$$\left[\frac{2\,(a-2\,b)fx+2\,a\cos(fx+e)\sin(fx+e)+b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,a^2f},\ \frac{(a-2\,b)fx+a\cos(fx+e)\sin(fx+e)-b\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{2\,a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))*
log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqr
t(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^
2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) - b*sq
rt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*
cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.43, size = 94, normalized size = 1.25

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}\,a^2}+\frac{(fx+e)(a-2b)}{a^2}+\frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x +
e)/((tan(f*x + e)^2 + 1)*a))/f
```

Mupad [B]
time = 5.24, size = 373, normalized size = 4.97

$$-\frac{2b^2\,\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)-a\left(\frac{b\sin(2e+2fx)}{2}-b\,\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)\right)-a^2\left(\frac{\sin(2e+2fx)}{2}+\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)\right)+\mathrm{atan}\left(\frac{a\sin(e+fx)\left(-b^4-ab^3\right)^{3/2}4\mathrm{i}+b\sin(e+fx)\left(-b^4-ab^3\right)^{3/2}8\mathrm{i}+b^5\sin(e+fx)\sqrt{-b^4-ab^3}\,8\mathrm{i}+ab^4\sin(e+fx)\sqrt{-b^4-ab^3}\,12\mathrm{i}+a^4b\sin(e+fx)\sqrt{-b^4-ab^3}\,1\mathrm{i}+a^2b^3\sin(e+fx)\sqrt{-b^4-ab^3}\,1\mathrm{i}-a^3b^2\sin(e+fx)\sqrt{-b^4-ab^3}\,2\mathrm{i}}{-\cos(e+fx)a^5b^2+\cos(e+fx)a^4b^3+5\cos(e+fx)a^3b^4+3\cos(e+fx)a^2b^5}\right)\sqrt{-b^4-ab^3}\,2\mathrm{i}}{f\left(2a^3+2ba^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(atan((a*sin(e + f*x)*(- a*b^3 - b^4)^(3/2)*4i + b*sin(e + f*x)*(- a*b^3 -
 b^4)^(3/2)*8i + b^5*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*8i + a*b^4*sin(e +
```

```
f*x)*(- a*b^3 - b^4)^(1/2)*12i + a^4*b*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1
i + a^2*b^3*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1i - a^3*b^2*sin(e + f*x)*(-
 a*b^3 - b^4)^(1/2)*2i)/(3*a^2*b^5*cos(e + f*x) + 5*a^3*b^4*cos(e + f*x) +
a^4*b^3*cos(e + f*x) - a^5*b^2*cos(e + f*x)))*(- a*b^3 - b^4)^(1/2)*2i + 2*
b^2*atan(sin(e + f*x)/cos(e + f*x)) - a*((b*sin(2*e + 2*f*x))/2 - b*atan(si
n(e + f*x)/cos(e + f*x))) - a^2*(sin(2*e + 2*f*x)/2 + atan(sin(e + f*x)/cos
(e + f*x))))/(f*(2*a^2*b + 2*a^3))
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{(3a^2-4ab+8b^2)\,x}{8a^3}-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,f}+\frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f}+\frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/4
*cos(f*x+e)^3*sin(f*x+e)/a/f-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))
/a^3/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}}+\frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{x(3a^2-4ab+8b^2)}{8a^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a^3*Sqrt[a + b]*f) + ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x
])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-3bx^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+}{(1+x}\right.}{}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{b^3\text{Subst}\left(\int \frac{1}{a+b+b}\right.}{}$$

$$= \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,f} + \frac{(3a-4b)\cos(e+fx)\sin(e+}{8a^2f}$$

Mathematica [A]
time = 0.46, size = 95, normalized size = 0.81

$$\frac{4(3a^2-4ab+8b^2)(e+fx)-\frac{32b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sin(2(e+fx))+a^2\sin(4(e+fx))}{32a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[4*
(e + f*x)])/(32*a^3*f)
```

Maple [A]
time = 0.17, size = 116, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\left(\tan^3(fx+e)\right)+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
default	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\left(\tan^3(fx+e)\right)+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
risch	$\frac{3x}{8a}-\frac{xb}{2a^2}+\frac{x\,b^2}{a^3}-\frac{ie^{2i(fx+e)}}{8af}+\frac{ie^{2i(fx+e)}b}{8a^2f}+\frac{ie^{-2i(fx+e)}}{8af}-\frac{ie^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-(a+b)\,b}\,b^2\ln\left(e^{2i(\right.}{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^3/a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^3*((
(3/8*a^2-1/2*a*b)*tan(f*x+e)^3+(-1/2*a*b+5/8*a^2)*tan(f*x+e))/(tan(f*x+e)^2
+1)^2+1/8*(3*a^2-4*a*b+8*b^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.48, size = 132, normalized size = 1.13

$$-\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}-\frac{(3\,a-4\,b)\tan(fx+e)^3+(5\,a-4\,b)\tan(fx+e)}{a^2\tan(fx+e)^4+2\,a^2\tan(fx+e)^2+a^2}-\frac{(3\,a^2-4\,ab+8\,b^2)(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")

[Out] -1/8*(8*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3) - ((3*a - 4*b)*tan(f*x + e)^3 + (5*a - 4*b)*tan(f*x + e))/(a^2*tan(f*x + e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a^3)/f

Fricas [**A**]
time = 2.67, size = 359, normalized size = 3.07

$$\left[\frac{2\,b^{2}\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\,\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}+4\,\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{3}-\left(ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^{2}}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}}\right)+\left(3\,a^{2}-4\,ab+8\,b^{2}\right)fx+\left(2\,a^{2}\cos\left(fx+e\right)^{3}+\left(3\,a^{2}-4\,ab\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,a^{3}f},\ \frac{4\,b^{2}\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left(\left(a+2\,b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+\left(3\,a^{2}-4\,ab+8\,b^{2}\right)fx+\left(2\,a^{2}\cos\left(fx+e\right)^{3}+\left(3\,a^{2}-4\,ab\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,a^{3}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")

[Out] [1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)

[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2), x)

Giac [**A**]
time = 0.43, size = 141, normalized size = 1.21

$$-\frac{\frac{8\,\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)b^{3}}{\sqrt{ab+b^{2}}\,a^{3}}-\frac{\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\left(fx+e\right)}{a^{3}}-\frac{3\,a\tan\left(fx+e\right)^{3}-4\,b\tan\left(fx+e\right)^{3}+5\,a\tan\left(fx+e\right)-4\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{2}a^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a
^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*tan(
f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B]
time = 5.26, size = 1114, normalized size = 9.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((tan(e + f*x)*(5*a - 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a - 4*b))/(8*a^2))
/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((((-b^5*(a + b))^(1/2)
*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))
/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2
)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b))^(1/2))
/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((-b^5*(a
 + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 +
 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (
3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a
+ b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4)
)/(((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((-b^5*(a + b
))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a
^4*b^3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^
8*b^2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b)
)^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^4) - ((-b
^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*
b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/
2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^
5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*b + a^
4)))*(-b^5*(a + b))^(1/2)*1i)/(f*(a^3*b + a^4)) - (atan((63*b^4*tan(e + f*x
))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35*b^5)/(32*a) +
 (5*b^6)/(4*a^2))) - (81*b^3*tan(e + f*x))/(256*((27*a*b^2)/256 - (81*b^3)/
256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3))) - (35*b^5*tan
(e + f*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)/256 + (27*a^3*b^
2)/256 + (5*b^6)/(4*a))) + (5*b^6*tan(e + f*x))/(4*((5*b^6)/4 - (35*a*b^5)/
32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(
e + f*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b^4)/(64*a^2) - (35*b
^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(a^2*3i - a*b*4i + b^2*8i)*1i)/(8*a^3*f)
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$\frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}\,f}+\frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f}+\frac{(5a$$

```
[Out] 1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*x/a^4+1/16*(5*a^2-6*a*b+8*b^2)*cos(f*x+
e)*sin(f*x+e)/a^3/f+1/24*(5*a-6*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(f*
x+e)^5*sin(f*x+e)/a/f+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^4/f/
(a+b)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}}+\frac{(5a-6b)\sin(e+fx)\cos^3(e+fx)}{24a^2f}+\frac{(5a^2-6ab+8b^2)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x(5a^3-6a^2b+8ab^2-16b^3)}{16a^4}+\frac{\sin(e+fx)\cos^5(e+fx)}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*x)/(16*a^4) + (b^(7/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*Sqrt[a + b]*f) + ((5*a^2 - 6*a*b + 8*b^2
)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((5*a - 6*b)*Cos[e + f*x]^3*Sin[e
 + f*x])/(24*a^2*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-5bx^2}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af} + \frac{\text{Subst}\left(\int \frac{3(5a^2-\cdots}{(1\cdots}\right.}{\cdots}$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} - \cdots$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} - \cdots$$

$$= \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}\,f} + \frac{(5a^2-6ab+8b^2)\,\mathrm{c}\cdots}{1\cdots}$$

Mathematica [A]

time = 0.99, size = 133, normalized size = 0.82

$$\frac{12(5a^3-6a^2b+8ab^2-16b^3)(e+fx) + \frac{192b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 3a(15a^2-16ab+16b^2)\sin(2(e+fx)) + 3a^2(3a-2b)\sin(4(e+fx)) + a^3\sin(6(e+fx))}{192a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 16*
b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(e +
 f*x)])/(192*a^4*f)
```

Maple [A]

time = 0.22, size = 165, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}} + \frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^5(fx+e)\right)+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\mathrm{ta}\cdots}{\left(\tan^2(fx+e)+1\right)^3}}{a^4}}{f}$

default	$\frac{\frac{b^{4} \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)\, b}}\right)}{a^{4} \sqrt{(a+b)\, b}}+\frac{\frac{\left(\frac{5}{16} a^{3}-\frac{3}{8} a^{2} b+\frac{1}{2} a\, b^{2}\right)\left(\tan^{5}(fx+e)\right)+\left(a\, b^{2}+\frac{5}{6} a^{3}-a^{2} b\right)\left(\tan^{3}(fx+e)\right)+\left(-\frac{5}{8} a^{2} b+\frac{1}{2} a\, b^{2}+\frac{11}{16} a^{3}\right)}{\left(\tan^{2}(fx+e)+1\right)^{3}}}{a^{4}}}{f}$
risch	$\frac{5x}{16a}-\frac{3xb}{8a^{2}}+\frac{x\, b^{2}}{2a^{3}}-\frac{x\, b^{3}}{a^{4}}-\frac{15i\mathrm{e}^{2i(fx+e)}}{128af}+\frac{i\mathrm{e}^{2i(fx+e)} b}{8a^{2} f}-\frac{i\mathrm{e}^{2i(fx+e)} b^{2}}{8a^{3} f}+\frac{15i\mathrm{e}^{-2i(fx+e)}}{128af}-\frac{i\mathrm{e}^{-2i(fx+e)} b}{8a^{2} f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^4/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^4*(((
5/16*a^3-3/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^5+(a*b^2+5/6*a^3-a^2*b)*tan(f*x+e)
^3+(-5/8*a^2*b+1/2*a*b^2+11/16*a^3)*tan(f*x+e))/(tan(f*x+e)^2+1)^3+1/16*(5*
a^3-6*a^2*b+8*a*b^2-16*b^3)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.48, size = 197, normalized size = 1.21

$$\frac{\frac{48\, b^{4} \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\, a^{4}}+\frac{3\left(5\, a^{2}-6\, ab+8\, b^{2}\right) \tan(fx+e)^{5}+8\left(5\, a^{2}-6\, ab+6\, b^{2}\right) \tan(fx+e)^{3}+3\left(11\, a^{2}-10\, ab+8\, b^{2}\right) \tan(fx+e)}{a^{3} \tan(fx+e)^{6}+3\, a^{3} \tan(fx+e)^{4}+3\, a^{3} \tan(fx+e)^{2}+a^{3}}+\frac{3\left(5\, a^{3}-6\, a^{2} b+8\, ab^{2}-16\, b^{3}\right)(fx+e)}{a^{4}}}{48\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4) +
 (3*(5*a^2 - 6*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 - 6*a*b + 6*b^2)*tan(
f*x + e)^3 + 3*(11*a^2 - 10*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x + e)^6
+ 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) + 3*(5*a^3 - 6*a^2*b +
 8*a*b^2 - 16*b^3)*(f*x + e)/a^4)/f
```

Fricas [A]
time = 3.12, size = 442, normalized size = 2.71

$$\left[\frac{12\, b^{3} \sqrt{-\frac{b}{a+b}} \log\left(\frac{\left(a^{2}+8\, ab+8\, b^{2}\right) \cos(fx+e)^{4}-2\left(3\, ab+4\, b^{2}\right) \cos(fx+e)^{2}-4\left(\left(a^{2}+3\, ab+2\, b^{2}\right) \cos(fx+e)^{3}-\left(ab+b^{2}\right) \cos(fx+e)\right) \sqrt{-\frac{b}{a+b}} \sin(fx+e)+b^{2}}{a^{2} \cos(fx+e)^{4}+2\, ab \cos(fx+e)^{2}+b^{2}}\right)+3\left(5\, a^{3}-6\, a^{2} b+8\, ab^{2}-16\, b^{3}\right) fx+\left(8\, a^{3} \cos(fx+e)^{5}+2\left(5\, a^{3}-6\, a^{2} b\right) \cos(fx+e)^{3}+3\left(5\, a^{3}-6\, a^{2} b+8\, ab^{2}\right) \cos(fx+e)\right) \sin(fx+e)}{48\, a^{4} f}, -\frac{24\, b^{3} \sqrt{\frac{b}{a+b}} \arctan\left(\frac{\left((a+2\, b) \cos(fx+e)^{2}-b\right) \sqrt{\frac{b}{a+b}}}{2\, b \cos(fx+e) \sin(fx+e)}\right)-3\left(5\, a^{3}-6\, a^{2} b+8\, ab^{2}-16\, b^{3}\right) fx-\left(8\, a^{3} \cos(fx+e)^{5}+2\left(5\, a^{3}-6\, a^{2} b\right) \cos(fx+e)^{3}+3\left(5\, a^{3}-6\, a^{2} b+8\, ab^{2}\right) \cos(fx+e)\right) \sin(fx+e)}{48\, a^{4} f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
```

```
6*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/48*(24
*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)
*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3*(5*a^
3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.44, size = 216, normalized size = 1.33

$$\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{\sqrt{ab+b^2}\,a^4}+\frac{3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)(fx+e)}{a^4}+\frac{15\,a^2\tan(fx+e)^5-18\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3-48\,ab\tan(fx+e)^3+48\,b^2\tan(fx+e)^3+33\,a^2\tan(fx+e)-30\,ab\tan(fx+e)+24\,b^2\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3a^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*
b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24*b^
2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b^2*t
an(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan(f*x
+ e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

Mupad [B]
time = 6.15, size = 1979, normalized size = 12.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((tan(e + f*x)*(11*a^2 - 10*a*b + 8*b^2))/(16*a^3) + (tan(e + f*x)^3*(5*a^2
 - 6*a*b + 6*b^2))/(6*a^3) + (tan(e + f*x)^5*(5*a^2 - 6*a*b + 8*b^2))/(16*a
^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (atan
((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(
```

e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*
16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (ta
n(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 -
 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*1
6i)*1i)/(32*a^4) - (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^
2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6
i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i
))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^
6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i
 + a^3*5i - b^3*16i)*1i)/(32*a^4))/((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^
3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a
*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i +
 a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2
*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*
b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - ((5*a*b^10)/4 - b^11 - (1
1*a^2*b^9)/8 + (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 +
(((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 + (tan(e
+ f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16
i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) + (tan(
e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 6
0*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i
))/(32*a^4)))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(16*a^4*f) + (at
an((((((-b^7*(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a
^11*b^2)/4)/(2*a^9) - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a
+ b))^(1/2))/(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) - (tan(e + f*x)*(5
12*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 +
 25*a^6*b^3))/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5) - ((((-b^7*
(a + b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(
2*a^9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/
(512*a^6*(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*
a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))
/(256*a^6))*(-b^7*(a + b))^(1/2)*1i)/(a^4*b + a^5))/(((((-b^7*(a + b))^(1/2
)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^9) - (tan
(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512*a^6*(a^4
b + a^5))))/(2(a^4*b + a^5)) - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a
^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(256*a^6))*(
-b^7*(a + b))^(1/2))/(a^4*b + a^5) - ((5*a*b^10)/4 - b^11 - (11*a^2*b^9)/8
+ (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (25*a^5*b^6)/128)/a^9 + ((((-b^7*(a +
 b))^(1/2)*((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/(2*a^
9) + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(-b^7*(a + b))^(1/2))/(512
a^6(a^4*b + a^5))))/(2*(a^4*b + a^5)) + (tan(e + f*x)*(512*b^9 - 256*a*b^
8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(25
6*a^6))*(-b^7*(a + b))^(1/2))/(a^4*b + a^5)))*(-b^7*(a + b))^(1/2)*1i)/(f*(
a^4*b + a^5))

3.193 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f}-\frac{a\sin(e+fx)}{2b(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] arctanh(sin(f*x+e))/b^2/f-1/2*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)-1
/2*(2*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3/2
)/f
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 425, 536, 212, 214}

$$-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}}-\frac{a\sin(e+fx)}{2bf(a+b)\,(-a\sin^2(e+fx)+a+b)}+\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b^2*f) - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*f) - (a*Sin[e + f*x])/(2*b*(a +
 b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\,(a+b-a\sin^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{-a-2b-ax^2}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+\right.}{2b(a+b)f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\,(a+b-a\sin^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{b^2 f} - \frac{(}{}$$

$$= \frac{\tanh^{-1}(\sin(e+fx))}{b^2 f} - \frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a}{2b(a+b)f\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.56, size = 980, normalized size = 9.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
```

```
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*
(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos
[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin
[e])^2] + 2*a*(2*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*
x])]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) - 8*a^
(3/2)*b*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x]))/(32*Sqrt[a]*
b^2*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(Cos[e] - I*Sin[e])^2])
```

Maple [A]
time = 0.32, size = 110, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^{2}(fx+e)\right)\right)}-\frac{(2a+3b)\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}\right)}{b^{2}}-\frac{\ln(\sin(fx+e)-1)}{2b^{2}}+\frac{\ln(\sin(fx+e)+1)}{2b^{2}}}{f}$
default	$\frac{\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^{2}(fx+e)\right)\right)}-\frac{(2a+3b)\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}\right)}{b^{2}}-\frac{\ln(\sin(fx+e)-1)}{2b^{2}}+\frac{\ln(\sin(fx+e)+1)}{2b^{2}}}{f}$
risch	$\frac{ia\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{b(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}+i\right)}{b^{2}f}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{b^{2}f}+\frac{\sqrt{(a+b)\,a}\,\ln\left(e^{2i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a/b^2*(1/2/(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(2*a+3*b)/(a+b
)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))-1/2/b^2*ln(sin(f*x
+e)-1)+1/2/b^2*ln(sin(f*x+e)+1))
```

Maxima [A]
time = 0.49, size = 152, normalized size = 1.49

$$\frac{\frac{(2\,a+3\,b)a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}}-\frac{2\,a\sin(fx+e)}{a^2b+2\,ab^2+b^3-(a^2b+ab^2)\sin(fx+e)^2}+\frac{2\,\log(\sin(fx+e)+1)}{b^2}-\frac{2\,\log(\sin(fx+e)-1)}{b^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + 3*b)*a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) +
 sqrt((a + b)*a)))/((a*b^2 + b^3)*sqrt((a + b)*a)) - 2*a*sin(f*x + e)/(a^2*
b + 2*a*b^2 + b^3 - (a^2*b + a*b^2)*sin(f*x + e)^2) + 2*log(sin(f*x + e) +
1)/b^2 - 2*log(sin(f*x + e) - 1)/b^2)/f
```

Fricas [A]
time = 2.81, size = 410, normalized size = 4.02

$$\left[-\frac{2\,ab\sin(fx+e)-\left((2\,a^2+3\,ab)\cos(fx+e)^2+2\,ab+3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-2\left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\log(\sin(fx+e)+1)+2\left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\log(-\sin(fx+e)+1)}{4\left((a^2b^2+ab^3)f\cos(fx+e)^2+(ab^3+b^4)f\right)},\ -\frac{ab\sin(fx+e)-\left((2\,a^2+3\,ab)\cos(fx+e)^2+2\,ab+3\,b^2\right)\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin(fx+e)\right)-\left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\log(\sin(fx+e)+1)+\left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\log(-\sin(fx+e)+1)}{2\left((a^2b^2+ab^3)f\cos(fx+e)^2+(ab^3+b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^2
)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(f*
x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^2 +
 a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b + b
^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 +
b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b +
 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a^2 + a*
b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x +
 e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.47, size = 126, normalized size = 1.24

$$\frac{\frac{(2a^2+3ab)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(ab^2+b^3)\sqrt{-a^2-ab}}+\frac{a\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(ab+b^2)}+\frac{\log(|\sin(fx+e)+1|)}{b^2}-\frac{\log(|\sin(fx+e)-1|)}{b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/2*((2*a^2 + 3*a*b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a*b^2 + b^3)
*sqrt(-a^2 - a*b)) + a*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a*b + b^2)
) + log(abs(sin(f*x + e) + 1))/b^2 - log(abs(sin(f*x + e) - 1))/b^2)/f
```

Mupad [B]
time = 5.87, size = 2039, normalized size = 19.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
[Out] (atan((((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3))
 - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^
2*(2*a*b^3 + b^4 + a^2*b^2)))*1i)/(2*b^2) + (sin(e + f*x)*(20*a^4*b + 8*a^5
 + 13*a^3*b^2)*1i)/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e + f*x)*(16*a^2*b
^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2
)))*1i)/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2)*1i)/(4*(2*a
*b^3 + b^4 + a^2*b^2)))/b^2)/((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a
*b^4 + b^5 + a^2*b^3)) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^
5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) + (sin(e + f*x)
*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 + (((4
*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e +
f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 +
 b^4 + a^2*b^2)))/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/
(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a
^2*b^3)))*1i)/(b^2*f) - (atan((((sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b
^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a
^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x)*(a*(a + b)^3)
```

```
^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*
(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*
b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)^(1/2)*(2*a + 3
*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e + f*x)*(20*a^4
*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(a + b)^3)^(1
/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) + (sin(e
 + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b
^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 +
 a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a +
 b)^3)^(1/2)*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))/(((
3*a^3*b)/2 + a^4)/(2*a*b^4 + b^5 + a^2*b^3) - (((sin(e + f*x)*(20*a^4*b + 8
*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*((
4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e + f*x
)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 3
2*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(a*(a + b)^3)
^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)) + (((sin(e +
f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) - ((a*(
a + b)^3)^(1/2)*((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b
^3) + (sin(e + f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^
6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^
2)))*(a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b
^2))))*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*1i)/(2*f*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)) - (a*sin(e + f*x))/(2*b*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

3.194 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}f}+\frac{\sin(e+fx)}{2(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] 1/2*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)+1/2*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/(a+b)^(3/2)/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4232, 205, 214}

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*f) + S
in[e + f*x]/(2*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
```

```
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2(a+b)f}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}f} + \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 88, normalized size = 1.19

$$\frac{\sqrt{a}\,\sqrt{a+b}\,\sin(e+fx) + \tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\left(a+b-a\sin^2(e+fx)\right)}{\sqrt{a}\,(a+b)^{3/2}f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
[Out] (Sqrt[a]*Sqrt[a + b]*Sin[e + f*x] + ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]]*(a + b - a*Sin[e + f*x]^2))/(Sqrt[a]*(a + b)^(3/2)*f*(a + 2*b + a*Cos[
2*(e + f*x)]))
```

Maple [A]
time = 0.17, size = 68, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}}{f}$
default	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}}{f}$

risch	$-\frac{i(e^{3i(fx+e)}-e^{i(fx+e)})}{f(a+b)(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{4\sqrt{a^2+ab}\,(a+b)f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b}{\sqrt{a^2}}\right.}{4\sqrt{a^2+ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.38

$$-\frac{\frac{2\sin(fx+e)}{(a^2+ab)\sin(fx+e)^2-a^2-2\,ab-b^2}+\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(f*x + e)/((a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*b - b^2) + log
((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sq
rt((a + b)*a)*(a + b)))/f
```

Fricas [A]
time = 3.07, size = 272, normalized size = 3.68

$$\left[\frac{(a\cos(fx+e)^2+b)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\,\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2(a^2+ab)\sin(fx+e)}{4\left((a^4+2a^3b+a^2b^2)f\cos(fx+e)^2+(a^3b+2a^2b^2+ab^3)f\right)},-\frac{(a\cos(fx+e)^2+b)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)-(a^2+ab)\sin(fx+e)}{2\left((a^4+2a^3b+a^2b^2)f\cos(fx+e)^2+(a^3b+2a^2b^2+ab^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqr
t(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b
)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^
2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan(sqr
t(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4 + 2*a
^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.49, size = 76, normalized size = 1.03

$$-\frac{\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,(a+b)}+\frac{\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a+b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

Mupad [B]
time = 0.14, size = 62, normalized size = 0.84

$$\frac{\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a\sin\left(e+f\,x\right)^2+a+b\right)}+\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{2\,\sqrt{a}\,f\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] sin(e + f*x)/(2*f*(a + b)*(a + b - a*sin(e + f*x)^2)) + atanh((a^(1/2)*sin(
e + f*x))/(a + b)^(1/2))/(2*a^(1/2)*f*(a + b)^(3/2))
```

3.195 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f}-\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/f-1
/2*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4232, 393, 214}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2af(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*f) - (b*Sin[e + f*x])/(2*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
```

```
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= -\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e$$
$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A]
time = 0.44, size = 82, normalized size = 0.99

$$\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{2\sqrt{a}\,b\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (2
*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(3/
2)*f)
```

Maple [A]
time = 0.18, size = 80, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{b\sin(fx+e)}{2(a+b)a\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{(2a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)a\sqrt{(a+b)\,a}}}{f}$
default	$\frac{\frac{b\sin(fx+e)}{2(a+b)a\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{(2a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)a\sqrt{(a+b)\,a}}}{f}$

risch	$\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,(a+b)f}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e}{\sqrt{a^2}}\right.}{4\sqrt{a^2+ab}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b/(a+b)/a*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)+1/2*(2*a+b)/(a+b)/a/((a
+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.48, size = 115, normalized size = 1.39

$$-\frac{\frac{2\,b\sin(fx+e)}{a^3+2\,a^2b+ab^2-(a^3+a^2b)\sin(fx+e)^2}+\frac{(2\,a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+ab)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*sin(f*x + e)/(a^3 + 2*a^2*b + a*b^2 - (a^3 + a^2*b)*sin(f*x + e)^
2) + (2*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqr
t((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)))/f
```

Fricas [A]
time = 3.79, size = 311, normalized size = 3.75

$$\left[\frac{\left((2\,a^2+ab)\cos(fx+e)^2+2\,ab+b^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{a^2+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-2\,(a^2b+ab^2)\sin(fx+e)}{4\,\left((a^5+2\,a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2\,a^3b^2+a^2b^3)f\right)},-\frac{\left((2\,a^2+ab)\cos(fx+e)^2+2\,ab+b^2\right)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)+(a^2b+ab^2)\sin(fx+e)}{2\,\left((a^5+2\,a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2\,a^3b^2+a^2b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^
2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(
f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos(f*x
 + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e
)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos
(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.45, size = 91, normalized size = 1.10

$$-\frac{\frac{(2a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+ab)\sqrt{-a^2-ab}} - \frac{b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a^2+ab)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(-
a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

Mupad [B]
time = 4.43, size = 71, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,f\,(a+b)^{3/2}} - \frac{b\,\sin\left(e+f\,x\right)}{2\,a\,f\,\left(a+b\right)\,\left(-a\sin\left(e+f\,x\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*f*(a + b
)^(3/2)) - (b*sin(e + f*x))/(2*a*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{a^2f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] -1/2*b*(4*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/2
)/f+sin(f*x+e)/a^2/f+1/2*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 398, 393, 214}

$$-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}}+\frac{b^2\sin(e+fx)}{2a^2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*(
a + b)^(3/2)*f) + Sin[e + f*x]/(a^2*f) + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*
f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2}-\frac{b(2a+b)-2abx^2}{a^2\left(a+b-ax^2\right)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sin(e+fx)}{a^2f}-\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{a^2f}\\ &= \frac{\sin(e+fx)}{a^2f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}-\frac{(b(4a+3b))\text{Subst}\left(\int \frac{}{a}\right.}{2a^2(}\\ &= -\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{a^2f}+\frac{b^2\sin(e+f}{2a^2(a+b)f\left(a+b-a\right.}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.76, size = 945, normalized size = 9.36

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*b*(4*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
```

```
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) + 8*Sqrt[a]*(a + b)^(3/2)*Cos[f*x]*(a + 2*b +
 a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[e] + 2*b*
(4*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b
+ a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) + 8*Sqrt[a]*(a + b)^
(3/2)*Cos[e]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 8*Sqrt[a]*b^2*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*
Tan[e + f*x]))/(32*a^(5/2)*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(C
os[e] - I*Sin[e])^2])
```

Maple [A]

time = 0.20, size = 92, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}-\frac{(4a+3b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{2(a+b)\sqrt{(a+b)a}}\right)}{a^2}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}-\frac{(4a+3b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{2(a+b)\sqrt{(a+b)a}}\right)}{a^2}}{f}$
risch	$-\frac{i e^{i(fx+e)}}{2f\,a^2}+\frac{i e^{-i(fx+e)}}{2f\,a^2}-\frac{i b^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}\right.}{\sqrt{a^2+ab}\,(a+b)fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*sin(f*x+e)+1/a^2*b*(-1/2/(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2
)-1/2*(4*a+3*b)/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))
))
```

Maxima [A]
time = 0.48, size = 138, normalized size = 1.37

$$\frac{\frac{2\,b^{2}\sin\left(fx+e\right)}{a^{4}+2\,a^{3}b+a^{2}b^{2}-\left(a^{4}+a^{3}b\right)\sin\left(fx+e\right)^{2}}+\frac{\left(4\,ab+3\,b^{2}\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{(a+b)a}}{a\sin\left(fx+e\right)+\sqrt{(a+b)a}}\right)}{\left(a^{3}+a^{2}b\right)\sqrt{(a+b)a}}+\frac{4\,\sin\left(fx+e\right)}{a^{2}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*sin(f*x + e)/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x +
e)^2) + (4*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x +
 e) + sqrt((a + b)*a)))/((a^3 + a^2*b)*sqrt((a + b)*a)) + 4*sin(f*x + e)/a^
2)/f
```

Fricas [A]
time = 3.78, size = 403, normalized size = 3.99

$$\left[\frac{\left(4\,ab^{2}+3\,b^{3}+\left(4\,a^{2}b+3\,ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a^{2}+ab}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,\sqrt{a^{2}+ab}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right)+2\,\left(2\,a^{3}b+5\,a^{2}b^{2}+3\,ab^{3}+2\,\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}{4\,\left(\left(a^{6}+2\,a^{5}b+a^{4}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b+2\,a^{4}b^{2}+a^{3}b^{3}\right)f\right)},\ \frac{\left(4\,ab^{2}+3\,b^{3}+\left(4\,a^{2}b+3\,ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a^{2}-ab}\arctan\left(\frac{\sqrt{-a^{2}-ab}\sin\left(fx+e\right)}{a+b}\right)+\left(2\,a^{3}b+5\,a^{2}b^{2}+3\,ab^{3}+2\,\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}{2\,\left(\left(a^{6}+2\,a^{5}b+a^{4}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b+2\,a^{4}b^{2}+a^{3}b^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*b
)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x
+ e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*a^2*b
 + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a^2*b
^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x + e)
^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.44, size = 113, normalized size = 1.12

$$-\frac{\frac{b^2\sin(fx+e)}{(a^3+a^2b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4\,ab+3\,b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+a^2b)\sqrt{-a^2-ab}}-\frac{2\,\sin(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
+ 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2 -
 a*b)) - 2*sin(f*x + e)/a^2)/f
```

Mupad [B]
time = 0.18, size = 94, normalized size = 0.93

$$\frac{\sin\left(e+f\,x\right)}{a^2\,f}+\frac{b^2\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^3\,\sin\left(e+f\,x\right)^2+a^3+b\,a^2\right)}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(4\,a+3\,b\right)}{2\,a^{5/2}\,f\,\left(a+b\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)/(a^2*f) + (b^2*sin(e + f*x))/(2*f*(a + b)*(a^2*b + a^3 - a^3*s
in(e + f*x)^2)) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + 3*b
))/(2*a^(5/2)*f*(a + b)^(3/2))
```

3.197 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}-\frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3/
2)/f+(a-2*b)*sin(f*x+e)/a^3/f-1/3*sin(f*x+e)^3/a^2/f-1/2*b^3*sin(f*x+e)/a^3
/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 398, 393, 214}

$$\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}}-\frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*f) + ((a - 2*b)*Sin[e + f*x])/(a^3*f) - Sin[e + f*x]^3/(3*a^2*f
) - (b^3*Sin[e + f*x])/(2*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{a^3} - \frac{x^2}{a^2} + \frac{b^2(3a+2b)-3ab^2x^2}{a^3(a+b-ax^2)^2}\right) dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f} + \frac{\text{Subst}\left(\int \frac{b^2(3a+2b)-3ab^2x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e+f\right.}{a^3f}$$
$$= \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f} - \frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))} +$$
$$= \frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f} + \frac{(a-2b)\sin(e+fx)}{a^3f} - \frac{\sin^3(e+fx)}{3a^2f} -$$

Mathematica [A]
time = 1.30, size = 139, normalized size = 1.10

$$\frac{-\frac{3b^2(6a+5b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\,\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\,\sin(e+fx)\right)\right)}{(a+b)^{3/2}} + 3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(e+fx) + a^{3/2}\sin(3(e+fx))}{12a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^
3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[3*(
e + f*x)])/(12*a^(7/2)*f)
```

Maple [A]
time = 0.26, size = 120, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\sin^{3}(fx+e)\right)}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^{3}}-\frac{b^{2}\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^{2}(fx+e)\right)\right)}-\frac{(6a+5b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}\right)}{a^{3}}}{f}$
default	$\frac{-\frac{\frac{a\left(\sin^{3}(fx+e)\right)}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^{3}}-\frac{b^{2}\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^{2}(fx+e)\right)\right)}-\frac{(6a+5b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)\sqrt{(a+b)\,a}}\right)}{a^{3}}}{f}$
risch	$-\frac{i\mathrm{e}^{3i(fx+e)}}{24f\,a^{2}}-\frac{3i\mathrm{e}^{i(fx+e)}}{8f\,a^{2}}+\frac{i\mathrm{e}^{i(fx+e)}b}{a^{3}f}+\frac{3i\mathrm{e}^{-i(fx+e)}}{8f\,a^{2}}-\frac{i\mathrm{e}^{-i(fx+e)}b}{a^{3}f}+\frac{i\mathrm{e}^{-3i(fx+e)}}{24f\,a^{2}}+\frac{ib^{3}\left(\mathrm{e}^{3i\cdots}\right.}{a^{3}(a+b)f\left(a\,\mathrm{e}^{4i(fx+e)}\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+2*sin(f*x+e)*b)-b^2/a^3*(-1/2/
(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(6*a+5*b)/(a+b)/((a+b)*a)^(1/2
)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [A]

time = 0.49, size = 160, normalized size = 1.27

$$-\frac{\frac{6\,b^{3}\sin(fx+e)}{a^{5}+2\,a^{4}b+a^{3}b^{2}-(a^{5}+a^{4}b)\sin(fx+e)^{2}}+\frac{3\,(6\,ab^{2}+5\,b^{3})\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^{4}+a^{3}b)\sqrt{(a+b)a}}+\frac{4\left(a\sin(fx+e)^{3}-3\,(a-2\,b)\sin(fx+e)\right)}{a^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^3*sin(f*x + e)/(a^5 + 2*a^4*b + a^3*b^2 - (a^5 + a^4*b)*sin(f*x
+ e)^2) + 3*(6*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin
(f*x + e) + sqrt((a + b)*a)))/((a^4 + a^3*b)*sqrt((a + b)*a)) + 4*(a*sin(f*
x + e)^3 - 3*(a - 2*b)*sin(f*x + e))/a^3)/f
```

Fricas [A]

time = 3.90, size = 504, normalized size = 4.00

$$\left[\frac{3\,(6\,ab^{3}+5\,b^{4}+(6\,a^{2}b^{2}+5\,ab^{3})\cos(fx+e)^{2})\sqrt{a^{2}+ab}\,\log\left(-\frac{a\cos(fx+e)^{2}-2\sqrt{a^{2}+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^{2}+b}\right)+2\,(4\,a^{4}b-4\,a^{3}b^{2}-23\,a^{2}b^{3}-15\,ab^{4}+2\,(a^{5}+2\,a^{4}b+a^{3}b^{2})\cos(fx+e)^{4}+2\,(2\,a^{5}-a^{4}b-8\,a^{3}b^{2}-5\,a^{2}b^{3})\cos(fx+e)^{2})\sin(fx+e)}{12\,((a^{7}+2\,a^{6}b+a^{5}b^{2})f\cos(fx+e)^{2}+(a^{6}b+2\,a^{5}b^{2}+a^{4}b^{3})f)},\ -\frac{3\,(6\,ab^{3}+5\,b^{4}+(6\,a^{2}b^{2}+5\,ab^{3})\cos(fx+e)^{2})\sqrt{-a^{2}-ab}\,\arctan\left(\frac{\sqrt{-a^{2}-ab}\,\sin(fx+e)}{a+b}\right)-(4\,a^{4}b-4\,a^{3}b^{2}-23\,a^{2}b^{3}-15\,ab^{4}+2\,(a^{5}+2\,a^{4}b+a^{3}b^{2})\cos(fx+e)^{4}+2\,(2\,a^{5}-a^{4}b-8\,a^{3}b^{2}-5\,a^{2}b^{3})\cos(fx+e)^{2})\sin(fx+e)}{6\,((a^{7}+2\,a^{6}b+a^{5}b^{2})f\cos(fx+e)^{2}+(a^{6}b+2\,a^{5}b^{2}+a^{4}b^{3})f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
+ a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(
a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2
*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 -
5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^4 + (6
*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b
)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2*
(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 - 5
*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 146, normalized size = 1.16

$$\frac{\frac{3\,b^3\sin(fx+e)}{(a^4+a^3b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{3\,(6\,ab^2+5\,b^3)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+a^3b)\sqrt{-a^2-ab}}-\frac{2\left(a^4\sin(fx+e)^3-3\,a^4\sin(fx+e)+6\,a^3b\sin(fx+e)\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*a
*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt(-
a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f*x
+ e))/a^6)/f
```

Mupad [**B**]
time = 4.62, size = 124, normalized size = 0.98

$$\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,f\,{(a+b)}^{3/2}}-\frac{{\sin\left(e+f\,x\right)}^3}{3\,a^2\,f}-\frac{b^3\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^4\,{\sin\left(e+f\,x\right)}^2+a^4+b\,a^3\right)}-\frac{\sin\left(e+f\,x\right)\,\left(\frac{2\,(a+b)}{a^3}-\frac{3}{a^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(6*a + 5*b))/(2*a^(7/2)*f*
(a + b)^(3/2)) - sin(e + f*x)^3/(3*a^2*f) - (b^3*sin(e + f*x))/(2*f*(a + b)
*(a^3*b + a^4 - a^4*sin(e + f*x)^2)) - (sin(e + f*x)*((2*(a + b))/a^3 - 3/a
^2))/f
```

3.198 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f}+\frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f}-\frac{2(a-b)\sin^3(e+fx)}{3a^3f}+\frac{\sin^5(e+fx)}{5a^2f}+\ldots$$

```
[Out] -1/2*b^3*(8*a+7*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9/2)/(a+b)^(3
/2)/f+(a^2-2*a*b+3*b^2)*sin(f*x+e)/a^4/f-2/3*(a-b)*sin(f*x+e)^3/a^3/f+1/5*s
in(f*x+e)^5/a^2/f+1/2*b^4*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 398, 393, 214}

$$-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}}+\frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{2(a-b)\sin^3(e+fx)}{3a^3f}+\frac{\sin^5(e+fx)}{5a^2f}+\frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(9/2)
*(a + b)^(3/2)*f) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/(a^4*f) - (2*(a -
b)*Sin[e + f*x]^3)/(3*a^3*f) + Sin[e + f*x]^5/(5*a^2*f) + (b^4*Sin[e + f*x]
)/(2*a^4*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-2ab+3b^2}{a^4} - \frac{2(a-b)x^2}{a^3} + \frac{x^4}{a^2} - \frac{b^3(4a+3b)-4ab^3x^2}{a^4(a+b-ax^2)^2}\right)\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} - \frac{\text{Subs}}{}$$

$$= \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{}{2a^4(}$$

$$= -\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-}{}$$

Mathematica [A]
time = 2.29, size = 171, normalized size = 1.09

$$\frac{\frac{60b^3(8a+7b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}} + 30\sqrt{a}\left(5a^2-12ab+8b^2\left(3+\frac{b^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\right)\sin(e+fx) + 5a^{3/2}(5a-8b)\sin(3(e+fx)) + 3a^{5/2}\sin(5(e+fx))}{240a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b +
 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))*Sin[e + f*x] + 5
*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(240*a^
(9/2)*f)
```

Maple [A]
time = 0.35, size = 158, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+\frac{2ab\left(\sin^3(fx+e)\right)}{3}+a^2\sin(fx+e)-2ab\sin(fx+e)+3b^2\sin(fx+e)}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2\right.\right.}\right.}{}}{f}$
default	$\frac{\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+\frac{2ab\left(\sin^3(fx+e)\right)}{3}+a^2\sin(fx+e)-2ab\sin(fx+e)+3b^2\sin(fx+e)}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\left(\sin^2\right.\right.}\right.}{}}{f}$
risch	$-\frac{5ie^{i(fx+e)}}{16f\,a^2}-\frac{3ie^{i(fx+e)}b^2}{2a^4f}+\frac{5ie^{-3i(fx+e)}}{96f\,a^2}+\frac{3ie^{i(fx+e)}b}{4a^3f}+\frac{ie^{3i(fx+e)}b}{12a^3f}-\frac{ie^{-3i(fx+e)}b}{12a^3f}+\frac{5ie^{-i(fx+e)}}{16f\,a^2}-3i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^4*(1/5*sin(f*x+e)^5*a^2-2/3*a^2*sin(f*x+e)^3+2/3*a*b*sin(f*x+e)^3+
a^2*sin(f*x+e)-2*a*b*sin(f*x+e)+3*b^2*sin(f*x+e))+b^3/a^4*(-1/2/(a+b)*b*sin
(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(8*a+7*b)/(a+b)/((a+b)*a)^(1/2)*arctanh(a
*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [A]

time = 0.47, size = 190, normalized size = 1.21

$$\frac{\frac{30\,b^4\sin(fx+e)}{a^6+2\,a^5b+a^4b^2-(a^6+a^5b)\sin(fx+e)^2}+\frac{15\,(8\,ab^3+7\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+a^4b)\sqrt{(a+b)a}}+\frac{4\left(3\,a^2\sin(fx+e)^5-10\,(a^2-ab)\sin(fx+e)^3+15\,(a^2-2\,ab+3\,b^2)\sin(fx+e)\right)}{a^4}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^4*sin(f*x + e)/(a^6 + 2*a^5*b + a^4*b^2 - (a^6 + a^5*b)*sin(f*x
+ e)^2) + 15*(8*a*b^3 + 7*b^4)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*si
n(f*x + e) + sqrt((a + b)*a)))/((a^5 + a^4*b)*sqrt((a + b)*a)) + 4*(3*a^2*s
in(f*x + e)^5 - 10*(a^2 - a*b)*sin(f*x + e)^3 + 15*(a^2 - 2*a*b + 3*b^2)*si
n(f*x + e))/a^4)/f
```

Fricas [A]

time = 3.91, size = 599, normalized size = 3.82

$$\left[\frac{15\,(8\,ab^4+7\,b^5+(8\,a^2b^3+7\,ab^4)\cos(fx+e)^2)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(6\,(a^6+2\,a^5b+a^4b^2)\cos(fx+e)^6+16\,a^5b-8\,a^4b^2+26\,a^3b^3+155\,a^2b^4+105\,ab^5+2\,(4\,a^6+a^5b-10\,a^4b^2-7\,a^3b^3)\cos(fx+e)^4+2\,(8\,a^6+11\,a^4b^2+54\,a^3b^3+35\,a^2b^4)\cos(fx+e)^2)\sin(fx+e)}{60\,((a^8+2\,a^7b+a^6b^2)f\cos(fx+e)^2+(a^7b+2\,a^6b^2+a^5b^3)f)},\ \frac{15\,(8\,ab^4+7\,b^5+(8\,a^2b^3+7\,ab^4)\cos(fx+e)^2)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)+(6\,(a^6+2\,a^5b+a^4b^2)\cos(fx+e)^6+16\,a^5b-8\,a^4b^2+26\,a^3b^3+155\,a^2b^4+105\,ab^5+2\,(4\,a^6+a^5b-10\,a^4b^2-7\,a^3b^3)\cos(fx+e)^4+2\,(8\,a^6+11\,a^4b^2+54\,a^3b^3+35\,a^2b^4)\cos(fx+e)^2)\sin(fx+e)}{30\,((a^8+2\,a^7b+a^6b^2)f\cos(fx+e)^2+(a^7b+2\,a^6b^2+a^5b^3)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/
(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6 + 1
6*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 + a^5
*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 + 54*a^
3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2
)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a*b^4 +
7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x +
e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a
^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2
 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b +
 a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 188, normalized size = 1.20

$$-\frac{\frac{15\,b^4\sin(fx+e)}{(a^5+a^4b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{15\,(8\,ab^3+7\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+a^4b)\sqrt{-a^2-ab}}-\frac{2\left(3\,a^8\sin(fx+e)^5-10\,a^8\sin(fx+e)^3+10\,a^7b\sin(fx+e)^3+15\,a^8\sin(fx+e)-30\,a^7b\sin(fx+e)+45\,a^6b^2\sin(fx+e)\right)}{a^{10}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15*
(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*sq
rt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a^7*
b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6*b^2
*sin(f*x + e))/a^10)/f
```

Mupad [**B**]
time = 0.18, size = 173, normalized size = 1.10

$$\frac{\sin\left(e+f\,x\right)^5}{5\,a^2\,f}+\frac{\sin\left(e+f\,x\right)^3\,\left(\frac{2\,(a+b)}{3\,a^3}-\frac{4}{3\,a^2}\right)}{f}+\frac{\sin\left(e+f\,x\right)\,\left(\frac{6}{a^2}-\frac{(a+b)^2}{a^4}+\frac{2\,(a+b)\,\left(\frac{2\,(a+b)}{a^3}-\frac{4}{a^2}\right)}{a}\right)}{f}+\frac{b^4\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^5\,\sin\left(e+f\,x\right)^2+a^5+b\,a^4\right)}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(8\,a+7\,b\right)}{2\,a^{9/2}\,f\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] sin(e + f*x)^5/(5*a^2*f) + (sin(e + f*x)^3*((2*(a + b))/(3*a^3) - 4/(3*a^2)
))/f + (sin(e + f*x)*(6/a^2 - (a + b)^2/a^4 + (2*(a + b)*((2*(a + b))/a^3 -
 4/a^2))/a))/f + (b^4*sin(e + f*x))/(2*f*(a + b)*(a^4*b + a^5 - a^5*sin(e +
 f*x)^2)) - (b^3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(8*a + 7*b))/(
2*a^(9/2)*f*(a + b)^(3/2))
```

3.199 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{a(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{b^2f}+\frac{a^2\tan(e+fx)}{2b^2(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -1/2*a*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2)
/f+tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 398, 393, 211}

$$\frac{a^2\tan(e+fx)}{2b^2f(a+b)\,(a+b\tan^2(e+fx)+b)}-\frac{a(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*(a
 + b)^(3/2)*f) + Tan[e + f*x]/(b^2*f) + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a(a+2b)+2abx^2}{b^2(a+b+bx^2)^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{b^2 f} - \frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{b^2 f}$$
$$= \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{(a(3a+4b))\text{Subst}\left(\int \frac{}{a+} \right.}{2b^2(a}$$
$$= -\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx}{2b^2(a+b)f\,(a+b+b\,\text{ta}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.72, size = 248, normalized size = 2.48

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{a(3a+4b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}} + 2(a+2b+a\cos(2(e+fx)))\sec(e)\sec(e+fx)\sin(fx) + \frac{a(-((a+2b)\sin(2e))+a\sin(2fx))}{(a+b)(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8b^2 f\,(a+b\sec^2(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec[
f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])
*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2
*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((a + 2
```

```
*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.16, size = 89, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^2}}{f}$
risch	$\frac{i\left(3a^2\mathrm{e}^{4i(fx+e)}+4ab\,\mathrm{e}^{4i(fx+e)}+6a^2\mathrm{e}^{2i(fx+e)}+14ab\,\mathrm{e}^{2i(fx+e)}+8b^2\mathrm{e}^{2i(fx+e)}+3a^2+2ab\right)}{(a+b)b^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{3a^2\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2iba+2i}{\ldots}\right.}{4\sqrt{-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b^2*tan(f*x+e)-a/b^2*(-1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1
/2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.47, size = 114, normalized size = 1.14

$$\frac{\frac{a^2\tan(fx+e)}{a^2b^2+2\,ab^3+b^4+(ab^3+b^4)\tan(fx+e)^2}-\frac{(3\,a^2+4\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}}+\frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/(a^2*b^2 + 2*a*b^3 + b^4 + (a*b^3 + b^4)*tan(f*x + e)
^2) - (3*a^2 + 4*a*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a*b^2 + b^3)
*sqrt((a + b)*b)) + 2*tan(f*x + e)/b^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(92) = 184$.

time = 3.78, size = 538, normalized size = 5.38

$$\left[-\frac{\left(\left(3a^3+4a^2b\right)\cos\left(fx+e\right)^3+\left(3a^2b+4ab^2\right)\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(fx+e)^4-2\left(3ab+4b^2\right)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)-4\left(2a^2b^2+4ab^3+2b^4+\left(3a^3b+5a^2b^2+2ab^3\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{8\left(\left(a^3b^3+2a^2b^4+ab^5\right)f\cos\left(fx+e\right)^3+\left(a^2b^4+2ab^5+b^6\right)f\cos\left(fx+e\right)\right)}, \frac{\left(\left(3a^3+4a^2b\right)\cos\left(fx+e\right)^3+\left(3a^2b+4ab^2\right)\cos\left(fx+e\right)\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)+2\left(2a^2b^2+4ab^3+2b^4+\left(3a^3b+5a^2b^2+2ab^3\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{4\left(\left(a^3b^3+2a^2b^4+ab^5\right)f\cos\left(fx+e\right)^3+\left(a^2b^4+2ab^5+b^6\right)f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e)
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*c
os(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^
3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*b)*cos(
f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))
+ 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^3 + (a^
2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.44, size = 120, normalized size = 1.20

$$\frac{\frac{a^2\tan(fx+e)}{(ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\left(3a^2+4ab\right)}{(ab^2+b^3)\sqrt{ab+b^2}}+\frac{2\tan(fx+e)}{b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*floo
r((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a
^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

Mupad [B]
time = 4.79, size = 113, normalized size = 1.13

$$\frac{\tan(e+fx)}{b^2 f}+\frac{a^2\tan(e+fx)}{2f\,(a+b)\,\left(b^3\tan\left(e+fx\right)^2+b^3+a\,b^2\right)}-\frac{a\,\mathrm{atan}\left(\frac{a\sqrt{b}\,\tan(e+fx)\,(3\,a+4\,b)}{\sqrt{a+b}\,(3\,a^2+4\,b\,a)}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,f\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*f*(a + b)*(a*b^2 + b^3 + b^3*t
an(e + f*x)^2)) - (a*atan((a*b^(1/2)*tan(e + f*x)*(3*a + 4*b))/((a + b)^(1/
2)*(4*a*b + 3*a^2))))*(3*a + 4*b))/(2*b^(5/2)*f*(a + b)^(3/2))
```

3.200 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f}-\frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/f-1/
2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 393, 211}

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2bf(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(
3/2)*f) - (a*Tan[e + f*x])/(2*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

m/2] && IntegerQ[n/2]

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e$$
$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A]
time = 0.32, size = 84, normalized size = 1.02

$$\frac{\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
[Out] (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*
Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^(3
/2)*f)
```

Maple [A]
time = 0.24, size = 76, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$

risch	$-\frac{i\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{fb(a+b)\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2iba+2ib^{2}+a\sqrt{-ab-b^{2}}+2b\sqrt{-ab-}}{a\sqrt{-ab-b^{2}}}\right.}{4\sqrt{-ab-b^{2}}\,(a+b)fb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*a/(a+b)/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/(a+b)/b/((a
+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 91, normalized size = 1.11

$$-\frac{\frac{a\tan\left(fx+e\right)}{a^{2}b+2\,ab^{2}+b^{3}+\left(ab^{2}+b^{3}\right)\tan\left(fx+e\right)^{2}}-\frac{(a+2\,b)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,\left(ab+b^{2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*tan(f*x + e)/(a^2*b + 2*a*b^2 + b^3 + (a*b^2 + b^3)*tan(f*x + e)^2)
 - (a + 2*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a*b +
 b^2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(73) = 146$.
time = 3.25, size = 424, normalized size = 5.17

$$\left[-\frac{4\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}+ab+2\,b^{2}\right)\sqrt{-ab-b^{2}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos(fx+e)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos(fx+e)^{2}+4\left(\left(a+2\,b\right)\cos(fx+e)^{3}-b\cos(fx+e)\right)\sqrt{-ab-b^{2}}\sin(fx+e)+b^{2}}{a^{2}\cos(fx+e)^{4}+2\,ab\cos(fx+e)^{2}+b^{2}}\right)}{8\left(\left(a^{3}b^{2}+2\,a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{3}+2\,ab^{4}+b^{5}\right)f\right)},\ -\frac{2\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}+ab+2\,b^{2}\right)\sqrt{ab+b^{2}}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^{2}-b}{2\sqrt{ab+b^{2}}\cos(fx+e)\sin(fx+e)}\right)}{4\left(\left(a^{3}b^{2}+2\,a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{3}+2\,ab^{4}+b^{5}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x
 + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b
*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^
2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b^3 + 2*a*
b^4 + b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.44, size = 89, normalized size = 1.09

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2b)}{(ab+b^2)^{\frac{3}{2}}}-\frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(ab+b^2)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)*(a*b + b^2)))/f
```

Mupad [B]
time = 4.41, size = 70, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)\,(a+2\,b)}{2\,b^{3/2}\,f\,(a+b)^{3/2}}-\frac{a\tan(e+f\,x)}{2\,b\,f\,\left(a+b\right)\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*f*(a + b)
^(3/2)) - (a*tan(e + f*x))/(2*b*f*(a + b)*(a + b + b*tan(e + f*x)^2))
```

3.201 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}f}+\frac{\tan(e+fx)}{2(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 1/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(3/2)/f/b^(1/2)+1/2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*f) + Ta
n[e + f*x]/(2*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}f} + \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 211, normalized size = 2.89

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{\operatorname{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{-((a+2b)\sin(2e))+a\sin(2fx)}{a(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8(a+b)f\left(a+b\sec^2(e+fx)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]
*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] -
 I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a +
b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.15, size = 64, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}}{f}$

default	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}}{f}$
risch	$\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right.}{4\sqrt{-ab-b^2}\,(a+b)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 74, normalized size = 1.01

$$\frac{\frac{\tan(fx+e)}{(ab+b^2)\tan(fx+e)^2+a^2+2\,ab+b^2}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(a+b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)/((a*b + b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2) + arctan
(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.
time = 3.65, size = 386, normalized size = 5.29

$$\left[\frac{4\,(ab+b^2)\cos(fx+e)\sin(fx+e)-\left(a\cos(fx+e)^2+b\right)\sqrt{-ab-b^2}\,\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\,\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{8\left((a^3b+2\,a^2b^2+ab^3)f\cos(fx+e)^2+(a^2b^2+2\,ab^3+b^4)f\right)},\ \frac{2\,(ab+b^2)\cos(fx+e)\sin(fx+e)-\left(a\cos(fx+e)^2+b\right)\sqrt{ab+b^2}\,\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\,\cos(fx+e)\sin(fx+e)}\right)}{4\left((a^3b+2\,a^2b^2+ab^3)f\cos(fx+e)^2+(a^2b^2+2\,ab^3+b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqrt
(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b -
b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))
)/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)
*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*
sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*
```

```
cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2
+ (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.45, size = 83, normalized size = 1.14

$$\frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}\,(a+b)}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a+b)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a +
b)*(a + b)))/f
```

Mupad [B]
time = 4.47, size = 69, normalized size = 0.95

$$\frac{\tan(e+f\,x)}{2\,f\,(a+b)\,\left(b\tan(e+f\,x)^2+a+b\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(2\,a+2\,b)}{2\,(a+b)^{3/2}}\right)}{2\,\sqrt{b}\,f\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

```
[Out] tan(e + f*x)/(2*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + atan((b^(1/2)*tan(e
 + f*x)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(2*b^(1/2)*f*(a + b)^(3/2))
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{x}{a^2}-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4213, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e-\right.}{2a(a+b)f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} - \\
&= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+f\right.}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 240, normalized size = 2.61

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8a^2\left(a+b\sec^2(e+fx)\right)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
```

)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e + f*x]^2)^2)

Maple [A]
time = 0.07, size = 90, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)\,b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)}}{a}\right.}{4(a+b)^2fa}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/a^2*b*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(f*x+e)))

Maxima [A]
time = 0.49, size = 110, normalized size = 1.20

$$-\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(83) = 166$.
time = 2.73, size = 455, normalized size = 4.95

$$\left[\frac{8\,(a^2+ab)fx\cos(fx+e)^2-4\,ab\cos(fx+e)\sin(fx+e)+8\,(ab+b^2)fx+\left((3\,a^2+2\,ab)\cos(fx+e)^2+3\,ab+2\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{8\left((a^4+a^3b)f\cos(fx+e)^2+(a^3b+a^2b^2)f\right)}, \frac{4\,(a^2+ab)fx\cos(fx+e)^2-2\,ab\cos(fx+e)\sin(fx+e)+4\,(ab+b^2)fx+\left((3\,a^2+2\,ab)\cos(fx+e)^2+3\,ab+2\,b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{4\left((a^4+a^3b)f\cos(fx+e)^2+(a^3b+a^2b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A]
time = 0.45, size = 114, normalized size = 1.24

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B]
time = 6.67, size = 2056, normalized size = 22.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
```

```
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.203 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=142

$$\frac{(a-4b)x}{2a^3}+\frac{b^{3/2}(5a+4b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)}+\frac{b(a+2b)\tan(e+f}{2a^2(a+b)f\left(a+b+b\tan^2\right.}$$

```
[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/
2))/a^3/(a+b)^(3/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1/
2*b*(a+2*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{b^{3/2}(5a+4b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{x(a-4b)}{2a^3}+\frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*
(a + b + b*Tan[e + f*x]^2)) + (b*(a + 2*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subs}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx}{2af\left(a+b+b\tan^2(e+f\right.}$$

Mathematica [A]
time = 1.48, size = 103, normalized size = 0.73

$$\frac{2(a-4b)(e+fx)+\frac{2b^{3/2}(5a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\left(a+\frac{2ab^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(2(e+fx))}{4a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

Maple [A]
time = 0.21, size = 120, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
default	$\frac{\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
risch	$\frac{x}{2a^2}-\frac{2xb}{a^3}-\frac{i e^{2i(fx+e)}}{8a^2f}+\frac{i e^{-2i(fx+e)}}{8a^2f}+\frac{i b^2\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^3(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{5\sqrt{-(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^3*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+4*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*(1/2*a*tan(f
*x+e)/(tan(f*x+e)^2+1)+1/2*(a-4*b)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.49, size = 181, normalized size = 1.27

$$\frac{\frac{(5\,ab^2+4\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}}+\frac{(ab+2\,b^2)\tan(fx+e)^3+(a^2+2\,ab+2\,b^2)\tan(fx+e)}{(a^3b+a^2b^2)\tan(fx+e)^4+a^4+2\,a^3b+a^2b^2+(a^4+3\,a^3b+2\,a^2b^2)\tan(fx+e)^2}+\frac{(fx+e)(a-4\,b)}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + a^3*b
)*sqrt((a + b)*b)) + ((a*b + 2*b^2)*tan(f*x + e)^3 + (a^2 + 2*a*b + 2*b^2)*
tan(f*x + e))/((a^3*b + a^2*b^2)*tan(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 +
 (a^4 + 3*a^3*b + 2*a^2*b^2)*tan(f*x + e)^2) + (f*x + e)*(a - 4*b)/a^3)/f
```

Fricas [**A**]
time = 3.13, size = 566, normalized size = 3.99

$$\left[\frac{4\,(a^3-3\,a^2b-4\,ab^2)fx\cos(fx+e)^2+4\,(a^2b-3\,ab^2-4\,b^3)fx+\left(5\,ab^2+4\,b^3+(5\,a^2b+4\,ab^2)\cos(fx+e)^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+4\left((a^3+a^2b)\cos(fx+e)^3+(a^2b+2\,ab^2)\cos(fx+e)\right)\sin(fx+e)}{8\left((a^5+a^4b)f\cos(fx+e)^2+(a^4b+a^3b^2)f\right)}, \frac{2\,(a^3-3\,a^2b-4\,ab^2)fx\cos(fx+e)^2+2\,(a^2b-3\,ab^2-4\,b^3)fx-\left(5\,ab^2+4\,b^3+(5\,a^2b+4\,ab^2)\cos(fx+e)^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)+2\left((a^3+a^2b)\cos(fx+e)^3+(a^2b+2\,ab^2)\cos(fx+e)\right)\sin(fx+e)}{4\left((a^5+a^4b)f\cos(fx+e)^2+(a^4b+a^3b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2 -
 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2)*
cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a^3*
b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a^2*b - 3
*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2
)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b
+ 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a
^4*b + a^3*b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [**A**]
time = 0.44, size = 192, normalized size = 1.35

$$\frac{\frac{(5\,ab^2+4\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+a^3b)\sqrt{ab+b^2}}+\frac{ab\tan(fx+e)^3+2\,b^2\tan(fx+e)^3+a^2\tan(fx+e)+2\,ab\tan(fx+e)+2\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)(a^3+a^2b)}+\frac{(fx+e)(a-4\,b)}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x +
 e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2*b^
2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2
+ a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

Mupad [B]
time = 7.69, size = 2401, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) + (b*tan(e + f*x)^3*(
a + 2*b))/(2*a^2*(a + b)))/(f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*(a
 + 2*b))) - (atan(((((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*
b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 +
2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i - b*
4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3) + (((tan
(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5
*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2
*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*
b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i -
b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)
/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((tan(e + f*x)*(64*a*b^6 +
 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2))
- (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^
2) - (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16
*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i
 - b*4i))/(4*a^3) + (((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3
*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 +
 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b
*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b +
 a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i))/(4*a^3)))*(a*1i -
b*4i)*1i)/(2*a^3*f) - (atan(((((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e
+ f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b
+ a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^
7*b + a^8 + a^6*b^2) + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(
32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4
*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3
```

)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) + (((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) + (((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) - (((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(-b^3*(a + b)^3)^(1/2)*2i)/(f*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=203

$$\frac{(3a^2-8ab+24b^2)\,x}{8a^4}-\frac{b^{5/2}(7a+6b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f}+\frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\cos^3(e+}{4af\,(a+b}$$

```
[Out] 1/8*(3*a^2-8*a*b+24*b^2)*x/a^4-1/2*b^(5/2)*(7*a+6*b)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f+3/8*(a-2*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/
(a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1
/8*(a-3*b)*b*(3*a+4*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$-\frac{b^{5/2}(7a+6b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3(a-2b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(3a^2-8ab+24b^2)}{8a^4}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 8*a*b + 24*b^2)*x)/(8*a^4) - (b^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*(a + b)^(3/2)*f) + (3*(a - 2*b)*Cos[e +
 f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*
Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) + ((a - 3*b)*b*(3*a + 4*b)
*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{-3a+b-5bx^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$
$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} + \frac{(a-3b}{8a^3(a+b}$$
$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} + \frac{(a-3b}{8a^3(a+b}$$
$$= \frac{(3a^2-8ab+24b^2)\,x}{8a^4} - \frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e}{8a^2f\,(a+b+}$$

Mathematica [A]
time = 1.79, size = 138, normalized size = 0.68

$$\frac{4(3a^2-8ab+24b^2)(e+fx) - \frac{16b^{5/2}(7a+6b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + 8a(a-2b)\sin(2(e+fx)) - \frac{16ab^3\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))} + a^2\sin(4(e+fx))}{32a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
[Out] (4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e + f
*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))
 + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

Maple [A]
time = 0.28, size = 160, normalized size = 0.79

method	result
derivativedivides	$\frac{-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(7a+6b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^4}+\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\left(\tan^3(fx+e)\right)+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(\tan^2(fx+e)+1\right)^2}+}{a^4}}{f}$

default	$\frac{-\frac{b^{3}\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)}+\frac{(7a+6b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^{4}}+\frac{\frac{\left(\frac{3}{8}a^{2}-ab\right)\left(\tan^{3}(fx+e)\right)+\left(-ab+\frac{5}{8}a^{2}\right)\tan(fx+e}{\left(\tan^{2}(fx+e)+1\right)^{2}}}{a^{4}}}{f}$
risch	$\frac{3x}{8a^{2}}-\frac{xb}{a^{3}}+\frac{3x\,b^{2}}{a^{4}}-\frac{i\mathrm{e}^{4i(fx+e)}}{64a^{2}f}-\frac{i\mathrm{e}^{2i(fx+e)}}{8a^{2}f}+\frac{i\mathrm{e}^{2i(fx+e)}b}{4a^{3}f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^{2}f}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^{3}f}+\frac{i\mathrm{e}^{-4i(fx+e)}}{64a^{2}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^3/a^4*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+6*b)/(a
+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(((3/8*a^2-
a*b)*tan(f*x+e)^3+(-a*b+5/8*a^2)*tan(f*x+e))/(tan(f*x+e)^2+1)^2+1/8*(3*a^2-
8*a*b+24*b^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.49, size = 276, normalized size = 1.36

$$-\frac{\frac{4\left(7ab^{3}+6b^{4}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^{5}+a^{4}b\right)\sqrt{(a+b)b}}-\frac{\left(3a^{2}b-5ab^{2}-12b^{3}\right)\tan(fx+e)^{5}+\left(3a^{3}+3a^{2}b-16ab^{2}-24b^{3}\right)\tan(fx+e)^{3}+\left(5a^{3}+2a^{2}b-11ab^{2}-12b^{3}\right)\tan(fx+e)}{\left(a^{4}b+a^{3}b^{2}\right)\tan(fx+e)^{6}+a^{5}+2a^{4}b+a^{3}b^{2}+\left(a^{5}+4a^{4}b+3a^{3}b^{2}\right)\tan(fx+e)^{4}+\left(2a^{5}+5a^{4}b+3a^{3}b^{2}\right)\tan(fx+e)^{2}}-\frac{\left(3a^{2}-8ab+24b^{2}\right)(fx+e)}{a^{4}}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(7*a*b^3 + 6*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + a^
4*b)*sqrt((a + b)*b)) - ((3*a^2*b - 5*a*b^2 - 12*b^3)*tan(f*x + e)^5 + (3*a
^3 + 3*a^2*b - 16*a*b^2 - 24*b^3)*tan(f*x + e)^3 + (5*a^3 + 2*a^2*b - 11*a*
b^2 - 12*b^3)*tan(f*x + e))/((a^4*b + a^3*b^2)*tan(f*x + e)^6 + a^5 + 2*a^4
*b + a^3*b^2 + (a^5 + 4*a^4*b + 3*a^3*b^2)*tan(f*x + e)^4 + (2*a^5 + 5*a^4*
b + 3*a^3*b^2)*tan(f*x + e)^2) - (3*a^2 - 8*a*b + 24*b^2)*(f*x + e)/a^4)/f
```

Fricas [A]
time = 3.23, size = 680, normalized size = 3.35

$$\left[\frac{\left(3a^{4}-5a^{3}b+16a^{2}b^{2}+24ab^{3}\right)fx\cos(fx+e)^{2}+\left(3a^{3}b-5a^{2}b^{2}+16ab^{3}+24b^{4}\right)fx+\left(7ab^{3}+6b^{4}+\left(7a^{2}b^{2}+6ab^{3}\right)\cos(fx+e)^{2}\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos(fx+e)^{4}-2\left(3ab+4b^{2}\right)\cos(fx+e)^{2}+4\left(\left(a^{2}+3ab+2b^{2}\right)\cos(fx+e)^{3}-\left(ab+b^{2}\right)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^{2}}{a^{2}\cos(fx+e)^{4}+2ab\cos(fx+e)^{2}+b^{2}}\right)+\left(2\left(a^{4}+a^{3}b\right)\cos(fx+e)^{5}+3\left(a^{4}-a^{3}b-2a^{2}b^{2}\right)\cos(fx+e)^{3}+\left(3a^{3}b-5a^{2}b^{2}-12ab^{3}\right)\cos(fx+e)\right)\sin(fx+e)}{8\left(\left(a^{6}+a^{5}b\right)f\cos(fx+e)^{2}+\left(a^{5}b+a^{4}b^{2}\right)f\right)},\frac{\left(3a^{4}-5a^{3}b+16a^{2}b^{2}+24ab^{3}\right)fx\cos(fx+e)^{2}+\left(3a^{3}b-5a^{2}b^{2}+16ab^{3}+24b^{4}\right)fx+2\left(7ab^{3}+6b^{4}+\left(7a^{2}b^{2}+6ab^{3}\right)\cos(fx+e)^{2}\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2b)\cos(fx+e)^{2}-b\right)\sqrt{\frac{b}{a+b}}}{2b\cos(fx+e)\sin(fx+e)}\right)+\left(2\left(a^{4}+a^{3}b\right)\cos(fx+e)^{5}+3\left(a^{4}-a^{3}b-2a^{2}b^{2}\right)\cos(fx+e)^{3}+\left(3a^{3}b-5a^{2}b^{2}-12ab^{3}\right)\cos(fx+e)\right)\sin(fx+e)}{8\left(\left(a^{6}+a^{5}b\right)f\cos(fx+e)^{2}+\left(a^{5}b+a^{4}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^3
*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6
```

```
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)*cos(f
*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b
^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2
+ (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*
cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + 2*(7*a*b^3
 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e
))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x
 + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.45, size = 195, normalized size = 0.96

$$-\frac{\frac{4\,b^3\tan(fx+e)}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)}+\frac{4\,(7\,ab^3+6\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}-\frac{(3\,a^2-8\,ab+24\,b^2)(fx+e)}{a^4}-\frac{3\,a\tan(fx+e)^3-8\,b\tan(fx+e)^3+5\,a\tan(fx+e)-8\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7*
a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24*b^
2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f*x +
 e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B]
time = 11.91, size = 2880, normalized size = 14.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)

[Out] (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 +
121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((
6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2
)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(
512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7
*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*
8i + b^2*24i)*1i)/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
 a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b
^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(a^2*3i
 - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11
*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(1
6*a^4))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(16*a^4))/(((135*a*b^9)/4 + 27*b^10
 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(
2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a
^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b +
 a^8 + a^6*b^2)) - (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b
^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i
 - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11
*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(1
6*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8
 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b
^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^
10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (t
an(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*
a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*
b*8i + b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4)))*(a^2*3i
- a*b*8i + b^2*24i)*1i)/(8*a^4*f) - ((tan(e + f*x)^3*(16*a*b^2 - 3*a^2*b -
3*a^3 + 24*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)*(11*a*b^2 - 2*a^2*b - 5*a^
3 + 12*b^3))/(8*a^3*(a + b)) + (tan(e + f*x)^5*(5*a*b^2 - 3*a^2*b + 12*b^3)
)/(8*a^3*(a + b)))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan(e + f*x)^
6 + tan(e + f*x)^4*(a + 3*b))) + (atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*
b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32
*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9
*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11
+ a^9*b^2) - (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5
+ 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b
^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7
+ a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b
 + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 8
00*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7
*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2
 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b
```

```
^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*
a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*
a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b
^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2)*(7*a + 6*b)*1i)/(4*(3*a^6*b + a^7
+ a^4*b^3 + 3*a^5*b^2)))/(((135*a*b^9)/4 + 27*b^10 - (9*a^2*b^8)/2 - (149*a
^3*b^7)/32 + (219*a^4*b^6)/64 - (63*a^5*b^5)/64)/(2*a^10*b + a^11 + a^9*b^2
) - (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121
*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - ((-b^5
*(a + b)^3)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3
)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(-b^5*(a
+ b)^3)^(1/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256
*a^11*b^2))/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5
*b^2)))*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a +
b)^3)^(1/2)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)) + (((tan
(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 -
 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + ((-b^5*(a + b)^3
)^(1/2)*((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a
^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*x)*(-b^5*(a + b)^3)^(1
/2)*(7*a + 6*b)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2)
)/(128*(2*a^7*b + a^8 + a^6*b^2)*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(7
*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2)))*(-b^5*(a + b)^3)^(1/2
)*(7*a + 6*b))/(4*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))))*(-b^5*(a + b)^3)
^(1/2)*(7*a + 6*b)*1i)/(2*f*(3*a^6*b + a^7 + a^4*b^3 + 3*a^5*b^2))
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=278

$$\frac{(5a^3-12a^2b+24ab^2-64b^3)\,x}{16a^5}+\frac{b^{7/2}(9a+8b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f}+\frac{(15a^2-26ab+48b^2)\cos(e+fx)\,\mathrm{s}}{48a^3f\,(a+b+b\tan^2(e+f}$$

```
[Out] 1/16*(5*a^3-12*a^2*b+24*a*b^2-64*b^3)*x/a^5+1/2*b^(7/2)*(9*a+8*b)*arctan(b^
(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(3/2)/f+1/48*(15*a^2-26*a*b+48*b^2)
*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(5*a-8*b)*cos(f*x+e)
^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)+1/16*b*(5*a^3-7*a^2*b+12*a*b^2+32*b^3)*tan(f*x+e)/a^4/(a
+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.25, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{b^{7/2}(9a+8b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}}+\frac{(5a-8b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\,(a+b\tan^2(e+fx)+b)}+\frac{(15a^2-26ab+48b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\,(a+b\tan^2(e+fx)+b)}+\frac{x(5a^3-12a^2b+24ab^2-64b^3)}{16a^5}+\frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4f(a+b)\,(a+b\tan^2(e+fx)+b)}+\frac{\sin(e+fx)\cos^5(e+fx)}{6af\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 - 12*a^2*b + 24*a*b^2 - 64*b^3)*x)/(16*a^5) + (b^(7/2)*(9*a + 8*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*(a + b)^(3/2)*f) + ((15*
a^2 - 26*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[
e + f*x]^2)) + ((5*a - 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b +
 b*Tan[e + f*x]^2)) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e
 + f*x]^2)) + (b*(5*a^3 - 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(16*a^
4*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-5a+b-7bx^2}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}(}{}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+\right.}$$

$$= \frac{(5a^3-12a^2b+24ab^2-64b^3)\,x}{16a^5} + \frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} + (15$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.88, size = 499, normalized size = 1.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*a
*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*ArcTa
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e]
)^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[6*e
])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2*(e + f*x
)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f
*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4*e]*(a + 2*
b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]

time = 0.19, size = 210, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\left(\tan^5(fx+e)\right)+\left(3ab^2+\frac{5}{6}a^3-2\right.}{\left(\tan^2(f\right.}}{f}$
default	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\left(\tan^5(fx+e)\right)+\left(3ab^2+\frac{5}{6}a^3-2\right.}{\left(\tan^2(f\right.}}{f}$
risch	$\frac{5x}{16a^2}-\frac{3xb}{4a^3}+\frac{3x\,b^2}{2a^4}-\frac{4x\,b^3}{a^5}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^3f}+\frac{i\mathrm{e}^{4i(fx+e)}b}{32a^3f}+\frac{3i\mathrm{e}^{-2i(fx+e)}b^2}{8a^4f}-\frac{3i\mathrm{e}^{4i(fx+e)}}{128a^2f}-\frac{15i\mathrm{e}^{2i(fx+e)}}{128a^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^5*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+8*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((5/16*a^3-
3/4*a^2*b+3/2*a*b^2)*tan(f*x+e)^5+(3*a*b^2+5/6*a^3-2*a^2*b)*tan(f*x+e)^3+(-
5/4*a^2*b+3/2*a*b^2+11/16*a^3)*tan(f*x+e))/(tan(f*x+e)^2+1)^3+1/16*(5*a^3-1
2*a^2*b+24*a*b^2-64*b^3)*arctan(tan(f*x+e))))
```

Maxima [A]

time = 0.49, size = 379, normalized size = 1.36

$$\frac{\frac{24\,(9\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+a^5b)\sqrt{(a+b)b}}+\frac{3\,(5\,a^3b-7\,a^2b^2+12\,ab^3+32\,b^4)\tan(fx+e)^7+(15\,a^4+34\,a^3b-41\,a^2b^2+156\,ab^3+288\,b^4)\tan(fx+e)^5+(40\,a^4+17\,a^3b-35\,a^2b^2+204\,ab^3+288\,b^4)\tan(fx+e)^3+3\,(11\,a^4+2\,a^3b-5\,a^2b^2+28\,ab^3+32\,b^4)\tan(fx+e)}{(a^5b+a^4b^2)\tan(fx+e)^8+(a^6+5\,a^5b+4\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b^2+3\,(a^6+3\,a^5b+2\,a^4b^2)\tan(fx+e)^4+(3\,a^6+7\,a^5b+4\,a^4b^2)\tan(fx+e)^2}+\frac{3\,(5\,a^3-12\,a^2b+24\,ab^2-64\,b^3)(fx+e)}{a^5}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(24*(9*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + a
^5*b)*sqrt((a + b)*b)) + (3*(5*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 32*b^4)*tan(f
*x + e)^7 + (15*a^4 + 34*a^3*b - 41*a^2*b^2 + 156*a*b^3 + 288*b^4)*tan(f*x
+ e)^5 + (40*a^4 + 17*a^3*b - 35*a^2*b^2 + 204*a*b^3 + 288*b^4)*tan(f*x + e
)^3 + 3*(11*a^4 + 2*a^3*b - 5*a^2*b^2 + 28*a*b^3 + 32*b^4)*tan(f*x + e))/((
a^5*b + a^4*b^2)*tan(f*x + e)^8 + (a^6 + 5*a^5*b + 4*a^4*b^2)*tan(f*x + e)^
6 + a^6 + 2*a^5*b + a^4*b^2 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x + e)^4
```

```
+ (3*a^6 + 7*a^5*b + 4*a^4*b^2)*tan(f*x + e)^2) + 3*(5*a^3 - 12*a^2*b + 24*
a*b^2 - 64*b^3)*(f*x + e)/a^5)/f
```

Fricas [A]
time = 4.51, size = 815, normalized size = 2.93

$$\left[\frac{3\left(5a^{5}-7a^{4}b+12a^{3}b^{2}-40a^{2}b^{3}-64ab^{4}\right)fx\cos\left(fx+e\right)^{2}+3\left(5a^{4}b-7a^{3}b^{2}+12a^{2}b^{3}-40ab^{4}-64b^{5}\right)fx+6\left(9ab^{4}+8b^{5}+\left(9a^{2}b^{3}+8ab^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3ab+4b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a^{2}+3ab+2b^{2}\right)\cos\left(fx+e\right)^{3}-\left(ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^{2}}{a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}}\right)+\left(8\left(a^{5}+a^{4}b\right)\cos\left(fx+e\right)^{7}+2\left(5a^{5}-3a^{4}b-8a^{3}b^{2}\right)\cos\left(fx+e\right)^{5}+\left(15a^{5}-11a^{4}b+22a^{3}b^{2}+48a^{2}b^{3}\right)\cos\left(fx+e\right)^{3}+3\left(5a^{4}b-7a^{3}b^{2}+12a^{2}b^{3}+32ab^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\left(\left(a^{7}+a^{6}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{6}b+a^{5}b^{2}\right)f\right)}, \frac{3\left(5a^{5}-7a^{4}b+12a^{3}b^{2}-40a^{2}b^{3}-64ab^{4}\right)fx\cos\left(fx+e\right)^{2}+3\left(5a^{4}b-7a^{3}b^{2}+12a^{2}b^{3}-40ab^{4}-64b^{5}\right)fx-12\left(9ab^{4}+8b^{5}+\left(9a^{2}b^{3}+8ab^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left(\left(a+2b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{b}{a+b}}}{2b\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+\left(8\left(a^{5}+a^{4}b\right)\cos\left(fx+e\right)^{7}+2\left(5a^{5}-3a^{4}b-8a^{3}b^{2}\right)\cos\left(fx+e\right)^{5}+\left(15a^{5}-11a^{4}b+22a^{3}b^{2}+48a^{2}b^{3}\right)\cos\left(fx+e\right)^{3}+3\left(5a^{4}b-7a^{3}b^{2}+12a^{2}b^{3}+32ab^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\left(\left(a^{7}+a^{6}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{6}b+a^{5}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
 + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x + 6*
(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt
(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2
)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x +
e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*
x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1/48*(3*(5*
a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x + e)^2 + 3*
(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12*(9*a*b^4 +
8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))
 + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2)*cos(f*
x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x + e)^3 + 3
*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*x + e))/
((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 270, normalized size = 0.97

$$\frac{\frac{24\,b^{4}\tan\left(fx+e\right)}{\left(a^{5}+a^{4}b\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)}+\frac{24\left(9\,ab^{4}+8\,b^{5}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{6}+a^{5}b\right)\sqrt{ab+b^{2}}}+\frac{3\left(5\,a^{3}-12\,a^{2}b+24\,ab^{2}-64\,b^{3}\right)\left(fx+e\right)}{a^{5}}+\frac{15\,a^{2}\tan\left(fx+e\right)^{5}-36\,ab\tan\left(fx+e\right)^{5}+72\,b^{2}\tan\left(fx+e\right)^{5}+40\,a^{2}\tan\left(fx+e\right)^{3}-96\,ab\tan\left(fx+e\right)^{3}+144\,b^{2}\tan\left(fx+e\right)^{3}+33\,a^{2}\tan\left(fx+e\right)-60\,ab\tan\left(fx+e\right)+72\,b^{2}\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}a^{4}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*(
9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2*b
+ 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*tan(f*
x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(f*x +
 e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x + e)
+ 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B]
time = 8.56, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)*(28*a*b^3 + 2*a^3*b + 11*a^4 + 32*b^4 - 5*a^2*b^2))/(16*a^4*
(a + b)) + (tan(e + f*x)^5*(156*a*b^3 + 34*a^3*b + 15*a^4 + 288*b^4 - 41*a^
2*b^2))/(48*a^4*(a + b)) + (tan(e + f*x)^3*(204*a*b^3 + 17*a^3*b + 40*a^4 +
 288*b^4 - 35*a^2*b^2))/(48*a^4*(a + b)) + (b*tan(e + f*x)^7*(12*a*b^2 - 7*
a^2*b + 5*a^3 + 32*b^3))/(16*a^4*(a + b)))/(f*(a + b + tan(e + f*x)^2*(3*a
+ 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)^6*(a
+ 4*b))) - (atan(-((((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^1
3*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) -
(tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5
120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 +
a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) - (tan(e +
f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 5
68*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 +
 a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5) - (((((
8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/
4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e + f*x)*(a*b^2*24i
 - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12
*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) + (tan(e + f*x)*(14336*a*b^10 + 819
2*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5
 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i -
 a^2*b*12i + a^3*5i - b^3*64i)*1i)/(32*a^5))/((72*a*b^12 + 64*b^13 - 11*a^2
*b^11 + (19*a^3*b^10)/8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^
7)/256 - (225*a^7*b^6)/256)/(2*a^13*b + a^14 + a^12*b^2) + (((((8*a^10*b^7
+ 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15
*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x)*(a*b^2*24i - a^2*b*12
i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024
*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i +
```

a^3*5i - b^3*64i))/(32*a^5) - (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 52
48*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b
^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i
+ a^3*5i - b^3*64i))/(32*a^5) + (((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^
5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 +
a^12*b^2) + (tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048
*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^
9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5
) + (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 6
4*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a
^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^
5)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(16*a^5*f) - (atan((((-
b^7*(a + b)^3)^(1/2)*((tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^
9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a
^8*b^3))/(128*(2*a^9*b + a^10 + a^8*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (
23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*
b + a^14 + a^12*b^2) - (tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(20
48*a^10*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b
+ a^10 + a^8*b^2)*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)
^(1/2)*(9*a + 8*b))/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*
1i)/(4*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-b^7*(a + b)^3)^(1/2)*((t
an(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*
b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b +
a^10 + a^8*b^2)) + (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13
*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (
tan(e + f*x)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*
b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^
7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b))/(4*(
3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(9*a + 8*b)*1i)/(4*(3*a^7*b + a^8 +
a^5*b^3 + 3*a^6*b^2)))/((72*a*b^12 + 64*b^13 - 11*a^2*b^11 + (19*a^3*b^10)/
8 + (267*a^4*b^9)/32 - (101*a^5*b^8)/16 + (655*a^6*b^7)/256 - (225*a^7*b^6)
/256)/(2*a^13*b + a^14 + a^12*b^2) - ((-b^7*(a + b)^3)^(1/2)*((tan(e + f*x)
*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 568*a
^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a^9*b + a^10 + a^8
*b^2)) - (((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4 - (
3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) - (tan(e + f*x
)*(-b^7*(a + b)^3)^(1/2)*(9*a + 8*b)*(2048*a^10*b^5 + 5120*a^11*b^4 + 4096*
a^12*b^3 + 1024*a^13*b^2))/(512*(2*a^9*b + a^10 + a^8*b^2)*(3*a^7*b + a^8 +
a^5*b^3 + 3*a^6*b^2)))*(-b^7*(a + b)^3)^(1/2)*...

3.206 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2}f}+\frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

```
[Out] 1/4*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+3/8*sin(f*x+e)/(a+b)^2/f/(a+b
-a*sin(f*x+e)^2)+3/8*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f/
a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 205, 214}

$$\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*f)
 + Sin[e + f*x]/(4*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*Sin[e + f*x
])/(8*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{3\sin(e+fx)}{8(a+b)^2 f\,(a+b-a\sin^2(e+fx))} + $$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2} f} + \frac{\sin(e+fx)}{4(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{}{8(a+b)^2 f}$$

Mathematica [A]
time = 0.58, size = 128, normalized size = 1.19

$$\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}} + \frac{4\sin(e+fx)(5(a+b)-3a\sin^2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}\right)}{64f\,(a+b\sec^2(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((3*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b)
 - 3*a*Sin[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(64*
f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.16, size = 108, normalized size = 1.00

method	result

derivativedivides	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{8(a+b)\sqrt{(a+b)\,a}}}{a+b}}{f}$
default	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(-a-b+a\left(\sin^2(fx+e)\right)\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{8(a+b)\sqrt{(a+b)\,a}}}{a+b}}{f}$
risch	$-\frac{i\left(3a\,e^{7i(fx+e)}+11a\,e^{5i(fx+e)}+20b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-20b\,e^{3i(fx+e)}-3a\,e^{i(fx+e)}\right)}{4(a+b)^2 f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}+\frac{3\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e}{\sqrt{a^2}}\right.}{16\sqrt{a^2+ab}\,(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)^2+3/4/(a+b)*(-1/2*sin(f*x+e
)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e
)/((a+b)*a)^(1/2))))
```

Maxima [A]
time = 0.49, size = 185, normalized size = 1.71

$$-\frac{\frac{2\left(3\,a\sin(fx+e)^3-5\,(a+b)\sin(fx+e)\right)}{(a^4+2\,a^3b+a^2b^2)\sin(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sin(fx+e)^2}+\frac{3\,\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a*sin(f*x + e)^3 - 5*(a + b)*sin(f*x + e))/((a^4 + 2*a^3*b + a^
2*b^2)*sin(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sin(f*x + e)^2) + 3*log((a*sin(f*x + e) - sq
rt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 +
2*a*b + b^2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(105) = 210.
time = 3.36, size = 488, normalized size = 4.52

$$\left[\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\left(2\,a^3+7\,a^2b+5\,ab^2+3\,(a^3+a^2b)\cos(fx+e)^2\right)\sin(fx+e)}{16\left((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cos(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(fx+e)^2+(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)f\right)},\ -\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)-\left(2\,a^3+7\,a^2b+5\,ab^2+3\,(a^3+a^2b)\cos(fx+e)^2\right)\sin(fx+e)}{8\left((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cos(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(fx+e)^2+(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f
*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)^2)*sin(
f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 + 2*(a^5*
b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2 + 3*a^3*b^
3 + 3*a^2*b^4 + a*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.47, size = 117, normalized size = 1.08

$$-\frac{\frac{3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+2ab+b^2)\sqrt{-a^2-ab}}+\frac{3a\sin(fx+e)^3-5a\sin(fx+e)-5b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)^2(a^2+2ab+b^2)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(-
a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/((
a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

Mupad [B]
time = 0.22, size = 113, normalized size = 1.05

$$\frac{\frac{5\sin(e+fx)}{8(a+b)}-\frac{3a\sin(e+fx)^3}{8(a+b)^2}}{f\left(2ab+a^2+b^2-\sin(e+fx)^2(2a^2+2ba)+a^2\sin(e+fx)^4\right)}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*sin(e + f*x))/(8*(a + b)) - (3*a*sin(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2) + a^2*sin(e + f*x)^4)) + (3
*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(8*a^(1/2)*f*(a + b)^(5/2))
```

3.207 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f}-\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx\right.}$$

```
[Out] 1/8*(4*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/f-1
/4*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+1/8*(4*a+b)*sin(f*x+e)/a/(
a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 393, 205, 214}

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}}+\frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*f) - (b*Sin[e + f*x])/(4*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) +
((4*a + b)*Sin[e + f*x])/(8*a*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{(a+b-ax^2)^2}\,dx, x, \sin(\right.}{4a(a+b)f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\,(a+b-a\sin^2(e+fx))}$$

$$= \frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+fx))^2} + \frac{}{8a(}$$

Mathematica [A]
time = 0.69, size = 163, normalized size = 1.30

$$-\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(\frac{8\sin(e+fx)}{(a+b-a\sin^2(e+fx))^2}-(4a+b)\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{4\sin(e+fx)(5(a+b)-3a\sin^2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}\right)\right)}{192af\,(a+b\sec^2(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/192*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/(
a + b - a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a*Si
n[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(a*f*(a + b*
Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.40, size = 124, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{(4a+b)\left(\sin^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8(a+b)a}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{(4a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{(a+b)\,a}}}{f}$
default	$\frac{\frac{-\frac{(4a+b)\left(\sin^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8(a+b)a}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{(4a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)\,a}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{(a+b)\,a}}}{f}$
risch	$-\frac{i(4a^2\mathrm{e}^{7i(fx+e)}+ab\,\mathrm{e}^{7i(fx+e)}+4a^2\mathrm{e}^{5i(fx+e)}+9ab\,\mathrm{e}^{5i(fx+e)}-4b^2\mathrm{e}^{5i(fx+e)}-4a^2\mathrm{e}^{3i(fx+e)}-9ab\,\mathrm{e}^{3i(fx+e)}+4b^2\mathrm{e}^{3i(fx+e}}{4a(a+b)^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/(a+b)/a*sin(f*x
+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.49, size = 218, normalized size = 1.74

$$-\frac{\frac{(4\,a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)a}}+\frac{2\left((4\,a^2+ab)\sin(fx+e)^3-(4\,a^2+3\,ab-b^2)\sin(fx+e)\right)}{a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+2\,a^4b+a^3b^2)\sin(fx+e)^4-2\,(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sin(fx+e)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((4*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + s
qrt((a + b)*a)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + 2*((4*a^2 + a*
b)*sin(f*x + e)^3 - (4*a^2 + 3*a*b - b^2)*sin(f*x + e))/(a^5 + 4*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 2*a^4*b + a^3*b^2)*sin(f*x + e)^4 - 2*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(122) = 244.
time = 3.68, size = 560, normalized size = 4.48

$$\left[\frac{\left((4\,a^3+a^2b)\cos(fx+e)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{a^2+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\left(2\,a^3b+a^2b^2-ab^3+(4\,a^4+5\,a^3b+a^2b^2)\cos(fx+e)^2\right)\sin(fx+e)}{16\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos(fx+e)^4+2\,(a^6b+3\,a^5b^2+3\,a^4b^3+a^3b^4)f\cos(fx+e)^2+(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)f\right)},-\frac{\left((4\,a^3+a^2b)\cos(fx+e)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)-\left(2\,a^3b+a^2b^2-ab^3+(4\,a^4+5\,a^3b+a^2b^2)\cos(fx+e)^2\right)\sin(fx+e)}{8\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos(fx+e)^4+2\,(a^6b+3\,a^5b^2+3\,a^4b^3+a^3b^4)f\cos(fx+e)^2+(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^2
)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b
)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2 -
a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3
*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a
^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2
*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*
b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f
*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 +
(a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.50, size = 155, normalized size = 1.24

$$-\frac{\frac{(4a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{-a^2-ab}}+\frac{4a^2\sin(fx+e)^3+ab\sin(fx+e)^3-4a^2\sin(fx+e)-3ab\sin(fx+e)+b^2\sin(fx+e)}{(a^3+2a^2b+ab^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b + a
*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*a^
2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b + a
*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B]
time = 4.68, size = 129, normalized size = 1.03

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)(4\,a+b)}{8\,a^{3/2}\,f\,(a+b)^{5/2}}-\frac{\frac{\sin(e+f\,x)^3\,(4\,a+b)}{8\,(a+b)^2}-\frac{\sin(e+f\,x)\,(4\,a-b)}{8\,a\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\,\sin\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + b))/(8*a^(3/2)*f*(a + b
)^(5/2)) - ((sin(e + f*x)^3*(4*a + b))/(8*(a + b)^2) - (sin(e + f*x)*(4*a -
 b))/(8*a*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2)
+ a^2*sin(e + f*x)^4))
```

3.208 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=144

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f}-\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}-\frac{3b(2a+b)\sin(e+\ldots}{8a^2(a+b)^2f\left(a+b-a\sin\ldots\right.}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+
b)^(5/2)/f-1/4*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-3
/8*b*(2*a+b)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 424, 393, 214}

$$-\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}}-\frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*f) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(4*a*(a + b)*f*(a
+ b - a*Sin[e + f*x]^2)^2) - (3*b*(2*a + b)*Sin[e + f*x])/(8*a^2*(a + b)^2*
f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{-4a-b+(4a+3b)x^2}{(a+b-ax^2)^2}\,dx, x, \sin(e\right.}{4a(a+b)f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+fx))^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\,(a+b-a\sin^2(e+fx}$$

$$= \frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\,(a+b-a\sin^2(e+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.49, size = 2256, normalized size = 15.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((
(I/128)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] +
a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Si
n[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]
*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
```

+ f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Co
s[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e]
- I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(5/2)*Sqrt[a + b]*
f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*
Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e -
f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt
[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*S
qrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I
*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e]
+ a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*
a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin
[e])/(128*a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(
a + b*Sec[e + f*x]^2)^3) + ((-8*a^2 - 8*a*b - 3*b^2)*(a + 2*b + a*Cos[2*e +
2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (
2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]
] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a
+ b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(5/2)*Sqr
t[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Sin[e
])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqr
t[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e]
- I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*S
in[e])/(a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a
+ b*Sec[e + f*x]^2)^3) + ((8*a^2 + 8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*e + 2*
f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos
[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]
*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e]
- I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] -
I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2
*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin
[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin
[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((-8*a^2 - 8*a*b - 3*b^2)*
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[-a - 2*a*Cos[2
*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*
e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]
*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(5/2)*Sqrt
[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[-a - 2*a*Cos[2*e] - 2
*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a
+ b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(5/2)*Sqrt[a +
b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) +

```
((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(-8*a*b*Sin[e + f*x] - 5*
b^2*Sin[e + f*x]))/(32*a^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (b^2*(a
+ 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f*x])/(8*a^2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.32, size = 142, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{-\frac{b(8a+5b)\left(\sin^3(fx+e)\right)}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{(a+b)a}}}{f}$
default	$\frac{-\frac{-\frac{b(8a+5b)\left(\sin^3(fx+e)\right)}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{(a+b)a}}}{f}$
risch	$\frac{ib(8a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+8a^2e^{5i(fx+e)}+29ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}-8a^2e^{3i(fx+e)}-29ab\,e^{3i(fx+e)}-12b^2e^{3i(}}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b/
(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*
b+b^2)/a^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [A]
time = 0.49, size = 239, normalized size = 1.66

$$-\frac{\frac{(8a^2+8ab+3b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)a}}-\frac{2\left((8a^2b+5ab^2)\sin(fx+e)^3-(8a^2b+11ab^2+3b^3)\sin(fx+e)\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+(a^6+2a^5b+a^4b^2)\sin(fx+e)^4-2(a^6+3a^5b+3a^4b^2+a^3b^3)\sin(fx+e)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2 + 8*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*si
n(f*x + e) + sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a))
- 2*((8*a^2*b + 5*a*b^2)*sin(f*x + e)^3 - (8*a^2*b + 11*a*b^2 + 3*b^3)*sin(
f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6 + 2*a^5*b
 + a^4*b^2)*sin(f*x + e)^4 - 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*
x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(142) = 284$.
time = 3.89, size = 629, normalized size = 4.37

$$\left[\frac{\left(\left(8\,a^4+8\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\left(8\,a^3b+8\,a^2b^2+3\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-2\left(6\,a^3b^2+9\,a^2b^3+3\,ab^4+\left(8\,a^4b+13\,a^3b^2+5\,a^2b^3\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{16\left(\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3\right)f\cos\left(fx+e\right)^4+2\left(a^7b+3\,a^6b^2+3\,a^5b^3+a^4b^4\right)f\cos\left(fx+e\right)^2+\left(a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5\right)f\right)},-\frac{\left(\left(8\,a^4+8\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\left(8\,a^3b+8\,a^2b^2+3\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)+\left(6\,a^3b^2+9\,a^2b^3+3\,ab^4+\left(8\,a^4b+13\,a^3b^2+5\,a^2b^3\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{8\left(\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3\right)f\cos\left(fx+e\right)^4+2\left(a^7b+3\,a^6b^2+3\,a^5b^3+a^4b^4\right)f\cos\left(fx+e\right)^2+\left(a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3
+ 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*b)
*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(
f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a^3*b
^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*b^2 +
 a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*
cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/8*(((8*
a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3*b^4 + 2
*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sq
rt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (
8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5
*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b
^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.49, size = 178, normalized size = 1.24

$$-\frac{\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{-a^2-ab}}-\frac{8\,a^2b\sin(fx+e)^3+5\,ab^2\sin(fx+e)^3-8\,a^2b\sin(fx+e)-11\,ab^2\sin(fx+e)-3\,b^3\sin(fx+e)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4
 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b^2
*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*sin(
f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B]
time = 0.25, size = 149, normalized size = 1.03

$$\frac{\frac{\sin(e+f\,x)^3\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,{\left(a+b\right)}^2}-\frac{\sin(e+f\,x)\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,\left(a+b\right)}}{f\,\left(2\,a\,b+a^2+b^2-{\sin\left(e+f\,x\right)}^2\,\left(2\,a^2+2\,b\,a\right)+a^2\,{\sin\left(e+f\,x\right)}^4\right)}+\frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,f\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((sin(e + f*x)^3*(8*a*b + 5*b^2))/(8*a*(a + b)^2) - (sin(e + f*x)*(8*a*b +
3*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*
a^2) + a^2*sin(e + f*x)^4)) + (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*
(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*f*(a + b)^(5/2))
```

3.209 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f}+\frac{\sin(e+fx)}{a^3f}-\frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{}{8a}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/f+sin(f*x+e)/a^3/f-1/4*b^3*sin(f*x+e)/a^3/(a+b)/f/(a+b-a*sin(
f*x+e)^2)^2+3/8*b^2*(4*a+3*b)*sin(f*x+e)/a^3/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4232, 398, 1171, 393, 214}

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}f(a+b)^{5/2}}-\frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*f) + Sin[e + f*x]/(a^3*f) - (b^3*Sin[e + f*x])
/(4*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e
+ f*x])/(8*a^3*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{b(3a^2+3ab+b^2)-3ab(2a+b)x^2+3a^2bx^4}{a^3(a+b-ax^2)^3}\right)\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)-3ab(2a+b)x^2+3a^2bx^4}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{a^3f}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2+12}{(a+b-ax}\right.}{4a^3}$$

$$= \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3b^2(4a+3b)\sin}{8a^3(a+b)^2f\left(a+b-\right.}$$

$$= -\frac{3b(4(a+b)^2+(2a+b)^2)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3f} - \frac{}{4a^3(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.70, size = 2382, normalized size = 15.27

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]

[Out] (Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[e])/(8*a^3*f*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((((-3*I)/128)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (3*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (((3*I)/128)*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Sin[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (((3*I)/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Si

```
n[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*
e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a
 + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b
+ a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*Cos[e]*Log[-a - 2*a*Cos[2*e] - 2
*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a
 + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (((3*I)/256)*Log[-a - 2*a*Cos[2*e] - 2*b*
Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqr
t[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a +
b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + (C
os[e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(8*a^3*f*(a
 + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^
6*(4*a*b^2*Sin[e + f*x] + 3*b^3*Sin[e + f*x]))/(32*a^3*(a + b)^2*f*(a + b*S
ec[e + f*x]^2)^3) - (b^3*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[
e + f*x])/(8*a^3*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.34, size = 149, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\left(\sin^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}-\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)a}}\right)}{a^3}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\left(\sin^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(-a-b+a\left(\sin^2(fx+e)\right)\right)^2}-\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{(a+b)a}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)a}}\right)}{a^3}}{f}$
risch	$-\frac{i e^{i(fx+e)}}{2f a^3}+\frac{i e^{-i(fx+e)}}{2f a^3}-\frac{i b^2\left(12a^2 e^{7i(fx+e)}+9ab e^{7i(fx+e)}+12a^2 e^{5i(fx+e)}+49ab e^{5i(fx+e)}+28b^2 e^{5i(fx+e)}-12a\right)}{4a^3(a+b)^2 f\left(a e^{4i(fx+e)}+2a e^{2i(fx+}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*sin(f*x+e)+1/a^3*b*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+
e)^3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-3/8*(8*a^2+
12*a*b+5*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a
)^(1/2))))
```

Maxima [A]
time = 0.48, size = 260, normalized size = 1.67

$$\frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)a}}-\frac{2\left(3\,(4\,a^2b^2+3\,ab^3)\sin(fx+e)^3-(12\,a^2b^2+19\,ab^3+7\,b^4)\sin(fx+e)\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sin(fx+e)^2}+\frac{16\,\sin(fx+e)}{a^3}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))
/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b
)*a)) - 2*(3*(4*a^2*b^2 + 3*a*b^3)*sin(f*x + e)^3 - (12*a^2*b^2 + 19*a*b^3
+ 7*b^4)*sin(f*x + e))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (
a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*sin(f*x + e)^2) + 16*sin(f*x + e)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(154) = 308.
time = 3.29, size = 745, normalized size = 4.78

$$\left[\frac{3\,(8\,a^2b^3+12\,ab^4+5\,b^5+(8\,a^4b+12\,a^3b^2+5\,a^2b^3)\cos(fx+e)^4+2\,(8\,a^3b^2+12\,a^2b^3+5\,ab^4)\cos(fx+e)^2)\sqrt{a^2+ab}\,\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\,\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(8\,a^4b^2+34\,a^3b^3+41\,a^2b^4+15\,ab^5+8\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\cos(fx+e)^4+(16\,a^5b+60\,a^4b^2+69\,a^3b^3+25\,a^2b^4)\cos(fx+e)^2)\sin(fx+e)}{16\,((a^9+3\,a^8b+3\,a^7b^2+a^6b^3)f\cos(fx+e)^4+2\,(a^8b+3\,a^7b^2+3\,a^6b^3+a^5b^4)f\cos(fx+e)^2+(a^7b^2+3\,a^6b^3+3\,a^5b^4+a^4b^5)f)},\ \frac{3\,(8\,a^2b^3+12\,ab^4+5\,b^5+(8\,a^4b+12\,a^3b^2+5\,a^2b^3)\cos(fx+e)^4+2\,(8\,a^3b^2+12\,a^2b^3+5\,ab^4)\cos(fx+e)^2)\sqrt{-a^2-ab}\,\arctan\left(\frac{\sqrt{-a^2-ab}\,\sin(fx+e)}{a+b}\right)+(8\,a^4b^2+34\,a^3b^3+41\,a^2b^4+15\,ab^5+8\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\cos(fx+e)^4+(16\,a^5b+60\,a^4b^2+69\,a^3b^3+25\,a^2b^4)\cos(fx+e)^2)\sin(fx+e)}{8\,((a^9+3\,a^8b+3\,a^7b^2+a^6b^3)f\cos(fx+e)^4+2\,(a^8b+3\,a^7b^2+3\,a^6b^3+a^5b^4)f\cos(fx+e)^2+(a^7b^2+3\,a^6b^3+3\,a^5b^4+a^4b^5)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)
*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sqrt
(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a -
 b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a
*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b +
 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 +
3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 +
a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 +
 5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x +
 e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (8*
a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b
^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)
^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 197, normalized size = 1.26

$$\frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{-a^2-ab}}-\frac{12\,a^2b^2\sin(fx+e)^3+9\,ab^3\sin(fx+e)^3-12\,a^2b^2\sin(fx+e)-19\,ab^3\sin(fx+e)-7\,b^4\sin(fx+e)}{(a^5+2\,a^4b+a^3b^2)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{8\,\sin(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^3
+ 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x + e)
- 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a - b)
^2) + 8*sin(f*x + e)/a^3)/f
```

Mupad [B]
time = 4.74, size = 175, normalized size = 1.12

$$\frac{\sin\left(e+f\,x\right)}{a^3\,f}+\frac{\frac{\sin(e+f\,x)\,\left(7\,b^3+12\,a\,b^2\right)}{8\,(a+b)}-\frac{3\sin(e+f\,x)^3\,\left(4\,a^2\,b^2+3\,a\,b^3\right)}{8\,(a+b)^2}}{f\,\left(2\,a^4\,b-\sin\left(e+f\,x\right)^2\,\left(2\,a^5+2\,b\,a^4\right)+a^5+a^3\,b^2+a^5\sin\left(e+f\,x\right)^4\right)}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{8\,a^{7/2}\,f\,\left(a+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] sin(e + f*x)/(a^3*f) + ((sin(e + f*x)*(12*a*b^2 + 7*b^3))/(8*(a + b)) - (3*
sin(e + f*x)^3*(3*a*b^3 + 4*a^2*b^2))/(8*(a + b)^2))/(f*(2*a^4*b - sin(e +
f*x)^2*(2*a^4*b + 2*a^5) + a^5 + a^3*b^2 + a^5*sin(e + f*x)^4)) - (3*b*atan
h((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2
)*f*(a + b)^(5/2))
```

3.210 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{b^2(48a^2+80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+}{4a^4(a+b)f\,(a+b-a}$$

```
[Out] 1/8*b^2*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9
/2)/(a+b)^(5/2)/f+(a-3*b)*sin(f*x+e)/a^4/f-1/3*sin(f*x+e)^3/a^3/f+1/4*b^4*s
in(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b^3*(16*a+13*b)*sin(f*x+e)
/a^4/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]

time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 398, 1171, 393, 214}

$$\frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^2(48a^2+80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(8*a^(9/2)*(a + b)^(5/2)*f) + ((a - 3*b)*Sin[e + f*x])/(a^4*f) - Sin[e +
f*x]^3/(3*a^3*f) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*f*(a + b - a*Sin[e + f
*x]^2)^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*f*(a + b - a*
Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{a^4}-\frac{x^2}{a^3}+\frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{a^4\left(a+b-ax^2\right)^3}\right)\,dx, x, \sin(e+fx)\right.}{f}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{\text{Subst}\left(\int \frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6}{\left(a+b-ax^2\right)^3}\right.}{a^4f}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\
&= \frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\
&= \frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin}{}\end{aligned}$$

Mathematica [A]

time = 4.85, size = 194, normalized size = 1.07

$$-\frac{-\frac{3b^2(48a^2+80ab+35b^2)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}}+12\sqrt{a}\left(-12b-\frac{b^4(9a+22b+13a\cos(2(e+fx)))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}+a\left(3-\frac{16b^3}{(a+b)^2(a+2b+a\cos(2(e+fx)))}\right)\right)\sin(e+fx)+4a^{3/2}\sin(3(e+fx))}{48a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]
] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*(-
12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)]))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

Maple [A]

time = 0.44, size = 177, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\frac{a(\sin^3(fx+e))}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)(\sin^3(fx+e))}{8(a^2+2ab+b^2)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{(-a-b+a(\sin^2(fx+e)))^2}-\frac{(48a^2+80ab+35b^2)\operatorname{arctan}}{8(a^2+2ab+b^2)\sqrt{}}\right.}{a^4}}{f}$
default	$\frac{-\frac{\frac{a(\sin^3(fx+e))}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)(\sin^3(fx+e))}{8(a^2+2ab+b^2)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{(-a-b+a(\sin^2(fx+e)))^2}-\frac{(48a^2+80ab+35b^2)\operatorname{arctan}}{8(a^2+2ab+b^2)\sqrt{}}\right.}{a^4}}{f}$
risch	$-\frac{ie^{3i(fx+e)}}{24f\,a^3}-\frac{3ie^{i(fx+e)}}{8f\,a^3}+\frac{3ie^{i(fx+e)}b}{2a^4f}+\frac{3ie^{-i(fx+e)}}{8f\,a^3}-\frac{3ie^{-i(fx+e)}b}{2a^4f}+\frac{ie^{-3i(fx+e)}}{24f\,a^3}+\frac{ib^3(16a^2e^{7i(fx+e)}+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+3*sin(f*x+e)*b)-b^2/a^4*((-1/8
*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin(f
*x+e))/(-a-b+a*sin(f*x+e)^2)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2)/(
(a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [A]

time = 0.49, size = 280, normalized size = 1.55

$$-\frac{\frac{3\,(48\,a^2b^2+80\,ab^3+35\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{(a+b)a}}-\frac{6\left((16\,a^2b^3+13\,ab^4)\sin(fx+e)^3-(16\,a^2b^3+27\,ab^4+11\,b^5)\sin(fx+e)\right)}{a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+(a^8+2\,a^7b+a^6b^2)\sin(fx+e)^4-2\,(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)\sin(fx+e)^2}+\frac{16\left(a\sin(fx+e)^3-3\,(a-3\,b)\sin(fx+e)\right)}{a^4}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*log((a*sin(f*x + e) - sqrt((a + b
)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((
a + b)*a)) - 6*((16*a^2*b^3 + 13*a*b^4)*sin(f*x + e)^3 - (16*a^2*b^3 + 27*a
*b^4 + 11*b^5)*sin(f*x + e))/(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b
^4 + (a^8 + 2*a^7*b + a^6*b^2)*sin(f*x + e)^4 - 2*(a^8 + 3*a^7*b + 3*a^6*b^
2 + a^5*b^3)*sin(f*x + e)^2) + 16*(a*sin(f*x + e)^3 - 3*(a - 3*b)*sin(f*x +
 e))/a^4)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(178) = 356$.
time = 2.69, size = 876, normalized size = 4.84

$$\left[\frac{3\left(48a^2b^4+80ab^5+35b^6+\left(48a^4b^2+80a^3b^3+35a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48a^3b^3+80a^2b^4+35ab^5\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(16a^5b^2-24a^4b^3-210a^3b^4-275a^2b^5-105ab^6+8\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\cos\left(fx+e\right)^6+8\left(2a^7-a^6b-15a^5b^2-19a^4b^3-7a^3b^4\right)\cos\left(fx+e\right)^4+\left(32a^6b-40a^5b^2-360a^4b^3-463a^3b^4-175a^2b^5\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{48\left(\left(a^{10}+3a^9b+3a^8b^2+a^7b^3\right)f\cos\left(fx+e\right)^4+2\left(a^9b+3a^8b^2+3a^7b^3+a^6b^4\right)f\cos\left(fx+e\right)^2+\left(a^8b^2+3a^7b^3+3a^6b^4+a^5b^5\right)f\right)}, -\frac{3\left(48a^2b^4+80ab^5+35b^6+\left(48a^4b^2+80a^3b^3+35a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48a^3b^3+80a^2b^4+35ab^5\right)\cos\left(fx+e\right)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin\left(fx+e\right)}{a+b}\right)-\left(16a^5b^2-24a^4b^3-210a^3b^4-275a^2b^5-105ab^6+8\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\cos\left(fx+e\right)^6+8\left(2a^7-a^6b-15a^5b^2-19a^4b^3-7a^3b^4\right)\cos\left(fx+e\right)^4+\left(32a^6b-40a^5b^2-360a^4b^3-463a^3b^4-175a^2b^5\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{24\left(\left(a^{10}+3a^9b+3a^8b^2+a^7b^3\right)f\cos\left(fx+e\right)^4+2\left(a^9b+3a^8b^2+3a^7b^3+a^6b^4\right)f\cos\left(fx+e\right)^2+\left(a^8b^2+3a^7b^3+3a^6b^4+a^5b^5\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^
2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)
^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e)
 - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*a^3*
b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos
(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)*cos(f
*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175*a^2*b^
5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*f*
cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f*x + e)
^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48*a^2*b^4 +
 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4
+ 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*a
rctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 24*a^4*b^3 - 2
10*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4
)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*
b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f
*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 229, normalized size = 1.27

$$-\frac{\frac{3\,(48\,a^2b^2+80\,ab^3+35\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{-a^2-ab}}-\frac{3\left(16\,a^2b^3\sin(fx+e)^3+13\,ab^4\sin(fx+e)^3-16\,a^2b^3\sin(fx+e)-27\,ab^4\sin(fx+e)-11\,b^5\sin(fx+e)\right)}{(a^6+2\,a^5b+a^4b^2)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{8\left(a^6\sin(fx+e)^3-3\,a^6\sin(fx+e)+9\,a^5b\sin(fx+e)\right)}{a^9}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 -
 a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f*x
 + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*sin(
f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x + e)^
2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*sin(f*
x + e))/a^9)/f
```

Mupad [B]
time = 0.39, size = 256, normalized size = 1.41

$$\frac{b^2\ln\left(\sqrt{a+b}+\sqrt{a}\,\sin\left(e+f\,x\right)\right)\,\left(3\,a^2+5\,a\,b+\frac{35\,b^2}{16}\right)}{a^{9/2}\,f\,{\left(a+b\right)}^{5/2}}-\frac{\frac{\sin(e+f\,x)\,\left(11\,b^4+16\,a\,b^3\right)}{8\,(a+b)}-\frac{\sin(e+f\,x)^3\,\left(16\,a^2\,b^3+13\,a\,b^4\right)}{8\,{(a+b)}^2}}{f\,\left(2\,a^5\,b-{\sin\left(e+f\,x\right)}^2\,\left(2\,a^6+2\,b\,a^5\right)+a^6+a^4\,b^2+a^6\,{\sin\left(e+f\,x\right)}^4\right)}-\frac{{\sin\left(e+f\,x\right)}^3}{3\,a^3\,f}-\frac{b^2\ln\left(\sqrt{a}\,\sin\left(e+f\,x\right)-\sqrt{a+b}\right)\,\left(48\,a^2+80\,a\,b+35\,b^2\right)}{16\,a^{9/2}\,f\,{\left(a+b\right)}^{5/2}}-\frac{\sin\left(e+f\,x\right)\,\left(\frac{3\,(a+b)}{a^4}-\frac{4}{a^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (b^2*log((a + b)^(1/2) + a^(1/2)*sin(e + f*x))*(5*a*b + 3*a^2 + (35*b^2)/16
))/(a^(9/2)*f*(a + b)^(5/2)) - ((sin(e + f*x)*(16*a*b^3 + 11*b^4))/(8*(a +
b)) - (sin(e + f*x)^3*(13*a*b^4 + 16*a^2*b^3))/(8*(a + b)^2))/(f*(2*a^5*b -
 sin(e + f*x)^2*(2*a^5*b + 2*a^6) + a^6 + a^4*b^2 + a^6*sin(e + f*x)^4)) -
sin(e + f*x)^3/(3*a^3*f) - (b^2*log(a^(1/2)*sin(e + f*x) - (a + b)^(1/2))*(
80*a*b + 48*a^2 + 35*b^2))/(16*a^(9/2)*f*(a + b)^(5/2)) - (sin(e + f*x)*((3
*(a + b))/a^4 - 4/a^3))/f
```

3.211 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=214

$$-\frac{b^3(80a^2+140ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f}+\frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4f}+$$

```
[Out] -1/8*b^3*(80*a^2+140*a*b+63*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^
(11/2)/(a+b)^(5/2)/f+(a^2-3*a*b+6*b^2)*sin(f*x+e)/a^5/f-1/3*(2*a-3*b)*sin(f
*x+e)^3/a^4/f+1/5*sin(f*x+e)^5/a^3/f-1/4*b^5*sin(f*x+e)/a^5/(a+b)/f/(a+b-a*
sin(f*x+e)^2)^2+1/8*b^4*(20*a+17*b)*sin(f*x+e)/a^5/(a+b)^2/f/(a+b-a*sin(f*x
+e)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 398, 1171, 393, 214}

$$-\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4f}+\frac{\sin^5(e+fx)}{5a^3f}-\frac{b^3(80a^2+140ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}f(a+b)^{5/2}}+\frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
 + b]])/(a^(11/2)*(a + b)^(5/2)*f) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/(
a^5*f) - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4*f) + Sin[e + f*x]^5/(5*a^3*f)
- (b^5*Sin[e + f*x])/(4*a^5*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*
(20*a + 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^5}{(a+b-ax^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{a^5} - \frac{(2a-3b)x^2}{a^4} + \frac{x^4}{a^3} - \frac{b^3(10a^2+15ab+6b^2)-5ab^3(4a+3b)x^2+10a^2b^3x^4}{a^5(a+b-ax^2)^3}\right)\right.}{f}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5 f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4 f} + \frac{\sin^5(e+fx)}{5a^3 f} - \frac{\text{Subs}}{}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5 f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4 f} + \frac{\sin^5(e+fx)}{5a^3 f} - \frac{}{4a^5(a}$$

$$= \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5 f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4 f} + \frac{\sin^5(e+fx)}{5a^3 f} - \frac{}{4a^5(a}$$

$$= -\frac{b^3(80a^2+140ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2} f} + \frac{(a^2-3ab+6b^2)\sin(e+}{a^5 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.76, size = 2670, normalized size = 12.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*
a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128)*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
 Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
 I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[C
os[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e]
 + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sq
rt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e
] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[
2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e
]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos
[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*
e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(12
8*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*S
ec[e + f*x]^2)^3) + ((80*a^2*b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] -
 (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sq
rt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt
[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(11/2)*
Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Si
n[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*
Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2
*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*a*b^4 - 63*b^5)*(a + 2*b +
 a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*C
os[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt
```

```
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e]
 - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqr
t[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt
[Cos[2*e] - I*Sin[2*e]]))))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((80*a^2*
b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(Cos[e]*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a
*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[
2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e
+ f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256
)*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*
Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]))))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((5*a - 12*b)*Cos[3*f*x]*(a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*Sin[3*e])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) +
 (Cos[5*f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*e])/(640
*a^3*f*(a + b*Sec[e + f*x]^2)^3) + ((5*a^2 - 18*a*b + 48*b^2)*Cos[e]*(a + 2
*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(64*a^5*f*(a + b*Sec[e
+ f*x]^2)^3) + ((5*a - 12*b)*Cos[3*e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[
e + f*x]^6*Sin[3*f*x])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) + (Cos[5*e]*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*f*x])/(640*a^3*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(20*a
*b^4*Sin[e + f*x] + 17*b^5*Sin[e + f*x]))/(32*a^5*(a + b)^2*f*(a + b*Sec[e
+ f*x]^2)^3) - (b^5*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f
*x])/(8*a^5*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.28, size = 214, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+ab\left(\sin^3(fx+e)\right)+a^2\sin(fx+e)-3ab\sin(fx+e)+6b^2\sin(fx+e)}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\left(\sin^3(fx+e\right.}{8\left(a^2+2ab+b^2\right)}}{\left(-a-b+a\left(\sin\right.\right.}\right.}{f}$
default	$\frac{\frac{\left(\sin^5(fx+e)\right)a^2}{5}-\frac{2a^2\left(\sin^3(fx+e)\right)}{3}+ab\left(\sin^3(fx+e)\right)+a^2\sin(fx+e)-3ab\sin(fx+e)+6b^2\sin(fx+e)}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\left(\sin^3(fx+e\right.}{8\left(a^2+2ab+b^2\right)}}{\left(-a-b+a\left(\sin\right.\right.}\right.}{f}$

risch	$-\frac{ie^{5i(fx+e)}}{160a^3f}-\frac{5ie^{i(fx+e)}}{16f\,a^3}-\frac{5ie^{3i(fx+e)}}{96f\,a^3}+\frac{ie^{-5i(fx+e)}}{160a^3f}-\frac{3ie^{i(fx+e)}b^2}{f\,a^5}+\frac{5ie^{-3i(fx+e)}}{96f\,a^3}-\frac{9ie^{-i(fx+e)}b}{8a^4f}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a^5*(1/5*sin(f*x+e)^5*a^2-2/3*a^2*sin(f*x+e)^3+a*b*sin(f*x+e)^3+a^2*sin(f*x+e)-3*a*b*sin(f*x+e)+6*b^2*sin(f*x+e))+b^3/a^5*((-1/8*a*b*(20*a+17*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))

Maxima [A]
time = 0.49, size = 312, normalized size = 1.46

$$\frac{\frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)a}}-\frac{30\left((20\,a^2b^4+17\,ab^5)\sin(fx+e)^3-5\,(4\,a^2b^4+7\,ab^5+3\,b^6)\sin(fx+e)\right)}{a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4+(a^9+2\,a^8b+a^7b^2)\sin(fx+e)^4-2\,(a^9+3\,a^8b+3\,a^7b^2+a^6b^3)\sin(fx+e)^2}+\frac{16\left(3\,a^2\sin(fx+e)^5-5\,(2\,a^2-3\,ab)\sin(fx+e)^3+15\,(a^2-3\,ab+6\,b^2)\sin(fx+e)\right)}{a^5}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] 1/240*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*a)) - 30*((20*a^2*b^4 + 17*a*b^5)*sin(f*x + e)^3 - 5*(4*a^2*b^4 + 7*a*b^5 + 3*b^6)*sin(f*x + e))/(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4 + (a^9 + 2*a^8*b + a^7*b^2)*sin(f*x + e)^4 - 2*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*sin(f*x + e)^2) + 16*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 - 3*a*b)*sin(f*x + e)^3 + 15*(a^2 - 3*a*b + 6*b^2)*sin(f*x + e))/a^5)/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(210) = 420.
time = 2.56, size = 1017, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] [1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^

```
5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a
^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2
 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2
)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 +
 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 +
 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 140*a*b^6 + 6
3*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*
b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a^3*b^5 +
2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^5*b^3 - 9
*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a^5*b^3 +
167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2 + 360*a^
5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2)*sin(f*x
 + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 + 2*(a^10*
b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 + 3*a^8*b^
3 + 3*a^7*b^4 + a^6*b^5)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 271, normalized size = 1.27

$$\frac{\frac{15\left(80a^2b^3+140ab^4+63b^5\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^7+2a^6b+a^5b^2\right)\sqrt{-a^2-ab}}-\frac{15\left(20a^2b^4\sin(fx+e)^3+17ab^5\sin(fx+e)^3-20a^2b^4\sin(fx+e)-35ab^5\sin(fx+e)-15b^6\sin(fx+e)\right)}{\left(a^7+2a^6b+a^5b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{8\left(3a^{12}\sin(fx+e)^5-10a^{12}\sin(fx+e)^3+15a^{11}b\sin(fx+e)^3+15a^{12}\sin(fx+e)-45a^{11}b\sin(fx+e)+90a^{10}b^2\sin(fx+e)\right)}{a^{15}}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^2
 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*sin(
f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^5*s
in(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*x +
e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 + 15*a
^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e) + 90*a
^10*b^2*sin(f*x + e))/a^15)/f
```

Mupad [B]
time = 0.33, size = 257, normalized size = 1.20

$$\frac{\frac{5\sin(e+fx)\left(3b^5+4ab^4\right)}{8(a+b)}-\frac{\sin(e+fx)^3\left(20a^2b^4+17ab^5\right)}{8(a+b)^2}}{f\left(2a^6b-\sin\left(e+fx\right)^2\left(2a^7+2ba^6\right)+a^7+a^5b^2+a^7\sin\left(e+fx\right)^4\right)}+\frac{\sin\left(e+fx\right)^5}{5a^3f}+\frac{\sin\left(e+fx\right)^3\left(\frac{a+b}{a^4}-\frac{5}{3a^3}\right)}{f}+\frac{\sin\left(e+fx\right)\left(\frac{10}{a^3}-\frac{3(a+b)^2}{a^5}+\frac{3(a+b)\left(\frac{3(a+b)}{a^4}-\frac{5}{a^3}\right)}{a}\right)}{f}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sin(e+fx)\,1\mathrm{i}}{\sqrt{a+b}}\right)\left(80a^2+140ab+63b^2\right)\,1\mathrm{i}}{8a^{11/2}f\left(a+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((5*sin(e + f*x)*(4*a*b^4 + 3*b^5))/(8*(a + b)) - (sin(e + f*x)^3*(17*a*b^5
 + 20*a^2*b^4))/(8*(a + b)^2))/(f*(2*a^6*b - sin(e + f*x)^2*(2*a^6*b + 2*a^
7) + a^7 + a^5*b^2 + a^7*sin(e + f*x)^4)) + sin(e + f*x)^5/(5*a^3*f) + (sin
(e + f*x)^3*((a + b)/a^4 - 5/(3*a^3)))/f + (sin(e + f*x)*(10/a^3 - (3*(a +
b)^2)/a^5 + (3*(a + b)*((3*(a + b))/a^4 - 5/a^3))/a))/f + (b^3*atan((a^(1/2
)*sin(e + f*x)*1i)/(a + b)^(1/2))*(140*a*b + 80*a^2 + 63*b^2)*1i)/(8*a^(11/
2)*f*(a + b)^(5/2))
```

3.212 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{(3a^2+8ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f}-\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b
)^(5/2)/f-1/4*a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-3/
8*a*(a+2*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 424, 393, 211}

$$\frac{(3a^2+8ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}}-\frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)\sec^2(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(
5/2)*(a + b)^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(4*b*(a + b)*f*(a +
 b + b*Tan[e + f*x]^2)^2) - (3*a*(a + 2*b)*Tan[e + f*x])/(8*b^2*(a + b)^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b+(3a+4b)x^2}{(a+b+bx^2)^2}\,dx, x, \tan(e\right.}{4b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= \frac{(3a^2+8ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx\right.}$$

Mathematica [A]
time = 1.11, size = 125, normalized size = 0.88

$$\frac{\frac{(3a^2+8ab+8b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\,(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+fx)))\sin(2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f
*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8*b^
(5/2)*f)
```

Maple [A]
time = 0.20, size = 137, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{-\frac{a(5a+8b)\left(\tan^3(fx+e)\right)}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)\,b}}}{f}$
default	$\frac{\frac{-\frac{a(5a+8b)\left(\tan^3(fx+e)\right)}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)\,b}}}{f}$
risch	$-\frac{i(3a^3e^{6i(fx+e)}+8a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+9a^3e^{4i(fx+e)}+42a^2b\,e^{4i(fx+e)}+72a\,b^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}+9a^3}{4(a+b)^2b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((-1/8*a*(5*a+8*b)/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(3*a+8*b)*a/b^2/(
a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+8*a*b+8*b^2)/(a^2+2*a*b+
b^2)/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 215, normalized size = 1.51

$$\frac{\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^2b^2+2\,ab^3+b^4\right)\sqrt{(a+b)b}}-\frac{\left(5\,a^2b+8\,ab^2\right)\tan(fx+e)^3+\left(3\,a^3+11\,a^2b+8\,ab^2\right)\tan(fx+e)}{a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6+\left(a^2b^4+2\,ab^5+b^6\right)\tan(fx+e)^4+2\left(a^3b^3+3\,a^2b^4+3\,ab^5+b^6\right)\tan(fx+e)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b
^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)) - ((5*a^2*b + 8*a*b^2)*tan(f*x + e)^3
+ (3*a^3 + 11*a^2*b + 8*a*b^2)*tan(f*x + e))/(a^4*b^2 + 4*a^3*b^3 + 6*a^2*b
^4 + 4*a*b^5 + b^6 + (a^2*b^4 + 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3
+ 3*a^2*b^4 + 3*a*b^5 + b^6)*tan(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(134) = 268.
time = 3.48, size = 746, normalized size = 5.25

$$\left[-\frac{\left(\left(3\,a^4+8\,a^3b+8\,a^2b^2\right)\cos(fx+e)^4+3\,a^2b^2+8\,ab^3+8\,b^4+2\left(3\,a^3b+8\,a^2b^2+8\,ab^3\right)\cos(fx+e)^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+4\left(3\left(a^4b+3\,a^3b^2+2\,a^2b^3\right)\cos(fx+e)^3+\left(5\,a^3b^2+13\,a^2b^3+8\,ab^4\right)\cos(fx+e)\right)\sin(fx+e)}{32\left(\left(a^5b^3+3\,a^4b^4+3\,a^3b^5+a^2b^6\right)f\cos(fx+e)^4+2\left(a^4b^4+3\,a^3b^5+3\,a^2b^6+ab^7\right)f\cos(fx+e)^2+\left(a^3b^5+3\,a^2b^6+3\,ab^7+b^8\right)f\right)},\ -\frac{\left(\left(3\,a^4+8\,a^3b+8\,a^2b^2\right)\cos(fx+e)^4+3\,a^2b^2+8\,ab^3+8\,b^4+2\left(3\,a^3b+8\,a^2b^2+8\,ab^3\right)\cos(fx+e)^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)+2\left(3\left(a^4b+3\,a^3b^2+2\,a^2b^3\right)\cos(fx+e)^3+\left(5\,a^3b^2+13\,a^2b^3+8\,ab^4\right)\cos(fx+e)\right)\sin(fx+e)}{16\left(\left(a^5b^3+3\,a^4b^4+3\,a^3b^5+a^2b^6\right)f\cos(fx+e)^4+2\left(a^4b^4+3\,a^3b^5+3\,a^2b^6+ab^7\right)f\cos(fx+e)^2+\left(a^3b^5+3\,a^2b^6+3\,ab^7+b^8\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3
 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(3*(a
^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8*
a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(f
*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a^4 + 8*a^
3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2*(3*a^3*b
+ 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b
)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(
a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8
*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2
*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(
f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [**A**]
time = 0.46, size = 185, normalized size = 1.30

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+8ab+8b^2)}{(a^2b^2+2ab^3+b^4)\sqrt{ab+b^2}}-\frac{5a^2b\tan(fx+e)^3+8ab^2\tan(fx+e)^3+3a^3\tan(fx+e)+11a^2b\tan(fx+e)+8ab^2\tan(fx+e)}{(a^2b^2+2ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^2)
) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) +
 11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)^2))/f
```

Mupad [B]
time = 5.21, size = 149, normalized size = 1.05

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{8\,b^{5/2}\,f\,{(a+b)}^{5/2}}-\frac{\frac{{\tan(e+f\,x)}^3\,\left(5\,a^2+8\,b\,a\right)}{8\,b\,{(a+b)}^2}+\frac{\tan(e+f\,x)\,\left(3\,a^2+8\,b\,a\right)}{8\,b^2\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+{\tan\left(e+f\,x\right)}^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,{\tan\left(e+f\,x\right)}^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(8*a*b + 3*a^2 + 8*b^2))/(8*b^(
5/2)*f*(a + b)^(5/2)) - ((tan(e + f*x)^3*(8*a*b + 5*a^2))/(8*b*(a + b)^2) +
 (tan(e + f*x)*(8*a*b + 3*a^2))/(8*b^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + ta
n(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.213 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{(a+4b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f}-\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx\right.}$$

```
[Out] 1/8*(a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/f-1/
4*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)*tan(f*x+e)/b/(a
+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 393, 205, 211}

$$\frac{(a+4b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}}+\frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(
5/2)*f) - (a*Tan[e + f*x])/(4*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (
(a + 4*b)*Tan[e + f*x])/(8*b*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(a+4b)\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^2}\,dx,x,\tan\right.}{4b(a+b)f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f}-\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{}{8b(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.02, size = 283, normalized size = 2.30

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{(a+4b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))^2(\cos(2e)-i\sin(2e))}{b\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}-\frac{4(a+b)((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\cos(e)+\sin(e))}+\frac{(a+2b+a\cos(2(e+fx)))((a+4b)\sin(2e)-(a-2b)\sin(2fx))}{b(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{64(a+b)^2f\left(a+b\sec^2(e+fx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f*
x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2
*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) -
(4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e
] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] - (a - 2
*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(64*(a + b)^2*f*
(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.16, size = 118, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\frac{(a+4b)\left(\tan^{3}(fx+e)\right)}{8a^{2}+16ab+8b^{2}}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^{2}+2ab+b^{2}\right)b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\frac{(a+4b)\left(\tan^{3}(fx+e)\right)}{8a^{2}+16ab+8b^{2}}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^{2}+2ab+b^{2}\right)b\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(-a^{3}e^{6i(fx+e)}-4a^{2}b\,e^{6i(fx+e)}-3a^{3}e^{4i(fx+e)}-2a^{2}b\,e^{4i(fx+e)}+8a\,b^{2}e^{4i(fx+e)}+16b^{3}e^{4i(fx+e)}-3a^{3}e^{2i(fx+e)}+4a^{2}b\,e^{2i}\right.}{4a(a+b)^{2}f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^{2}b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((1/8*(a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(a-4*b)/(a+b)/b*tan(f*x+
e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 192, normalized size = 1.56

$$\frac{\frac{(a+4\,b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^{2}b+2\,ab^{2}+b^{3})\sqrt{(a+b)b}}+\frac{(ab+4\,b^{2})\tan(fx+e)^{3}-(a^{2}-3\,ab-4\,b^{2})\tan(fx+e)}{a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+b^{5}+(a^{2}b^{3}+2\,ab^{4}+b^{5})\tan(fx+e)^{4}+2\,(a^{3}b^{2}+3\,a^{2}b^{3}+3\,ab^{4}+b^{5})\tan(fx+e)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a + 4*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b + 2*a*b^2 + b
^3)*sqrt((a + b)*b)) + ((a*b + 4*b^2)*tan(f*x + e)^3 - (a^2 - 3*a*b - 4*b^2
)*tan(f*x + e))/(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 + (a^2*b^3 +
 2*a*b^4 + b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*ta
n(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(114) = 228.
time = 2.73, size = 678, normalized size = 5.51

$$\left[-\frac{\left((a^{3}+4\,a^{2}b)\cos(fx+e)^{4}+ab^{2}+4\,b^{3}+2\,(a^{2}b+4\,ab^{2})\cos(fx+e)^{2}\right)\sqrt{-ab-b^{2}}\log\left(\frac{(a^{2}+8\,ab+8\,b^{2})\cos(fx+e)^{4}-2\,(3\,ab+4\,b^{2})\cos(fx+e)^{2}+4\,\left((a+2\,b)\cos(fx+e)^{3}-b\cos(fx+e)\right)\sqrt{-ab-b^{2}}\sin(fx+e)+b^{2}}{a^{2}\cos(fx+e)^{4}+2\,ab\cos(fx+e)^{2}+b^{2}}\right)+4\,\left((a^{3}b-a^{2}b^{2}-2\,ab^{3})\cos(fx+e)^{3}-(a^{2}b^{2}+5\,ab^{3}+4\,b^{4})\cos(fx+e)\right)\sin(fx+e)}{32\,\left((a^{5}b^{2}+3\,a^{4}b^{3}+3\,a^{3}b^{4}+a^{2}b^{5})f\cos(fx+e)^{4}+2\,(a^{4}b^{3}+3\,a^{3}b^{4}+3\,a^{2}b^{5}+ab^{6})f\cos(fx+e)^{2}+(a^{3}b^{4}+3\,a^{2}b^{5}+3\,ab^{6}+b^{7})f\right)},\ -\frac{\left((a^{3}+4\,a^{2}b)\cos(fx+e)^{4}+ab^{2}+4\,b^{3}+2\,(a^{2}b+4\,ab^{2})\cos(fx+e)^{2}\right)\sqrt{ab+b^{2}}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^{2}-b}{2\,\sqrt{ab+b^{2}}\cos(fx+e)\sin(fx+e)}\right)+2\,\left((a^{3}b-a^{2}b^{2}-2\,ab^{3})\cos(fx+e)^{3}-(a^{2}b^{2}+5\,ab^{3}+4\,b^{4})\cos(fx+e)\right)\sin(fx+e)}{16\,\left((a^{5}b^{2}+3\,a^{4}b^{3}+3\,a^{3}b^{4}+a^{2}b^{5})f\cos(fx+e)^{4}+2\,(a^{4}b^{3}+3\,a^{3}b^{4}+3\,a^{2}b^{5}+ab^{6})f\cos(fx+e)^{2}+(a^{3}b^{4}+3\,a^{2}b^{5}+3\,ab^{6}+b^{7})f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^
2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos
(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^3 - (
a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^
3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f), -
1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^2)
*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b^2 - 2*a*b^
3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3
 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 +
 3*a*b^6 + b^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.48, size = 163, normalized size = 1.33

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+4b)}{(a^2b+2ab^2+b^3)\sqrt{ab+b^2}}+\frac{ab\tan(fx+e)^3+4b^2\tan(fx+e)^3-a^2\tan(fx+e)+3ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^2b+2ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) + 4*
b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

Mupad [B]
time = 5.10, size = 125, normalized size = 1.02

$$\frac{\frac{\tan(e+fx)^3\,(a+4\,b)}{8\,(a+b)^2}-\frac{\tan(e+fx)\,(a-4\,b)}{8\,b\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)\,(a+4\,b)}{8\,b^{3/2}\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((tan(e + f*x)^3*(a + 4*b))/(8*(a + b)^2) - (tan(e + f*x)*(a - 4*b))/(8*b*(
a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e
 + f*x)^4)) + (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 4*b))/(8*b^(
3/2)*f*(a + b)^(5/2))
```

3.214 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}f}+\frac{\tan(e+fx)}{4(a+b)f\,(a+b+b\tan^2(e+fx))^2}+\frac{3\tan(e+fx)}{8(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] 3/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f/b^(1/2)+1/4*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x
+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,f(a+b)^{5/2}}+\frac{3\tan(e+fx)}{8f(a+b)^2\,(a+b\tan^2(e+fx)+b)}+\frac{\tan(e+fx)}{4f(a+b)\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*f)
+ Tan[e + f*x]/(4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (3*Tan[e + f*x]
)/(8*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{3\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx,x,\tan(e+fx)\right)}{4(a+b)f}$$

$$=\frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{3\tan(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}+$$

$$=\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}f}+\frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{3}{8(a+b)^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.89, size = 265, normalized size = 2.50

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{3\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))^2(\cos(2e)-i\sin(2e))}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{4b(a+b)\sec(2e)((a+2b)\sin(2e)-a\sin(2fx))}{a^2}+\frac{(a+2b+a\cos(2(e+fx)))\sec(2e)\left(-\left(\left(5a^2+16ab+8b^2\right)\sin(2e)\right)+a(5a+2b)\sin(2fx)\right)}{a^2}\right)}{64(a+b)^2f\left(a+b\sec^2(e+fx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[2
*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a + b)
*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2*b)*Si
n[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.28, size = 100, normalized size = 0.94

method	result

derivativedivides	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
default	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
risch	$\frac{i(5a^3e^{6i(fx+e)}+16a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+46a^2b\,e^{4i(fx+e)}+56a\,b^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}+15a^3}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)^2+3/4/(a+b)*(1/2*tan(f*x+e)/
(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((
a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 161, normalized size = 1.52

$$\frac{\frac{3\,b\tan(fx+e)^3+5\,(a+b)\tan(fx+e)}{(a^2b^2+2\,ab^3+b^4)\tan(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+2\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b*tan(f*x + e)^3 + 5*(a + b)*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^
4)*tan(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 2*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^2) + 3*arctan(b*tan(f*x + e)/sqrt((
a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(97) = 194.
time = 2.44, size = 604, normalized size = 5.70

$$\left[-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)-4\left((5\,a^2b+7\,ab^2+2\,b^3)\cos(fx+e)^3+3\,(ab^2+b^3)\cos(fx+e)\right)\sin(fx+e)}{32\left((a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(fx+e)^4+2\,(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)f\cos(fx+e)^2+(a^3b^3+3\,a^2b^4+3\,ab^5+b^6)f\right)},\ -\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2\left((5\,a^2b+7\,ab^2+2\,b^3)\cos(fx+e)^3+3\,(ab^2+b^3)\cos(fx+e)\right)\sin(fx+e)}{16\left((a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(fx+e)^4+2\,(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)f\cos(fx+e)^2+(a^3b^3+3\,a^2b^4+3\,ab^5+b^6)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^2
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^
2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f
*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a
^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2*b^3)*cos(
f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.50, size = 124, normalized size = 1.17

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^3+5\,a\tan(fx+e)+5\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 + 2*a
*b + b^2)))/f
```

Mupad [B]
time = 5.00, size = 112, normalized size = 1.06

$$\frac{\frac{5\tan(e+f\,x)}{8\,(a+b)}+\frac{3\,b\tan(e+f\,x)^3}{8\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{8\,\sqrt{b}\,f\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

```
[Out] ((5*tan(e + f*x))/(8*(a + b)) + (3*b*tan(e + f*x)^3)/(8*(a + b)^2))/(f*(2*a
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (3
*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(8*b^(1/2)*f*(a + b)^(5/2))
```

3.215 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f}-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e\right.}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+f}{4a(a+b)f\left(a+b+b\tan\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.32, size = 332, normalized size = 2.31

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))^2(\cos(2e)-i\sin(2e))}{(a+b)^{5/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}-\frac{4b^2((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}+\frac{b(a+2b+a\cos(2(e+fx)))((9a^2+28ab+16b^2)\sin(2e)-3a(3a+2b)\sin(2fx))}{(a+b)^2f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{64a^3\left(a+b\sec^2(e+fx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.00, size = 147, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2b\,e^{4i(fx+e)}+120a\,b^2e^{4i(fx+e)}+48b^3e^{4i\cdots}\right.}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan
(f*x+e)^3+1/8*(9*a+4*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*
a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 237, normalized size = 1.65

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{(a+b)b}}+\frac{\left(7\,ab^{2}+4\,b^{3}\right)\tan(fx+e)^{3}+\left(9\,a^{2}b+13\,ab^{2}+4\,b^{3}\right)\tan(fx+e)}{a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}\right)\tan(fx+e)^{4}+2\left(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4}\right)\tan(fx+e)^{2}}-\frac{8\,(fx+e)}{a^{3}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(135) = 270.
time = 3.31, size = 847, normalized size = 5.88

$$\left[\frac{32\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)fx\cos(fx+e)^{4}+64\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)fx\cos(fx+e)^{2}+32\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)fx+\left(\left(15\,a^{4}+20\,a^{3}b+8\,a^{2}b^{2}\right)\cos(fx+e)^{4}+15\,a^{2}b^{2}+20\,ab^{3}+8\,b^{4}+2\left(15\,a^{3}b+20\,a^{2}b^{2}+8\,ab^{3}\right)\cos(fx+e)^{2}\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos(fx+e)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos(fx+e)^{2}+4\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\cos(fx+e)^{3}-\left(ab+b^{2}\right)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^{2}}{a^{2}\cos(fx+e)^{4}+2\,ab\cos(fx+e)^{2}+b^{2}}\right)-4\left(3\left(3\,a^{3}b+2\,a^{2}b^{2}\right)\cos(fx+e)^{3}+\left(7\,a^{2}b^{2}+4\,ab^{3}\right)\cos(fx+e)\right)\sin(fx+e)}{32\left(\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)f\cos(fx+e)^{4}+2\left(a^{6}b+2\,a^{5}b^{2}+a^{4}b^{3}\right)f\cos(fx+e)^{2}+\left(a^{5}b^{2}+2\,a^{4}b^{3}+a^{3}b^{4}\right)f\right)},\ \frac{16\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)fx\cos(fx+e)^{4}+32\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)fx\cos(fx+e)^{2}+16\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)fx+\left(\left(15\,a^{4}+20\,a^{3}b+8\,a^{2}b^{2}\right)\cos(fx+e)^{4}+15\,a^{2}b^{2}+20\,ab^{3}+8\,b^{4}+2\left(15\,a^{3}b+20\,a^{2}b^{2}+8\,ab^{3}\right)\cos(fx+e)^{2}\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^{2}-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-2\left(3\left(3\,a^{3}b+2\,a^{2}b^{2}\right)\cos(fx+e)^{3}+\left(7\,a^{2}b^{2}+4\,ab^{3}\right)\cos(fx+e)\right)\sin(fx+e)}{16\left(\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)f\cos(fx+e)^{4}+2\left(a^{6}b+2\,a^{5}b^{2}+a^{4}b^{3}\right)f\cos(fx+e)^{2}+\left(a^{5}b^{2}+2\,a^{4}b^{3}+a^{3}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Giac [A]
time = 0.44, size = 196, normalized size = 1.36

$$-\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan\left(fx+e\right)^3+4\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)+13\,ab^2\tan\left(fx+e\right)+4\,b^3\tan\left(fx+e\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}-\frac{8\left(fx+e\right)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B]
time = 9.33, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
```

56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a
^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b
^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 +
a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b
^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e +
f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(
64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6
 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9
*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7
+ 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2
))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3)
 - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^
4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a
^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*
x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2
*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6
 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^
8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3
+ 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)
*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3
+ 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*
a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^
5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^
2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2))
 + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*
a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6
*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b
+ a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1
/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 +
3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4
 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7
*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^
(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^
4 + 10*a^5*b^3 + 10*a^6*b^2)))/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105
*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e
 + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(
32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*
a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^
6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b
 + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^

```
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 2...
```

3.216 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=201

$$\frac{(a-6b)x}{2a^4}+\frac{b^{3/2}(35a^2+56ab+24b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2}+\frac{b(2a+}{4a^2(a+b)f\,(a}$$

```
[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x
+e)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan
(f*x+e)^2)^2+1/4*b*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+
1/8*b*(4*a+3*b)*(a+4*b)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{x(a-6b)}{2a^4}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\,(a+b\tan^2(e+fx)+b)}+\frac{b(2a+3b)\tan(e+fx)}{4a^2f(a+b)\,(a+b\tan^2(e+fx)+b)^2}+\frac{b^{3/2}(35a^2+56ab+24b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{5/2}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*f) + (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(2*a + 3*b)*Tan[e + f*x])
/(4*a^2*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*
Tan[e + f*x])/(8*a^3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{-a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subs}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{b(}{8a^3(}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{b(}{8a^3(}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)}{2af\,(a+b+b}$$

Mathematica [A]
time = 4.17, size = 156, normalized size = 0.78

$$\frac{4(a-6b)(e+fx) + \frac{b^{3/2}(35a^2+56ab+24b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\left(2 + \frac{13ab^2}{(a+b)^2(a+2b+a\cos(2(e+fx)))} + \frac{2b^3(3a+8b+5a\cos(2(e+fx)))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}\right)\sin(2(e+fx))}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^2*
(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x)])
)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^4*f)
```

Maple [A]
time = 0.36, size = 177, normalized size = 0.88

method	result

derivativedivides	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{(35a^2+56ab+24b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a^2+2ab+b^2)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+}{}}{f}$
default	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{(35a^2+56ab+24b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a^2+2ab+b^2)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2\left(\tan^2(fx+e)\right)+2}+}{}}{f}$
risch	$\frac{x}{2a^3}-\frac{3xb}{a^4}-\frac{i e^{2i(fx+e)}}{8fa^3}+\frac{i e^{-2i(fx+e)}}{8fa^3}+\frac{ib^2(13a^3e^{6i(fx+e)}+40a^2b\,e^{6i(fx+e)}+24a\,b^2e^{6i(fx+e)}+39a^3e^{4i(fx+e)}+}{4a^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^4*((1/8*a*b*(11*a+8*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(13*a+8*
b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(35*a^2+56*a*b+24*b^2)/(a
^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(
1/2*a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2*(a-6*b)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.50, size = 346, normalized size = 1.72

$$\frac{\frac{(35a^2b^2+56ab^3+24b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+2a^5b+a^4b^2)\sqrt{(a+b)b}}+\frac{(4a^2b^2+19ab^3+12b^4)\tan(fx+e)^5+(8a^3b+37a^2b^2+56ab^3+24b^4)\tan(fx+e)^3+(4a^4+16a^3b+37a^2b^2+37ab^3+12b^4)\tan(fx+e)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+(a^5b^2+2a^4b^3+a^3b^4)\tan(fx+e)^6+(2a^6b+7a^5b^2+8a^4b^3+3a^3b^4)\tan(fx+e)^4+(a^7+6a^6b+12a^5b^2+10a^4b^3+3a^3b^4)\tan(fx+e)^2}+\frac{4(fx+e)(a-6b)}{a^4}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)) + ((4*a^2*b^2 + 19*a*b^3 + 12
*b^4)*tan(f*x + e)^5 + (8*a^3*b + 37*a^2*b^2 + 56*a*b^3 + 24*b^4)*tan(f*x +
 e)^3 + (4*a^4 + 16*a^3*b + 37*a^2*b^2 + 37*a*b^3 + 12*b^4)*tan(f*x + e))/(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^
3*b^4)*tan(f*x + e)^6 + (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 3*a^3*b^4)*tan(f
*x + e)^4 + (a^7 + 6*a^6*b + 12*a^5*b^2 + 10*a^4*b^3 + 3*a^3*b^4)*tan(f*x +
 e)^2) + 4*(f*x + e)*(a - 6*b)/a^4)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(191) = 382$.

time = 3.78, size = 1000, normalized size = 4.98

$$\left[\frac{16\,(a^5-4\,a^4b-11\,a^3b^2-6\,a^2b^3)fx\cos(fx+e)^4+32\,(a^4b-4\,a^3b^2-11\,a^2b^3-6\,ab^4)fx\cos(fx+e)^2+16\,(a^3b^2-4\,a^2b^3-11\,ab^4-6\,b^5)fx+(35\,a^2b^3+56\,ab^4+24\,b^5+(35\,a^4b+56\,a^3b^2+24\,a^2b^3)\cos(fx+e)^4+2\,(35\,a^3b^2+56\,a^2b^3+24\,ab^4)\cos(fx+e)^2)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+4\,(4\,(a^5+2\,a^4b+a^3b^2)\cos(fx+e)^5+(8\,a^4b+29\,a^3b^2+18\,a^2b^3)\cos(fx+e)^3+(4\,a^3b^2+19\,a^2b^3+12\,ab^4)\cos(fx+e))\sin(fx+e)}{32\,((a^8+2\,a^7b+a^6b^2)f\cos(fx+e)^4+2\,(a^7b+2\,a^6b^2+a^5b^3)f\cos(fx+e)^2+(a^6b^2+2\,a^5b^3+a^4b^4)f)}, \frac{8\,(a^5-4\,a^4b-11\,a^3b^2-6\,a^2b^3)fx\cos(fx+e)^4+16\,(a^4b-4\,a^3b^2-11\,a^2b^3-6\,ab^4)fx\cos(fx+e)^2+8\,(a^3b^2-4\,a^2b^3-11\,ab^4-6\,b^5)fx-(35\,a^2b^3+56\,ab^4+24\,b^5+(35\,a^4b+56\,a^3b^2+24\,a^2b^3)\cos(fx+e)^4+2\,(35\,a^3b^2+56\,a^2b^3+24\,ab^4)\cos(fx+e)^2)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{((a+2\,b)\cos(fx+e)^2-b)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)+2\,(4\,(a^5+2\,a^4b+a^3b^2)\cos(fx+e)^5+(8\,a^4b+29\,a^3b^2+18\,a^2b^3)\cos(fx+e)^3+(4\,a^3b^2+19\,a^2b^3+12\,ab^4)\cos(fx+e))\sin(fx+e)}{16\,((a^8+2\,a^7b+a^6b^2)f\cos(fx+e)^4+2\,(a^7b+2\,a^6b^2+a^5b^3)f\cos(fx+e)^2+(a^6b^2+2\,a^5b^3+a^4b^4)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32*
(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b^2
 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (3
5*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*
b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(a^5 + 2*
a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f
*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16*(8*(a^5 - 4
*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*b - 4*a^3*b^2
 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*a^2*b^3 - 11*a
*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^4*b + 56*a^3*b^
2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3 + 24*a*b^4)*cos
(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 + 2*a^4*b + a^3*b^2)
*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + (4*a
^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^8 + 2*a^7*b
 + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x +
e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 228, normalized size = 1.13

$$\frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan(fx+e)^3+8\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+21\,ab^3\tan(fx+e)+8\,b^4\tan(fx+e)}{(a^5+2\,a^4b+a^3b^2)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{4\,(fx+e)(a-6\,b)}{a^4}+\frac{4\,\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")

[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^2*tan(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f

Mupad [B]
time = 9.97, size = 2500, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)

[Out] ((tan(e + f*x)^5*(19*a*b^3 + 12*b^4 + 4*a^2*b^2))/(8*a^3*(a + b)^2) + (tan(e + f*x)*(25*a*b^2 + 12*a^2*b + 4*a^3 + 12*b^3))/(8*a^3*(a + b)) + (b*tan(e + f*x)^3*(56*a*b^2 + 37*a^2*b + 8*a^3 + 24*b^3))/(8*a^3*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4*(2*a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4) - ((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4))/(((405*a*b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3*b^6)/32 - (49*a^4*b^5)/64 - (35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 +

1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 +
4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (((((6*a^8*b^7 + (49*a^9*
b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b
 + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e + f*x)*(a*1i - b*6i)*
(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3
 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^
2)))*(a*1i - b*6i))/(4*a^4) + (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a
^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^
9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4))*(a
*1i - b*6i)*1i)/(2*a^4*f) - (atan(((((tan(e + f*x)*(4800*a*b^8 + 1152*b^9 +
 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(3
2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*
a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^
12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(-b^3*(a +
 b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*
a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(512*(4*a^9*b + a
^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4
 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*
b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))
(-b^3(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*(5*a^8*b + a^9 +
 a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)) + (((tan(e + f*x)*(4800*a*
b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 +
 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (((
6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2
*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) + (tan(e +
 f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*(512*a^8*b^7 + 2304
*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(
512*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)*(5*a^8*b + a^9 + a^4
*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*
b + 35*a^2 + 24*b^2))/(16*(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3
 + 10*a^7*b^2)))*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2)*1i)/(16*
(5*a^8*b + a^9 + a^4*b^5 + 5*a^5*b^4 + 10*a^6*b^3 + 10*a^7*b^2)))/(((405*a*
b^8)/4 + 27*b^9 + (261*a^2*b^7)/2 + (1877*a^3*b^6)/32 - (49*a^4*b^5)/64 - (
35*a^5*b^4)/16)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (((
tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^
4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^
3 + 6*a^8*b^2)) - (((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^
4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6
*a^11*b^2) - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(56*a*b + 35*a^2 + 24*b^2
)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 +...

3.217 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=269

$$\frac{3(a^2-4ab+16b^2)\,x}{8a^5}-\frac{3b^{5/2}(21a^2+36ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f}+\frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2}$$

```
[Out] 3/8*(a^2-4*a*b+16*b^2)*x/a^5-3/8*b^(5/2)*(21*a^2+36*a*b+16*b^2)*arctan(b^(1
/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(5/2)/f+1/8*(3*a-8*b)*cos(f*x+e)*sin(
f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*
tan(f*x+e)^2)^2+1/8*b*(3*a^2-7*a*b-12*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*ta
n(f*x+e)^2)^2+3/8*b*(a+2*b)*(a^2-4*a*b-4*b^2)*tan(f*x+e)/a^4/(a+b)^2/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.27, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{(3a-8b)\sin(e+fx)\cos(e+fx)}{8a^2f\,(a+b\tan^2(e+fx)+b)^2}-\frac{3b^{5/2}(21a^2+36ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}}+\frac{3x(a^2-4ab+16b^2)}{8a^5}+\frac{3b(a+2b)\,(a^2-4ab-4b^2)\tan(e+fx)}{8a^4f(a+b)^2\,(a+b\tan^2(e+fx)+b)}+\frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3f(a+b)\,(a+b\tan^2(e+fx)+b)^2}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 4*a*b + 16*b^2)*x)/(8*a^5) - (3*b^(5/2)*(21*a^2 + 36*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*(a + b)^(5/2)*f) + ((3
*a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*
(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e +
f*x]^2)^2) + (3*b*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Tan[e + f*x])/(8*a^4*(a +
 b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-3a+b-7bx^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a}{8a^3(a+}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a}{8a^3(a+}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a}{8a^3(a+}$$

$$= \frac{3(a^2-4ab+16b^2)\,x}{8a^5} - \frac{3b^{5/2}(21a^2+36ab+16b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} + \frac{(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.62, size = 1430, normalized size = 5.32

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((21*a^2 + 36*a*b + 16*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((3*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin
[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^5*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)/64)*b^3*ArcTan[Sec[f*x]*(
Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])
/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e +
 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(144*a^6*f*x*Cos[2*e] + 96*a^5*b*f*x*Cos[2
*e] + 912*a^4*b^2*f*x*Cos[2*e] + 6720*a^3*b^3*f*x*Cos[2*e] + 16512*a^2*b^4*
f*x*Cos[2*e] + 16896*a*b^5*f*x*Cos[2*e] + 6144*b^6*f*x*Cos[2*e] + 96*a^6*f*
```

```
x*Cos[2*f*x] + 480*a^4*b^2*f*x*Cos[2*f*x] + 4416*a^3*b^3*f*x*Cos[2*f*x] + 6
912*a^2*b^4*f*x*Cos[2*f*x] + 3072*a*b^5*f*x*Cos[2*f*x] + 96*a^6*f*x*Cos[4*e
 + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*f*x*Cos[4*e + 2
*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 24*a^6*f*x*Cos[2*e + 4*f*x] - 48*a^5*b*f*x*Cos[2*e + 4*f*x] + 216*a^4*b^
2*f*x*Cos[2*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[2*e + 4*f*x] + 384*a^2*b^4*f*x
*Cos[2*e + 4*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5*b*f*x*Cos[6*e + 4*
f*x] + 216*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[6*e + 4*f*x]
 + 384*a^2*b^4*f*x*Cos[6*e + 4*f*x] + 816*a^3*b^3*Sin[2*e] + 2848*a^2*b^4*Si
n[2*e] + 3968*a*b^5*Sin[2*e] + 1792*b^6*Sin[2*e] + 44*a^6*Sin[2*f*x] + 104*
a^5*b*Sin[2*f*x] - 180*a^4*b^2*Sin[2*f*x] - 1696*a^3*b^3*Sin[2*f*x] - 3264*
a^2*b^4*Sin[2*f*x] - 1664*a*b^5*Sin[2*f*x] + 44*a^6*Sin[4*e + 2*f*x] + 104*
a^5*b*Sin[4*e + 2*f*x] - 180*a^4*b^2*Sin[4*e + 2*f*x] - 608*a^3*b^3*Sin[4*e
 + 2*f*x] - 192*a^2*b^4*Sin[4*e + 2*f*x] + 128*a*b^5*Sin[4*e + 2*f*x] + 38*
a^6*Sin[2*e + 4*f*x] + 60*a^5*b*Sin[2*e + 4*f*x] - 170*a^4*b^2*Sin[2*e + 4*
f*x] - 640*a^3*b^3*Sin[2*e + 4*f*x] - 400*a^2*b^4*Sin[2*e + 4*f*x] + 38*a^6
*Sin[6*e + 4*f*x] + 60*a^5*b*Sin[6*e + 4*f*x] - 170*a^4*b^2*Sin[6*e + 4*f*x
] - 368*a^3*b^3*Sin[6*e + 4*f*x] - 176*a^2*b^4*Sin[6*e + 4*f*x] + 12*a^6*Si
n[4*e + 6*f*x] + 8*a^5*b*Sin[4*e + 6*f*x] - 20*a^4*b^2*Sin[4*e + 6*f*x] - 1
6*a^3*b^3*Sin[4*e + 6*f*x] + 12*a^6*Sin[8*e + 6*f*x] + 8*a^5*b*Sin[8*e + 6*
f*x] - 20*a^4*b^2*Sin[8*e + 6*f*x] - 16*a^3*b^3*Sin[8*e + 6*f*x] + a^6*Sin[
6*e + 8*f*x] + 2*a^5*b*Sin[6*e + 8*f*x] + a^4*b^2*Sin[6*e + 8*f*x] + a^6*Si
n[10*e + 8*f*x] + 2*a^5*b*Sin[10*e + 8*f*x] + a^4*b^2*Sin[10*e + 8*f*x]))/(
2048*a^5*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.23, size = 215, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\left(\tan^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+12b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{3\left(21a^2+36ab+16b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\left(\tan^3\right.}{}}{f}$
default	$\frac{-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\left(\tan^3(fx+e)\right)}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+12b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{3\left(21a^2+36ab+16b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\left(\tan^3\right.}{}}{f}$
risch	$\frac{3x}{8a^3}-\frac{3xb}{2a^4}+\frac{6xb^2}{a^5}-\frac{3i\mathrm{e}^{-2i(fx+e)}b}{8a^4f}-\frac{ib^3\left(17a^3\mathrm{e}^{6i(fx+e)}+52a^2b\,\mathrm{e}^{6i(fx+e)}+32a\,b^2\mathrm{e}^{6i(fx+e)}+51a^3\mathrm{e}^{4i(fx+e)}+178a\right.}{4a^5(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^5*b^3*((3/8*a*b*(5*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(17*a+
12*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+3/8*(21*a^2+36*a*b+16*b^2)
/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^
5*(((3/8*a^2-3/2*a*b)*tan(f*x+e)^3+(-3/2*a*b+5/8*a^2)*tan(f*x+e))/(tan(f*x+
e)^2+1)^2+3/8*(a^2-4*a*b+16*b^2)*arctan(tan(f*x+e))))
```

Maxima [**A**]
time = 0.49, size = 474, normalized size = 1.76

$$-\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)b}}-\frac{3\,(a^3b^2-2\,a^2b^3-12\,ab^4-8\,b^5)\tan(fx+e)^7+(6\,a^4b-a^3b^2-73\,a^2b^3-144\,ab^4-72\,b^5)\tan(fx+e)^5+(3\,a^5+10\,a^4b-24\,a^3b^2-136\,a^2b^3-180\,ab^4-72\,b^5)\tan(fx+e)^3+(5\,a^5+8\,a^4b-18\,a^3b^2-69\,a^2b^3-72\,ab^4-24\,b^5)\tan(fx+e)}{(a^6b^2+2\,a^5b^3+a^4b^4)\tan(fx+e)^8+a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+2\,(a^7b+4\,a^6b^2+5\,a^5b^3+2\,a^4b^4)\tan(fx+e)^6+(a^8+8\,a^7b+19\,a^6b^2+18\,a^5b^3+6\,a^4b^4)\tan(fx+e)^4+2\,(a^8+5\,a^7b+9\,a^6b^2+7\,a^5b^3+2\,a^4b^4)\tan(fx+e)^2}-\frac{3\,(a^2-4\,ab+16\,b^2)(fx+e)}{a^5}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)) - (3*(a^3*b^2 - 2*a^2*b^3
- 12*a*b^4 - 8*b^5)*tan(f*x + e)^7 + (6*a^4*b - a^3*b^2 - 73*a^2*b^3 - 144*
a*b^4 - 72*b^5)*tan(f*x + e)^5 + (3*a^5 + 10*a^4*b - 24*a^3*b^2 - 136*a^2*b
^3 - 180*a*b^4 - 72*b^5)*tan(f*x + e)^3 + (5*a^5 + 8*a^4*b - 18*a^3*b^2 - 6
9*a^2*b^3 - 72*a*b^4 - 24*b^5)*tan(f*x + e))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^
4)*tan(f*x + e)^8 + a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 + 2*(a^
7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^6 + (a^8 + 8*a^7*b +
19*a^6*b^2 + 18*a^5*b^3 + 6*a^4*b^4)*tan(f*x + e)^4 + 2*(a^8 + 5*a^7*b + 9*
a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^2) - 3*(a^2 - 4*a*b + 16*b^2)
*(f*x + e)/a^5)/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(260) = 520$.
time = 4.30, size = 1161, normalized size = 4.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x
 + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*co
s(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x
 + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^
4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*
sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
```

```
a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7
+ (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10
*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3
 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*a^8*b + a^7*
b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 +
(a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28
*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^4*b^2 + 9*a^3*b
^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2 - 2*a^3*b^3 + 9
*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21
*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2
*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^6
+ 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3
*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos
(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*
sin(f*x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^
7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 464, normalized size = 1.72

$$-\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{ab+b^2}}-\frac{3\,a^3b^2\tan(fx+e)^7-6\,a^2b^3\tan(fx+e)^7-36\,ab^4\tan(fx+e)^7-24\,b^5\tan(fx+e)^7+6\,a^4b\tan(fx+e)^5-a^3b^2\tan(fx+e)^5-73\,a^2b^3\tan(fx+e)^5-144\,ab^4\tan(fx+e)^5-72\,b^5\tan(fx+e)^5+3\,a^5\tan(fx+e)^3+10\,a^4b\tan(fx+e)^3-24\,a^3b^2\tan(fx+e)^3-136\,a^2b^3\tan(fx+e)^3-180\,ab^4\tan(fx+e)^3-72\,b^5\tan(fx+e)^3+5\,a^5\tan(fx+e)+8\,a^4b\tan(fx+e)-18\,a^3b^2\tan(fx+e)-69\,a^2b^3\tan(fx+e)-72\,ab^4\tan(fx+e)-24\,b^5\tan(fx+e)}{(a^6+2\,a^5b+a^4b^2)\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)^2}-\frac{3\,(a^2-4\,ab+16\,b^2)(fx+e)}{a^5}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*sqr
t(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 36*a
*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 - a^3*
b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e)^5 -
 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)^3 - 2
4*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*tan(f*x +
 e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*x + e) -
 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(f*x + e)
- 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^4 + a*tan
```

```
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b + 16*b^2)*(f*
x + e)/a^5)/f
```

Mupad [B]
time = 9.59, size = 2500, normalized size = 9.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^
3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))
/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b
^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^1
5*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^1
4*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*
(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b
^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^
2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5
*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5
 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b +
21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3
 + 10*a^8*b^2)) + (((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9
 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 +
9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*
((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b
^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*
b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2
 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 +
1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3
+ 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b
^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^1
0 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)
*(36*a*b + 21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 +
 10*a^7*b^3 + 10*a^8*b^2)))/((756*a*b^11 + 216*b^12 + (1755*a^2*b^10)/2 + (
1215*a^3*b^9)/4 - (1701*a^4*b^8)/32 - (351*a^5*b^7)/64 + (1215*a^6*b^6)/128
 - (567*a^7*b^5)/256)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2
) - (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3
*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/
(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^10*b^
8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15
*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14
*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*(
512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^
```

3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)) + (3*((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) + (3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2))))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2)*3i)/(8*f*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)) - (atan(((((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) - (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i))/(16*a^5) - (tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)))*(a^2*1i - a*b*4i + b^2*16i)*3i)/(16*a^5) - (((3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(a^2*1i - a*b*4i + b^2*16i)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*a^5*(4*a^11*b + a^12 + a^8*b^4 + ...

3.218 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=352

$$\frac{(5a^3-18a^2b+48ab^2-160b^3)\,x}{16a^6}+\frac{b^{7/2}(99a^2+176ab+80b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f}+\frac{(15a^2-34ab+80b^2}{48a^3f\,(a+b}$$

[Out] 1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*x/a^6+1/8*b^(7/2)*(99*a^2+176*a*b+80*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/(a+b)^(5/2)/f+1/48*(15*a^2-34*a*b+80*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+5/24*(a-2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/48*b*(15*a^3-29*a^2*b+64*a*b^2+120*b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/16*b*(5*a^4-8*a^3*b+17*a^2*b^2+116*a*b^3+80*b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)

Rubi [A]
time = 0.34, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 209, 211}

$$\frac{5(a-2b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{b^{7/2}(99a^2+176ab+80b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}}+\frac{(15a^2-34ab+80b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{x(5a^3-18a^2b+48ab^2-160b^3)}{16a^6}+\frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sin(e+fx)\cos^5(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]

[Out] ((5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*x)/(16*a^6) + (b^(7/2)*(99*a^2 + 176*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*(a + b)^(5/2)*f) + ((15*a^2 - 34*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) + (5*(a - 2*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(15*a^3 - 29*a^2*b + 64*a*b^2 + 120*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(5*a^4 - 8*a^3*b + 17*a^2*b^2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(16*a^5*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{-5a+b-9bx^2}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}}{}$$

$$= \frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= \frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= \frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= \frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e}{24a^2f\,(a+b+b\tan^2(e+}$$

$$= \frac{(5a^3-18a^2b+48ab^2-160b^3)\,x}{16a^6} + \frac{b^{7/2}(99a^2+176ab+80b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan}{\sqrt{a-}}\right.}{8a^6(a+b)^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.69, size = 1770, normalized size = 5.03

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^
6*(-1/64*(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^6*Sqrt[
a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Si
n[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2
```

```
*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*f*x*Cos[2*
e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 64128*a^3*b^4*
f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*Cos[2*e] - 61
440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*Cos[2*f*x] +
96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 41856*a^3*b^4*f*x
*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x*Cos[2*f*x] + 4
80*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x] + 96*a^5*b^2*f
*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] - 41856*a^3*b^4*f*x
*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] - 30720*a*b^6*f*x*Co
s[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*b*f*x*Cos[2*e + 4*f
*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[2*e + 4*f*x]
- 6528*a^3*b^4*f*x*Cos[2*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[2*e + 4*f*x] + 1
20*a^7*f*x*Cos[6*e + 4*f*x] - 192*a^6*b*f*x*Cos[6*e + 4*f*x] + 408*a^5*b^2*
f*x*Cos[6*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[6*e + 4*f*x] - 6528*a^3*b^4*f*x
*Cos[6*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[6*e + 4*f*x] - 6048*a^3*b^4*Sin[2*
e] - 21312*a^2*b^5*Sin[2*e] - 29952*a*b^6*Sin[2*e] - 13824*b^7*Sin[2*e] + 2
62*a^7*Sin[2*f*x] + 524*a^6*b*Sin[2*f*x] - 26*a^5*b^2*Sin[2*f*x] + 1728*a^4
*b^3*Sin[2*f*x] + 14976*a^3*b^4*Sin[2*f*x] + 28416*a^2*b^5*Sin[2*f*x] + 145
92*a*b^6*Sin[2*f*x] + 262*a^7*Sin[4*e + 2*f*x] + 524*a^6*b*Sin[4*e + 2*f*x]
 - 26*a^5*b^2*Sin[4*e + 2*f*x] + 1728*a^4*b^3*Sin[4*e + 2*f*x] + 6912*a^3*b
^4*Sin[4*e + 2*f*x] + 5376*a^2*b^5*Sin[4*e + 2*f*x] + 768*a*b^6*Sin[4*e + 2
*f*x] + 238*a^7*Sin[2*e + 4*f*x] + 304*a^6*b*Sin[2*e + 4*f*x] - 250*a^5*b^2
*Sin[2*e + 4*f*x] + 1556*a^4*b^3*Sin[2*e + 4*f*x] + 5904*a^3*b^4*Sin[2*e +
4*f*x] + 3744*a^2*b^5*Sin[2*e + 4*f*x] + 238*a^7*Sin[6*e + 4*f*x] + 304*a^6
*b*Sin[6*e + 4*f*x] - 250*a^5*b^2*Sin[6*e + 4*f*x] + 1556*a^4*b^3*Sin[6*e +
 4*f*x] + 3888*a^3*b^4*Sin[6*e + 4*f*x] + 2016*a^2*b^5*Sin[6*e + 4*f*x] + 8
7*a^7*Sin[4*e + 6*f*x] + 46*a^6*b*Sin[4*e + 6*f*x] - 9*a^5*b^2*Sin[4*e + 6*
f*x] + 192*a^4*b^3*Sin[4*e + 6*f*x] + 160*a^3*b^4*Sin[4*e + 6*f*x] + 87*a^7
*Sin[8*e + 6*f*x] + 46*a^6*b*Sin[8*e + 6*f*x] - 9*a^5*b^2*Sin[8*e + 6*f*x]
+ 192*a^4*b^3*Sin[8*e + 6*f*x] + 160*a^3*b^4*Sin[8*e + 6*f*x] + 13*a^7*Sin[
6*e + 8*f*x] + 16*a^6*b*Sin[6*e + 8*f*x] - 7*a^5*b^2*Sin[6*e + 8*f*x] - 10*
a^4*b^3*Sin[6*e + 8*f*x] + 13*a^7*Sin[10*e + 8*f*x] + 16*a^6*b*Sin[10*e + 8
*f*x] - 7*a^5*b^2*Sin[10*e + 8*f*x] - 10*a^4*b^3*Sin[10*e + 8*f*x] + a^7*Si
n[8*e + 10*f*x] + 2*a^6*b*Sin[8*e + 10*f*x] + a^5*b^2*Sin[8*e + 10*f*x] + a
^7*Sin[12*e + 10*f*x] + 2*a^6*b*Sin[12*e + 10*f*x] + a^5*b^2*Sin[12*e + 10*
f*x]))/(12288*a^6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.24, size = 267, normalized size = 0.76

method	result

derivativedivides	$\frac{b^{4}\left(\frac{\frac{ab(19a+16b)\left(\tan^{3}(fx+e)\right)}{8a^{2}+16ab+8b^{2}}+\frac{(21a+16b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{\left(99a^{2}+176ab+80b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\left(a^{2}+2ab+b^{2}\right)\sqrt{(a+b)\,b}}\right)}{a^{6}}+\frac{\left(\frac{5}{16}a^{3}-\frac{9}{8}a^{2}b+3\right.}{}$
default	$\frac{b^{4}\left(\frac{\frac{ab(19a+16b)\left(\tan^{3}(fx+e)\right)}{8a^{2}+16ab+8b^{2}}+\frac{(21a+16b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{\left(99a^{2}+176ab+80b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{8\left(a^{2}+2ab+b^{2}\right)\sqrt{(a+b)\,b}}\right)}{a^{6}}+\frac{\left(\frac{5}{16}a^{3}-\frac{9}{8}a^{2}b+3\right.}{}$
risch	$\frac{5x}{16a^{3}}-\frac{9xb}{8a^{4}}+\frac{3x\,b^{2}}{a^{5}}-\frac{10x\,b^{3}}{a^{6}}+\frac{3i\mathrm{e}^{4i(fx+e)}b}{64a^{4}f}-\frac{15i\mathrm{e}^{2i(fx+e)}}{128f\,a^{3}}+\frac{3i\mathrm{e}^{-2i(fx+e)}b^{2}}{4a^{5}f}-\frac{3i\mathrm{e}^{-2i(fx+e)}b}{8a^{4}f}-\frac{3i\mathrm{e}^{4i(fx}}{128a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^6*((1/8*a*b*(19*a+16*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(21*a+1
6*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+176*a*b+80*b^2)
/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^
6*(((5/16*a^3-9/8*a^2*b+3*a*b^2)*tan(f*x+e)^5+(6*a*b^2+5/6*a^3-3*a^2*b)*tan
(f*x+e)^3+(-15/8*a^2*b+3*a*b^2+11/16*a^3)*tan(f*x+e))/(tan(f*x+e)^2+1)^3+1/
16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*arctan(tan(f*x+e))))
```

Maxima [A]

time = 0.50, size = 623, normalized size = 1.77

$$\frac{\frac{6\left(99a^{2}b^{4}+176ab^{5}+80b^{6}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^{8}+2a^{7}b+a^{6}b^{2}\right)\sqrt{(a+b)b}}+\frac{3\left(5a^{4}b^{2}-8a^{3}b^{3}+17a^{2}b^{4}+116ab^{5}+80b^{6}\right)\tan(fx+e)^{9}+2\left(15a^{5}b+11a^{4}b^{2}-5a^{3}b^{3}+368a^{2}b^{4}+876ab^{5}+480b^{6}\right)\tan(fx+e)^{7}+\left(15a^{6}+86a^{5}b+3a^{4}b^{2}+240a^{3}b^{3}+1982a^{2}b^{4}+3168ab^{5}+1440b^{6}\right)\tan(fx+e)^{5}+2\left(20a^{6}+41a^{5}b-15a^{4}b^{2}+197a^{3}b^{3}+980a^{2}b^{4}+1236ab^{5}+480b^{6}\right)\tan(fx+e)^{3}+3\left(11a^{6}+14a^{5}b-6a^{4}b^{2}+56a^{3}b^{3}+221a^{2}b^{4}+236ab^{5}+80b^{6}\right)\tan(fx+e)}{\left(a^{7}b^{2}+2a^{6}b^{3}+a^{5}b^{4}\right)\tan(fx+e)^{10}+a^{9}+4a^{8}b+6a^{7}b^{2}+4a^{6}b^{3}+a^{5}b^{4}+\left(2a^{8}b+9a^{7}b^{2}+12a^{6}b^{3}+5a^{5}b^{4}\right)\tan(fx+e)^{8}+\left(a^{9}+10a^{8}b+27a^{7}b^{2}+28a^{6}b^{3}+10a^{5}b^{4}\right)\tan(fx+e)^{6}+\left(3a^{9}+18a^{8}b+37a^{7}b^{2}+32a^{6}b^{3}+10a^{5}b^{4}\right)\tan(fx+e)^{4}+\left(3a^{9}+14a^{8}b+24a^{7}b^{2}+18a^{6}b^{3}+5a^{5}b^{4}\right)\tan(fx+e)^{2}}+\frac{3\left(5a^{3}-18a^{2}b+48ab^{2}-160b^{3}\right)(fx+e)}{a^{6}}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 2*a^7*b + a^6*b^2)*sqrt((a + b)*b)) + (3*(5*a^4*b^2 - 8*a^3*b
^3 + 17*a^2*b^4 + 116*a*b^5 + 80*b^6)*tan(f*x + e)^9 + 2*(15*a^5*b + 11*a^4
*b^2 - 5*a^3*b^3 + 368*a^2*b^4 + 876*a*b^5 + 480*b^6)*tan(f*x + e)^7 + (15*
a^6 + 86*a^5*b + 3*a^4*b^2 + 240*a^3*b^3 + 1982*a^2*b^4 + 3168*a*b^5 + 1440
*b^6)*tan(f*x + e)^5 + 2*(20*a^6 + 41*a^5*b - 15*a^4*b^2 + 197*a^3*b^3 + 98
0*a^2*b^4 + 1236*a*b^5 + 480*b^6)*tan(f*x + e)^3 + 3*(11*a^6 + 14*a^5*b - 6
*a^4*b^2 + 56*a^3*b^3 + 221*a^2*b^4 + 236*a*b^5 + 80*b^6)*tan(f*x + e))/((a
^7*b^2 + 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^10 + a^9 + 4*a^8*b + 6*a^7*b^2 +
 4*a^6*b^3 + a^5*b^4 + (2*a^8*b + 9*a^7*b^2 + 12*a^6*b^3 + 5*a^5*b^4)*tan(f
```

```
*x + e)^8 + (a^9 + 10*a^8*b + 27*a^7*b^2 + 28*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^6 + (3*a^9 + 18*a^8*b + 37*a^7*b^2 + 32*a^6*b^3 + 10*a^5*b^4)*tan(f*x
 + e)^4 + (3*a^9 + 14*a^8*b + 24*a^7*b^2 + 18*a^6*b^3 + 5*a^5*b^4)*tan(f*x
+ e)^2) + 3*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*(f*x + e)/a^6)/f
```

Fricas [A]
time = 3.11, size = 1330, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2
*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^
4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b^3
+ 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 + 176*
a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2
*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log
(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 -
 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-
b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^
2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^
3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 26
6*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3
*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b
 + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x +
e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 - 8*a^6*b + 17*a^
5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*x + e)^4 + 6*(5*a
^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*c
os(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*
b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176
*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*
b^6)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (8*(a^7 + 2*a^6*b + a^
5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (
15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 +
2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x
+ e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*co
s(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*
(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6
*b^4)*f)]
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 350, normalized size = 0.99

$$\frac{\frac{6\left(99\,a^{2}b^{4}+176\,ab^{5}+80\,b^{6}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{8}+2\,a^{7}b+a^{6}b^{2}\right)\sqrt{ab+b^{2}}}+\frac{6\left(19\,ab^{5}\tan\left(fx+e\right)^{3}+16\,b^{6}\tan\left(fx+e\right)^{3}+21\,a^{2}b^{4}\tan\left(fx+e\right)+37\,ab^{5}\tan\left(fx+e\right)+16\,b^{6}\tan\left(fx+e\right)\right)}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{2}}+\frac{3\left(5\,a^{3}-18\,a^{2}b+48\,ab^{2}-160\,b^{3}\right)\left(fx+e\right)}{a^{6}}+\frac{15\,a^{2}\tan\left(fx+e\right)^{5}-54\,ab\tan\left(fx+e\right)^{5}+144\,b^{2}\tan\left(fx+e\right)^{5}+40\,a^{2}\tan\left(fx+e\right)^{3}-144\,ab\tan\left(fx+e\right)^{3}+288\,b^{2}\tan\left(fx+e\right)^{3}+33\,a^{2}\tan\left(fx+e\right)-90\,ab\tan\left(fx+e\right)+144\,b^{2}\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}a^{5}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*sq
rt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21*a^
2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7 + 2
*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b + 48*
a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(f*x +
e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(f*x + e
)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x + e) +
144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

Mupad [B]
time = 10.11, size = 2500, normalized size = 7.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(156*a*b^4 + 3*a^4*b + 11*a^5 + 80*b^5 + 65*a^2*b^3 - 9*a^3*
b^2))/(16*a^5*(a + b)) + (tan(e + f*x)^7*(876*a*b^5 + 15*a^5*b + 480*b^6 +
368*a^2*b^4 - 5*a^3*b^3 + 11*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^3
*(1236*a*b^5 + 41*a^5*b + 20*a^6 + 480*b^6 + 980*a^2*b^4 + 197*a^3*b^3 - 15
*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^5*(3168*a*b^5 + 86*a^5*b + 15
*a^6 + 1440*b^6 + 1982*a^2*b^4 + 240*a^3*b^3 + 3*a^4*b^2))/(48*a^5*(a + b)^
2) + (b*tan(e + f*x)^9*(116*a*b^4 + 5*a^4*b + 80*b^5 + 17*a^2*b^3 - 8*a^3*b
^2))/(16*a^5*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^6*(8*a*b + a^2 + 10*b^2)
+ a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2) + b^2*tan(e + f*x)^10 + tan(e
+ f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e + f*x)^4*(12*a*b + 3*a^2 + 10*b^2)
```

)) - (atan(-((((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i)*1i)/(32*a^6) - (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a
^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)
/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2)
 + (tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7
 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a
^17*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2))
)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19
9680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^
9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 2
5*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*
(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*1i)/(32*a^6))/((3350*a*b^14 + 1
000*b^15 + (7315*a^2*b^13)/2 + (4597*a^3*b^12)/4 - (4325*a^4*b^11)/32 + (10
281*a^5*b^10)/128 + (8973*a^6*b^9)/512 - (25551*a^7*b^8)/1024 + (4235*a^8*b
^7)/512 - (2475*a^9*b^6)/1024)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6
*a^17*b^2) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*
b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4
*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e + f*x)*(a*b^2
*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a^13*b^6 + 1638
4*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(4096*a^6*(4*
a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18
i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a*b^12 + 51200*b^1
3 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a
^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^
13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^2*48i - a^2*b*18i
+ a^3*5i - b^3*160i))/(32*a^6) + (((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14
*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4
- (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) +
(tan(e + f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 +
9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17
*b^2))/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(
a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) + (tan(e + f*x)*(19968
0*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 -
 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a
^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*
b^2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6)))*(a*b^2*48i - a^2*b*18i

+ a^3*5i - b^3*160i)*1i)/(16*a^6*f) - (atan((((-b^7*(a + b)^5)^(1/2)*((tan(e + f*x)*(199680*a*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 36*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)) + (((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 + (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^19*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) + (tan(e + f*x)*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2)*(2048*a^12*b^7 + 9216*a^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)*(5*a^10*b + a^11 + a^6*b^5 + 5*a^7*b^4 + 10*a^8*b^3 + 10*a^9*b^2)))*(-b^7*(a + b)^5)^(1/2)*(11*a*b + (99*a^2)/16 + 5*b^2))/(5*a^10*b + a^11 +...

3.219 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{x}{a^4}-\frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d}-\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3}-\frac{}{24a^2}$$

[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctan(b^(1/2)*tan(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^3-1/24*b*(11*a+6*b)*tan(d*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)

Rubi [A]
time = 0.24, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\frac{x}{a^4}-\frac{b(11a+6b)\tan(c+dx)}{24a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^2}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\tan(c+dx)}{6ad(a+b)\left(a+b\tan^2(c+dx)+b\right)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x]^2)^(-4),x]

[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(6*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tan[c + d*x])/(24*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tan[c + d*x])/(16*a^3*(a + b)^3*d*(a + b + b*Tan[c + d*x]^2))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c - a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\,(a+b+b\tan^2(c+dx))^3} + \frac{\text{Subst}\left(\int \frac{6a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(c+\right.}{6a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\,(a+b+b\tan^2(c+dx))^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\,(a+b+b\tan^2(c+dx)}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\,(a+b+b\tan^2(c+dx))^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\,(a+b+b\tan^2(c+dx)}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\,(a+b+b\tan^2(c+dx))^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\,(a+b+b\tan^2(c+dx)}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\,(35a^3+70a^2b+56ab^2+16b^3)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b}{6a(a+b)d\,(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.93, size = 1411, normalized size = 6.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*S
ec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c]
- I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[
4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(256*a
^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[Sec[d
*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2)*Sin[
2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x]) - 2*b*S
in[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] -
 I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b + a*Cos[
2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 3168*a^5*b*d*
x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2*c] + 13248
*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*Cos[2*c] + 3
60*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b^2*d*x*Cos[2*
d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d*x] + 1152*a*b
^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*b*d*x*Cos[4*c +
```

```
 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cos[4*c + 2*
d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*Cos[4*c + 2*d*x]
+ 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c + 4*d*x] + 1296*a^4*
b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*d*x] + 288*a^2*b^4*
d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] + 720*a^5*b*d*x*Cos[6*c
 + 4*d*x] + 1296*a^4*b^2*d*x*Cos[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[6*c +
4*d*x] + 288*a^2*b^4*d*x*Cos[6*c + 4*d*x] + 24*a^6*d*x*Cos[4*c + 6*d*x] + 7
2*a^5*b*d*x*Cos[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cos[4*c + 6*d*x] + 24*a^3*b^3
*d*x*Cos[4*c + 6*d*x] + 24*a^6*d*x*Cos[8*c + 6*d*x] + 72*a^5*b*d*x*Cos[8*c
+ 6*d*x] + 72*a^4*b^2*d*x*Cos[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cos[8*c + 6*d*x
] + 870*a^5*b*Sin[2*c] + 4292*a^4*b^2*Sin[2*c] + 8792*a^3*b^3*Sin[2*c] + 99
36*a^2*b^4*Sin[2*c] + 5824*a*b^5*Sin[2*c] + 1408*b^6*Sin[2*c] - 870*a^5*b*S
in[2*d*x] - 3792*a^4*b^2*Sin[2*d*x] - 6432*a^3*b^3*Sin[2*d*x] - 4608*a^2*b^
4*Sin[2*d*x] - 1248*a*b^5*Sin[2*d*x] + 435*a^5*b*Sin[4*c + 2*d*x] + 2124*a^
4*b^2*Sin[4*c + 2*d*x] + 3972*a^3*b^3*Sin[4*c + 2*d*x] + 3072*a^2*b^4*Sin[4
*c + 2*d*x] + 864*a*b^5*Sin[4*c + 2*d*x] - 435*a^5*b*Sin[2*c + 4*d*x] - 137
4*a^4*b^2*Sin[2*c + 4*d*x] - 1248*a^3*b^3*Sin[2*c + 4*d*x] - 384*a^2*b^4*Si
n[2*c + 4*d*x] + 87*a^5*b*Sin[6*c + 4*d*x] + 366*a^4*b^2*Sin[6*c + 4*d*x] +
 408*a^3*b^3*Sin[6*c + 4*d*x] + 144*a^2*b^4*Sin[6*c + 4*d*x] - 87*a^5*b*Sin
[4*c + 6*d*x] - 116*a^4*b^2*Sin[4*c + 6*d*x] - 44*a^3*b^3*Sin[4*c + 6*d*x])
)/(3072*a^4*(a + b)^3*d*(a + b*Sec[c + d*x]^2)^4)
```

Maple [A]
time = 0.40, size = 229, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\left(\tan^5(dx+c)\right)}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\left(\tan^3(dx+c)\right)}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\left(\tan^2(dx+c)\right)\right)^3}\right.}{a^4}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\left(\tan^5(dx+c)\right)}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\left(\tan^3(dx+c)\right)}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\left(\tan^2(dx+c)\right)\right)^3}\right.}{a^4}}{d}$
risch	$\frac{x}{a^4}-\frac{ib\left(870a^5e^{4i(dx+c)}+435a^5e^{2i(dx+c)}+87a^5e^{10i(dx+c)}+435a^5e^{8i(dx+c)}+870a^5e^{6i(dx+c)}+1408b^5e^{6i(dx+c)}+1248a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*arctan(tan(d*x+c))-b/a^4*((1/16*b^2*a*(19*a^2+22*a*b+8*b^2)/(a^3
```

```
+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^5+1/6*(17*a^2+18*a*b+6*b^2)*a*b/(a^2+2*a*b
+b^2)*tan(d*x+c)^3+1/16*(29*a^2+26*a*b+8*b^2)*a/(a+b)*tan(d*x+c))/(a+b+b*ta
n(d*x+c)^2)^3+1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b^2+b
^3)/((a+b)*b)^(1/2)*arctan(b*tan(d*x+c)/((a+b)*b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(188) = 376$.
time = 0.51, size = 401, normalized size = 1.97

$$-\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{(a+b)b}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{(a+b)b}}+\frac{3\,(19\,a^2b^3+22\,ab^4+8\,b^5)\tan(dx+c)^5+8\,(17\,a^3b^2+35\,a^2b^3+24\,ab^4+6\,b^5)\tan(dx+c)^3+3\,(29\,a^4b+84\,a^3b^2+89\,a^2b^3+42\,ab^4+8\,b^5)\tan(dx+c)}{a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6+(a^6b^3+3\,a^5b^4+3\,a^4b^5+a^3b^6)\tan(dx+c)^6+3\,(a^7b^2+4\,a^6b^3+6\,a^5b^4+4\,a^4b^5+a^3b^6)\tan(dx+c)^4+3\,(a^8b+5\,a^7b^2+10\,a^6b^3+10\,a^5b^4+5\,a^4b^5+a^3b^6)\tan(dx+c)^2}-\frac{48\,(dx+c)}{a^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(b*tan(d*x + c)/
sqrt((a + b)*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)) +
(3*(19*a^2*b^3 + 22*a*b^4 + 8*b^5)*tan(d*x + c)^5 + 8*(17*a^3*b^2 + 35*a^2*
b^3 + 24*a*b^4 + 6*b^5)*tan(d*x + c)^3 + 3*(29*a^4*b + 84*a^3*b^2 + 89*a^2*
b^3 + 42*a*b^4 + 8*b^5)*tan(d*x + c))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*
b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6 + (a^6*b^3 + 3*a^5*b^4 + 3*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^6 + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*tan(d*x + c)^4 + 3*(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a^5*b^4 +
 5*a^4*b^5 + a^3*b^6)*tan(d*x + c)^2) - 48*(d*x + c)/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(188) = 376$.
time = 2.48, size = 1323, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b^2
 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3
)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*
b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 +
70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))*log(((
a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c)^2 + 4*
((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*sqrt(-b/(
a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^2 + b^
2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b
^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 +
```

```
 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^
3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x
 + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3
 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b
^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*(
(35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 +
 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 1
6*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*
b^5)*cos(d*x + c)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(d*x + c)^2 -
 b)*sqrt(b/(a + b))/(b*cos(d*x + c)*sin(d*x + c))) - 2*((87*a^5*b + 116*a^4
*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4
)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(
d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9
*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7
*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5
*b^5 + a^4*b^6)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 324, normalized size = 1.59

$$-\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab+b^2}}+\frac{57\,a^2b^3\tan(dx+c)^5+66\,ab^4\tan(dx+c)^5+24\,b^5\tan(dx+c)^5+136\,a^3b^2\tan(dx+c)^3+280\,a^2b^3\tan(dx+c)^3+192\,ab^4\tan(dx+c)^3+48\,b^5\tan(dx+c)^3+87\,a^4b\tan(dx+c)+252\,a^3b^2\tan(dx+c)+267\,a^2b^3\tan(dx+c)+126\,ab^4\tan(dx+c)+24\,b^5\tan(dx+c)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\left(b\tan(dx+c)^2+a+b\right)^3}-\frac{48\,(dx+c)}{a^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/pi
 + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b +
3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*a*b
^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3 + 28
0*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x + c)^3
 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan(d*x +
 c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b + 3*a^4
*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

Mupad [B]
time = 9.46, size = 2500, normalized size = 12.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cos(c + d*x)^2)^4,x)

[Out] atan((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 - ((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/(2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4)/((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + (((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))*1i)/(2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3)*1i)/(256*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)))/a^4 + ((25*a*b^7)/4 + b^8 + (131*a^2*b^6)/8 + (721*a^3*b^5)/32 + (525*a^4*b^4)/32 + (66

5*a^5*b^3)/128)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)))/(a^4*d) - ((tan(c + d*x)^3*(18*a*b^3 + 6*b^4 + 17*a^2*b^2))/(6*a^3*(a + b)^2) + (tan(c + d*x)^5*(22*a*b^4 + 8*b^5 + 19*a^2*b^3))/(16*a^3*(a + b)^3) + (tan(c + d*x)*(26*a*b^2 + 29*a^2*b + 8*b^3))/(16*a^3*(a + b)))/(d*(tan(c + d*x)^4*(3*a*b^2 + 3*b^3) + 3*a*b^2 + 3*a^2*b + tan(c + d*x)^2*(6*a*b^2 + 3*a^2*b + 3*b^3) + a^3 + b^3 + b^3*tan(c + d*x)^6)) + (atan((((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)) - ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2) - (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(4096*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3))/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)))*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*1i)/(32*(7*a^10*b + a^11 + a^4*b^7 + 7*a^5*b^6 + 21*a^6*b^5 + 35*a^7*b^4 + 35*a^8*b^3 + 21*a^9*b^2)) + ((-b*(a + b)^7)^(1/2)*((tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(128*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2)) + ((-b*(a + b)^7)^(1/2)*((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)/(6*a^14*b + a^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2) + (tan(c + d*x)*(-b*(a + b)^7)^(1/2)*(56*a*b^2 + 70*a^2*b + 35*a^3 + 16*b^3)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + ...

3.220 $\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^3\tan^3(c+dx)\sqrt{}}{4}$$

```
[Out] -a^3*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^3*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d-1
/6*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] -((a^3*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^3*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^3*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d) - (a^3*Tan[c + d*x]^5*Sqrt[-(a*Tan[c + d*x]^2)])/
(6*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx &= \int \left(-a\tan^2(c+dx)\right)^{7/2}\,dx\\ &= -\left(\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^7(c+dx)\,dx\right)\\ &= -\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\Big(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\\ &= \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}\\ &= -\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}\\ &= -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a^3\tan(c+dx)\sqrt{-}}{2d}\end{aligned}$$

Mathematica [A]
time = 2.36, size = 70, normalized size = 0.52

$$\frac{\cot^7(c+dx)\left(-a\tan^2(c+dx)\right)^{7/2}\left(12\log(\cos(c+dx))+6\tan^2(c+dx)-3\tan^4(c+dx)+2\tan^6(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] (Cot[c + d*x]^7*(-(a*Tan[c + d*x]^2))^(7/2)*(12*Log[Cos[c + d*x]] + 6*Tan[c
 + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A]
time = 0.71, size = 167, normalized size = 1.25

method	result

default	$\frac{\left(12\left(\cos^6(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+12\left(\cos^6(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-12\left(\cos^6(dx+c)\right)\ln\left(\frac{2}{\cos(dx+c)+1}\right)-11}{12d\sin(dx+c)^7}$
risch	$\frac{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{2ia^3\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(9\,e^{10i(dx}\right.}{3\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/d*(12*cos(d*x+c)^6*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*cos(d*x
+c)^6*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-12*cos(d*x+c)^6*ln(2/(cos(
d*x+c)+1))-11*cos(d*x+c)^6+18*cos(d*x+c)^4-9*cos(d*x+c)^2+2)*cos(d*x+c)*(-s
in(d*x+c)^2*a/cos(d*x+c)^2)^(7/2)/sin(d*x+c)^7
```

Maxima [A]
time = 0.49, size = 81, normalized size = 0.60

$$-\frac{2\sqrt{-a}\,a^3\tan\left(dx+c\right)^6-3\sqrt{-a}\,a^3\tan\left(dx+c\right)^4+6\sqrt{-a}\,a^3\tan\left(dx+c\right)^2-6\sqrt{-a}\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*sq
rt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.25, size = 100, normalized size = 0.75

$$-\frac{\left(12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)+18\,a^3\cos\left(dx+c\right)^4-9\,a^3\cos\left(dx+c\right)^2+2\,a^3\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{12\,d\cos\left(dx+c\right)^5\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 - 9
*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d
*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.70, size = 217, normalized size = 1.62

$$-\frac{6\sqrt{-a}\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-6\sqrt{-a}\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{11\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)^3\sqrt{-a}\,a^3-90\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)^2\sqrt{-a}\,a^3+276\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)\sqrt{-a}\,a^3-408\sqrt{-a}\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(-a)
*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a^
3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^3 -
408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^3
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^2}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(7/2), x)
```

3.221 $\int (a - a\sec^2(c+dx))^{5/2}\,dx$

Optimal. Leaf size=101

$$-\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^2\tan^3(c+dx)\sqrt{\ldots}}{4\ldots}$$

```
[Out] -a^2*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^2*(-a*tan(d*
x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^2*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -((a^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^2*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^2*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(-a\tan^2(c+dx)\right)^{5/2}\,dx$$
$$=\left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx$$
$$=\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)$$
$$=-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}$$
$$=-\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a^2\tan(c+dx)\sqrt{-}}{2d}$$

Mathematica [A]
time = 0.61, size = 60, normalized size = 0.59

$$-\frac{\cot^5(c+dx)\left(-a\tan^2(c+dx)\right)^{5/2}\left(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
[Out] -1/4*(Cot[c + d*x]^5*(-(a*Tan[c + d*x]^2))^(5/2)*(4*Log[Cos[c + d*x]] + 2*T
an[c + d*x]^2 - Tan[c + d*x]^4))/d
```

Maple [A]
time = 0.18, size = 157, normalized size = 1.55

method	result
default	$-\frac{\left(4\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)+4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)-4\ln\left(\frac{2}{\cos(dx+c)+1}\right)\left(\cos^4(dx+c)\right)-3}{4d\sin(dx+c)^5}$
risch	$\frac{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ia^2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{6i(dx-}\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+4*ln(-(-1+c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-4*ln(2/(cos(d*x+c)+1))*cos(d
*x+c)^4-3*cos(d*x+c)^4+4*cos(d*x+c)^2-1)*cos(d*x+c)*(-sin(d*x+c)^2*a/cos(d*
x+c)^2)^(5/2)/sin(d*x+c)^5
```

Maxima [A]
time = 0.48, size = 62, normalized size = 0.61

$$\frac{\sqrt{-a}\,a^2\tan\left(dx+c\right)^4-2\,\sqrt{-a}\,a^2\tan\left(dx+c\right)^2+2\,\sqrt{-a}\,a^2\log\left(\tan\left(dx+c\right)^2+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(-
a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.21, size = 87, normalized size = 0.86

$$-\frac{\left(4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{4\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2)
*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(5/2), x)
```

Giac [A]
time = 0.58, size = 182, normalized size = 1.80

$$-\frac{2\sqrt{-a}\,a^{2}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}+2\right)-2\sqrt{-a}\,a^{2}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-2\right)+\frac{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}\right)^{2}\sqrt{-a}\,a^{2}-20\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}\right)\sqrt{-a}\,a^{2}+44\sqrt{-a}\,a^{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-2\right)^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
+ 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a
^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^2 +
44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^2)
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^{2}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(5/2), x)
```

3.222 $\int (a-a\sec^2(c+dx))^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}$$

```
[Out] -a*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a*(-a*tan(d*x+c)
^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a*Tan[
c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\int \left(-a\tan^2(c+dx)\right)^{3/2}\,dx$$
$$=-\left(\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx\right)$$
$$=-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)$$
$$=-\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a\tan(c+dx)\sqrt{-a\,t}}{2d}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.75

$$\frac{\cot^3(c+dx)\left(-a\tan^2(c+dx)\right)^{3/2}\left(2\log(\cos(c+dx))+\tan^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
[Out] (Cot[c + d*x]^3*(-(a*Tan[c + d*x]^2))^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c +
d*x]^2))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(58) = 116$.
time = 0.14, size = 147, normalized size = 2.30

method	result
default	$\frac{\left(2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))+2\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-2\ln\left(\frac{2}{\cos(dx+c)+1}\right)(\cos^2(dx+c))-(\cos}{2d\sin(dx+c)^3}$
risch	$\frac{a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{2ia\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+2*ln((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-2*ln(2/(cos(d*x+c)+1))*cos(d*
x+c)^2-cos(d*x+c)^2+1)*cos(d*x+c)*(-sin(d*x+c)^2*a/cos(d*x+c)^2)^(3/2)/sin(
d*x+c)^3
```

Maxima [A]
time = 0.48, size = 40, normalized size = 0.62

$$-\frac{\sqrt{-a}\,a\tan\left(dx+c\right)^2-\sqrt{-a}\,a\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.68, size = 68, normalized size = 1.06

$$-\frac{\left(2\,a\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 - a
)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.
time = 0.53, size = 137, normalized size = 2.14

$$-\frac{\sqrt{-a}\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-\sqrt{-a}\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)\sqrt{-a}\,a-6\,\sqrt{-a}\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2)
 - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2) +
((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt(-a
)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(3/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(3/2), x)
```

3.223 $\int \sqrt{a - a\sec^2(c + dx)}\, dx$

Optimal. Leaf size=33

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4206, 3739, 3556}

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = \int \sqrt{-a\tan^2(c+dx)}\,dx$$
$$= \left(\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 1.00

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(31) = 62$.
time = 0.14, size = 109, normalized size = 3.30

method	result
default	$\frac{\left(\ln\left(\frac{2}{\cos(dx+c)+1}\right)-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\right)\cos(dx+c)\sqrt{-\frac{(\sin^2(dx+c))a}{\cos(dx+c)^2}}}{d\sin(dx+c)}$
risch	$\frac{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)x}{e^{2i(dx+c)}-1}-\frac{2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{i\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)\,\mathrm{l}}{\left(e^{2i(dx+c)}-1\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(ln(2/(cos(d*x+c)+1))-ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-ln((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(-sin(d*x+c)^2*a/cos(d*x+c)^
2)^(1/2)/sin(d*x+c)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.64

$$\frac{\sqrt{-a}\,\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

Fricas [A]
time = 3.22, size = 53, normalized size = 1.61

$$-\frac{\sqrt{\frac{a\cos\left(dx+c\right)^{2}-a}{\cos\left(dx+c\right)^{2}}}\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c)
)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec^{2}\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.
time = 0.49, size = 141, normalized size = 4.27

$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{-a}\,\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x
 + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)^3
 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1/2*
d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x)^2)^(1/2),x)
```

```
[Out] int((a - a/cos(c + d*x)^2)^(1/2), x)
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] ln(sin(d*x+c))*tan(d*x+c)/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4206, 3739, 3556}

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\tan^2(c+dx)}}\,dx$$
$$= \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 1.25

$$\frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c +
d*x]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.
time = 0.15, size = 75, normalized size = 2.34

method	result	size
default	$\frac{\left(-\ln\left(\frac{2}{\cos(dx+c)+1}\right)+\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)\sin(dx+c)}{d\sqrt{-\frac{(\sin^2(dx+c))a}{\cos(dx+c)^2}}\,\cos(dx+c)}$	75
risch	$\frac{(e^{2i(dx+c)}-1)x}{\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(dx+c)}+1)}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(dx+c)}+1)d}-\frac{i(e^{2i(dx+c)}-1)\ln(e^{2i(dx+c)}-1)}{\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(dx+c)}+1)d}$	194

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-ln(2/(cos(d*x+c)+1))+ln(-(-1+cos(d*x+c))/sin(d*x+c)))*sin(d*x+c)/(-si
n(d*x+c)^2*a/cos(d*x+c)^2)^(1/2)/cos(d*x+c)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.16

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}}-\frac{2\,\log(\tan(dx+c))}{\sqrt{-a}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

Fricas [A]
time = 2.06, size = 56, normalized size = 1.75

$$-\frac{\sqrt{\frac{a\cos\left(dx+c\right)^{2}-a}{\cos\left(dx+c\right)^{2}}}\cos\left(dx+c\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x +
 c))/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a\sec^{2}\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
[Out] Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(1/2),x)
[Out] int(1/(a - a/cos(c + d*x)^2)^(1/2), x)
```

3.225 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a/d/(-
a*tan(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4206, 3739, 3554, 3556}

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(2*a*d*Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{3/2}}\,dx\\ &= -\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}}\\ &= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}}\\ &= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 57, normalized size = 0.85

$$-\frac{\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)\tan^3(c+dx)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
[Out] -1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
d*x]^3)/(d*(-(a*Tan[c + d*x]^2))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(61) = 122.
time = 0.13, size = 139, normalized size = 2.07

method	result
default	$-\frac{\left(4\ln\left(\frac{2}{\cos(dx+c)+1}\right)\left(\cos^2(dx+c)\right)-4\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\cos^2(dx+c)-4\ln\left(\frac{2}{\cos(dx+c)+1}\right)+4\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\right.}{4d\cos(dx+c)^3\left(-\frac{\left(\sin^2(dx+c)\right)a}{\cos(dx+c)^2}\right)^{\frac{3}{2}}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{2ie^{2i(dx+c)}}{a\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-\right.}{\left(e^{2i(dx+c)}+1\right.}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(4*ln(2/(cos(d*x+c)+1))*cos(d*x+c)^2-4*cos(d*x+c)^2*ln(-(-1+cos(d*x+
c))/sin(d*x+c))+cos(d*x+c)^2-4*ln(2/(cos(d*x+c)+1))+4*ln(-(-1+cos(d*x+c))/s
in(d*x+c))+1)*sin(d*x+c)/cos(d*x+c)^3/(-sin(d*x+c)^2*a/cos(d*x+c)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 60, normalized size = 0.90

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}\,a}-\frac{2\log(\tan(dx+c))}{\sqrt{-a}\,a}+\frac{\sqrt{-a}}{a^2\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)*
a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

Fricas [A]
time = 2.42, size = 94, normalized size = 1.40

$$-\frac{\left(2\left(\cos\left(dx+c\right)^3-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x + c
))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a^2
*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)
```

[Out] Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)

Giac [A]
time = 0.58, size = 108, normalized size = 1.61

$$-\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\sqrt{-a}\,a}-\frac{8\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}\,a}+\frac{4\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}\,a}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{\sqrt{-a}\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")

[Out] -1/8*(tan(1/2*d*x + 1/2*c)^2/(sqrt(-a)*a) - 8*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a) + 4*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a) - (4*tan(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*tan(1/2*d*x + 1/2*c)^2))/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-\frac{a}{{\cos\left(c+d\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a - a/cos(c + d*x)^2)^(3/2),x)

[Out] int(1/(a - a/cos(c + d*x)^2)^(3/2), x)

3.226 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^2/d/(-a*tan
(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^2*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^2*d*Sqrt[-
(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{5/2}}\,dx$$
$$= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+$$
$$= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan($$

Mathematica [**A**]
time = 0.29, size = 69, normalized size = 0.69

$$\frac{\left(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx))\right)\tan^5(c+dx)}{4d\left(-a\tan^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d
*x]])*Tan[c + d*x]^5)/(4*d*(-(a*Tan[c + d*x]^2))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(90) = 180.
time = 0.14, size = 203, normalized size = 2.03

method	result

default	$\frac{\left(32\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-32\ln\left(\frac{2}{\cos(dx+c)+1}\right)\left(\cos^4(dx+c)\right)-13\left(\cos^4(dx+c)\right)-64\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\right.}{32d\cos(dx+c)^5\left(-\frac{\left(\sin^2(dx\right.}{\cos(dx+}\right.}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i\left(e^{6i(dx+c)}-e^{4i(dx+c)}+e^{2i(dx+\right.}{a^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(d\right.}{\left(e^{2i(dx\right.}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(32*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(d*x+c))-32*ln(2/(cos(d*x+c)
+1))*cos(d*x+c)^4-13*cos(d*x+c)^4-64*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d
*x+c))+64*ln(2/(cos(d*x+c)+1))*cos(d*x+c)^2-6*cos(d*x+c)^2+32*ln(-(-1+cos(d
*x+c))/sin(d*x+c))-32*ln(2/(cos(d*x+c)+1))+11)*sin(d*x+c)/cos(d*x+c)^5/(-si
n(d*x+c)^2*a/cos(d*x+c)^2)^(5/2)
```

Maxima [A]
time = 0.50, size = 79, normalized size = 0.79

$$-\frac{\frac{2\,\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}\,a^2}-\frac{4\,\log(\tan(dx+c))}{\sqrt{-a}\,a^2}+\frac{2\,\sqrt{-a}\,\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt(
-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d
```

Fricas [A]
time = 3.01, size = 125, normalized size = 1.25

$$\frac{\left(4\,\cos\left(dx+c\right)^3-4\,\left(\cos\left(dx+c\right)^5-2\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-3\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{4\,\left(a^3d\cos\left(dx+c\right)^4-2\,a^3d\cos\left(dx+c\right)^2+a^3d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)
)*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d
*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)
```

Giac [A]
time = 0.71, size = 147, normalized size = 1.47

$$\frac{\frac{64\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}\,a^2}-\frac{32\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}\,a^2}-\frac{\sqrt{-a}\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,\sqrt{-a}\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^5}+\frac{48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}{\sqrt{-a}\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2) - 32*log(tan(1/2*d*
x + 1/2*c)^2)/(sqrt(-a)*a^2) - (sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*sq
rt(-a)*a^2*tan(1/2*d*x + 1/2*c)^2)/a^5 + (48*tan(1/2*d*x + 1/2*c)^4 - 12*ta
n(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-\frac{a}{{\cos\left(c+d\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(5/2), x)
```

3.227 $\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^3/d/(-a*tan
(d*x+c)^2)^(1/2)+1/6*cot(d*x+c)^5/a^3/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+
c))*tan(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4206, 3739, 3554, 3556}

$$\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^3*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + Cot[c + d*x]^5/(6*a^3*d*Sqrt[-(a*Tan[c + d*x]^
2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(-a\tan^2(c+dx))^{7/2}}\,dx$$
$$= -\frac{\tan(c+dx)\int \cot^7(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$= \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c}{a^3\sqrt{-a\tan^2(c}$$
$$= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+d}$$
$$= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+d}$$

Mathematica [A]
time = 0.38, size = 79, normalized size = 0.59

$$-\frac{\left(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx))\right)\tan^7(c+dx)}{12d\left(-a\tan^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] -1/12*((6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos
[c + d*x]] + 12*Log[Tan[c + d*x]])*Tan[c + d*x]^7)/(d*(-(a*Tan[c + d*x]^2))
^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(119) = 238.
time = 0.17, size = 265, normalized size = 1.99

method	result
default	$\frac{\left(48\left(\cos^6(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-48\left(\cos^6(dx+c)\right)\ln\left(\frac{2}{\cos(dx+c)+1}\right)-25\left(\cos^6(dx+c)\right)-144\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cdots\right.}{}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{2i\left(9\,e^{10i(dx+c)}-18\,e^{8i(dx+c)}+34\,e^{6i(dx+c)}\cdots\right.}{3a^3\left(e^{2i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(48*cos(d*x+c)^6*ln(-(-1+cos(d*x+c))/sin(d*x+c))-48*cos(d*x+c)^6*ln(
2/(cos(d*x+c)+1))-25*cos(d*x+c)^6-144*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(
d*x+c))+144*ln(2/(cos(d*x+c)+1))*cos(d*x+c)^4+3*cos(d*x+c)^4+144*cos(d*x+c)
^2*ln(-(-1+cos(d*x+c))/sin(d*x+c))-144*ln(2/(cos(d*x+c)+1))*cos(d*x+c)^2+33
*cos(d*x+c)^2-48*ln(-(-1+cos(d*x+c))/sin(d*x+c))+48*ln(2/(cos(d*x+c)+1))-19
)*sin(d*x+c)/cos(d*x+c)^7/(-sin(d*x+c)^2*a/cos(d*x+c)^2)^(7/2)
```

Maxima [A]
time = 0.50, size = 94, normalized size = 0.71

$$-\frac{\frac{6\,\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}\,a^3}-\frac{12\,\log(\tan(dx+c))}{\sqrt{-a}\,a^3}+\frac{6\,\sqrt{-a}\,\tan(dx+c)^4-3\,\sqrt{-a}\,\tan(dx+c)^2+2\,\sqrt{-a}}{a^4\tan(dx+c)^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sqr
t(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*sqr
t(-a))/(a^4*tan(d*x + c)^6))/d
```

Fricas [A]
time = 3.34, size = 162, normalized size = 1.22

$$\frac{\left(18\,\cos\left(dx+c\right)^5-27\,\cos\left(dx+c\right)^3-12\,\left(\cos\left(dx+c\right)^7-3\,\cos\left(dx+c\right)^5+3\,\cos\left(dx+c\right)^3-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+11\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)^2-a}{\cos\left(dx+c\right)^2}}}{12\,\left(a^4d\cos\left(dx+c\right)^6-3\,a^4d\cos\left(dx+c\right)^4+3\,a^4d\cos\left(dx+c\right)^2-a^4d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d*
x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*cos(
```

```
d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x + c)^
6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-7/2), x)
```

Giac [A]
time = 0.68, size = 171, normalized size = 1.29

$$-\frac{\frac{384\sqrt{-a}\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a^4}+\frac{192\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}\,a^3}-\frac{352\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-87\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{\sqrt{-a}\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}+\frac{a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-12\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+87\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\sqrt{-a}\,a^{10}}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/384*(384*sqrt(-a)*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^4 + 192*log(tan(1/2*
d*x + 1/2*c)^2)/(sqrt(-a)*a^3) - (352*tan(1/2*d*x + 1/2*c)^6 - 87*tan(1/2*d
*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a^3*tan(1/2*d*x +
1/2*c)^6) + (a^7*tan(1/2*d*x + 1/2*c)^6 - 12*a^7*tan(1/2*d*x + 1/2*c)^4 + 8
7*a^7*tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^10))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-\frac{a}{{\cos\left(c+d\,x\right)}^2}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x)^2)^(7/2), x)
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=372

$$-\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15b^2f}+\frac{(2a^2-3ab-8b^2)\,\sqrt{\cos^2(e+fx)}$$

```
[Out] -1/15*(2*a^2-3*a*b-8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b^2/f+1/15*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^2/f/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)-1/15*(a-8*b)*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1
/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*si
n(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/15*(a+4*b)*sec(f*x+e)*(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/b/f+1/5*sec(f*x+e)^3*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.39, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{15b^2f}-\frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15bf\,(-a\sin^2(e+fx)+a+b)}+\frac{(a+4b)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{15bf}+\frac{\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/15*((2*a^2 - 3*a*b - 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)])/(b^2*f) + ((2*a^2 - 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)])/(15*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b
)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*S
qrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)])/(15*b*f*(a + b - a*Sin[e + f*x]^2)) + ((a + 4*b)*Sec[e + f*x]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*b*f) + (Sec[
e + f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5
*f)
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
```

```
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a\,(1-x^2}}{(1-x^2)^{7/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{7/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e}{5f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a}}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a}}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a}}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{(2a^2-3ab-8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a}}{15b^2f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]

time = 29.02, size = 0, normalized size = 0.00

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 1.31, size = 6560, normalized size = 17.63

method	result	size
default	Expression too large to display	6560

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\sec^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\cos\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=288

$$\frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3bf}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,E\big(\mathrm{ArcSin}(\sin(e+f}{3bf\sqrt{1-}}$$

```
[Out] 1/3*(a+2*b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f-1/3*(a
+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+2/3*(a+b
)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x
+e)^2)+1/3*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/
f
```

Rubi [A]
time = 0.29, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,F\big(\mathrm{ArcSin}(\sin(e+fx))\,\big|\,\frac{a}{a+b}\big)}{3f\,(-a\sin^2(e+fx)+a+b)}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,E\big(\mathrm{ArcSin}(\sin(e+fx))\,\big|\,\frac{a}{a+b}\big)}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{3bf}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3
*b*f) - ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(
a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*b*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2)) +
(Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])
/(3*f)
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a\,(1-x^2}}{(1-x^2)^{5/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{5/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{3bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{3bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{3bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{3bf\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]
time = 12.15, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 4739, normalized size = 16.45

method	result	size
default	Expression too large to display	4739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-4*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2*b+2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^3*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-4*cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a^2*b+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a*b^2+6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^2*b+6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+8*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^(3/2)*b^(3/2)-4*I*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^(5/2)*b^(1/2)-8*I*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
```

```
))^(1/2)*a^(3/2)*b^(3/2)-4*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^(3/2)*b^(3/2)-8*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^(1/2)*b^(5/2)+4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^(1/2)*b^(5/2)+4*I*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^(5/2)*b^(1/2)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b
^2-5*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a*b^2-4*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*b-5*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-4
*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*I*sin(f*x+e)*cos(f*x+e)^3*a^(5/2)*b^(1/2)*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*I*sin(f*x+e)*cos(f*x+e)^3*
a^(3/2)*b^(3/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+co
s(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-4*I*sin(f*x+
e)*cos(f*x+e)^3*a^(1/2)*b^(5/2)*2^(1/2)*((I*cos...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Fricas [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\sec^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f}-\frac{\sqrt{\cos^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-EllipticE(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+(a+b)*EllipticF(sin(f*x+e),(a/(a+b))
^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a
*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.22, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 423, 12, 507, 437, 435, 432, 430}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{f\left(-a\sin^2(e+fx)+a+b\right)}-\frac{\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f - (Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{1-x^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x}}{(1-x^2)^{3/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{3/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{(a}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{(\sqrt{}}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{(\sqrt{}}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\sqrt{}}{}$$

Mathematica [F]
time = 12.66, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 3454, normalized size = 15.84

method	result	size
default	Expression too large to display	3454

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(4*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^
(1/2)+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2
)-4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)+
2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x+e)^2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f
*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*sin(f*x+e)*cos(f*x+e)^2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b-2*sin(f*x+e)*cos(f*x+
e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*((2*I*a^(1/2)*b^(
```

```
1/2)+a-b)/(a+b))^(1/2)*b^2-4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1
/2)*b^(3/2)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*I*sin(f*x+e)*co
s(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I
*sin(f*x+e)*cos(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))+2*I*sin(f*x+e)*cos(f*x+e)^2*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*sin(f*x+e)*cos(f*x+e)^2*a^(1/2)*b^(3/2)*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,\sec\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{\cos^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 437, 435}

$$\frac{\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e
 + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a\,(1-x^2}}{\sqrt{1-x^2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}\\
&= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}\\
&= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\right)}{f\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin}{}}}\\
&= \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{}}{f\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}
\end{aligned}$$

Mathematica [A]

time = 0.30, size = 69, normalized size = 0.86

$$\frac{\sqrt{2}\,\cos(e+fx)E\left(e+fx\middle|\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 3408, normalized size = 42.60

method	result	size
default	Expression too large to display	3408

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
E((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-sin(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+4*I*cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)+4*I*cos(f*x+e)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2)-4*I*cos(f*x+e)^2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)+2*cos(f*x+e)^3*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-
2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(
f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+co
```

s(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+
e)-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)*b
^2+2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b+2*I*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip
ticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f
*x+e)+2*I*sin(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-4*I*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b+2*I*sin(f*x+e)*cos(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))+2*I*sin(f*x+e)*cos(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\cos{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\big(\mathrm{ArcSin}(\sin(e\cdots}{3af\sqrt{1\cdots}}$$

```
[Out] 1/3*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+1/3
*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*
sin(f*x+e)^2)
```

Rubi [A]
time = 0.22, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4233, 1985, 1986, 428, 538, 437, 435, 432, 430}

$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3af\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])/(3*f) + ((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*a*f*Sqrt[1 -
 (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[A
rcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*f*(a + b - a*Sin[e + f*x]^2
))
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int (1-x^2)\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{b+\cdots}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{a+\cdots}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e\cdots}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e\cdots}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e\cdots}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e\cdots}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.70, size = 539, normalized size = 2.19

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{(2a+2b)\sqrt{\frac{a+2b+a\cos(2e+2fx)}{2a+2b}}E\left(\frac{1}{2}(2e+2fx)\middle|\frac{2a}{2a+2b}\right)}{f\sqrt{a+2b+a\cos(2e+2fx)}}+\frac{\cos(2(e+fx))\left(-\sqrt{-\frac{1}{a+b}}(-a+a\cos(2e+2fx))(a+a\cos(2e+2fx))\sqrt{a+2b+a\cos(2e+2fx)}-ib(a+2b)\sqrt{\frac{a-a\cos(2e+2fx)}{a+b}}\sqrt{4-\frac{2(a+2b+a\cos(2e+2fx))}{b}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)-iab\sqrt{\frac{4a+4b-2(a+2b+a\cos(2e+2fx))}{a+b}}\sqrt{2-\frac{a+2b+a\cos(2e+2fx)}{b}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)\right)\sec\left(2\left(e+\frac{1}{2}(-2e+\text{ArcCos}(\cos(2e+2fx)))\right)\right)\sin(2e+2fx)}{3a^2\sqrt{-\frac{1}{a+b}}f\sqrt{\frac{(a-a\cos(2e+2fx))(a+a\cos(2e+2fx))}{a^2}}\sqrt{1-\cos^2(2e+2fx)}}\right)}{2\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(((2*a + 2*b)*Sqrt[(a + 2*b + a*Co
s[2*e + 2*f*x])/(2*a + 2*b)]*EllipticE[(2*e + 2*f*x)/2, (2*a)/(2*a + 2*b)])
/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + (Cos[2*(e + f*x)]*(-(Sqrt[-(a + b
)^(-1)]*(-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x])*Sqrt[a + 2*b + a
*Cos[2*e + 2*f*x]]) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*
Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(
a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - I*a
*b*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a
 + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sq
rt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e +
ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(3*a^2*Sqrt[-(a + b)^(-1)]*
f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Co
s[2*e + 2*f*x]^2])))/(2*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 4623, normalized size = 18.79

method	result	size
default	Expression too large to display	4623

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+4*
I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)*sin(f*x+e)
+2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*I*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
```

)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)*sin(f*x+e)+6*sin(f
*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+6*sin
(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+4*I
*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)+4*I*c
os(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)+8*I*cos
(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)-8*I*cos(f
x+e)^2((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)-4*I*cos(f*x
+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)-2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-8*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e)*a^(5/2)*b^(1/2)*2^(1/2)*((I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)+2*I*cos(f*x+e)*a^(3/2)*b^(3/2)*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2*I*cos(f*x+e)*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
x+e))/(a+b))^(1/2)(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)+4*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-2*I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^

```
(5/2)*sin(f*x+e)-5*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=338

$$\frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af}+\frac{\cos^2(e+fx)\sin(e+fx)\,(a+$$

```
[Out] 2/15*(2*a-b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a/f+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)*(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)/a/f+1/15*(8*a^2+3*a*b-2*b^2)*EllipticE(sin(f*x+e),(
a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/a^2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-2/15*(2*a-b)*b*(a+b)*EllipticF(sin
(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+
e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]
time = 0.34, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^2f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{2(2a-b)\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15af}+\frac{\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(2*a - b)*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])/(15*a*f) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f
*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*a*f) + ((8*a^2 +
 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a^2*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^2*f*(a + b - a*Si
n[e + f*x]^2))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int (1-x^2)^2\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int (1-x^2)^{3/2}\sqrt{b}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int (1-x^2)^{3/2}\sqrt{a}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\left(a+b-a\sin^2(e+\right.}{5af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]
time = 12.51, size = 0, normalized size = 0.00

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 6392, normalized size = 18.91

method	result	size
default	Expression too large to display	6392

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=186

$$\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}+\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\,\mathrm{ta}}}{16b^2f}$$

```
[Out] 1/16*(a+b)*(a^2-2*a*b+5*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/b^(5/2)/f+1/16*(a^2-2*a*b+5*b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e)/b^2/f-1/24*(3*a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/6*s
ec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A]
time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 427, 396, 201, 223, 212}

$$\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24b^2f}+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) + ((a^2 - 2*a*b + 5*b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) - ((3*a - 5*b)*Tan[e + f*x]*(a + b
 + b*Tan[e + f*x]^2)^(3/2))/(24*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*(a +
b + b*Tan[e + f*x]^2)^(3/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int (1+x^2)^2\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+}{}$$

$$= \frac{(a^2-2ab+5b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-}{}$$

$$= \frac{(a^2-2ab+5b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-}{}$$

$$= \frac{(a+b)(a^2-2ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.34, size = 407, normalized size = 2.19

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a^2\left(1+e^{2i(e+fx)}\right)^4+4ab\left(1+e^{2i(e+fx)}\right)^2\left(1+4e^{2i(e+fx)}+e^{4i(e+fx)}\right)+b^2\left(15+100e^{2i(e+fx)}+298e^{4i(e+fx)}+100e^{6i(e+fx)}+15e^{8i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^6}-\frac{3(a^3-a^2b+3ab^2+5b^3)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{24\sqrt{2}b^{5/2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-3*a^2*(1 + E
^((2*I)*(e + f*x)))^4 + 4*a*b*(1 + E^((2*I)*(e + f*x)))^2*(1 + 4*E^((2*I)*(
e + f*x)) + E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x)) + 298*
E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)))))/(
1 + E^((2*I)*(e + f*x)))^6 - (3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*Log[(-4*Sqr
t[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e + f*x]^2])
/(24*Sqrt[2]*b^(5/2)*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 2518, normalized size = 13.54

method	result	size
default	Expression too large to display	2518

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a*b^2+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b^2+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^3+10*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^3-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+6*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-18*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-3*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+9*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^2-30*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a

^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3+15*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3+3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
x+e))/(a+b))^(1/2)(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a^3+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-4*cos(f*x+e)^7*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-14*cos(f*x+e)^5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^3-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-
8*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/(b
+a*cos(f*x+e)^2)/cos(f*x+e)^5/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)

Maxima [A]
time = 0.28, size = 335, normalized size = 1.80

$$\frac{\frac{8\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)^3}{b}+\frac{3(a+b)^2a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3(a+b)^2\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{12(a+b)a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{12(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+\frac{24a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+24\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+24\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)-\frac{6\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}{b^2}+\frac{3\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2\tan(fx+e)}{b^2}+\frac{24\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)}{b}-\frac{12\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)}{b}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/48*(8*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)^3/b + 3*(a + b)^2*a*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)^2*arcsinh(b*tan(
f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*a*arcsinh(b*tan(f*x + e)/sqr
t((a + b)*b))/b^(3/2) - 12*(a + b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/
sqrt(b) + 24*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*sqrt(b)
*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 24*sqrt(b*tan(f*x + e)^2 + a + b
)*tan(f*x + e) - 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b^
2 + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b^2 + 24*(b*tan
(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)/b - 12*sqrt(b*tan(f*x + e)^2 + a +
b)*(a + b)*tan(f*x + e)/b)/f

Fricas [A]
time = 5.01, size = 496, normalized size = 2.67

$$\left[\frac{3\left(a^3-a^2b+3ab^2+5b^3\right)\sqrt{b}\cos(fx+e)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)-4\left(\left(3a^2b-4ab^2-15b^3\right)\cos(fx+e)^4-8b^3-2\left(ab^2+5b^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{192b^3f\cos(fx+e)^5}, \frac{3\left(a^3-a^2b+3ab^2+5b^3\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^5-2\left(\left(3a^2b-4ab^2-15b^3\right)\cos(fx+e)^4-8b^3-2\left(ab^2+5b^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{96b^3f\cos(fx+e)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^2 -
15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1
/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^
2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\,\sec^6{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{\cos\left(e + f\,x\right)^2}}}{\cos\left(e + f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6, x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf}+\frac{t}{}$$

```
[Out] -1/8*(a-3*b)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b
^(3/2)/f-1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*tan(f*x+
e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 396, 201, 223, 212}

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf}-\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/8*((a - 3*b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(b^(3/2)*f) - ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f
*x]^2])/(8*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int (1+x^2)\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-3b)\text{Subst}\left(\int \sqrt{a+\right.}{}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\,(a}{}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\,(a}{}$$

$$= -\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.03, size = 322, normalized size = 2.64

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(a\left(1+e^{2i(e+fx)}\right)^2+b\left(3+14e^{2i(e+fx)}+3e^{4i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^4}+\frac{(a^2-2ab-3b^2)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{4\sqrt{2}\,b^{3/2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(a*(1 + E^((2*
I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)) + 3*E^((4*I)*(e + f*x)))))
/(1 + E^((2*I)*(e + f*x)))^4 + ((a^2 - 2*a*b - 3*b^2)*Log[(-4*Sqrt[b]*(-1 +
 E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt[2
]*b^(3/2)*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 1769, normalized size = 14.50

method	result	size
default	Expression too large to display	1769

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*
x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*cos(f*x+e)^4*si
n(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*cos(f*x+e)^4*
sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-2*sin(f*x+e)*
cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+4*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
```

```
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*
b+6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b-cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b+3*cos(f*x+e)^3*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^2+2*cos(f*x+
e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/cos(f*x+e)^3/b/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.27, size = 183, normalized size = 1.50

$$-\frac{\frac{(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4\,a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-4\sqrt{b\tan(fx+e)^2+a+b}\,\tan(fx+e)-\frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)}{b}+\frac{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)}{b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*a
rcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsinh(b*tan(f*x + e)
/sqrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b
)) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 2*(b*tan(f*x + e)^2 +
a + b)^(3/2)*tan(f*x + e)/b + sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*
x + e)/b)/f
```

Fricas [A]
time = 3.51, size = 416, normalized size = 3.41

$$\left[-\frac{(a^2-2ab-3b^2)\sqrt{b}\cos(fx+e)^3\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)-4\left((ab+3b^2)\cos(fx+e)^2+2b^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{32b^2f\cos(fx+e)^3},\ -\frac{(a^2-2ab-3b^2)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^3-2\left((ab+3b^2)\cos(fx+e)^2+2b^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{16b^2f\cos(fx+e)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*
```

```
b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f
*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*((a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\sec^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{{\cos\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}+\frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+
1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 201, 223, 212}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Subst}\left(\int \frac{}{\sqrt{a+\cdots}}\right.}{}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1-bx^2}\cdots\right.}{}$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a\cdots}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(76) = 152.
time = 1.83, size = 210, normalized size = 2.76

$$\frac{\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(\sqrt{2}(a+b)\tanh^{-1}\left(\frac{\sqrt{\frac{b\sin^2(e+fx)}{a+b}}}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)\cos^2(e+fx)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}+(a+2b+a\cos(2(e+fx)))\sqrt{\frac{b\sin^2(e+fx)}{a+b}}\right)\tan(e+fx)}{\sqrt{2}f(a+2b+a\cos(2(e+fx)))^{3/2}\sqrt{\frac{b\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b)
*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (a +
 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*x])/
(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2)/(a
+ b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.12, size = 1096, normalized size = 14.42

method	result	size
default	Expression too large to display	1096

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f
*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+sin(f*x+e)*cos(f*x
+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b-2*sin(f*x+e)*cos(f*x
+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(cos(f*x+e)-1)/(b+a*cos(f*x+
e)^2)/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.26, size = 73, normalized size = 0.96

$$\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \sqrt{b\tan\left(fx+e\right)^2 + a + b}\,\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) + sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)
)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(68) = 136$.
time = 3.50, size = 344, normalized size = 4.53

$$\left[\frac{(a+b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)+4b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{8bf\cos(fx+e)}, \frac{(a+b)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)+2b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{4bf\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\sec^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\right.}{f}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \right.}{}$$
$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e}\right.}{f}$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 588, normalized size = 7.44

method	result
default	$-\frac{\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{(\cos(fx+e)-1)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin(fx+e)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^{2}+6ab-b^{2}}{(a+b)^{2}}}\right)a+\mathrm{EllipticF}\left(\frac{(\cos(fx+e)}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a+EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*Ellipti
cPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)*sin(f*x+e)^2*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)/(cos(f*x
+e)-1)/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(71) = 142$.

time = 4.96, size = 1293, normalized size = 16.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{(a+b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}\,f}+\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 386, 385, 209}

$$\frac{(a+b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}\,f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Subs}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\text{Sub}}{}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}\,f} + \frac{\cos(e+fx)\sin(e}{}$$

Mathematica [A]
time = 0.75, size = 136, normalized size = 1.66

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{2\sqrt{2}\sqrt{a}\,f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
```

```
])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*(e
+ f*x)])/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 1066, normalized size = 13.00

method	result	size
default	Expression too large to display	1066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/f*(2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)-2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*b)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(1/2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(75) = 150$.
time = 4.10, size = 526, normalized size = 6.41

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{-a}\left(a+b\right)\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{16\,af},\ \frac{4\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(a+b\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{8\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^{2}\left(e + fx\right)}\,\cos^{2}\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{(3a-b)(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af}$$

[Out] 1/8*(3*a-b)*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 390, 386, 385, 209}

$$\frac{(3a-b)(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}+\frac{(3a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8af}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] ((3*a - b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(8*a^(3/2)*f) + ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*a*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*a*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af} + \frac{(3a-b)\text{Su}}{}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3}{}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3}{}$$

$$= \frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\,\text{c}}{}$$

Mathematica [A]

time = 1.35, size = 152, normalized size = 1.09

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\,(3a-b)\sqrt{a+b}\,\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}\,(4a+b+a\cos(2(e+fx)))\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\sin(e+fx)\right)}{8a^{3/2}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e + f
*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(3/2
)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1713, normalized size = 12.24

method	result	size
default	Expression too large to display	1713

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(1/2)
*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
```

```
x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a*b*sin(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f
*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^
(1/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/a/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Fricas [A]
time = 4.54, size = 596, normalized size = 4.26

$$\left[\frac{\left(3a^2+2ab-b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5a^4-14a^3b+5a^2b^2\right)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32\left(a^4-7a^3b+7a^2b^2-ab^3\right)\cos(fx+e)^2-8\left(16a^3\cos(fx+e)^7-24\left(a^3-a^2b\right)\cos(fx+e)^5+2\left(5a^3-14a^2b+5ab^2\right)\cos(fx+e)^3-\left(a^3-7a^2b+7ab^2-b^3\right)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)+8\left(2a^2\cos(fx+e)^3+\left(3a^2+ab\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{64a^2f}, -\frac{\left(3a^2+2ab-b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\left(2a^2\cos(fx+e)^3+\left(3a^2+ab\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{32a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f
```

```
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f),
 -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*
(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x + e
)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\cos^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{(a+b)\left(5a^2-2ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}+\frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{}}{48a^2f}$$

```
[Out] 1/16*(a+b)*(5*a^2-2*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)
^(1/2))/a^(5/2)/f+1/48*(3*a-b)*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/24*(5*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)
^2)^(1/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 423, 541, 12, 385, 209}

$$\frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^2f}+\frac{(a+b)\left(5a^2-2ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}+\frac{(5a+b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + ((3*a - b)*(5*a + 3*b)*Cos[e + f*x]*Sin
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^2*f) + ((5*a + b)*Cos[e + f
*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) + (Cos[e + f*x]
^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos\ldots}{\ldots}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{48a^2f}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{48a^2f}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx\ldots}}{48a^2f}$$

$$= \frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{(3\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 16.84, size = 1902, normalized size = 9.70

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Sec[
e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a +
```

```
 b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*AppellF1[1/
2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1
[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)
*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) + (3*(a + b)*Cos[e
 + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]*(-1/3*(a*f*Appell
F1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f
*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(f*(3*(a + b)
*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*Sin[e + f*x]*(-2*f*(a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*
(a + b)*(-1/3*(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/3) - Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b))
- (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/5) + 4*(a + b)*((-3*a*f*AppellF1[5/2, -1, 1
/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f
*x])/(5*(a + b)) - (6*f*Cos[e + f*x]*Sin[e + f*x]*(1 - (a*Sin[e + f*x]^2)/(
a + b))^(3/2)*(5/(6*(1 - (a*Sin[e + f*x]^2)/(a + b))) + (5*(a + b)^3*Csc[e
+ f*x]^6*((-2*a*Sin[e + f*x]^2)/(a + b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)
^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(
Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])))/(32*a^3*(1 - (a*Sin[e +
 f*x]^2)/(a + b)))))/5))))/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) - (3
*a*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e +
```

```
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 2436, normalized size = 12.43

method	result	size
default	Expression too large to display	2436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*(30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)+10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b+14*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*b-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^2*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2*b+4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^
2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a^3*sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b
^3*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+18*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-6*2
```

```
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*a*b^2*sin(f*x+e)-9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2*b*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b
^2*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*cos(f*x+e)^7*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^3+10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^3-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15
*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-15*cos(f*x+e)^2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*b^3)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2
)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Fricas [A]
time = 5.43, size = 672, normalized size = 3.43

$$\left[-\frac{3\,(5a^3+3a^2b-ab^2+b^3)\sqrt{-a}\,\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b^2)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32(a^4-7a^3b+7a^2b^2-ab^3)\cos(fx+e)^2+8\left(16a^3\cos(fx+e)^7-24(a^3-a^2b)\cos(fx+e)^5+2(5a^3-14a^2b+5ab^2)\cos(fx+e)^3-(a^3-7a^2b+7ab^2-b^3)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)-8\left(8a^3\cos(fx+e)^5+2(5a^3+a^2b)\cos(fx+e)^3+(15a^3+4a^2b-3ab^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{384a^3f},\ -\frac{3\,(5a^3+3a^2b-ab^2+b^3)\sqrt{a}\,\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\left(8a^3\cos(fx+e)^5+2(5a^3+a^2b)\cos(fx+e)^3+(15a^3+4a^2b-3ab^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{192a^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
```

```
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 + 2*(5*
a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -1/192*(3*
(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x +
 e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f
)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=450

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f}+\frac{2(a+2b)\left(a^2-4ab-4b\right.}{}$$

```
[Out] -2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)/b^2/f+2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b
))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^
2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/35*(a+b)*(a^2-16*a*b-16*b^2)*EllipticF
(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/
35*(a^2+11*a*b+8*b^2)*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*
tan(f*x+e)/b/f+2/35*(4*a+3*b)*sec(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^5*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.51, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}F\left(\text{ArcSin}(\sin(e+fx))|\frac{a}{a+b}\right)}{35bf\left(-a\sin^2(e+fx)+a+b\right)}+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\text{ArcSin}(\sin(e+fx))|\frac{a}{a+b}\right)}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35b^2f}+\frac{\left(a^2+11ab+8b^2\right)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35bf}+\frac{b\tan(e+fx)\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{7f}+\frac{2(4a+3b)\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])/(35*b^2*f) + (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*
x]^2*(a + b - a*Sin[e + f*x]^2)])/(35*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) - ((a + b)*(a^2 - 16*a*b - 16*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(35*b*f*(a + b - a*Sin[e + f*x]^2
)) + ((a^2 + 11*a*b + 8*b^2)*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]*Tan[e + f*x])/(35*b*f) + (2*(4*a + 3*b)*Sec[e + f*x]^3*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(35*f) + (b*Sec[e
+ f*x]^5*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(7*f
)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(b+a(1-x^2))^{3/2}}{(1-x^2)^{9/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{9/2}}\,d\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\,\text{ta}}{7f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(4a+3b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e}}{35f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(a^2+11ab+8b^2)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a}}{35bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)(a^2-4ab-4b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)(a^2-4ab-4b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)(a^2-4ab-4b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)(a^2-4ab-4b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]

time = 10.46, size = 0, normalized size = 0.00

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.81, size = 7996, normalized size = 17.77

method	result	size
default	Expression too large to display	7996

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}} \sec^5\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=371

$$\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf}-\frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}}{}$$

```
[Out] 1/15*(3*a^2+13*a*b+8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)/b/f-1/15*(3*a^2+13*a*b+8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(co
s(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*
x+e)^2/(a+b))^(1/2)+1/15*(a+b)*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/
2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin
(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)+2/15*(3*a+2*b)*sec(f*x+e)*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^3*(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.40, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$-\frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15bf}+\frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{2(3a+2b)\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15f}+\frac{b\tan(e+fx)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a^2 + 13*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(9*a
+ 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(15*f*(a + b - a*Sin[e + f*x]^2)) + (2*(3*a + 2*b)*Sec[e + f*x]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*f) + (b*Sec[e
 + f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5*
f)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(b+a(1-x^2))^{3/2}}{(1-x^2)^{7/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{7/2}}\,d\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}\ \text{ta}}{5f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(3a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{(3a^2+13ab+8b^2)\,\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]
time = 17.67, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 6562, normalized size = 17.69

method	result	size
default	Expression too large to display	6562

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}} \sec^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=290

$$\frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}-\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,E\big(\text{ArcSin}(\sin(e+\cdots}{3f\sqrt{1-\cdots}}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-2/3*(2*a
+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^
2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(a+b)*(3
*a+2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x
+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*s
in(f*x+e)^2)+1/3*b*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan
(f*x+e)/f
```

Rubi [A]
time = 0.31, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3f\left(-a\sin^2(e+fx)+a+b\right)}-\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}+\frac{b\tan(e+fx)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/
(3*f) - (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(3*a + 2*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f
*x]^2)) + (b*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*T
an[e + f*x])/(3*f)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{1-x^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(b+a(1-x^2))^{3/2}}{(1-x^2)^{5/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{5/2}}dx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F]
time = 12.43, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 5185, normalized size = 17.88

method	result	size
default	Expression too large to display	5185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\sec\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f}+\frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right.}{f\sqrt{1-\frac{a\sin^2(e+}{a+}}}$$

```
[Out] b*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+(a-b)*EllipticE(si
n(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+b*(a+b)*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]

time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4233, 1985, 1986, 424, 538, 437, 435, 432, 430}

$$\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a-b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f + ((a -
b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) + (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/
(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e
+ f*x]^2)/(a + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(b+a(1-x^2))^{3/2}}{(1-x^2)^{3/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{(1-x^2)^{3/2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} -$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} -$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} -$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} +$$

Mathematica [F]
time = 13.56, size = 0, normalized size = 0.00

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 3632, normalized size = 16.21

method	result	size
default	Expression too large to display	3632

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(2*cos(f*x+e)^4
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*b-4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*I*cos(f*x+e
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)-4*I*cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)+4*I*cos(f*x+e)^3*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e)^4*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)-sin(f*x+e)*cos(f*x+e)^2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+sin(f*x+e)*cos(f*x+e)^2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-sin(f*x+e)*cos(f*x+e)^2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+sin(f*x+e)*cos(f*x+e)
^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+2*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b^2-cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
```

```
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*sin(f*x+e)*a^2*b+cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*sin(f*x+e)*a*b^2-cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*sin(f*x+e)*a^3+cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*sin(f*x+e)*b^3-4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2
)*b^(5/2)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-4*I*cos(f*x+e)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)-2*cos(f*x+e)^4*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^3+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
3+2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^(5/2)*b^(1/2)+4*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a^(3/2)*b^(3/2)+2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
e)^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\cos\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=241

$$\frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}+\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\text{ArcSin}(\text{si}\right.}{3f}$$

```
[Out] 1/3*a*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+2
/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b
*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*si
n(f*x+e)^2)
```

Rubi [A]
time = 0.23, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4233, 1985, 1986, 427, 538, 437, 435, 432, 430}

$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{a\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])/(3*f) + (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x
]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1
 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2
))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] && !IntegerQ[p]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int (1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(b+a(1-x^2))^{3}}{\sqrt{1-x^2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+b-ax^2)^{3/2}}{\sqrt{1-x^2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{3f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [A]

time = 2.08, size = 179, normalized size = 0.74

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(4\sqrt{2}\,(a^2+3ab+2b^2)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,E\!\left(e+fx\middle|\frac{a}{a+b}\right)-2\sqrt{2}\,b(a+b)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,F\!\left(e+fx\middle|\frac{a}{a+b}\right)+a(a+2b+a\cos(2(e+fx)))\sin(2(e+fx))\right)}{6f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*S
qrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] -
 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF
[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/
(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 5069, normalized size = 21.03

method	result	size
default	Expression too large to display	5069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=319

$$-\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f}+\frac{a\cos^4(e+fx)\sin(e+f$$

```
[Out] -2/15*(-2*a-3*b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/f+1/5*a*cos(f*x+e)^4*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)/f+1/15*(8*a^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(c
os(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f
*x+e)^2/(a+b))^(1/2)-1/15*b*(a+b)*(4*a+3*b)*EllipticF(sin(f*x+e),(a/(a+b))^
(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*sin(f*x+e)^2)
```

Rubi [A]

time = 0.37, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15af\left(-a\sin^2(e+fx)+a+b\right)}+\frac{a\sin(e+fx)\cos^4(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{5f}-\frac{2(a-3(a+b))\sin(e+fx)\cos^2(e+fx)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - 3*(a + b))*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)])/(15*f) + (a*Cos[e + f*x]^4*Sin[e + f*x]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[
Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b
)]) - (b*(a + b)*(4*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e +
f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)])/(15*a*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\,(b\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\,(a\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{5f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+f}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+f}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+f}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\sin(e+f}{15f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.98, size = 858, normalized size = 2.69

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(-1/3*((Sqrt[-(a + b)^(-1)]*(-
a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x])*Sqrt[a + 2*b + a*Cos[2*e +
```

```
 2*f*x]] + (4*I)*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4
- (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^
(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - (2*I)*b*(a
 + 3*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a + 2*b + a*Cos
[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b +
a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sin[2*e + 2*f*x])/(a*Sqrt[-(a +
b)^(-1)]*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sq
rt[1 - Cos[2*e + 2*f*x]^2]) + (Cos[2*(e + f*x)]*(-(Sqrt[-(a + b)^(-1)]*(-a
- a*Cos[2*e + 2*f*x])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(2*a^2 + 4*b^2 - (
a + 2*b + a*Cos[2*e + 2*f*x])^2 + a*(a + 8*b + a*Cos[2*e + 2*f*x]))) - (4*I
)*b*(a^2 + a*b + b^2)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a
 + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*S
qrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] + (2*I)*a*(a - b)*b
*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e +
2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2
*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f
*x]])/2)]*Sin[2*e + 2*f*x])/(5*a^2*Sqrt[-(a + b)^(-1)]*f*Sqrt[((a - a*Cos[2
*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Cos[2*e + 2*f*x]^2]))
/(2*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 6396, normalized size = 20.05

method	result	size
default	Expression too large to display	6396

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=243

$$\frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)}{128b^2f}$$

```
[Out] 1/128*(a+b)^2*(3*a^2-10*a*b+35*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/b^(5/2)/f+1/128*(a+b)*(3*a^2-10*a*b+35*b^2)*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a+b+b*
tan(f*x+e)^2)^(3/2)/b^2/f-1/48*(3*a-7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5
/2)/b^2/f+1/8*sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f
```

Rubi [A]
time = 0.15, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 427, 396, 201, 223, 212}

$$\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{128b^2f}+\frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{128b^{5/2}f}-\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{48b^2f}+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) + ((a + b)*(3*a^2 - 10*a*b + 35*b
^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(128*b^2*f) + ((3*a^2 - 10
*a*b + 35*b^2)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(192*b^2*f) -
 ((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(48*b^2*f) + (
Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int (1+x^2)^2\,(a+b+bx^2)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+}{}$$

$$= \frac{(3a^2-10ab+35b^2)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} -$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{128b^2f}$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{128b^2f}$$

$$= \frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{128b^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.60, size = 512, normalized size = 2.11

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-9a^3\left(1+e^{2i(e+fx)}\right)^6+3a^2b\left(1+e^{2i(e+fx)}\right)^4\left(5+18e^{2i(e+fx)}+5e^{4i(e+fx)}\right)+ab^2\left(1+e^{2i(e+fx)}\right)^2\left(145+948e^{2i(e+fx)}+2758e^{4i(e+fx)}+948e^{6i(e+fx)}+145e^{8i(e+fx)}\right)+b^3\left(105+910e^{2i(e+fx)}+3591e^{4i(e+fx)}+8644e^{6i(e+fx)}+3591e^{8i(e+fx)}+910e^{10i(e+fx)}+105e^{12i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^8}-\frac{3(a+b)^2\left(3a^2-10ab+35b^2\right)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{96\sqrt{2}b^{5/2}f\left(a+2b+a\cos(2e+2fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1 +
 E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^((2
*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)))^2
*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6*I)*(
e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f*x)) +
 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)*(e + f
*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 + E^((2*I)
*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sqrt[b]*(-1
```

```
 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(96*S
qrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.52, size = 3343, normalized size = 13.76

method	result	size
default	Expression too large to display	3343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/f*sin(f*x+e)*(-107*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e
)^7*a^2*b^2-215*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^3+
107*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b^2+215*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a*b^3-148*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a*b^3+148*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*cos(f*x+e)^4*a*b^3-105*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^4+105*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4
-56*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-108*sin(f*x+e)
*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((co
s(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2+3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^3*b-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^3*b-78*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+78
*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-120*cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+120*cos(f*x+e)^2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-70*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^5*b^4+70*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e
)^4*b^4+56*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^4-15*cos(
f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-145*cos(f*x+e)^9*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-105*cos(f*x+e)^9*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+15*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3*b+145*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2*b^2+105*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+9*
sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4+
```

```
105*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
b^4-18*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4-210*sin(f*x+e)*cos(f*x+e)
^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*b^4+48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-360*si
n(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3-12*sin(f*x+e)*cos(f*x+e)^8*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b+54*sin(f*x+e)*cos(f*x+e)^8*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2+180*sin(f*x+e)*cos(f
*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)...
```

Maxima [A]
time = 0.28, size = 439, normalized size = 1.81

$$\frac{\frac{48\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)^3}{b}+\frac{9(a+b)^3a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{9(a+b)^3\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48(a+b)^2a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48(a+b)^2\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+\frac{144(a+b)a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+144(a+b)\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+96\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)+144\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)-\frac{24\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}(a+b)\tan(fx+e)}{b^2}+\frac{6\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2\tan(fx+e)}{b^2}+\frac{9\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^3\tan(fx+e)}{b^2}+\frac{128\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)}{b}-\frac{32\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}{b}-\frac{48\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2\tan(fx+e)}{b}}{384f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(48*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)^3/b + 9*(a + b)^3*a
*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 9*(a + b)^3*arcsinh(b*ta
n(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*a*arcsinh(b*tan(f*x + e)
/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b))/sqrt(b) + 144*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(
b) + 144*(a + b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 96*(b*ta
n(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e) + 144*sqrt(b*tan(f*x + e)^2 + a +
b)*(a + b)*tan(f*x + e) - 24*(b*tan(f*x + e)^2 + a + b)^(5/2)*(a + b)*tan(f
*x + e)/b^2 + 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)/b^2
 + 9*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(f*x + e)/b^2 + 128*(b*tan
(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b - 32*(b*tan(f*x + e)^2 + a + b)^(
3/2)*(a + b)*tan(f*x + e)/b - 48*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*t
an(f*x + e)/b)/f
```

Fricas [A]
time = 18.89, size = 596, normalized size = 2.45

$$\left[\frac{3\,(3\,a^4-4\,a^3b+18\,a^2b^2+60\,ab^3+35\,b^4)\sqrt{b}\,\cos(fx+e)^7\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)-4\left((9\,a^3b-15\,a^2b^2-145\,ab^3-105\,b^4)\cos(fx+e)^6-2\,(3\,a^2b^2+46\,ab^3+35\,b^4)\cos(fx+e)^4-48\,b^4-8\,(9\,ab^3+7\,b^4)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{1536\,b^3f\cos(fx+e)^7},\ \frac{3\,(3\,a^4-4\,a^3b+18\,a^2b^2+60\,ab^3+35\,b^4)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^7-2\left((9\,a^3b-15\,a^2b^2-145\,ab^3-105\,b^4)\cos(fx+e)^6-2\,(3\,a^2b^2+46\,ab^3+35\,b^4)\cos(fx+e)^4-48\,b^4-8\,(9\,ab^3+7\,b^4)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{768\,b^3f\cos(fx+e)^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(f
*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x +
 e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((
9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 +
 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos
(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*
cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4
- 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\sec^{6}\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6,x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6, x)
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{16bf}$$

[Out] -1/16*(a-5*b)*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-1/16*(a-5*b)*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-1/24*(a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f+1/6*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f

Rubi [A]
time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 396, 201, 223, 212}

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf}-\frac{(a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24bf}-\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((a - 5*b)*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(b^(3/2)*f) - ((a - 5*b)*(a + b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(16*b*f) - ((a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*
x]^2)^(3/2))/(24*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*
b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int (1+x^2)\left(a+b+bx^2\right)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a-5b)\text{Subst}\left(\int (a\cdots\right.}{}$$

$$= -\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+f\cdots}{}$$

$$= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-\cdots}{}$$

$$= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-\cdots}{}$$

$$= -\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a\cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 9.77, size = 400, normalized size = 2.42

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(3a^2\left(1+e^{2i(e+fx)}\right)^4+2ab\left(1+e^{2i(e+fx)}\right)^2\left(11+50e^{2i(e+fx)}+11e^{4i(e+fx)}\right)+b^2\left(15+100e^{2i(e+fx)}+298e^{4i(e+fx)}+100e^{6i(e+fx)}+15e^{8i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^6}+\frac{3(a-5b)(a+b)^2\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{12\sqrt{2}b^{3/2}f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(3*a^2*(1 +
E^((2*I)*(e + f*x)))^4 + 2*a*b*(1 + E^((2*I)*(e + f*x)))^2*(11 + 50*E^((2*I
)*(e + f*x)) + 11*E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x))
+ 298*E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/(1 + E^((2*I)*(e + f*x)))^6 + (3*(a - 5*b)*(a + b)^2*Log[(-4*Sqrt[b]*(
-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(12
*Sqrt[2]*b^(3/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 2519, normalized size = 15.27

method	result	size
default	Expression too large to display	2519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*sin(f*x+e)*(-17*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2*b+17*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-22*co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+22*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+22*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^6*a^2*b+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)^6*a*b^2+32*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a
*b^2+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^3+10*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-15*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^3-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*b^3-18*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-54*cos(
f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
```

```
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+9*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+27*cos(f*x+e)^6*sin(f*x+e)*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^2-30*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*b^3+15*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+8*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^3-22*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b-32*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
^2-3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(f*x+e)^
6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(cos(f
*x+e)-1)/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)
```

Maxima [A]
time = 0.28, size = 257, normalized size = 1.56

$$-\frac{\frac{3\,(a+b)^2 a \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2 \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a \operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-12\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)-18\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)\tan(fx+e)-\frac{8\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)}{b}+\frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}{b}+\frac{3\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2\tan(fx+e)}{b}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a
 + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcs
inh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b)) - 12*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x
 + e) - 18*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 8*(b*tan(f
*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b + 2*(b*tan(f*x + e)^2 + a + b)^(3/2
)*(a + b)*tan(f*x + e)/b + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f
*x + e)/b)/f
```

Fricas [A]
time = 5.29, size = 498, normalized size = 3.02

$$\left[-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\cos(fx+e)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)-4\left(\left(3a^2b+22ab^2+15b^3\right)\cos(fx+e)^4+8b^3+2\left(7ab^2+5b^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{192\,b^2 f\cos(fx+e)^5},\ -\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^5-2\left(\left(3a^2b+22ab^2+15b^3\right)\cos(fx+e)^4+8b^3+2\left(7ab^2+5b^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{96\,b^2 f\cos(fx+e)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a
^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*a*b^
2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^2)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)
^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a -
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^2*f*cos(f*x + e)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}+\frac{\tan(e+\ldots}{}$$

```
[Out] 3/8*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+3/8*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*tan(f*x+e)*(a+b+b*t
an(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 201, 223, 212}

$$\frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(8*Sqrt[b]*f) + (3*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
8*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b))\text{Subst}\left(\int \sqrt{}\right.}{}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\,(}{}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\,(}{}$$

$$= \frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}\,f} + \frac{3(a+b)\tan(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.36, size = 84, normalized size = 0.76

$$\frac{(a+b)^2\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}\,\sin(2(e+fx))}{f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Sin
[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b + a*
Cos[2*(e + f*x)]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.17, size = 1768, normalized size = 15.93

method	result	size
default	Expression too large to display	1768

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*(6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+12*cos(f*x+e
)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos
(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
b^2-3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^2-6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a*b-3*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^2+5*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(
f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)^4*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
```

```
/2)*cos(f*x+e)^4*a*b+7*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-7*cos(f*x+e
)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^2*b^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(f*x+
e)^2)/cos(f*x+e)^2)^(3/2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.99

$$\frac{\frac{3\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + 3\,(a+b)\sqrt{b}\,\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + 2\,\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}\tan\left(fx+e\right) + 3\,\sqrt{b\tan\left(fx+e\right)^2+a+b}\,\left(a+b\right)\tan\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a + b
)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 2*(b*tan(f*x + e)^2 + a
 + b)^(3/2)*tan(f*x + e) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x
 + e))/f
```

Fricas [A]
time = 4.08, size = 416, normalized size = 3.75

$$\left[\frac{3\,(a^2+2\,ab+b^2)\sqrt{b}\,\cos\left(fx+e\right)^3\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\,\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)+4\,\left((5\,ab+3\,b^2)\cos\left(fx+e\right)^2+2\,b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin\left(fx+e\right)}{32\,bf\cos\left(fx+e\right)^3}, \frac{3\,(a^2+2\,ab+b^2)\sqrt{-b}\,\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\,\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\,\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^3+2\,\left((5\,ab+3\,b^2)\cos\left(fx+e\right)^2+2\,b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin\left(fx+e\right)}{16\,bf\cos\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2
)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*
x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^2\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

3.250 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+f}{}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,d\right.}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\text{t}}{\sqrt{a+b+}}\right.}{f}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+b+}}\right.}{2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.21, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+ia^{3/2}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-3a\sqrt{b}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)-b^{3/2}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.12, size = 1557, normalized size = 13.19

method	result	size
default	Expression too large to display	1557

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*
x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f
*x+e)^2*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((co
s(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+2*sin(f*x+e)*cos(f
*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*b^2+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((
```

```
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*
x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(106) = 212$.
time = 5.13, size = 1547, normalized size = 13.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
```

```
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.251 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{a}\,(a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{a\cos(e+f\text{:}}{}$$

[Out] b^(3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*a*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4231, 424, 537, 223, 212, 385, 209}

$$\frac{\sqrt{a}\,(a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f + (a*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\text{Subst}\Big(}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\text{Subst}\Big(}{}$$

$$= \frac{\sqrt{a}\,(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.65, size = 466, normalized size = 3.76

$$\frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-ia\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2a^{3/2}fx+6\sqrt{a}\,bfx-i\sqrt{a}\,(a+3b)\log\left(e^{-2ie}\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)\right)+i\sqrt{a}\,(a+3b)\log\left(e^{-2ie}\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)\right)-4b^{3/2}\log\left(-\frac{e^{3ie}\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{2b^2\left(1+e^{2i(e+fx)}\right)}\right)\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2\sqrt{2}\,f\left(a+2b+a\cos(2e+2fx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/2)
*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x)) + 2*
b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-1/2*(E^((3*I)*e)*(Sqrt[b]
*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))
/(2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 1511, normalized size = 12.19

method	result	size
default	Expression too large to display	1511

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f
*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+2*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-4*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(
f*x+e)-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b)*cos(
f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(cos(f*x+e)-1)/(b+a*cos(f*
x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(113) = 226$.
time = 5.35, size = 1485, normalized size = 11.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*
b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, 1/8*(4*a*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
```

```
s(f*x + e)^4))/f, 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b)*b*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/
f]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^2\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}$$

[Out] 3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
+3/8*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*cos(f*x+e
)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 386, 385, 209}

$$\frac{3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}\,f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*Sqrt[a]*f) + (3*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(4*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+}{}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + $$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + $$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a+b)\cos(}{}$$

Mathematica [A]
time = 1.08, size = 191, normalized size = 1.53

$$\frac{\cos(e+fx)\left(b+a\cos^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(3(a+b)^{3/2}\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}\left(4a+5b+a\cos(2(e+fx))\right)\sin(e+fx)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\right)}{2\sqrt{a}f\left(a+2b+a\cos(2(e+fx))\right)^{3/2}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
- a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1713, normalized size = 13.70

method	result	size
default	Expression too large to display	1713

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(-2*
cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-12*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a*b*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi
```

```
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2-7*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+7*cos(f*x+e)^2*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Fricas [A]
time = 5.41, size = 592, normalized size = 4.74

$$\left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-a}\,\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)^4+a^4-28\,a^3b+70\,a^2b^2-28\,ab^3+b^4-32\,(a^4-7\,a^3b+7\,a^2b^2-ab^3)\cos(fx+e)^2+8\,(16\,a^3\cos(fx+e)^7-24\,(a^3-a^2b)\cos(fx+e)^5+2\,(5\,a^3-14\,a^2b+5\,ab^2)\cos(fx+e)^3-(a^3-7\,a^2b+7\,ab^2-b^3)\cos(fx+e))\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)-8\,(2\,a^2\cos(fx+e)^3+(3\,a^2+5\,ab)\cos(fx+e))\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{64\,af},\ -\frac{3\,(a^2+2\,ab+b^2)\sqrt{a}\arctan\left(\frac{(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e))\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2)\sin(fx+e)}\right)-4\,(2\,a^2\cos(fx+e)^3+(3\,a^2+5\,ab)\cos(fx+e))\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{32\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)
, -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x +
e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=193

$$\frac{(5a-b)(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f}+\frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b}}{16af}$$

[Out] 1/16*(5*a-b)*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+1/16*(5*a-b)*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1/24*(5*a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/a/f

Rubi [A]
time = 0.12, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 390, 386, 385, 209}

$$\frac{(5a-b)(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af}+\frac{(5a-b)\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{24af}+\frac{(5a-b)(a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] ((5*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((5*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a - b)*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(24*a*f) + (Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{(5a-}{}$$

$$= \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}}{16af}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+f}}{16af}$$

$$= \frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-}{}$$

Mathematica [A]
time = 1.95, size = 165, normalized size = 0.85

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}\,(5a^2+4ab-b^2)\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\sqrt{a}\,(23a^2+29ab+3b^2+a(9a+7b)\cos(2(e+fx))+a^2\cos(4(e+fx)))\sin(e+fx)\right)}{48a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 + 4
*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a + 7*b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 2439, normalized size = 12.64

method	result	size

default	Expression too large to display	2439

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)+22*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-22*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+32*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+17*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-32*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-17*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+22*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+54*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I

```
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a*b^2*sin(f*x+e)-27*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^2*b*sin(f*x+e)-9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^
2*sin(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*cos(f*x+e)^7*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^3+10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^3-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-15*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/a/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Fricas [**A**]
time = 6.57, size = 678, normalized size = 3.51

$$\left[\frac{3\,(5a^3+9a^2b+3ab^2-b^3)\sqrt{-a}\,\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b^2)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32(a^4-7a^3b+7a^2b^2-ab^3)\cos(fx+e)^2-8\left(16a^3\cos(fx+e)^7-24(a^3-a^2b)\cos(fx+e)^5+2(5a^3-14a^2b+5ab^2)\cos(fx+e)^3-(a^3-7a^2b+7ab^2-b^3)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)+8\,(8a^3\cos(\ldots\text{[illegible]}}{384a^2f},\ \text{[illegible]}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
```

```
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 2*(5
*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), -1/192
*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e
)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos
(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.254 $\int (a+b\sec^2(c+dx))^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d}+$$

```
[Out] a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/d+1/8*(15*a^2
+10*a*b+3*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))*b^(1/
2)/d+1/8*b*(7*a+3*b)*(a+b+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d*x+
c)*(a+b+b*tan(d*x+c)^2)^(3/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4213, 427, 542, 537, 223, 212, 385, 209}

$$\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d}+\frac{b\tan(c+dx)\,(a+b\tan^2(c+dx)+b)^{3/2}}{4d}+\frac{b(7a+3b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)+b}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]])/d +
 (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a +
 b + b*Tan[c + d*x]^2]])/(8*d) + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b
*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))
/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{5/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\,((a+b)(4a}{1+x^2}\right.}{}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\,(a+b+b}{4d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\,(a+b+b}{4d}$$

$$= \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\,(a+b+b}{4d}$$

$$= \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tanh^{-1}}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 706, normalized size = 4.25

$$\frac{e^{i(c+dx)}\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\cos^5(c+dx)\left(-\frac{ib\left(-1+e^{2i(c+dx)}\right)\left(9a\left(1+e^{2i(c+dx)}\right)^2+b\left(3+14e^{2i(c+dx)}+3e^{4i(c+dx)}\right)\right)}{\left(1+e^{2i(c+dx)}\right)^4}+\frac{8a^{5/2}dx-4ia^{5/2}\log\left(a+2b+ae^{2i(c+dx)}+\sqrt{a}\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}\right)+4ia^{5/2}\log\left(a+ae^{2i(c+dx)}+2be^{2i(c+dx)}+\sqrt{a}\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}\right)-15a^2\sqrt{b}\log\left(\frac{-4\sqrt{b}\,d\left(-1+e^{2i(c+dx)}\right)+4id\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}}{b\left(15a^2+10ab+3b^2\right)\left(1+e^{2i(c+dx)}\right)}\right)-10ab^{3/2}\log\left(\frac{-4\sqrt{b}\,d\left(-1+e^{2i(c+dx)}\right)+4id\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}}{b\left(15a^2+10ab+3b^2\right)\left(1+e^{2i(c+dx)}\right)}\right)-3b^{5/2}\log\left(\frac{-4\sqrt{b}\,d\left(-1+e^{2i(c+dx)}\right)+4id\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}}{b\left(15a^2+10ab+3b^2\right)\left(1+e^{2i(c+dx)}\right)}\right)}{\sqrt{4be^{2i(c+dx)}+a\left(1+e^{2i(c+dx)}\right)^2}}\right)\left(a+b\sec^2(c+dx)\right)^{5/2}}{\sqrt{2}\,d(a+2b+a\cos(2c+2dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c + d
*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*I)
*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x)))))/(
1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b + a
*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*
I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*b*E^((
2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c
 + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)))
 + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(
b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^(3/2)*Log[
(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d
```

```
*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^
((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)
)) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])
/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/Sqrt[4*b*E^((2*I
)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c + d*x]^2)^(5/2)
)/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.62, size = 2231, normalized size = 13.44

method	result	size
default	Expression too large to display	2231

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(16*2^(1/2)*((I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+
c)*a+b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(d*x+c)*a-b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3+30*2^(1/2)*((I*cos(d
*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)+1)/(a+b
))^(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*x+c)*a-b
)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^2*b+20*2^(1/2)*((I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(-2*(I*cos(d*x+c)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*x+c)*a-b)/(cos(d*x+c)+1)/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
d*x+c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b
^2+6*2^(1/2)*((I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+
b)/(cos(d*x+c)+1)/(a+b))^(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(d*x+c)*a-b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3-8*2^(1/2)*((I*cos(d*x+c)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)+1)/(a+b))^(1/2
)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*x+c)*a-b)/(cos(
d*x+c)+1)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3-15*2^(1/2)*((I*cos(d*x+
c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)+1)/(a+b))^
```

```
(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*x+c)*a-b)/(
cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b-10*2^(1/2)*((I*co
s(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)+1)/(
a+b))^(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*x+c)*
a-b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2-3*2^(1/2)*(
(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(d*x+c)*a+b)/(cos(d*x+c)
+1)/(a+b))^(1/2)*(-2*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(d*
x+c)*a-b)/(cos(d*x+c)+1)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3+9*cos(d*
x+c)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*cos(d*x+c)^5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b-3*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2+11*cos(d*x+c)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*
cos(d*x+c)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-11*cos(d*x+c)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(d*x+c)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^3+2*cos(d*x+c)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*cos(d*x+c)*((a*cos
(d*x+c)^2+b)/cos(d*x+c)^2)^(5/2)*sin(d*x+c)/(-1+cos(d*x+c))/(a*cos(d*x+c)^2
+b)^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.
time = 6.51, size = 1611, normalized size = 9.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
- a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4
```

```
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*c
os(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos
(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d*x + c)
^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x + c)^2 +
 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos(d*x + c)
^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4*(3*(3*a*b
 + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x + c)^3*log(
128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
+ b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*c
os(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a
*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10
*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)
^2 + b^2)*sin(d*x + c)))*cos(d*x + c)^3 + 2*(3*(3*a*b + b^2)*cos(d*x + c)^2
 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^3), -1/32*(8*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a
*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c)^2 + b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(d*x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*
a*b + 3*b^2)*sqrt(b)*cos(d*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4
 + 8*(a*b - b^2)*cos(d*x + c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
 c))*sqrt(b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b
^2)/cos(d*x + c)^4) - 4*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*co
s(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), -1/16*(
4*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x + c)^3 +
 (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(d*
x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b
)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*
cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)^2 + b^2)*sin(d*x + c
)))*cos(d*x + c)^3 - 2*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(c + dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x)^2)^(5/2),x)
[Out] int((a + b/cos(c + d*x)^2)^(5/2), x)
```

3.255 $\int \left(1+\sec^2(x)\right)^{3/2} dx$

Optimal. Leaf size=42

$$2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)+\frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

```
[Out] 2*arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))+1/2*(2+tan(
x)^2)^(1/2)*tan(x)
```

Rubi [A]

time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4213, 427, 537, 221, 385, 209}

$$\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}+2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] 2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sqr
t[2 + Tan[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sec^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(2+x^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{6+4x^2}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}+2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\tan(x)\right)+\text{Subst}\left(\int \frac{}{(1+}\right.\\ &= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\tan(}{\sqrt{2+\tan}}\right.\\ &= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)+\frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 109, normalized size = 2.60

$$\frac{\left(1+\cos^2(x)\right)\sec(x)\sqrt{1+\sec^2(x)}\left(4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\cos^2(x)-2i\sqrt{2}\cos^2(x)\log\left(\sqrt{3+\cos(2x)}+i\sqrt{2}\sin(x)\right)+\sqrt{3+\cos(2x)}\sin(x)\right)}{(3+\cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] ((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[x
])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos[2
*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3/2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 429, normalized size = 10.21

method	result
default	$\left(-\frac{1}{4}+\frac{i}{4}\right)\left(8\sin(x)(\cos^2(x))\sqrt{\frac{i\cos(x)+1-i+\cos(x)}{\cos(x)+1}}\sqrt{-\frac{i\cos(x)-\cos(x)-1-i}{\cos(x)+1}}\,\mathrm{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(\cos(x)-1)}{\sin(x)},-i,i\right)(-1)^{\frac{3}{4}}+4\sin($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4+1/4*I)*(8*sin(x)*cos(x)^2*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-
(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(cos(x)-1)/si
n(x),-I,I)*(-1)^(3/4)+4*sin(x)*cos(x)^2*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^
(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(cos(
x)-1)/sin(x),I,I)*(-1)^(3/4)+6*sin(x)*cos(x)^2*((I*cos(x)+1-I+cos(x))/(cos(
x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticF((1/2+1/2*I
)*(cos(x)-1)*2^(1/2)/sin(x),I)*2^(1/2)-8*sin(x)*cos(x)^2*((I*cos(x)+1-I+cos
(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*EllipticPi
((-1)^(1/4)*(cos(x)-1)/sin(x),-I,I)*(-1)^(1/4)-4*sin(x)*cos(x)^2*((I*cos(x)
+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)*El
lipticPi((-1)^(1/4)*(cos(x)-1)/sin(x),I,I)*(-1)^(1/4)-I*cos(x)^3-cos(x)^3+I
*cos(x)^2+cos(x)^2-I*cos(x)-cos(x)+1+I)*sin(x)*cos(x)*((cos(x)^2+1)/cos(x)^
2)^(3/2)/(cos(x)-1)/(cos(x)^2+1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(35) = 70.
time = 2.66, size = 160, normalized size = 3.81

$$\frac{\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x)-\arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x)+2\cos(x)\log\left(\cos(x)^2+\cos(x)\sin(x)+\left(\cos(x)^2+\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)-2\cos(x)\log\left(\cos(x)^2-\cos(x)\sin(x)+\left(\cos(x)^2-\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)+\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos(x
)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2
+ 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - c
os(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)/cos(
x)^2)*sin(x))/cos(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2 + 1)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cos(x)^2}+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2 + 1)^(3/2),x)
```

```
[Out] int((1/cos(x)^2 + 1)^(3/2), x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\, dx$

Optimal. Leaf size=24

$$\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

[Out] arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4213, 399, 221, 385, 209}

$$\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sec[x]^2],x]

[Out] ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sec^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{2+x^2}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.
time = 0.06, size = 57, normalized size = 2.38

$$\frac{\sqrt{2}\left(\text{ArcSin}\left(\frac{\sin(x)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\right)\cos(x)\sqrt{1+\sec^2(x)}}{\sqrt{3+\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sec[x]^2],x]
```

```
[Out] (Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2*
x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 190, normalized size = 7.92

method	result

default	$\frac{(-1+i)\left((-1)^{\frac{3}{4}}\operatorname{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(\cos(x)-1)}{\sin(x)},-i,i\right)+(-1)^{\frac{3}{4}}\operatorname{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(\cos(x)-1)}{\sin(x)},i,i\right)-(-1)^{\frac{1}{4}}\operatorname{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(\cos(x)-1)}{\sin(x)},-i,\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+I)*((-1)^(3/4)*EllipticPi((-1)^(1/4)*(cos(x)-1)/sin(x),-I,I)+(-1)^(3/4)
*EllipticPi((-1)^(1/4)*(cos(x)-1)/sin(x),I,I)-(-1)^(1/4)*EllipticPi((-1)^(1
/4)*(cos(x)-1)/sin(x),-I,I)-(-1)^(1/4)*EllipticPi((-1)^(1/4)*(cos(x)-1)/sin
(x),I,I)+2^(1/2)*EllipticF((1/2+1/2*I)*(cos(x)-1)*2^(1/2)/sin(x),I))*cos(x)
*sin(x)^2*((cos(x)^2+1)/cos(x)^2)^(1/2)*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^
(1/2)*(-(I*cos(x)-cos(x)-1-I)/(cos(x)+1))^(1/2)/(cos(x)-1)/(cos(x)^2+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(21) = 42$.
time = 2.69, size = 131, normalized size = 5.46

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x)+\left(\cos(x)^2+\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)-\frac{1}{2}\log\left(\cos(x)^2-\cos(x)\sin(x)+\left(\cos(x)^2-\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2 +
 cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) +
 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqrt((c
os(x)^2 + 1)/cos(x)^2) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(x)^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{1}{\cos\left(x\right)^2}+1}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int((1/cos(x)^2 + 1)^(1/2), x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=330

$$\frac{2(a-b)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(a-2b)F\left(\text{ArcSin}(\sin(\right.}{3bf\sqrt{\cos^2(e+fx)}\sqrt{s}}$$

```
[Out] 2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/f/
(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x
+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*s
in(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(a-b)*sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b^2/f
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*sec(f*x+e)^3*(a+b-a*sin(f*x+
e)^2)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2(a-b)\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3b^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*
x]^2))/(3*b^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e +
 f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*f*S
qrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(
a - b)*Sec[e + f*x]*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x])/(3*b^2*f*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Sec[e + f*x]^3*(a + b - a*Si
n[e + f*x]^2)*Tan[e + f*x])/(3*b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f
*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec^3(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{3bf\sqrt{a+b\sec^2(e+fx)}} + \cdots$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e\cdots}{3b^2f\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e\cdots}{3b^2f\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e\cdots}{3b^2f\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{a+b-a\sin^2(e\cdots}}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Mathematica [F]
time = 12.25, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 4753, normalized size = 14.40

method	result	size
default	Expression too large to display	4753

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/f*(sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+4*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+4*I*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5

```
/2)*b^(1/2)+4*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2
)*b^(3/2)+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b^2-sin(f*
x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+sin(
f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
E((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-si
n(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-2*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-4*I*cos(f*x+e)^5*a^(3/2)*b^(3/2)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^4*a^(5/2)*b^(1/2)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*I*cos(f*x+e)^4*a^(3/2)*b^(3/2)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^3*a^(1/2)*b^(5/2)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*I*cos(f*x+e)^2*a^(3/2)*b^(3/2)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*I*cos(f*x+e)^2*a^(1/2)*b^(5/2)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^3+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+si
n(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-si
n(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{a}\,\sqrt{a+b}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2\right.}}$$

```
[Out] -EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*a^(1/2)*(a+b)^(1
/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4233, 1985, 1986, 425, 21, 438, 435}

$$\frac{\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{\sqrt{a}\,\sqrt{a+b}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{bf\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
, (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(b*f*Sqrt[Cos[e + f*x]^2
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])) + (Sec[e + f*x]*(a + b
- a*Sin[e + f*x]^2)*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} + \cdots$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \cdots$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\,\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \cdots$$

$$= -\frac{\sqrt{a}\,\sqrt{a+b}\,\sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a}{\cdots}}\cdots}{bf\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+f\cdots}}$$

Mathematica [F]
time = 11.59, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 3033, normalized size = 17.84

method	result	size
default	Expression too large to display	3033

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(2*I*sin(f*x+e)*cos(f*x+e)^2*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))-2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(3/2)-4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2)+4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^(1/2)*b^(3/2)+4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+
e)*cos(f*x+e)^2*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^2+2*I*sin(f*x+e)*cos(f*x+e)*a^(3/2)*b^(1/2)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
```

```
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+4*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^(3/2)*b^(1/2)-4*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^(3/2)*b^(1/2)+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*I*
sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1
/2)*b^(3/2)-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(cos
(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 432, 430}

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-\frac{ax}{a+}}}\right.}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,F\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.22, size = 69, normalized size = 0.86

$$\frac{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,F\left(e+fx\middle|\frac{a}{a+b}\right)\sec(e+fx)}{\sqrt{2}\,f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)]
*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 269, normalized size = 3.36

method	result
default	$\frac{(\sin^2(fx+e))\sqrt{2}\sqrt{\frac{i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}-\cos(\right.}{(1+\cos(fx+e))(a+b)}}}{f\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(fx+e)(\cos(fx+e)-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*sin(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((
cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))/((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.84, size = 305, normalized size = 3.81

$$-\frac{\left(2 i a^{\frac{3}{2}} \sqrt{\frac{ab+b^{2}}{a^{2}}}+\sqrt{a}\,(i a+2 i b)\right) \sqrt{\frac{2 a \sqrt{\frac{ab+b^{2}}{a^{2}}}-a-2 b}{a}}\, \mathrm{ellipticF}\left(\sqrt{\frac{2 a \sqrt{\frac{ab+b^{2}}{a^{2}}}-a-2 b}{a}}\,(\cos(fx+e)+i \sin(fx+e)), \frac{a^{2}+8ab+8b^{2}+4(a^{2}+2ab) \sqrt{\frac{ab+b^{2}}{a^{2}}}}{a^{2}}\right)+\left(-2 i a^{\frac{3}{2}} \sqrt{\frac{ab+b^{2}}{a^{2}}}+\sqrt{a}\,(-i a-2 i b)\right) \sqrt{\frac{2 a \sqrt{\frac{ab+b^{2}}{a^{2}}}-a-2 b}{a}}\, \mathrm{ellipticF}\left(\sqrt{\frac{2 a \sqrt{\frac{ab+b^{2}}{a^{2}}}-a-2 b}{a}}\,(\cos(fx+e)-i \sin(fx+e)), \frac{a^{2}+8ab+8b^{2}+4(a^{2}+2ab) \sqrt{\frac{ab+b^{2}}{a^{2}}}}{a^{2}}\right)}{a^{2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*ellipticF(sqrt((2*a*sqrt((a*b + b^2)/a^2)
- a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e)), (a^2 + 8*a*b + 8*b^2 + 4*(a^
2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b + b^2)/a^2
) + sqrt(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*e
llipticF(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*si
n(f*x + e)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/
a^2))/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^{2}(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{b}{{\cos(e+f\,x)}^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{a+b}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

```
[Out] EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*(a+b)^(1/2)*(1-a*
sin(f*x+e)^2/(a+b))^(1/2)/f/a^(1/2)/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 438, 435}

$$\frac{\sqrt{a+b}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)/
a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a]*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x,\right.}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sqrt{a+b}\,\sqrt{b+a\cos^2(e+fx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+}{a+b}}}{\sqrt{a}\,f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.97, size = 279, normalized size = 2.66

$$\frac{\csc(2(e+fx))\sin(e+fx)\left(a^2\sqrt{-\frac{1}{a+b}}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}F\left(e+fx\middle|\frac{a}{a+b}\right)-2i\sqrt{-\frac{a\cos^2(e+fx)}{b}}\sqrt{a+2b+a\cos(2(e+fx))}\csc(2(e+fx))\left(2bE\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2(e+fx))}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)+aF\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2(e+fx))}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)\right)\sqrt{\frac{a\sin^2(e+fx)}{a+b}}\right)}{\sqrt{2}a^2\sqrt{-\frac{1}{a+b}}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-(a + b)^(-1)]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*C
os[e + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*b
*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
])/Sqrt[2]], (a + b)/b] + a*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a
 + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a + b)/b])*Sqrt[(a*Sin[e + f*x]^2)
/(a + b)]))/(Sqrt[2]*a^2*Sqrt[-(a + b)^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 2985, normalized size = 28.43

method	result	size
default	Expression too large to display	2985

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*I*a^(3/2)*b^(1/2)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
sin(f*x+e)-2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a^(1/2)*b^(3/2)-4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^(3/2)*b^(1/2)+4*I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^(3/2)*b^(1/2)-4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^
(3/2)+4*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
```

```
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^2-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^
2-2*I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(3/2)*sin(f*
x+e)+4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-sin(f*
x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-sin(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+2*I*sin(f*x+e)*cos(f*x+e)*a^(3/2)*b^(1/2)*2
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*cos(f*x+e)^3*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
```

```
b))^(1/2)*a*b+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2
)*b^(3/2)+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-2*cos(
f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)}{\sqrt{a+\frac{b}{{\cos(e+f\,x)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=255

$$\frac{\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{2(a-b)E\left(\mathrm{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\,\mathrm{si}\right.}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] 1/3*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)+2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)
/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*
sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2
))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*
(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4233, 1985, 1986, 427, 538, 437, 435, 432, 430}

$$-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\mathrm{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3af\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(3*a*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) + (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) -
((a - 2*b)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
```

```
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e \cdots}{\cdots}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{2(a-b)\sqrt{b+\cdots}}{3a^2f\sqrt{\cdots}}$$

Mathematica [F]
time = 7.93, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

[Out] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 4637, normalized size = 18.18

method	result	size
default	Expression too large to display	4637

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/f*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)-cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)*sin(f*x+e)-2*I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)*cos(f*x+e)*sin(f*x+e)-3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)-2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2

```
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticE((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-2*I*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a^(5/2)*b^(1/2)*sin(f*x+e)+2*I*cos(f*x+e)*a^(3/2)*b^(3/2)*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2*I*cos(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)-2*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2+4*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)-
2*I*a^(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5-2*
I*a^(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3+2*I*
a^(3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3+4*I*a^
(5/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2-2*I*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)*cos(f*x+e)-2*I*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)+cos(f*x+e
)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=345

$$\frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{(8}{15a^3f\sqrt{}}$$

```
[Out] 4/15*(a-b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/f/(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f
*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(8*a^2-7*a*b+8*b^2)*EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15*
b*(4*a^2-3*a*b+8*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)
^2/(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 427, 542, 538, 437, 435, 432, 430}

$$\frac{4(a-b)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b(4a^2-3ab+8b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(8a^2-7ab+8b^2)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{5af\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (4*(a - b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(15*a^2*f*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b -
 a*Sin[e + f*x]^2))/(5*a*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
 + ((8*a^2 - 7*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*
(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a
^2 - 3*a*b + 8*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)])/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{5af\sqrt{a+b\sec^2(e+fx)}} - \cdots$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(\cdots}{\cdots}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(\cdots}{\cdots}$$

Mathematica [F]
time = 13.83, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 6382, normalized size = 18.50

method	result	size
default	Expression too large to display	6382

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=137

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}+\frac{\text{se}}{}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/b^(5/2)/f-3/8*(a-b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*sec
(f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A]

time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 427, 396, 223, 212}

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(8*b^(5/2)*f) - (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(4*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{\text{Subst}\left(\int \frac{-a+3b-3(a-b)}{\sqrt{a+b+bx}}\right.}{4b}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)}{4}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)}{4}$$

$$= \frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 8.21, size = 326, normalized size = 2.38

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\sqrt{a+2b+a\cos(2e+2fx)}\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a\left(1+e^{2i(e+fx)}\right)^2+b\left(3+14e^{2i(e+fx)}+3e^{4i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^4}-\frac{\left(3a^2-2ab+3b^2\right)\log\left(\frac{-4\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sec(e+fx)}{8\sqrt{2}\,b^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)) +
 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b + 3
*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[e + f*
x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.22, size = 1756, normalized size = 12.82

method	result	size
default	Expression too large to display	1756

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*sin(f*x+e)*(3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^2-2*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-
2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a*b+3*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
```

```
s(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+4*cos
(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipti
cPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*b^2+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(
f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^4*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*cos(f*x+e)^4*a*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-cos(f*x+e)^2*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)^2*b^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/cos
(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/b^2
```

Maxima [A]
time = 0.27, size = 169, normalized size = 1.23

$$\frac{\frac{2\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\tan(fx+e)^{3}}{b}+\frac{3\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3\,(a+b)\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{8\,a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{3\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)\tan(fx+e)}{b^{2}}+\frac{8\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\tan(fx+e)}{b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)^3/b + 3*(a + b)*a*arcsin
h(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)*arcsinh(b*tan(f*x + e
)/sqrt((a + b)*b))/b^(3/2) - 8*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^
(3/2) - 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b^2 + 8*sqrt(
b*tan(f*x + e)^2 + a + b)*tan(f*x + e)/b)/f
```

Fricas [A]
time = 5.15, size = 422, normalized size = 3.08

$$\left[\frac{(3\,a^{2}-2\,ab+3\,b^{2})\sqrt{b}\cos\left(fx+e\right)^{3}\log\left(\frac{(a^{2}-6\,ab+b^{2})\cos(fx+e)^{4}+8\,(ab-b^{2})\cos(fx+e)^{2}+4\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin(fx+e)+8\,b^{2}}{\cos(fx+e)^{4}}\right)-4\left(3\,(ab-b^{2})\cos\left(fx+e\right)^{2}-2\,b^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{32\,b^{3}f\cos\left(fx+e\right)^{3}},\ \frac{(3\,a^{2}-2\,ab+3\,b^{2})\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\left(ab\cos(fx+e)^{2}+b^{2}\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^{3}-2\left(3\,(ab-b^{2})\cos\left(fx+e\right)^{2}-2\,b^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{16\,b^{3}f\cos\left(fx+e\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2 - 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(
f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(3
*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f
+1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A]

time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 396, 223, 212}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/2*((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]
])/(b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x\right.}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+b}}\right.}{2bf}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan}}{2bf}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.29, size = 326, normalized size = 4.02

$$\frac{\sqrt{a+2b+a\cos(2e+2fx)}\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan(e+fx)\left(\frac{16b^2(b+a\cos^2(e+fx))\,{}_2F_1\left(2,3;\frac{7}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^4(e+fx)\sqrt{-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}}}{(a+b)^3}+\frac{15(3b+a(3-2\sin^2(e+fx)))\left(\text{ArcSin}\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)-\sqrt{-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}}\right)}{a+b}\right)}{30\sqrt{2}f\sqrt{a+b\sec^2(e+fx)}\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}\sqrt{a+b-a\sin^2(e+fx)}\left(-\frac{b\tan^2(e+fx)}{a+b}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)/
(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2,
3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^3 + (15
*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b)
)]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(
```

```
a + b)^2)]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[(a + b
*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]
^2)/(a + b)))^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 1086, normalized size = 13.41

method	result	size
default	Expression too large to display	1086

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*b-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+2*sin(f*x+e)*cos(f
*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*b)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/b
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.27, size = 78, normalized size = 0.96

$$-\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,\tan\left(fx+e\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsinh(b*tan(f*x
 + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e
)/b)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(73) = 146$.
time = 3.73, size = 348, normalized size = 4.30

$$\left[-\frac{(a-b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)-4b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{8b^2f\cos(fx+e)}, -\frac{(a-b)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)-2b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{4b^2f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a - b)*
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 2*b*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x +
e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4231, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 0.14, size = 87, normalized size = 2.23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{b}\,f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 379, normalized size = 9.72

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}-\cos(fx+e)a-b\right.}{(1+\cos(fx+e))(a+b)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2
)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.62

$$\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(35) = 70$.
time = 3.00, size = 229, normalized size = 5.87

$$\left[\frac{\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)}{4\sqrt{b}\,f}, \frac{\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)}{2bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)*f)
, 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))/(b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4213, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 0.09, size = 87, normalized size = 2.23

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\,\sec(e+fx)}{\sqrt{2}\,\sqrt{a}\,f\,\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 380, normalized size = 9.74

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}-\cos(fx+e)a-b\right)}{(1+\cos(fx+e))(a+b)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(35) = 70$.
time = 0.61, size = 1046, normalized size = 26.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e

```
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(35) = 70$.
time = 3.30, size = 427, normalized size = 10.95

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\left(fx+e\right)^4+a^4-28\,a^3b+70\,a^2b^2-28\,ab^3+b^4-32\,(a^4-7\,a^3b+7\,a^2b^2-ab^3)\cos\left(fx+e\right)^2+8\,\left(16\,a^3\cos\left(fx+e\right)^7-24\,(a^3-a^2b)\cos\left(fx+e\right)^5+2\,(5\,a^3-14\,a^2b+5\,ab^2)\cos\left(fx+e\right)^3-(a^3-7\,a^2b+7\,ab^2-b^3)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)\right)}{8\,af},-\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,\sqrt{a}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec^2\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f}+\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+1
/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4231, 390, 385, 209}

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\text{Subst}\left(\int \frac{}{(1+x^2)\sqrt{a}}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx\right.}{}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+}}{2af}$$

Mathematica [A]
time = 0.27, size = 126, normalized size = 1.45

$$\frac{\sqrt{a+2b+a\cos(2(e+fx))}\left((a-b)\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sec(e+fx)+\sqrt{a}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)\right)}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 1056, normalized size = 12.14

method	result
default	$\frac{\sin(fx+e)\left(-\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}-\cos(\right.}{(1+\cos(fx+e))(a+b)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*(-2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x
+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+2*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(cos(f*x+e)-1)/cos(f*x+e)/((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(80) = 160$.
time = 3.95, size = 529, normalized size = 6.08

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-a}\,(a-b)\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,(a^{4}-a^{3}b)\cos\left(fx+e\right)^{6}+32\,(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2})\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3})\cos\left(fx+e\right)^{2}-8\,(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,(a^{3}-a^{2}b)\cos\left(fx+e\right)^{5}+2\,(5\,a^{3}-14\,a^{2}b+5\,ab^{2})\cos\left(fx+e\right)^{3}-(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3})\cos\left(fx+e\right))\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{16\,a^{2}f}, \frac{4\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-(a-b)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos(fx+e)^{5}-8\,(a^{2}-ab)\cos(fx+e)^{3}+(a^{2}-6\,ab+b^{2})\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos(fx+e)^{4}-a^{2}b+ab^{2}-(a^{3}-3\,a^{2}b)\cos(fx+e)^{2}\right)\sin(fx+e)}\right)}{8\,a^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$\frac{(3a^2-2ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f}+\frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e}{8a^2f}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/a^(5/2)/f+3/8*(a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2
/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{(3a^2-2ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(5/2)*f) + (3*(a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-2ba}{(1+x^2)^2\sqrt{a+b}}\right.}{4}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 16.54, size = 1840, normalized size = 12.87

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*
Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)*
AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -
2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2,
```

```
 -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appe
llF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x
]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[
3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x])/3))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appel
lF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a +
 b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
)*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(a*AppellF1[3
/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos
[e + f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)
) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/
2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((3
*a*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (9*(a + b)^3*f*Cot[e + f*x]*Csc[
e + f*x]^4*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*((-2*a*Sin[e + f*x]^2)/(a +
 b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])))/(8*a^3)))))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b
)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2
*(e + f*x)])/((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2,
 -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/
2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Ap
pellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1701, normalized size = 11.90

method	result	size
default	Expression too large to display	1701

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*sin(f*x+e)*(-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x
+e)+4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos
(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/
```

```
2)+a-b)/(a+b))^(1/2)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/cos(f*x+e)/((b+a*
cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.73, size = 596, normalized size = 4.17

$$\left[-\frac{\left(3a^2-2ab+3b^2\right)\sqrt{-a}\log\left(128a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5a^4-14a^3b+5a^2b^2\right)\cos\left(fx+e\right)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32\left(a^4-7a^3b+7a^2b^2-ab^3\right)\cos\left(fx+e\right)^2+8\left(16a^3\cos\left(fx+e\right)^7-24\left(a^3-a^2b\right)\cos\left(fx+e\right)^5+2\left(5a^3-14a^2b+5ab^2\right)\cos\left(fx+e\right)^3-\left(a^3-7a^2b+7ab^2-b^3\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)\right)-8\left(2a^2\cos\left(fx+e\right)^3+3\left(a^2-ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)}{64a^3f}, -\frac{\left(3a^2-2ab+3b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\left(2a^2\cos\left(fx+e\right)^3+3\left(a^2-ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)}{32a^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
 + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*
x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{(a-b)\left(5a^2+2ab+5b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f}+\frac{\left(15a^2-14ab+15b^2\right)\cos(e+fx)\sin(e+
}{48a^3f}$$

```
[Out] 1/16*(a-b)*(5*a^2+2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/a^(7/2)/f+1/48*(15*a^2-14*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+
b*tan(f*x+e)^2)^(1/2)/a^3/f+5/24*(a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f
*x+e)^2)^(1/2)/a^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)
/a/f
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{5(a-b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24a^2f}+\frac{(a-b)\left(5a^2+2ab+5b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{\left(15a^2-14ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}+\frac{\sin(e+fx)\cos^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b +
 b*Tan[e + f*x]^2]])/(16*a^(7/2)*f) + ((15*a^2 - 14*a*b + 15*b^2)*Cos[e + f
*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + (5*(a - b)*Co
s[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos
[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-4}{(1+x^2)^3\sqrt{a+}}\right.}{}$$

$$= \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\,\mathrm{s}}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(}{}$$

$$= \frac{(a-b)\left(5a^2+2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} + \frac{(15a^2-14}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 16.81, size = 1739, normalized size = 8.52

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)
```

```
*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2,
-3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2
, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*App
ellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
 + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*
x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[
3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x]))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1
/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*AppellF1[3/2,
 -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appe
llF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e
+ f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/2, -3,
5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 6*(a + b)*((3*a*f*Ap
pellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(f*
Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2,
1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) +
 (3*a*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e + f*x)])/((a + 2*b + a*Cos[
2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 2425, normalized size = 11.89

method	result	size
default	Expression too large to display	2425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*sin(f*x+e)*(30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)-2*cos(f*x+e)^5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*c
os(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-5*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*cos(f*x+e)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b-14*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*b^3*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-18*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b
*sin(f*x+e)+18*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos
(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((c
```

```
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+9*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a*b^2*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
3+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^6
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+10*cos(f*x+e)^5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a^3-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^3+15*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-
15*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/cos(f*x+e)/((b+a
*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A]
time = 6.44, size = 674, normalized size = 3.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
```

```
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 10
*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/192
*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*c
os(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^4*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cos(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^6}{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.270 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}-\frac{(2a+b)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(a+b-a\sin\right.}}$$

```
[Out] a*(2*a+b)*sin(f*x+e)/b^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-
(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/(a+b
)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e
)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)+sec(f*x+e)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)
```

Rubi [A]
time = 0.32, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$-\frac{(2a+b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{b^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{a(2a+b)\sin(e+fx)}{b^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\tan(e+fx)\sec(e+fx)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*a + b)*Sin[e + f*x])/(b^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]) - ((2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a
+ b - a*Sin[e + f*x]^2))/(b^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
 b)])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]) + (Sec[e + f*x]*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sec(e+fx)\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{S}}{bf\sqrt{\text{co}}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(}}{bf\sqrt{a+b\sec^2}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(}}{bf\sqrt{a+b\sec^2}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(}}{bf\sqrt{a+b\sec^2}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{(2a+b)\sqrt{b+}}{b^2(a+b}$$

<u>Mathematica</u> [F]
time = 21.11, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 12510, normalized size = 43.29

method	result	size
default	Expression too large to display	12510

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.271 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}+\frac{E\left(\operatorname{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-}{b(a+b)f\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin}}$$

```
[Out] -a*sin(f*x+e)/b/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+EllipticE
(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b/(a+b)/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)
```

Rubi [A]

time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4233, 1985, 1986, 425, 21, 437, 435}

$$\frac{(-a\sin^2(e+fx)+a+b)\,E\left(\operatorname{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}-\frac{a\sin(e+fx)}{bf(a+b)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Sin[e + f*x])/(b*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + (EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x
]^2))/(b*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e}{b}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e}{b(a+}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(\sqrt{b+a\cos^2(\right.}{b(}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e}{b(a+b)f\sqrt{}}$$

Mathematica [A]
time = 2.51, size = 113, normalized size = 0.75

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{2}\,(a+b)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,E\left(e+fx\middle|\frac{a}{a+b}\right)-a\sin(2(e+fx))\right)}{4b(a+b)f\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a + 2
*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2*(
e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 6601, normalized size = 44.01

method	result	size
default	Expression too large to display	6601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.272 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}-\frac{E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-\right.}{a(a+b)f\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2\right.}}$$

```
[Out] sin(f*x+e)/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+E
llipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4233, 1985, 1986, 423, 507, 437, 435, 432, 430}

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{af\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{af(a+b)\sqrt{\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)}{f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Sin[e + f*x]/((a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) -
(EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(a*
(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]]
, a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*f*Sqrt[Cos[e + f*x]^2
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+}}{(a}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+}}{}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(\sqrt{b+a\cos^2(e}\right.}{a(a}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+}}{a(a+b)f\sqrt{\text{co}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 15.34, size = 822, normalized size = 3.59

$$\frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)\left(-\frac{\left(\frac{2(a-a\cos(2e+2fx))(a+a\cos(2e+2fx))}{b\sqrt{a+2b+a\cos(2e+2fx)}}+\frac{2i\sqrt{\frac{a-a\cos(2e+2fx)}{a+b}}\sqrt{4-\frac{2(a+2b+a\cos(2e+2fx))}{b}}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)\right)}{\sqrt{-\frac{1}{a+b}}}\right)\sin(2e+2fx)}{8a(a+b)f\sqrt{\frac{(a-a\cos(2e+2fx))(a+a\cos(2e+2fx))}{a^2}}\sqrt{1-\cos^2(2e+2fx)}}+\frac{\sqrt{-\frac{1}{a+b}}\cos(2(e+fx))\left(\sqrt{-\frac{1}{a+b}}(a+a\cos(2e+2fx))(-2a^2+2b(a+a\cos(2e+2fx))+a(a-4b+a\cos(2e+2fx)))-ib(a+2b)\sqrt{\frac{a-a\cos(2e+2fx)}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}\sqrt{4-\frac{2(a+2b+a\cos(2e+2fx))}{b}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)-iab\sqrt{a+2b+a\cos(2e+2fx)}\sqrt{\frac{4a+4b-2(a+2b+a\cos(2e+2fx))}{a+b}}\sqrt{2-\frac{a+2b+a\cos(2e+2fx)}{b}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{a+2b+a\cos(2e+2fx)}}{\sqrt{2}}\right)\middle|\frac{a+b}{b}\right)\right)\sec\left(2\left(e+\frac{1}{2}(-2e+\arccos(\cos(2e+2fx)))\right)\right)\sin(2e+2fx)}{4a^2bf\sqrt{\frac{(a-a\cos(2e+2fx))(a+a\cos(2e+2fx))}{a^2}}\sqrt{a+2b+a\cos(2e+2fx)}\sqrt{1-\cos^2(2e+2fx)}}\right)}{2(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3*(-1/8*(((2*(a - a*Cos[
2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/(b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x
]]) + ((2*I)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a + 2*b +
a*Cos[2*e + 2*f*x]))/b]*(EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - EllipticF[I*ArcSinh[(Sqrt
[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b]))/
Sqrt[-(a + b)^(-1)])*Sin[2*e + 2*f*x])/(a*(a + b)*f*Sqrt[((a - a*Cos[2*e +
2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Cos[2*e + 2*f*x]^2]) + (Sqr
t[-(a + b)^(-1)]*Cos[2*(e + f*x)]*(Sqrt[-(a + b)^(-1)]*(a + a*Cos[2*e + 2*f
*x])*(-2*a^2 + 2*b*(a + a*Cos[2*e + 2*f*x]) + a*(a - 4*b + a*Cos[2*e + 2*f*
x])) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b +
a*Cos[2*e + 2*f*x]]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*Elliptic
E[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2
]], (a + b)/b] - I*a*b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(4*a + 4*b -
 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e +
 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2
*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f
*x]])/2)]*Sin[2*e + 2*f*x])/(4*a^2*b*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a +
a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2
*e + 2*f*x]^2])))/(2*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 6593, normalized size = 28.79

method	result	size
default	Expression too large to display	6593

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{b}{{\cos(e+f\,x)}^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.273 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{(a+2b)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin\right.}}$$

```
[Out] -b*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+(a+2*b)*E
llipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(cos(
f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2
/(a+b))^(1/2)-2*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(
a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4233, 1985, 1986, 424, 538, 437, 435, 432, 430}

$$-\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + ((a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*S
in[e + f*x]^2))/(a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
+ b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*b*Ellip
ticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
/(a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e}{a}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(2b\sqrt{b+a\cos}\right.}{}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left((a+2b)\sqrt{b}\right.}{}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+2b)\sqrt{b+}}{a^2(a+b}$$

Mathematica [F]

time = 14.94, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 8684, normalized size = 36.18

method	result	size
default	Expression too large to display	8684

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.274 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{(a+4b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] -b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/3*(a+4*b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/
(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-8*
b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 424, 542, 538, 437, 435, 432, 430}

$$-\frac{b(a-8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+4b)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(2a^2-3ab-8b^2\right)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)\cos^2(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^2*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((a + 4*b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))
/(3*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((2*a^
2 - 3*a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Si
n[e + f*x]^2))/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
 + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b
)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)])/(3*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e}{a}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+}}{}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+}}{}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+}}{}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+}}{}$$

Mathematica [F]

time = 17.16, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.30, size = 11939, normalized size = 35.64

method	result	size
default	Expression too large to display	11939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=436

$$-\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(a+b-a\sin^2(e+fx\right.}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

```
[Out] -b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)+1/15*(4*a^2-5*a*b-24*b^2)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/5*(a+6*b)*cos(f*x+e)^2*sin(f*x+e
)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)+1/15*(8*a^3-9*a^2*b+16*a*b^2+48*b^3)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2)
)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-4/15*b*(a^2-2*a*b+12*
b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a
^4/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 424, 542, 538, 437, 435, 432, 430}

$$\frac{(a+6b)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{5a^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{4b(a^2-2ab+12b^2)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^3f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(8a^3-9a^2b+16ab^2+48b^3)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)\cos^4(e+fx)}{af(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^4*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((4*a^2 - 5*a*b - 24*b^2)*Sin[e + f*x]*(a + b - a*S
in[e + f*x]^2))/(15*a^3*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]) + ((a + 6*b)*Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)
)/(5*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((8*a
^3 - 9*a^2*b + 16*a*b^2 + 48*b^3)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)
]*(a + b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[S
ec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)]) - (4*b*(a^2 - 2*a*b + 12*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b
)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^4*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
```

```
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(b+a(1-x^2))^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e}{a}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+6b)\cos^2(e}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(4a^2-5ab-2}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(4a^2-5ab-2}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(4a^2-5ab-2}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(4a^2-5ab-2}{}$$

Mathematica [F]

time = 16.32, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.52, size = 15199, normalized size = 34.86

method	result	size
default	Expression too large to display	15199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^5}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.276 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(3a+b)\tan(e+fx)}{2b}$$

```
[Out] -1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)
/f-a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*(3*a+
b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/(a+b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 424, 396, 223, 212}

$$-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}+\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)}-\frac{a\tan(e+fx)\sec^2(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((3*a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^
2]])/(b^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b
+ b*Tan[e + f*x]^2]) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/(2*b^2*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+(3a+b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan}}{2b^2(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan}}{2b^2(a+b)f}$$

$$= -\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e}{b(a+b)f\sqrt{a+b+b\tan}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.32, size = 375, normalized size = 2.72

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\,(a+2b+a\cos(2e+2fx))^{3/2}\left(-\frac{i\sqrt{b}\,\left(-1+e^{2i(e+fx)}\right)\left(4b^2e^{2i(e+fx)}+3a^2\left(1+e^{2i(e+fx)}\right)^2+ab\left(1+6e^{2i(e+fx)}+e^{4i(e+fx)}\right)\right)}{(a+b)\left(1+e^{2i(e+fx)}\right)^2\left(4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}+\frac{(3a-b)\log\left(\frac{-4\sqrt{b}\,\left(-1+e^{2i(e+fx)}\right)f+4i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}{1+e^{2i(e+fx)}}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sec^3(e+fx)}{4\sqrt{2}\,b^{5/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x)))*(4*b^2*E^((2*I)*(e + f*x)) + 3*a^2*(1 + E^((2*I)*(e + f*x)))^2 +
a*b*(1 + 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/((a + b)*(1 + E^((2
*I)*(e + f*x)))^2*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)
) + ((3*a - b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e
+ f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*S
ec[e + f*x]^3)/(4*Sqrt[2]*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.21, size = 4338, normalized size = 31.43

method	result	size
default	Expression too large to display	4338

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(-8*cos(f*x+e)^4*b^(3/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+6*cos(f*x+e)^4
*b^(1/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-12*c
os(f*x+e)^4*b^(1/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
```

1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-13*cos(f*x+e)^2*sin(f*x+e
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1
/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*b^
3+2*cos(f*x+e)*b^(7/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-8*cos(f*x+e)
^2*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+6*cos(f*x+e)^2*b^(3/2)*sin(f*x+e)*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-12*cos(f*x+e)^2*b^(3/2)*si
n(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a^2-2*cos(f*x+e)^4*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/
(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)
*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+4*cos(f*x+e)^4*b^(5/2)*sin(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a+4*cos(f*x+e)^4*b^(3/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a^2+4*cos(f*x+e)^2*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a-2*cos(f*x+e)^2*b^(7/2)*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+

```
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+4*cos(f*x+e)^2*b^(7/2)*sin(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellip...
```

Maxima [A]
time = 0.26, size = 171, normalized size = 1.24

$$\frac{\frac{\tan(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a+b}\,b}-\frac{3(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{4\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)}+\frac{3(a+b)\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,b^2}-\frac{4\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)^3/(sqrt(b*tan(f*x + e)^2 + a + b)*b) - 3*(a + b)*arcsinh(
b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 4*arcsinh(b*tan(f*x + e)/sqrt((a
+ b)*b))/b^(3/2) + 2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b))
+ 3*(a + b)*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b^2) - 4*tan(f*x +
 e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b))/f
```

Fricas [A]
time = 4.08, size = 554, normalized size = 4.01

$$\left[-\frac{\left((3a^3+2a^2b-ab^2)\cos(fx+e)^3+(3a^2b+2ab^2-b^3)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)-4\left(ab^2+b^3+(3a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{8\left((a^2b^3+ab^4)f\cos(fx+e)^3+(ab^4+b^5)f\cos(fx+e)\right)}, -\frac{\left((3a^3+2a^2b-ab^2)\cos(fx+e)^3+(3a^2b+2ab^2-b^3)\cos(fx+e)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)-2\left(ab^2+b^3+(3a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{4\left((a^2b^3+ab^4)f\cos(fx+e)^3+(ab^4+b^5)f\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3
)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
+ e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x +
e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^2)*cos(
f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 2*(a*b^2
+ b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 + (a*b^4 + b^5
)*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^6\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.277 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+
e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4231, 393, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
- (a*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{bf}$$

$$= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b+b\tan}}\right.}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.18, size = 405, normalized size = 5.26

$$-\frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(15\text{ArcSin}\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\sec^2(e+fx)\left(3b^2+ab(6-5\sin^2(e+fx))+a^2(3-5\sin^2(e+fx)+2\sin^4(e+fx))\right)+15(a+b)\left(-3b+a(-3+2\sin^2(e+fx))\right)\sqrt{-\frac{b\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan^2(e+fx)}{(a+b)^2}}+4b(a+b)\,{}_2F_1\left(2,2;\frac{7}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\sin^2(e+fx)\left(-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}\right)^{3/2}\right)}{15(a+b)^2(2a+2b)f\left(a+b\sec^2(e+fx)\right)^{3/2}\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}\sqrt{2a+2b-2a\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\left(-\frac{b\tan^2(e+fx)}{a+b}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*Si
n[e + f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]^2
```

```
)/(a + b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3 -
5*Sin[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Sin[e
+ f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]
^2)/(a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)*Tan[e + f*x]^2)/(a + b)^2))^(3/2)))/((a + b)^2*(2*a + 2*b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2*b - 2*a
*Sin[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e + f*x]^2)
/(a + b)))^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 3068, normalized size = 39.84

method	result	size
default	Expression too large to display	3068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*b^(3/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a-4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*b^(3/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+2*sin(f*
x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(1/2)*El
lipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^
2-4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
b^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+2*sin(f*x+e)*2^(1/2)
*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(5/2)*EllipticF((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
```

```
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-4*sin(f*x+e)*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(5/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))+2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(3/2
)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a-4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*b^(3/2)*Ell
ipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-sin(f*x+e)*cos(f*x+e)^2*arctanh(1
/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-s
in(f*x+e)*cos(f*x+e)^2*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos
(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*co
s(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+sin(f*x+e)*cos(f*x+e
)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-1)*
(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b+2*cos(f*x+e)^3*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-sin(
f*x+e)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-
2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2)
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b-sin(f*x+e)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*
cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*b^(1/2)*4^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4
^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+si...
```

Maxima [**A**]

time = 0.27, size = 83, normalized size = 1.08

$$\frac{\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)}-\frac{\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + tan(f*x + e)/(sqrt(b*tan
(f*x + e)^2 + a + b)*(a + b)) - tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b
)*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(73) = 146$.
time = 3.82, size = 436, normalized size = 5.66

$$\left[-\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left((a^{2}+ab)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{b}\log\left(\frac{(a^{2}-6\,ab+b^{2})\cos(fx+e)^{4}+8\,(ab-b^{2})\cos(fx+e)^{2}+4\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin(fx+e)+8\,b^{2}}{\cos(fx+e)^{4}}\right)}{4\left((a^{2}b^{2}+ab^{3})f\cos\left(fx+e\right)^{2}+(ab^{3}+b^{4})f\right)},\ -\frac{2\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left((a^{2}+ab)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^{3}+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\left(ab\cos(fx+e)^{2}+b^{2}\right)\sin(fx+e)}\right)}{2\left((a^{2}b^{2}+ab^{3})f\cos\left(fx+e\right)^{2}+(ab^{3}+b^{4})f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*
b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f*x +
e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.278 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4231, 197}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.71, size = 57, normalized size = 1.78

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.10, size = 59, normalized size = 1.84

method	result	size
default	$\frac{\left(b+a\left(\cos^2(fx+e)\right)\right)\sin(fx+e)}{f\cos(fx+e)^3\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{3}{2}}(a+b)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b+a*cos(f*x+e)^2)*sin(f*x+e)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(3/2)/(a+b)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.00

$$\frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 3.02, size = 70, normalized size = 2.19

$$\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)}{(a^2+ab)f\cos(fx+e)^2+(ab+b^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^2
 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(30) = 60.
time = 0.77, size = 103, normalized size = 3.22

$$\frac{2\,a^2b^2\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(a^3b^2+a^2b^3)\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)/((a^3*b^2 + a^2*b^3)*sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*f)
```

Mupad [B]
time = 6.29, size = 199, normalized size = 6.22

$$\frac{\sqrt{\frac{a+2\,b+a\,\cos(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,(5\,a\,\sin(2\,e+2\,f\,x)+4\,a\,\sin(4\,e+4\,f\,x)+a\,\sin(6\,e+6\,f\,x)+8\,b\,\sin(2\,e+2\,f\,x)+4\,b\,\sin(4\,e+4\,f\,x))}{f\,(a+b)\,(24\,a\,b+10\,a^2+16\,b^2+15\,a^2\,\cos(2\,e+2\,f\,x)+6\,a^2\,\cos(4\,e+4\,f\,x)+a^2\,\cos(6\,e+6\,f\,x)+16\,b^2\,\cos(2\,e+2\,f\,x)+32\,a\,b\,\cos(2\,e+2\,f\,x)+8\,a\,b\,\cos(4\,e+4\,f\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

```
[Out] (((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2)*(5*a*sin(2*e
 + 2*f*x) + 4*a*sin(4*e + 4*f*x) + a*sin(6*e + 6*f*x) + 8*b*sin(2*e + 2*f*x
) + 4*b*sin(4*e + 4*f*x)))/(f*(a + b)*(24*a*b + 10*a^2 + 16*b^2 + 15*a^2*co
s(2*e + 2*f*x) + 6*a^2*cos(4*e + 4*f*x) + a^2*cos(6*e + 6*f*x) + 16*b^2*cos
(2*e + 2*f*x) + 32*a*b*cos(2*e + 2*f*x) + 8*a*b*cos(4*e + 4*f*x)))
```

3.279 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4213, 390, 385, 209}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e}{\sqrt{a+b+b\tan}}\right.}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

time = 1.52, size = 168, normalized size = 2.18

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\sqrt{a}\,b\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 1007, normalized size = 13.08

method	result
default	$-\frac{(b+a(\cos^2(fx+e)))\left(\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{\right.}}{(1+\cos(fx+e))(a+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*s
in(f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b)*sin(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(3/2)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. $2(73) = 146$.
time = 0.72, size = 2169, normalized size = 28.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
```

```
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(73) = 146$.
time = 4.50, size = 632, normalized size = 8.21

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\,\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\,\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{8\,\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)},-\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\,\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{4\,\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.280 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(a-3b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}}+\frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f
+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*b*(a+3*b)*tan
(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{(a-3b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) + (b*(a + 3*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e +
 f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a}{}$$

$$= \frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 15.63, size = 2059, normalized size = 15.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appell
F1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Ap
pellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(
a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]
```

^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((9*a*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (6*f*Cos[e + f*x]*Hypergeometric2F1[3/2, 5/2, 7/2, (a*Sin[e + f*x]^2)/(a + b)]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x

```
)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))
))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 1645, normalized size = 12.56

method	result	size
default	Expression too large to display	1645

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^2*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+co
s(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+
e)-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
```

```
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*b^2*sin(f*x+e)-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*sin
(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(122) = 244.
time = 4.08, size = 732, normalized size = 5.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
+ 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a
^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^3 + a^2*b)*c
os(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a
^3*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.281 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{3(a^2-2ab+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f}+\frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\cos^3(e+f$$

```
[Out] 3/8*(a^2-2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(7/2)/f+1/8*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(
1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(a-3*b)
*b*(3*a+5*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{3(a^2-2ab+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(7/2)*f) + ((3*a - 5*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2
*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*S
qrt[a + b + b*Tan[e + f*x]^2]) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e + f*x])/(8*
a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{4af}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a}{8a^3(a}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a}{8a^3(a}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a}{8a^3(a}$$

$$= \frac{3(a^2-2ab+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+}{8a^2f\sqrt{a+b+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 17.16, size = 2046, normalized size = 10.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[
1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF
```

```
1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*C
os[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/2, -3, 3/
2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3,
5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[
3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x
]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
- a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF
1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*
x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e +
 f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + ((a + b)*Cos[e + f*x]^6*Sin[e
 + f*x]*((a*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - 2*f*AppellF1[3/2, -2, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
)/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a +
b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
+ (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*Ap
pellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Cos[e + f*x]*Sin[e + f*x] + (a + b)*((a*f*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a +
 b) - 2*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2,
 -3, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin
[e + f*x])/(a + b) - (18*f*AppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 2*(a + b)*((9*a*f*A
ppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*A
ppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a
*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
```

```
 + b)])*Sin[e + f*x]^2)^2) + (a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e +
 f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.24, size = 2372, normalized size = 12.23

method	result	size
default	Expression too large to display	2372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^3+2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+6*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a^3*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a*b^2*sin(f*x+e)+30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-3*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
```

```
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-9*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a*b^2*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(
1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*b^3*sin(f*x+e)-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^3-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*cos(f*x+
e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-2*cos(f*x+e)^3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^3+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+5*cos(f*x
+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b^2-15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
^3-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a*b^2+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*sin
(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 9.27, size = 846, normalized size = 4.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5
*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3
*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*
b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^2*b^2 + 3*a
*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3*b - 5*a
^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos
(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos(e+f\,x)}^4}{{\left(a+\frac{b}{{\cos(e+f\,x)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.282 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{(5a^3-9a^2b+15ab^2-35b^3)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f}+\frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 1/16*(5*a^3-9*a^2*b+15*a*b^2-35*b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f
*x+e)^2)^(1/2))/a^(9/2)/f+1/48*(15*a^2-22*a*b+35*b^2)*cos(f*x+e)*sin(f*x+e)
/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/24*(5*a-7*b)*cos(f*x+e)^3*sin(f*x+e)/a^
2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*tan(f
*x+e)^2)^(1/2)+1/48*b*(15*a^3-17*a^2*b+25*a*b^2+105*b^3)*tan(f*x+e)/a^4/(a+
b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{(5a-7b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(15a^2-22ab+35b^2)\sin(e+fx)\cos(e+fx)}{48a^3f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(5a^3-9a^2b+15ab^2-35b^3)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{b(15a^3-17a^2b+25ab^2+105b^3)\tan(e+fx)}{48a^4f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos^5(e+fx)}{6af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((5*a^3 - 9*a^2*b + 15*a*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
 + b + b*Tan[e + f*x]^2]])/(16*a^(9/2)*f) + ((15*a^2 - 22*a*b + 35*b^2)*Cos
[e + f*x]*Sin[e + f*x])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((5*a -
 7*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f
*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-5a+b-6bx^2}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{6af}$$

$$= \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(}{24a^2f\sqrt{a+b+b\tan^2(e}}$$

$$= \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(}{24a^2f\sqrt{a+b+b\tan^2(e}}$$

$$= \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(}{24a^2f\sqrt{a+b+b\tan^2(e}}$$

$$= \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(}{24a^2f\sqrt{a+b+b\tan^2(e}}$$

$$= \frac{(5a^3-9a^2b+15ab^2-35b^3)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{(15a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 20.50, size = 2068, normalized size = 7.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appel
lF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*A
ppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*
(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1
/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Appell
F1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a +
b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3,
 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
+ b)*Cos[e + f*x]^8*Sin[e + f*x]*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8
*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a +
 b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*
AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^8*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a
+ b)*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8*f*AppellF1[3/2, -3, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3
) + Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*App
ellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
+ f*x]*Sin[e + f*x])/5) - 8*(a + b)*((9*a*f*AppellF1[5/2, -3, 5/2, 7/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a +
 b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
```

```
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e +
 f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3
, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)
 + (3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*
Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2,
 -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[
3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2))))
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 3171, normalized size = 11.70

method	result	size
default	Expression too large to display	3171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*(b+a*cos(f*x+e)^2)*(210*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^4+15*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4*sin(f*x+e)-30*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4
*sin(f*x+e)-8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+8*
cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+4*cos(f*x+e)^5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+14*cos(f*x+e)^5*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-4*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3*b-14*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2*b^2+7*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-13*co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-35*cos(f*x+e)^3*
```

```
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-7*cos(f*x+e)^2*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+13*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2*b^2+35*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a*b^3+17*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-25*co
s(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-15*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^4+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*a^4+105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+8*
cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+10*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^4-105*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b^4-8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-17*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+25*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b^3+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-105*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4-15*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a^3*b+24*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*
a^3*b-36*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*b^2+120*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*
a*b^3-12*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b+18
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 17.13, size = 978, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 - 4
*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^5 + a^
4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*
a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^
3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5
*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 +
(5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b
- 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*
cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*
b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.283 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}-\frac{a\sin(e+fx)}{3b(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx}}$$

```
[Out] -2/3*a*(a+2*b)*sin(f*x+e)/b^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)-1/3*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)+2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+
b-a*sin(f*x+e)^2)/b^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*s
in(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*EllipticF(sin(f*x+e)
,(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/(a+b)/f/(cos(f*x+e)^2)^(
1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 425, 541, 538, 437, 435, 432, 430}

$$\frac{2(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3b^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2a(a+2b)\sin(e+fx)}{3b^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{a\sin(e+fx)}{3bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a*(a + 2*b)*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) - (a*Sin[e + f*x])/(3*b*(a + b)*f*(a + b - a*Sin[e +
 f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (2*(a + 2*b)*El
lipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*b^2
*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (EllipticF[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*(a + b)*f*Sqrt[C
os[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(b+a(1-x^2))^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{\sqrt{b+a\cos^2}}{3}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2a(a}{3b^2(a+b)^2f}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2a(a}{3b^2(a+b)^2f}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2a(a}{3b^2(a+b)^2f}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2a(a}{3b^2(a+b)^2f}$$

Mathematica [A]
time = 2.89, size = 167, normalized size = 0.52

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\sqrt{2}\,(a+b)^2\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}\left(2(a+2b)E\left(e+fx\middle|\frac{a}{a+b}\right)-bF\left(e+fx\middle|\frac{a}{a+b}\right)\right)-2a(a^2+5ab+5b^2+a(a+2b)\cos(2(e+fx)))\sin(2(e+fx))\right)}{24b^2(a+b)^2f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*b
 + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(a
+ b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*(a
+ 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.37, size = 14353, normalized size = 44.71

method	result	size
default	Expression too large to display	14353

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.284 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=319

$$-\frac{(a-b)\sin(e+fx)}{3b(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}+\frac{\sin(e+fx)}{3(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\,(}}$$

```
[Out] -1/3*(a-b)*sin(f*x+e)/b/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
+1/3*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e
)^2))^(1/2)+1/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+
e)^2)/a/b/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*EllipticF(sin(f*x+e),(a/(a+b))^(
1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*
x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4233, 1985, 1986, 423, 541, 538, 437, 435, 432, 430}

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3af(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3abf(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{(a-b)\sin(e+fx)}{3bf(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)}{3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((a - b)*Sin[e + f*x])/(b*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*S
in[e + f*x]^2)]) + Sin[e + f*x]/(3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((a - b)*EllipticE[ArcSin[S
in[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a*b*(a + b)^2*f*Sqr
t[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*
Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*(a + b)*f*Sqrt[Cos[e + f*x]^2]*S
qrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(b+a(1-x^2))^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{\sqrt{b+a\cos^2(e}{3(a}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{(a-b}{3b(a+b)^2f\sqrt{a}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{(a-b}{3b(a+b)^2f\sqrt{a}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{(a-b}{3b(a+b)^2f\sqrt{a}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{(a-b}{3b(a+b)^2f\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.31, size = 1156, normalized size = 3.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-1/24*((-2*Sqrt[-(a +
 b)^(-1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) +
 b*(2*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2
*f*x])^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Co
s[2*e + 2*f*x])^2)) + (2*I)*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a +
b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e +
 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[
2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - I*b*(a + 3*b)*Sqrt[(a - a*Cos[2*e + 2
*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b +
 a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sin[2*e + 2*f*x])/(a*b^2*S
qrt[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*
e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*e + 2
*f*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-(a + b)^(-1)]*(-a - a*Cos[2*e + 2*f
*x])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2 + 2*a^3*(a + 3*b + a*Cos
[2*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b
+ a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x])
- (a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*b*(a^2 + a*b + b^2)*Sqrt[(a -
a*Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 -
(2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-
1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] + I*a*b*(-a +
b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqr
t[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*
Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(24*a^2*
b^2*Sqrt[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*C
os[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*
e + 2*f*x]^2])))/(2*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 10271, normalized size = 32.20

method	result	size
default	Expression too large to display	10271

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+f\,x)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.285 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=327

$$\frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}-\frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a}}$$

```
[Out] 2/3*(2*a+b)*sin(f*x+e)/a/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)-1/3*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)-2/3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*s
in(f*x+e)^2)/a^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(3*a+2*b)*EllipticF(sin(f
*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/(a+b)/f/(cos(f*x+
e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 424, 541, 538, 437, 435, 432, 430}

$$\frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}-\frac{2(2a+b)\,(-a\sin^2(e+fx)+a+b)\,E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}+\frac{2(2a+b)\sin(e+fx)}{3af(a+b)^2\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}-\frac{b\sin(e+fx)}{3af(a+b)\,(-a\sin^2(e+fx)+a+b)\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*(2*a + b)*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) - (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b - a*Sin[e + f*x]
^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(2*a + b)*Ellipti
cE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*(a +
 b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((3*a + 2*b)*EllipticF[ArcSin[Sin
[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^2*(a + b)
*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(b+a(1-x^2))^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{\sqrt{b+a\cos^2}}{3}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} + \frac{2(2a}{3a(a+b)^2f}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} + \frac{2(2a}{3a(a+b)^2f}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} + \frac{2(2a}{3a(a+b)^2f}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} + \frac{2(2a}{3a(a+b)^2f}$$

Mathematica [F]
time = 13.37, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 14353, normalized size = 43.89

method	result	size
default	Expression too large to display	14353

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e + fx)}{(a + b\sec^2(e + fx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.286 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}-\frac{b\cos^2(e+fx)\sin(e+fx}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx}}$$

```
[Out] -2/3*b*(3*a+2*b)*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)-1/3*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a^2+13*a*b+8*b^2)*EllipticE(sin
(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1
/2)-1/3*b*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2
/(a+b))^(1/2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4233, 1985, 1986, 424, 540, 538, 437, 435, 432, 430}

$$-\frac{b(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\,\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}-\frac{2b(3a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}+\frac{(3a^2+13ab+8b^2)\,(-a\sin^2(e+fx)+a+b)\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{3a^3f(a+b)^2\sqrt{\cos^2(e+fx)}\,\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}-\frac{b\sin(e+fx)\cos^2(e+fx)}{3af(a+b)\,(-a\sin^2(e+fx)+a+b)\,\sqrt{\sec^2(e+fx)\,(-a\sin^2(e+fx)+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(3*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*(a + b)*f*(
a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ ((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e
+ f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
- (b*(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\operatorname{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(b+a(1-x^2))^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\operatorname{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{\sqrt{b+a\cos^2}\cdots}{\cdots}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(3\cdots}{3a^2(a+b)^2\cdots}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(3\cdots}{3a^2(a+b)^2\cdots}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(3\cdots}{3a^2(a+b)^2\cdots}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(3\cdots}{3a^2(a+b)^2\cdots}$$

Mathematica [F]
time = 17.12, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 17494, normalized size = 50.13

method	result	size
default	Expression too large to display	17494

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*
b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{(e + fx)}}{(a + b\sec^2{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.287 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}-\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)}}$$

```
[Out] -2/3*b*(4*a+3*b)*cos(f*x+e)^2*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(a^2+11*a*b+8*b^2)*si
n(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)+2/3*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/
2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b*(a^2-16*a*b
-16*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/
2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)
```

Rubi [A]
time = 0.42, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4233, 1985, 1986, 424, 540, 542, 538, 437, 435, 432, 430}

$$-\frac{2b(4a+3b)\sin(e+fx)\cos^2(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{3a^4f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{3a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)\cos^4(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(4*a + 3*b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^4*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + ((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]*(a + b - a*Sin[e + f*
x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^4*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]) - (b*(a^2 - 16*a*b - 16*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^4*(a + b)*f*Sqrt[Cos[e + f
*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(b+a(1-x^2))^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{7/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{\sqrt{b+a\cos^2}}{}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(4a+3b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(4a+3b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(4a+3b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(4a+3b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))^{3/2}} - \frac{2b(4a+3b)}{3a^2(a+b)^2}$$

Mathematica [F]

time = 13.28, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 1.00, size = 20922, normalized size = 47.44

method	result	size
default	Expression too large to display	20922

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=559

$$-\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}-\frac{b\cos^6(e+fx)\sin(e+fx}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx}}$$

```
[Out] -2/3*b*(5*a+4*b)*cos(f*x+e)^4*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^6*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*
x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+2/15*(2*a^3-3*a^2*b-42*a*
b^2-32*b^3)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(sec(f*x+e)^2*(a+
b-a*sin(f*x+e)^2))^(1/2)+1/15*(3*a^2+61*a*b+48*b^2)*cos(f*x+e)^2*sin(f*x+e)
*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)+1/15*(8*a^4-11*a^3*b+27*a^2*b^2+184*a*b^3+128*b^4)*EllipticE(sin(f*x+e),
(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^5/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/15
*b*(4*a^3-9*a^2*b+120*a*b^2+128*b^3)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*
(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^5/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)
^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4233, 1985, 1986, 424, 540, 542, 538, 437, 435, 432, 430}

$$-\frac{2b(5a+4b)\sin(e+fx)\cos^4(e+fx)}{3a^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\cos^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\left(4a^3-9a^2b+120ab^2+128b^3\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^5f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2\left(2a^3-3a^2b-42ab^2-32b^3\right)\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{15a^4f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\left(-a\sin^2(e+fx)+a+b\right)E\left(\text{ArcSin}(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{15a^5f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{b\sin(e+fx)\cos^6(e+fx)}{3af(a+b)\left(-a\sin^2(e+fx)+a+b\right)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*b*(5*a + 4*b)*Cos[e + f*x]^4*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^6*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + (2*(2*a^3 - 3*a^2*b - 42*a*b^2 - 32*b^3)*Sin[e + f*x]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) + ((3*a^2 + 61*a*b + 48*b^2)*Cos[e + f*x]^2*Sin[e + f*x]*
(a + b - a*Sin[e + f*x]^2))/(15*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)]) + ((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b
^4)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/
(15*a^5*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^3 - 9*a^2*b +
```

```
 120*a*b^2 + 128*b^3)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)])/(15*a^5*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(b+a(1-x^2)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\,\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2}}{}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\,\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\,\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)}{3a^2(a+b)^2}$$

Mathematica [F]
time = 23.83, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] Result contains complex when optimal does not.
time = 1.64, size = 26983, normalized size = 48.27

method	result	size
default	Expression too large to display	26983

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.289 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(3a+5b)\tan(e+}{3b^2(a+b)^2f\sqrt{a+b+b\tan}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*(3*a
+5*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*a*sec(f*x+e)^
2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4231, 424, 393, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f)
- (a*Sec[e + f*x]^2*Tan[e + f*x])/(3*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (a*(3*a + 5*b)*Tan[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b + b*Ta
n[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+3b+3(a+b)x^2}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\,(a+b+b\tan^2(e+fx))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.33, size = 607, normalized size = 4.56

$$\frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^6(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(-\frac{24b\cos^2(e+fx)\,{}_3F_2\left(2,2,2;1,\frac{9}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\sin^2(e+fx)\left(-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}\right)^{5/2}}{a+b}-\frac{\sec^6(e+fx)\left(24b^3\,{}_2F_1\left(2,2;\frac{9}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\sin^6(e+fx)(4b^2+ab(8-7\sin^2(e+fx))+a^2(4-7\sin^2(e+fx)+3\sin^4(e+fx)))\sqrt{-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}}+35(a+b)\cos^2(e+fx)(15b^2+10ab(3-2\sin^2(e+fx))+a^2(15-20\sin^2(e+fx)+8\sin^4(e+fx)))\left(-3\mathrm{ArcSin}\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)(a+b-a\sin^2(e+fx))^2-\cos^2(e+fx)(-3a^2\cos^2(e+fx)+2ab(-3+\sin^2(e+fx))-b^2(3+\sin^2(e+fx)))\sqrt{-\frac{b\sec^2(e+fx)(a+b-a\sin^2(e+fx))\tan^2(e+fx)}{(a+b)^2}}\right)\right)}{(a+b)^5}\right)}{315(2a+2b)^2f(a+b\sec^2(e+fx))^{5/2}\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}\sqrt{2a+2b-2a\sin^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{3/2}\left(-\frac{b\tan^2(e+fx)}{a+b}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^6*Sqrt[1 - (2*a*Sin[e +
f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*((-24*b*Cos[e + f*x]^2*HypergeometricPFQ[
{2, 2, 2}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Se
c[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2))/
(a + b) - (Sec[e + f*x]^6*(24*b^3*Hypergeometric2F1[2, 2, 9/2, -((b*Tan[e +
 f*x]^2)/(a + b))]*Sin[e + f*x]^6*(4*b^2 + a*b*(8 - 7*Sin[e + f*x]^2) + a^2
*(4 - 7*Sin[e + f*x]^2 + 3*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + 35*(a + b)*Cos[e + f*x]^
2*(15*b^2 + 10*a*b*(3 - 2*Sin[e + f*x]^2) + a^2*(15 - 20*Sin[e + f*x]^2 + 8
*Sin[e + f*x]^4))*(-3*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(a + b -
a*Sin[e + f*x]^2)^2 - Cos[e + f*x]^2*(-3*a^2*Cos[e + f*x]^2 + 2*a*b*(-3 + S
in[e + f*x]^2) - b^2*(3 + Sin[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))/(a + b)^5))/(315*(2*a +
2*b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*
Sqrt[2*a + 2*b - 2*a*Sin[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2)
*(-((b*Tan[e + f*x]^2)/(a + b)))^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.26, size = 3018, normalized size = 22.69

method	result	size
default	Expression too large to display	3018

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2
```

)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
(-2(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)
^2*sin(f*x+e)*a^3-12*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*sin(f*x
+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ellipt
icF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*si
n(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f
x+e)-1)((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2

```
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+
e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^3+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2*b-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-5*cos(f*x+e
)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2...
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(126) = 252$.
time = 0.28, size = 293, normalized size = 2.20

$$-\frac{\left(\frac{3\tan\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b^{2}}+\frac{2}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}\right)\tan\left(fx+e\right)-\frac{3\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{\left(a+b\right)b}}\right)}{b^{\frac{5}{2}}}-\frac{2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\left(a+b\right)^{2}}-\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}\left(a+b\right)}+\frac{3\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,b^{2}}-\frac{2a\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\left(a+b\right)b^{2}}+\frac{2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}-\frac{4\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\left(a+b\right)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*tan(f*x + e)^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + 2*a/((b*tan(
f*x + e)^2 + a + b)^(3/2)*b^2) + 2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b))*ta
n(f*x + e) - 3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) - 2*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - tan(f*x + e)/((b*tan(f*x
+ e)^2 + a + b)^(3/2)*(a + b)) + 3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a
+ b)*b^2) - 2*a*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b^2) +
 2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) - 4*tan(f*x + e)/(sqrt
(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(126) = 252$.
time = 4.41, size = 720, normalized size = 5.41

$$\left[\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{b}\log\left(\frac{\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)+8b^{2}}{\cos\left(fx+e\right)^{4}}\right)-4\left(\left(3a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+2\left(2a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{12\left(\left(a^{4}b^{3}+2a^{3}b^{4}+a^{2}b^{5}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{3}b^{4}+2a^{2}b^{5}+ab^{6}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{5}+2ab^{6}+b^{7}\right)f\right)},\ \frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-b}\arctan\left(-\frac{\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+2b\cos\left(fx+e\right)\right)\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\left(ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sin\left(fx+e\right)}\right)-2\left(\left(3a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+2\left(2a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{6\left(\left(a^{4}b^{3}+2a^{3}b^{4}+a^{2}b^{5}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{3}b^{4}+2a^{2}b^{5}+ab^{6}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{5}+2ab^{6}+b^{7}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^
4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
```

```
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*cos(f*
x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*
b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 +
 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-b)*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 +
 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*
cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^6\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.290 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*
x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4231, 386, 197}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (2*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3(a+b)f}$$
$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A]
time = 4.64, size = 74, normalized size = 0.94

$$\frac{(a+2b+a\cos(2(e+fx)))(2a+3b+a\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{6(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.14, size = 76, normalized size = 0.96

method	result	size
default	$\frac{\sin(fx+e)\left(b+a\left(\cos^2(fx+e)\right)\right)\left(2a\left(\cos^2(fx+e)\right)+a+3b\right)}{3f\cos(fx+e)^5\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{5}{2}}(a+b)^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/cos(f*x+e)^5/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/(a+b)^2
```

Maxima [A]
time = 0.27, size = 125, normalized size = 1.58

$$\frac{\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)^2} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)} - \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b} + \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}\,(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x +
e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
b)*b))/f
```

Fricas [A]
time = 3.55, size = 141, normalized size = 1.78

$$\frac{\left(2\,a\cos\left(fx+e\right)^{3}+\left(a+3\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\sin\left(fx+e\right)}{3\left(\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(71) = 142.
time = 1.06, size = 336, normalized size = 4.25

$$\frac{2\left(\left(\left(\frac{3\left(a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))+2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))+a^{4}b^{6}\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}-\frac{2\left(a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))-2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))-3\,a^{4}b^{6}\mathrm{sgn}(\cos(fx+e))\right)}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\frac{3\left(a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))+2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))+a^{4}b^{6}\mathrm{sgn}(\cos(fx+e))\right)}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{3\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a+b\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) + a^4*b^6*
sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6
```

```
+ a^4*b^7) - 2*(a^6*b^4*sgn(cos(f*x + e)) - 2*a^5*b^5*sgn(cos(f*x + e)) - 3
*a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7))*ta
n(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x
 + e)) + a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*
b^7))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3
/2)*f)
```

Mupad [B]
time = 12.67, size = 153, normalized size = 1.94

$$-\frac{2\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\sqrt{a+\frac{b}{{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(a\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,4\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}\right)}{3\,f\,{\left(a+b\right)}^2\,{\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

```
[Out] -(2*(exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2)^2)^(1/2)*(a*1i + a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*1i
 + b*exp(e*2i + f*x*2i)*6i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i) + a
*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

3.291 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\tan(e+fx)}{3(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}+\frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] 2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a+b)/f/
(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {4231, 198, 197}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3f(a+b)\,(a+b\tan^2(e+fx)+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Tan[e + f*x]/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{3(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3(a+b)f}$$
$$= \frac{\tan(e+fx)}{3(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(71) = 142.
time = 6.14, size = 215, normalized size = 3.03

$$\frac{(3a+b)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(\frac{\sqrt{2}\sin(e+fx)}{(a+b)(a+b-a\sin^2(e+fx))^{3/2}}+\frac{2\sqrt{2}\sin(e+fx)}{(a+b)^2\sqrt{a+b-a\sin^2(e+fx)}}\right)}{48af\,(a+b\sec^2(e+fx))^{5/2}} - \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{8\sqrt{2}\,af\,(a+b\sec^2(e+fx))^{5/2}\,(a+b-a\sin^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a + b)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*((Sqrt[2]*Si
n[e + f*x])/((a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*Sqrt[2]*Sin[e +
 f*x])/((a + b)^2*Sqrt[a + b - a*Sin[e + f*x]^2])))/(48*a*f*(a + b*Sec[e +
f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e
 + f*x])/(8*Sqrt[2]*a*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x
]^2)^(3/2))
```

Maple [A]
time = 0.14, size = 85, normalized size = 1.20

method	result	size
default	$\frac{\sin(fx+e)\left(3a\left(\cos^2(fx+e)\right)+\left(\cos^2(fx+e)\right)b+2b\right)\left(b+a\left(\cos^2(fx+e)\right)\right)}{3f\cos(fx+e)^5\left(\frac{b+a\left(\cos^2(fx+e)\right)}{\cos(fx+e)^2}\right)^{\frac{5}{2}}(a+b)^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)*(3*a*cos(f*x+e)^2+cos(f*x+e)^2*b+2*b)*(b+a*cos(f*x+e)^2)/c
os(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/(a+b)^2
```

Maxima [**A**]
time = 0.27, size = 65, normalized size = 0.92

$$\frac{\frac{2\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^{2}+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 3.71, size = 141, normalized size = 1.99

$$\frac{\left((3\,a+b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\sin\left(fx+e\right)}{3\left((a^{4}+2\,a^{3}b+a^{2}b^{2})f\cos\left(fx+e\right)^{4}+2\,(a^{3}b+2\,a^{2}b^{2}+ab^{3})f\cos\left(fx+e\right)^{2}+(a^{2}b^{2}+2\,ab^{3}+b^{4})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(63) = 126.
time = 1.06, size = 337, normalized size = 4.75

$$\frac{2\left(\left(\frac{3\,(a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))+2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))+a^{4}b^{6}\mathrm{sgn}(\cos(fx+e)))\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^{2}}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}-\frac{2\,(3\,a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))+2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))-a^{4}b^{6}\mathrm{sgn}(\cos(fx+e)))}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\frac{3\,(a^{6}b^{4}\mathrm{sgn}(\cos(fx+e))+2\,a^{5}b^{5}\mathrm{sgn}(\cos(fx+e))+a^{4}b^{6}\mathrm{sgn}(\cos(fx+e)))}{a^{7}b^{4}+3\,a^{6}b^{5}+3\,a^{5}b^{6}+a^{4}b^{7}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{3\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a+b\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] 2/3*((3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) + a^4*b^6*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7) - 2*(3*a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) - a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7))*tan(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) + a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2)*f)

Mupad [B]
time = 13.90, size = 172, normalized size = 2.42

$$-\frac{\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\,\sqrt{a+\frac{b}{{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)}^2}}\,\left(a\,3\mathrm{i}+b\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,10\mathrm{i}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}\right)}{3\,f\,{\left(a+b\right)}^2\,{\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)

[Out] -((exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a*3i + b*1i + a*exp(e*2i + f*x*2i)*6i + a*exp(e*4i + f*x*4i)*3i + b*exp(e*2i + f*x*2i)*10i + b*exp(e*4i + f*x*4i)*1i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)

3.292 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)

Rubi [A]

time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 425, 541, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]

[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) - (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x,\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.58, size = 1927, normalized size = 15.42

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 3024, normalized size = 24.19

method	result	size
default	Expression too large to display	3024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a^3-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*cos(f*x+e)^2*s
in(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
```

```
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e
)-12*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+3*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+6*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-
4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(117) = 234$.

time = 5.03, size = 918, normalized size = 7.34

$$\left[-\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5a^{4}-14a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28a^{3}b+70a^{2}b^{2}-28ab^{3}+b^{4}-32\left(a^{4}-7a^{3}b+7a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5a^{3}-14a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7a^{2}b+7ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)+8\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{24\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)},\ -\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)+4\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{12\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.293 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{(a-5b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{b(3a+5b)\tan(e}{6a^2(a+b)f\left(a+b+b\tan\right.}$$

```
[Out] 1/2*(a-5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/f
+1/6*b*(3*a^2+22*a*b+15*b^2)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^
(1/2)+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*a+5
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{(a-5b)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a - 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) + (b*(3*a + 5*b)*Tan[e + f*x])/(6*a^2*(a + b)*f*(a + b + b*Tan[e +
 f*x]^2)^(3/2)) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(6*a^3*(a + b)
^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}$$

$$= \frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx}{2af\,(a+b+b\tan^2(e+fx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 17.80, size = 1775, normalized size = 9.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
```

```
-2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos
[e + f*x]^6*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*
AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x]))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(
a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2
*f*(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2,
-3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[
e + f*x])/(3*(a + b)) - 2*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(5*a*(
(21*a*f*AppellF1[5/2, -3, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 7/2,
7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
/5) - 6*(a + b)*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*AppellF1[5/
2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)
*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 4270, normalized size = 22.83

method	result	size
default	Expression too large to display	4270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*b^4-18*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3*b-54*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)
^2*sin(f*x+e)*a^2*b^2-30*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a*b^3+6*co
s(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+3*cos(f*x+e)^5*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-6*cos(f*x+e)^4*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2*b^2+6*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^3*b+30*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+20*cos
(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-6*cos(f*x+e)^2*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-30*cos(f*x+e)^2*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-20*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b^3+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b^2+22*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+9*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
```

```
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*EllipticF((co
s(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b+27*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2+
15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*E
llipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b^3-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-3*cos(f*x+e)^4*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^4+15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
^4-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-22*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a*b^3+15*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*sin(f*x+e)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*b^4+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^4-3*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*EllipticF((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(176) = 352$.

time = 10.06, size = 1062, normalized size = 5.68

$$\left[\frac{3\left(a^{3}b^{2}-3a^{2}b^{3}-9ab^{4}-5b^{5}+\left(a^{5}-3a^{4}b-9a^{3}b^{2}-5a^{2}b^{3}\right)\cos\left(fx+e\right)^{4}+2\left(a^{4}b-3a^{3}b^{2}-9a^{2}b^{3}-5ab^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5a^{4}-14a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28a^{3}b+70a^{2}b^{2}-28ab^{3}+b^{4}-32\left(a^{4}-7a^{3}b+7a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}-8\left(16a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5a^{3}-14a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7a^{2}b+7ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)+8\left(3\left(a^{5}+2a^{4}b+a^{3}b^{2}\right)\cos\left(fx+e\right)^{5}+2\left(3a^{4}b+15a^{3}b^{2}+10a^{2}b^{3}\right)\cos\left(fx+e\right)^{3}+\left(3a^{3}b^{2}+22a^{2}b^{3}+15ab^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{48\left(\left(a^{8}+2a^{7}b+a^{6}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{7}b+2a^{6}b^{2}+a^{5}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{6}b^{2}+2a^{5}b^{3}+a^{4}b^{4}\right)f\right)}, -\frac{3\left(a^{3}b^{2}-3a^{2}b^{3}-9ab^{4}-5b^{5}+\left(a^{5}-3a^{4}b-9a^{3}b^{2}-5a^{2}b^{3}\right)\cos\left(fx+e\right)^{4}+2\left(a^{4}b-3a^{3}b^{2}-9a^{2}b^{3}-5ab^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)-4\left(3\left(a^{5}+2a^{4}b+a^{3}b^{2}\right)\cos\left(fx+e\right)^{5}+2\left(3a^{4}b+15a^{3}b^{2}+10a^{2}b^{3}\right)\cos\left(fx+e\right)^{3}+\left(3a^{3}b^{2}+22a^{2}b^{3}+15ab^{4}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{24\left(\left(a^{8}+2a^{7}b+a^{6}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{7}b+2a^{6}b^{2}+a^{5}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{6}b^{2}+2a^{5}b^{3}+a^{4}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + 2*(3*a^4*
b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2*b^3 + 15*
a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), -1/24*(3*(a^3
*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^2 - 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5
+ 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^
2*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a
^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e + fx\right)}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.294 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{(3a^2-10ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f}+\frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e\cdots}{4af\,(a+b\cdots}$$

```
[Out] 1/8*(3*a^2-10*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(9/2)/f+1/24*b*(9*a^3-15*a^2*b-145*a*b^2-105*b^3)*tan(f*x+e)/a^4/(a+
b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(3*a-7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f
/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+
e)^2)^(3/2)+1/24*b*(9*a^2-18*a*b-35*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(
f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.23, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{(3a-7b)\sin(e+fx)\cos(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{(3a^2-10ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) + ((3*a - 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e +
 f*x])/(24*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^3 - 15
*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(24*a^4*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-6bx^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{4af}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\text{Sub}}{}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{}{24a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{}{24a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{}{24a}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{}{24a}$$

$$= \frac{(3a^2-10ab+35b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e}{8a^2f\,(a+b+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 21.01, size = 1777, normalized size = 6.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
```

```
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2,
 -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f
*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1
/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^
9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
 -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*Appe
llF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/((a + b - a*Sin
[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[
e + f*x]^8*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (8*f*
AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(
3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -4, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]
*(2*f*(5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/
2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(3*(a + b)) - (8*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2
*(5*a*((21*a*f*AppellF1[5/2, -4, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (24*f*AppellF1[5/2, -3
, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/5) - 8*(a + b)*((3*a*f*AppellF1[5/2, -3, 7/2, 7/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (18*f*Appe
llF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*
(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
```

```
(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.59, size = 5600, normalized size = 21.46

method	result	size
default	Expression too large to display	5600

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(249) = 498$.
time = 22.89, size = 1228, normalized size = 4.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6
 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^
5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
```

```
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(3*(3*a^4*b^2 -
4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5*b + 18*a^4*b^2
+ 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4*a^4*b^2 + 18*a^3
*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.295 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=332

$$\frac{5(a-3b)\left(a^2+7b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f}+\frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+$$

```
[Out] 5/16*(a-3*b)*(a^2+7*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/a^(11/2)/f+1/48*b*(15*a^4-20*a^3*b+38*a^2*b^2+420*a*b^3+315*b^4)*tan(f*
x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/16*(5*a^2-10*a*b+21*b^2)*co
s(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/24*(5*a-9*b)*cos(f*x
+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^5*sin(f*x+
e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/48*b*(15*a^3-25*a^2*b+49*a*b^2+105*b^3)
*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4231, 425, 541, 12, 385, 209}

$$\frac{(5a-9b)\sin(e+fx)\cos^3(e+fx)}{24a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{5(a-3b)\left(a^2+7b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}+\frac{\left(5a^2-10ab+21b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b\left(15a^3-25a^2b+49ab^2+105b^3\right)\tan(e+fx)}{48a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{b\left(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4\right)\tan(e+fx)}{48a^5f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos^5(e+fx)}{6af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a - 3*b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(16*a^(11/2)*f) + ((5*a^2 - 10*a*b + 21*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*a - 9*b)*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos
[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15
*a^3 - 25*a^2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15*a^4 - 20*a^3*b + 38*a^2*b^2 + 420*a*b
^3 + 315*b^4)*Tan[e + f*x])/(48*a^5*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-5a+b-8bx^2}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{6af}$$

$$= \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Su}}{}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= \frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e}{24a^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= \frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{(5a^2-10ab+}{16a^3f\,(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 23.88, size = 1776, normalized size = 5.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

[Out] (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2,
 -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e +
f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[
1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -
5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/
2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*Ap
pellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b -
a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*Cos[e + f*x]^10*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^
(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -5,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^10*Sin[
e + f*x]*(10*f*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*Appe
llF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/(3*(a + b)) - (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + 5*Sin[
e + f*x]^2*(a*((21*a*f*AppellF1[5/2, -5, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - 6*f*AppellF1[5
/2, -4, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*
Sin[e + f*x]) - 2*(a + b)*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*A
ppellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*

```
(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.03, size = 6934, normalized size = 20.89

method	result	size
default	Expression too large to display	6934

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [A]
time = 82.97, size = 1380, normalized size = 4.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
+ (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x
+ e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^
6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^
7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 +
 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^
```

```
4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^
3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*
x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*
a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b
^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4
 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4
 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^
7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a
^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*co
s(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^
2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5
 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*
b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.296 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=179

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d}-\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}}-\frac{b(9a+5b)\tan(c+}{15a^2(a+b)^2d\,(a+b+b\tan}$$

```
[Out] arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(33*
a^2+40*a*b+15*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)^(1/2)-1/5*
b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*tan(d*x+
c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 425, 541, 12, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^{3/2}}-\frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}}-\frac{b\tan(c+dx)}{5ad(a+b)\left(a+b\tan^2(c+dx)+b\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]]/(a^(7/2)*d) -
 (b*Tan[c + d*x])/(5*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^(5/2)) - (b*(9*
a + 5*b)*Tan[c + d*x])/(15*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^(3/2)
) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(15*a^3*(a + b)^3*d*Sqrt[a
+ b + b*Tan[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{5a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \text{t}\right.}{5a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 18.75, size = 1777, normalized size = 9.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^(7
/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2,
5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
```

-2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)*
((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^
2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Sin[c + d*
x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*S
in[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x
]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^7
)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3,
7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c +
 d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
(a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]
^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin
[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*Cos
[c + d*x]^6*Sin[c + d*x]*((7*a*d*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2
, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(3*(a + b)) - 2*d*
AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos
[c + d*x]*Sin[c + d*x]))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^6*Sin[c + d*x]*(2
d(7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x
]^2)/(a + b)])*Cos[c + d*x]*Sin[c + d*x] + 3*(a + b)*((7*a*d*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[
c + d*x])/(3*(a + b)) - 2*d*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*
Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]) + Sin[c + d*x]^2*(7*a*(
(27*a*d*AppellF1[5/2, -3, 11/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a
+ b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (18*d*AppellF1[5/2, -2, 9/2,
 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]
)/5) - 6*(a + b)*((21*a*d*AppellF1[5/2, -2, 9/2, 7/2, Sin[c + d*x]^2, (a*Si
n[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (12*d*Appel
lF1[5/2, -1, 7/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c +
d*x]*Sin[c + d*x])/5))))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)^2)))

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.11, size = 6116, normalized size = 34.17

method	result	size
default	Expression too large to display	6116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(161) = 322.
time = 13.09, size = 1241, normalized size = 6.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^3
 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*
cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c)^2
)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d
*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))
 + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a^4*b^2 + 9
4*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^4 + 15*a*b^
5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/
((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8
*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a
^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4
```

```
*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 +
 a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*
x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x
+ c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 +
 b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(d*x + c)^2)*sin(d*x + c))) + 4*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*
cos(d*x + c)^5 + (75*a^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (3
3*a^3*b^3 + 40*a^2*b^4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b
)/cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*c
os(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^
4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b
^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\cos\left(c+d\,x\right)}^2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x)^2)^(7/2), x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

Optimal. Leaf size=14

$$\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

[Out] arctan(tan(x)/(2+tan(x)^2)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4213, 385, 209}

$$\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Sec[x]^2],x]

[Out] ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\ &= \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.
time = 0.03, size = 37, normalized size = 2.64

$$\frac{\text{ArcSin}\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{3+\cos(2x)}\,\sec(x)}{\sqrt{2}\sqrt{1+\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sec[x]^2],x]
```

```
[Out] (ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x]
^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.13, size = 142, normalized size = 10.14

method	result
default	$\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sin^2(x)\right)\sqrt{\frac{i\cos(x)+1-i+\cos(x)}{\cos(x)+1}}\,\sqrt{-\frac{i\cos(x)-\cos(x)-1-i}{\cos(x)+1}}\,\left(2(-1)^{\frac{3}{4}}\,\text{EllipticPi}\left(\frac{(-1)^{\frac{1}{4}}(\cos(x)-1)}{\sin(x)},i,i\right)-2(-1)^{\frac{1}{4}}\,\text{Ellipt}\right.}{\sqrt{\frac{\cos^2(x)+1}{\cos(x)^2}}\,\cos(x)(\cos(x)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2+1/2*I)*sin(x)^2*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-
cos(x)-1-I)/(cos(x)+1))^(1/2)*(2*(-1)^(3/4)*EllipticPi((-1)^(1/4)*(cos(x)-1
)/sin(x),I,I)-2*(-1)^(1/4)*EllipticPi((-1)^(1/4)*(cos(x)-1)/sin(x),I,I)+2^(
1/2)*EllipticF((1/2+1/2*I)*(cos(x)-1)*2^(1/2)/sin(x),I))/((cos(x)^2+1)/cos(
x)^2)^(1/2)/cos(x)/(cos(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(12) = 24$.

time = 0.52, size = 388, normalized size = 27.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*
sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(
4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(
4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(2*(6*c
os(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*
x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*
x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6*cos(2*x)
 + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(
2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*
sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

time = 3.00, size = 53, normalized size = 3.79

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sec^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sec(x)**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sec(x)^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{\frac{1}{\cos\left(x\right)^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(1/cos(x)^2 + 1)^(1/2), x)
```

3.298 $\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=111

$$\frac{F_1\left(\frac{m}{2};\frac{1}{2},-p;\frac{2+m}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\cot(e+fx)(d\sec(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e}{a}\right.}{fm}$$

[Out] AppellF1(1/2*m,1/2,-p,1+1/2*m,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*cos(f*x+e)*(d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p*(-tan(f*x+e)^2)^(1/2)/f/m/((1+b*sec(f*x+e)^2/a)^p)/sin(f*x+e)

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

Verification is not applicable to the result.

[In] Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]

[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p, x]

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2195 vs. 2(111) = 222.
time = 18.69, size = 2195, normalized size = 19.77

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[e + f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T an[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1 + m/2 + p)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*(1 - m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*(1 - m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(1 - m/2)*AppellF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*(-2 + m)*((6*b*p*AppellF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(2 - m/2)*AppellF1[5/2, 3 - m/2,

```
-p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*
Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^m \left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
[Out] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*sec(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
[Out] int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

3.299 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{1}{2};2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right.}{f}$$

```
[Out] AppellF1(1/2,2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {4233, 1985, 1986, 441, 440}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p(a+\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p(b+a}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1989 vs. $2(103) = 206$.
time = 17.36, size = 1989, normalized size = 19.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]^3*(Sec[e + f*x]^
```

2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[
1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*
p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*
x)])^p*(Sec[e + f*x]^2)^(3/2 + p))/(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*App
ellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*T
an[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^(1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF
1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*
b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(1/2 + p)*AppellF1[1/2, -
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*
Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2,
 -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a
 + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2
, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
 a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*(2*(2*b*p*Ap
pellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2
, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan
[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, -1/2, 2 - p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b)) + (3*AppellF1[5/2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*((6*b*p*AppellF1[5/
2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*AppellF1[5/2, 3/2, -p, 7/2, -Tan[e +

```
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/
(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(fx+e\right)\right) \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{1}{2};1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)}{f}$$

```
[Out] AppellF1(1/2,1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p
*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4233, 1985, 1986, 441, 440}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+\frac{b}{1-x^2}\right)^p}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p(a+\right.}{}$$

$$=\frac{F_1\left(\frac{1}{2};1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\,(b+a}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1995 vs. $2(103) = 206$.
time = 17.11, size = 1995, normalized size = 19.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]*(Sec[e + f*x]^2)^
```

(-1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1/2 + p)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*(2*(2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 1/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*AppellF1[5/2, 3/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - (a + b)*((6*b*p*AppellF1[5/2, 3/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (9*AppellF1[5/2, 5/2, -p, 7/2, -Tan[e + f*x]^2, -((b*T

```
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sec \left(f x+e\right) \left(a+b\left(\sec^{2}\left(f x+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+f x\right)\right)^{p} \sec \left(e+f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x), x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)\right)^p\,(}{f}$$

```
[Out] AppellF1(1/2,p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*s
in(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b))
^p)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4233, 1985, 1986, 441, 440}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos
[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))^p)/
(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int\left(a+\frac{b}{1-x^2}\right)^p dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\operatorname{Subs}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\operatorname{Subs}}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p(a+b\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1983 vs. $2(101) = 202$.
time = 16.77, size = 1983, normalized size = 19.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2, -p,
```

```
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a
+ b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2
, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 3
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*AppellF1[1/2, 3/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))/(-3*(a + b)*A
ppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2, 3/2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[
2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*(2*(-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3
*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + Tan[e + f*x]
^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 3/2, 2 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (9
*AppellF1[5/2, 5/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 3*(a + b)*((6*b*p*AppellF1[5/2, 5/2,
 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2
*Tan[e + f*x])/(5*(a + b)) - 3*AppellF1[5/2, 7/2, -p, 7/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))))/(-3*(a + b)
```

```
*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^p\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{1}{2};-1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+fx\right.\right.}{f}$$

```
[Out] AppellF1(1/2,-1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+
b))^p)
```

Rubi [A]

time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 441, 440}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
)^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+\frac{b}{1-x^2}\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Su}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Su}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p(a-\right.}{}$$

$$=\frac{F_1\left(\frac{1}{2};-1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p(b-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1987 vs. 2(103) = 206.
time = 17.45, size = 1987, normalized size = 19.29

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2, -p,
```

3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a
+ b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2
, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 5
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*AppellF1[1/2, 5/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 5/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(-3*(a
+ b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2
, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*(2*(-2*b*p*A
ppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3
/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) +
Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 5/2, 2 - p, 7/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(
a + b)) - 3*AppellF1[5/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + 5*(a + b)*((6*b*p*AppellF1[5
/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, -p, 7/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)))

```
)/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(f x +e \right)\right) \left(a +b \left(\sec^{2}\left(f x +e \right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{1}{2};-2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e+f\right.\right.}{f}$$

[Out] AppellF1(1/2,-2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b))^p)

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4233, 1985, 1986, 441, 440}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)\right)^p}{f}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Sub}}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p(a+\right.}{}$$

$$=\frac{F_1\left(\frac{1}{2};-2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p(b+}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1997 vs. $2(103)=206$.
time = 17.58, size = 1997, normalized size = 19.39

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF
```

1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*
b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2,
 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a +
b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
 p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-7/2 + p)*Appell
F1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]^2)/(-3
*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]*((2*b*p*A
ppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a +
b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*
x]*(2*(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2
*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 7/2, 2 -
 p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 7*(a + b
)*((6*b*p*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (27*AppellF1[5/2,
11/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]

```
^2*Tan[e + f*x])/5))))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2,
 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2)^2))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(fx+e\right)\right) \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.304 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=216

$$-\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx\right.}{bf(5+2p)}$$

```
[Out] -(3*a-2*b*(2+p))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15
)+sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2-4*a
*b*(1+p)+4*b^2*(p^2+3*p+2))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b)
)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*tan(f*x+e)^
2/(a+b))^p)
```

Rubi [A]
time = 0.17, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4231, 427, 396, 252, 251}

$$\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{b^2f(2p+3)(2p+5)}-\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b^2f(2p+3)(2p+5)}+\frac{\tan(e+fx)\sec^2(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b^
2*f*(3 + 2*p)*(5 + 2*p))) + (Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e +
 f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*
p + p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Ta
n[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b
*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}(\ldots}{\ldots}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\text{se}\ldots}{\ldots}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\text{se}\ldots}{\ldots}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\text{se}\ldots}{\ldots}$$

Mathematica [A]
time = 2.32, size = 149, normalized size = 0.69

$$\frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left(15\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+10\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^2(e+fx)+3\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^4(e+fx)\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f*x]^
2)/(a + b))^p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{6}\left(fx+e\right)\right) \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6,x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6, x)
```

3.305 $\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=129

$$\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)}-\frac{(a-2b(1+p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\right.}{bf(3+2p)}$$

[Out] tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a-2*b*(1+p))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b/f/(3+2*p)/((1+b*tan(f*x+e)^2/(a+b))^p)

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 396, 252, 251}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{bf(2p+3)}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]

[Out] (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - 2*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\text{Subs}}{}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\Big((a-2b(1+p))\,(a}{}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,{}_2F_1\Big(}{}$$

Mathematica [A]
time = 2.41, size = 126, normalized size = 0.98

$$\frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((-a+2b(1+p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(a+b+b\tan^2(e+fx)\right)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^p\right)}{bf(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^2)
*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)\left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=72

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+
e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 252, 251}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^p\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\right.\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right.}{f}$$

Mathematica [A]
time = 1.10, size = 71, normalized size = 0.99

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec[
e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^2(fx+e)\right)\left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p}{{\cos\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

3.307 $\int (a+b\sec^2(e+fx))^p \, dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx,x,\tan(\right.}{f}$$
$$=\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2137 vs. $2(83)=166$.
time = 6.25, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
```

```
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right.}{(1+x^2)}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1914 vs. $2(83) = 166$.
time = 16.15, size = 1914, normalized size = 23.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-
2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(
a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*
Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/
2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (6*(a + b)*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*
```

```
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)
^(-2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan
[e + f*x]*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (
3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e +
 f*x]*(4*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p
*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b)) - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*(a + b)*((6*b*p*App
ellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.309 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,3,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right.}{(1+x^2)}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1912 vs. $2(83) = 166$.
time = 16.72, size = 1912, normalized size = 23.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^3*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-3 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-2 + p))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-3 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[
3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((
```

```
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-3 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2,
4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1
[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f
*x]^2*Tan[e + f*x]))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*
(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*Tan[e + f
*x]*(4*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*A
ppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1[3/2, 4, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) +
 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 3, 2 - p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a
+ b)) - (18*AppellF1[5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 3*(a + b)*((6*b*p*AppellF1[
5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (24*AppellF1[5/2, 5, -p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(
3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.310 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,4,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]

time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right.}{(1+x^2)}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1914 vs. $2(83) = 166$.
time = 17.52, size = 1914, normalized size = 23.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4
*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-4 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[
3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((
```

```
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-4 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2,
5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF
1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) -
 (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e
 + f*x]*(4*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b
*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF1[3/2, 5, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 4, 2 - p, 7/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/(5*(a + b)) - (24*AppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 4*(a + b)*((6*b*p*A
ppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - 6*AppellF1[5/2, 6, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])))
)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\cos^6\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p, x)
```

3.311 $\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx$

Optimal. Leaf size=72

$$-\frac{a\log(\cos(e+fx))}{f}-\frac{(2a-b)\sec^2(e+fx)}{2f}+\frac{(a-2b)\sec^4(e+fx)}{4f}+\frac{b\sec^6(e+fx)}{6f}$$

```
[Out] -a*ln(cos(f*x+e))/f-1/2*(2*a-b)*sec(f*x+e)^2/f+1/4*(a-2*b)*sec(f*x+e)^4/f+1
/6*b*sec(f*x+e)^6/f
```

Rubi [A]

time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 77}

$$\frac{(a-2b)\sec^4(e+fx)}{4f}-\frac{(2a-b)\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - ((2*a - b)*Sec[e + f*x]^2)/(2*f) + ((a - 2*b)*
Sec[e + f*x]^4)/(4*f) + (b*Sec[e + f*x]^6)/(6*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)}{x^7}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a-2b}{x^3}+\frac{-2a+b}{x^2}+\frac{a}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\log(\cos(e+fx))}{f} - \frac{(2a-b)\sec^2(e+fx)}{2f} + \frac{(a-2b)\sec^4(e+}{4f}$$

Mathematica [A]
time = 0.18, size = 55, normalized size = 0.76

$$\frac{b\tan^6(e+fx)}{6f} - \frac{a(4\log(\cos(e+fx))+2\tan^2(e+fx)-\tan^4(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
[Out] (b*Tan[e + f*x]^6)/(6*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan
[e + f*x]^4))/(4*f)
```

Maple [A]
time = 0.07, size = 57, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+\frac{b\left(\sin^6(fx+e)\right)}{6\cos(fx+e)^6}}{f}$
default	$\frac{a\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+\frac{b\left(\sin^6(fx+e)\right)}{6\cos(fx+e)^6}}{f}$
risch	$iax+\frac{2iae}{f}+\frac{-4a\,e^{10i(fx+e)}+2b\,e^{10i(fx+e)}-12a\,e^{8i(fx+e)}-16a\,e^{6i(fx+e)}+\frac{20b\,e^{6i(fx+e)}}{3}-12a\,e^{4i(fx+e)}-4a\,e^{2i(fx+}}{f\left(e^{2i(fx+e)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2-ln(cos(f*x+e)))+1/6*b*sin(f*x+e)^
6/cos(f*x+e)^6)
```

Maxima [A]
time = 0.26, size = 101, normalized size = 1.40

$$-\frac{6\,a\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{6\,(2\,a-b)\sin(fx+e)^{4}-3\,(7\,a-2\,b)\sin(fx+e)^{2}+9\,a-2\,b}{\sin(fx+e)^{6}-3\,\sin(fx+e)^{4}+3\,\sin(fx+e)^{2}-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a -
 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*si
n(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 5.01, size = 74, normalized size = 1.03

$$-\frac{12\,a\cos\left(fx+e\right)^{6}\log\left(-\cos\left(fx+e\right)\right)+6\,(2\,a-b)\cos\left(fx+e\right)^{4}-3\,(a-2\,b)\cos\left(fx+e\right)^{2}-2\,b}{12\,f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
- 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.
time = 0.76, size = 116, normalized size = 1.61

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{4}\left(e+fx\right)}{4f}-\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)\sec^{2}\left(e+fx\right)}{6f}-\frac{b\tan^{2}\left(e+fx\right)\sec^{2}\left(e+fx\right)}{6f}+\frac{b\sec^{2}\left(e+fx\right)}{6f} & \text{for}\ f\neq 0\\ x(a+b\sec^{2}\left(e\right))\tan^{5}\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e +
f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(a +
 b*sec(e)**2)*tan(e)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(66) = 132$.
time = 1.84, size = 280, normalized size = 3.89

$$\frac{6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{11\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^{3}+90\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^{2}+276\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+280\,a-128\,b}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/12*(6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) - 6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a*((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90*a*(
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
1))^2 + 276*a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) + 280*a - 128*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f
```

Mupad [B]
time = 4.70, size = 52, normalized size = 0.72

$$\frac{\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2}-\frac{a\tan(e+f\,x)^2}{2}+\frac{a\tan(e+f\,x)^4}{4}+\frac{b\tan(e+f\,x)^6}{6}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] ((a*log(tan(e + f*x)^2 + 1))/2 - (a*tan(e + f*x)^2)/2 + (a*tan(e + f*x)^4)/
4 + (b*tan(e + f*x)^6)/6)/f
```

3.312 $\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx$

Optimal. Leaf size=49

$$\frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*(a-b)*sec(f*x+e)^2/f+1/4*b*sec(f*x+e)^4/f
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 77}

$$\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + ((a - b)*Sec[e + f*x]^2)/(2*f) + (b*Sec[e + f*x]^
4)/(4*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$= \frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 0.88

$$\frac{b\tan^4(e+fx)}{4f}+\frac{a(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
[Out] (b*Tan[e + f*x]^4)/(4*f) + (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.92

method	result	size
derivativedivides	$\frac{a\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+\frac{b\left(\sin^4(fx+e)\right)}{4\cos(fx+e)^4}}{f}$	45
default	$\frac{a\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+\frac{b\left(\sin^4(fx+e)\right)}{4\cos(fx+e)^4}}{f}$	45
risch	$-iax-\frac{2iae}{f}-\frac{2\left(-a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}-a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}+\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(1/2*tan(f*x+e)^2+ln(cos(f*x+e)))+1/4*b*sin(f*x+e)^4/cos(f*x+e)^4)
```

Maxima [A]
time = 0.26, size = 68, normalized size = 1.39

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin(fx+e)^2-2\,a+b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.78, size = 54, normalized size = 1.10

$$\frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\,(a-b)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)/
(f*cos(f*x + e)^4)
```

Sympy [A]
time = 0.31, size = 80, normalized size = 1.63

$$\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0\\ x(a+b\sec^2\left(e\right))\tan^3\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)),
 (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(45) = 90.
time = 0.84, size = 236, normalized size = 4.82

$$-\frac{2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^2+20\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+28\,a-16\,b}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) - 2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a*((cos(f*x + e) +
 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 20*a*((
cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1
)) + 28*a - 16*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)^2)/f
```

Mupad [B]
time = 4.78, size = 46, normalized size = 0.94

$$\frac{a\tan(e+f\,x)^2}{2\,f}-\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*tan(e + f*x)^2)/(2*f) - (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f
*x)^4)/(4*f)
```

3.313 $\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx$

Optimal. Leaf size=30

$$-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/2*b*sec(f*x+e)^2/f
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {4223, 14}

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\left(\sec^2(fx+e)\right)}{2}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\left(\sec^2(fx+e)\right)}{2}+a\ln(\sec(fx+e))}{f}$	26
risch	$iax+\frac{2iae}{f}+\frac{2b\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 1.17

$$-\frac{a\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin(fx+e)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 3.55, size = 40, normalized size = 1.33

$$-\frac{2\,a\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-b}{2\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

Sympy [A]
time = 0.14, size = 42, normalized size = 1.40

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\sec^2\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x(a+b\sec^2\left(e\right))\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(f
, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(28) = 56.
time = 0.48, size = 190, normalized size = 6.33

$$\frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2a-4b}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 2)) - a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)
 - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (a*((cos(f*x + e) + 1)/(co
s(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b)/((cos
(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 2))/f
```

Mupad [B]
time = 4.93, size = 32, normalized size = 1.07

$$\frac{a\,\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)}{2\,f}+\frac{b\,{\mathrm{tan}\left(e+f\,x\right)}^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f*x)^2)/(2*f)
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}$$

[Out] -b*ln(cos(f*x+e))/f+(a+b)*ln(sin(f*x+e))/f

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4223, 457, 78}

$$\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]

[Out] -((b*Log[Cos[e + f*x]])/f) + ((a + b)*Log[Sin[e + f*x]])/f

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.57

$$-\frac{b(\log(\cos(e+fx))-\log(\sin(e+fx)))}{f}+\frac{a(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
[Out] -((b*(Log[Cos[e + f*x]] - Log[Sin[e + f*x]]))/f) + (a*(Log[Cos[e + f*x]] +
Log[Tan[e + f*x]]))/f
```

Maple [A]
time = 0.06, size = 24, normalized size = 0.86

method	result	size
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
default	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
risch	$-iax-\frac{2iae}{f}+\frac{\ln(e^{2i(fx+e)}-1)a}{f}+\frac{\ln(e^{2i(fx+e)}-1)b}{f}-\frac{b\ln(e^{2i(fx+e)}+1)}{f}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*ln(sin(f*x+e))+b*ln(tan(f*x+e)))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 1.25

$$-\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

Fricas [A]
time = 3.16, size = 37, normalized size = 1.32

$$-\frac{b\log\left(\cos\left(fx+e\right)^{2}\right)-(a+b)\log\left(-\frac{1}{4}\cos\left(fx+e\right)^{2}+\frac{1}{4}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(28) = 56.
time = 0.45, size = 98, normalized size = 3.50

$$-\frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+b\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
[Out] -1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 2)) + b*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1
) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)))/f
```

Mupad [B]
time = 4.87, size = 32, normalized size = 1.14

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+b\right)}{f}-\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2),x)
[Out] (log(tan(e + f*x))*(a + b))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.315 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=32

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*(a+b)*csc(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4223, 455, 45}

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + b)*Csc[e + f*x]^2)/f - (a*Log[Sin[e + f*x]])/f
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)\csc^2(e+fx)}{2f} - \frac{a\log(\sin(e+fx))}{f}$$

Mathematica [A]
time = 0.19, size = 52, normalized size = 1.62

$$-\frac{b\csc^2(e+fx)}{2f} - \frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
[Out] -1/2*(b*Csc[e + f*x]^2)/f - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Lo
g[Tan[e + f*x]]))/(2*f)
```

Maple [A]
time = 0.06, size = 39, normalized size = 1.22

method	result	size
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
default	$\frac{a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
risch	$iax+\frac{2iae}{f}+\frac{2(a+b)e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-1/2*b/sin(f*x+e)^2)
```

Maxima [A]
time = 0.25, size = 31, normalized size = 0.97

$$-\frac{a \log\left(\sin\left(fx+e\right)^{2}\right)+\frac{a+b}{\sin(fx+e)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

Fricas [A]
time = 3.86, size = 53, normalized size = 1.66

$$-\frac{2\left(a\cos\left(fx+e\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^{2}-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
e)^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(30) = 60.
time = 0.45, size = 146, normalized size = 4.56

$$-\frac{4\,a\log\left(\frac{\left|-\cos(fx+e)+1\right|}{\left|\cos(fx+e)+1\right|}\right)-8\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*log(abs(-
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f*x + e) -
```

```
1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

Mupad [B]
time = 6.14, size = 51, normalized size = 1.59

$$\frac{a\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{a\ln\left(\tan(e+f\,x)\right)}{f}-\frac{\cot(e+f\,x)^2\,\left(\frac{a}{2}+\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a*log(tan(e + f*x)))/f - (cot(e + f*x)
^2*(a/2 + b/2))/f
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$\frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] 1/2*(2*a+b)*csc(f*x+e)^2/f-1/4*(a+b)*csc(f*x+e)^4/f+a*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 78}

$$-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[e + f*x]^2)/(2*f) - ((a + b)*Csc[e + f*x]^4)/(4*f) + (a*Log[
Sin[e + f*x]])/f
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}\,dx,x,\cos(e+fx)\right)}{f}$$
$$=-\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=\frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

Mathematica [A]
time = 0.20, size = 64, normalized size = 1.25

$$-\frac{b\cot^4(e+fx)}{4f}+\frac{a(2\cot^2(e+fx)-\cot^4(e+fx)+4\log(\cos(e+fx))+4\log(\tan(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/4*(b*Cot[e + f*x]^4)/f + (a*(2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[C
os[e + f*x]] + 4*Log[Tan[e + f*x]]))/(4*f)
```

Maple [A]
time = 0.06, size = 55, normalized size = 1.08

method	result	size
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)-\frac{b\left(\cos^4(fx+e)\right)}{4\sin(fx+e)^4}}{f}$	55
default	$\frac{a\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)-\frac{b\left(\cos^4(fx+e)\right)}{4\sin(fx+e)^4}}{f}$	55
risch	$-iax-\frac{2iae}{f}-\frac{2\left(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/4*b/sin(f*x+e)
^4*cos(f*x+e)^4)
```

Maxima [A]
time = 0.25, size = 52, normalized size = 1.02

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^{2}\right)+\frac{2\,(2\,a+b)\sin(fx+e)^{2}-a-b}{\sin(fx+e)^{4}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*x
 + e)^4)/f
```

Fricas [A]
time = 4.30, size = 89, normalized size = 1.75

$$-\frac{2\,(2\,a+b)\cos\left(fx+e\right)^{2}-4\,\left(a\cos\left(fx+e\right)^{4}-2\,a\cos\left(fx+e\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-3\,a-b}{4\,\left(f\cos\left(fx+e\right)^{4}-2\,f\cos\left(fx+e\right)^{2}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2
 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)
^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cot^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(47) = 94.
time = 0.49, size = 238, normalized size = 4.67

$$\frac{32\,a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-64\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)(\cos(fx+e)+1)^{2}}{(\cos(fx+e)-1)^{2}}-\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(32*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a*log(abs
(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 12*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x +
 e) - 1)^2 - 12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - b*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

Mupad [B]
time = 6.10, size = 61, normalized size = 1.20

$$\frac{a\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{a\,\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f}-\frac{-\frac{a\,{\tan(e+f\,x)}^2}{2}+\frac{a}{4}+\frac{b}{4}}{f\,{\tan\left(e+f\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a/4 + b/4 -
(a*tan(e + f*x)^2)/2)/(f*tan(e + f*x)^4)
```

3.317 $\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx$

Optimal. Leaf size=64

$$-ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -a*x+a*tan(f*x+e)/f-1/3*a*tan(f*x+e)^3/f+1/5*a*tan(f*x+e)^5/f+1/7*b*tan(f*x
+e)^7/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4226, 1816, 209}

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (a*Tan[e + f*x]^5)
/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6(a+b(1+x^2))}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a-ax^2+ax^4+bx^6-\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{a\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} + \frac{b\tan^7(e+fx}{7f}$$
$$= -ax + \frac{a\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} + \frac{b\tan^7}{}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.14

$$-\frac{a\text{ArcTan}(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} + \frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*
f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Maple [A]
time = 0.06, size = 61, normalized size = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)-fx-e\right)+\frac{b\left(\sin^7(fx+e)\right)}{7\cos(fx+e)^7}}{f}$
default	$\frac{a\left(\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)-fx-e\right)+\frac{b\left(\sin^7(fx+e)\right)}{7\cos(fx+e)^7}}{f}$
risch	$-ax-\frac{2i(-315a\,\mathrm{e}^{12i(fx+e)}+105b\,\mathrm{e}^{12i(fx+e)}-1260a\,\mathrm{e}^{10i(fx+e)}-2555a\,\mathrm{e}^{8i(fx+e)}+525b\,\mathrm{e}^{8i(fx+e)}-3080a\,\mathrm{e}^{6i(fx+e)}-2}{105f\left(\mathrm{e}^{2i(fx+e)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+1/7*b*sin(f*x+e
)^7/cos(f*x+e)^7)
```

Maxima [A]
time = 0.46, size = 61, normalized size = 0.95

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}-35\,a\tan\left(fx+e\right)^{3}-105\,(fx+e)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

Fricas [A]
time = 5.15, size = 95, normalized size = 1.48

$$-\frac{105\,afx\cos\left(fx+e\right)^{7}-\left((161\,a-15\,b)\cos\left(fx+e\right)^{6}-(77\,a-45\,b)\cos\left(fx+e\right)^{4}+3\,(7\,a-15\,b)\cos\left(fx+e\right)^{2}+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a -
 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e))
/(f*cos(f*x + e)^7)
```

Sympy [A]
time = 1.85, size = 66, normalized size = 1.03

$$a\left(\begin{cases}-x+\frac{\tan^{5}\left(e+fx\right)}{5f}-\frac{\tan^{3}\left(e+fx\right)}{3f}+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^{6}\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x\tan^{6}\left(e\right)\sec^{2}\left(e\right) & \text{for}\ f=0\\ \frac{\tan^{7}\left(e+fx\right)}{7f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f
*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)**2
, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

Giac [A]
time = 2.33, size = 56, normalized size = 0.88

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}-35\,a\tan\left(fx+e\right)^{3}-105\,(fx+e)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
```

[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f

Mupad [B]
time = 4.99, size = 51, normalized size = 0.80

$$\frac{\frac{b\tan(e+f\,x)^7}{7}+\frac{a\tan(e+f\,x)^5}{5}-\frac{a\tan(e+f\,x)^3}{3}+a\tan(e+f\,x)-a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2),x)

[Out] (a*tan(e + f*x) - (a*tan(e + f*x)^3)/3 + (a*tan(e + f*x)^5)/5 + (b*tan(e +
f*x)^7)/7 - a*f*x)/f

3.318 $\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$

Optimal. Leaf size=48

$$ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] a*x-a*tan(f*x+e)/f+1/3*a*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4226, 1816, 209}

$$\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] a*x - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^5)/(5
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-a+ax^2+bx^4+\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+\frac{a\text{Subst}\left(\int \frac{1}{1}\right.}{}$$
$$= ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.19

$$\frac{a\text{ArcTan}(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
[Out] (a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
+ (b*Tan[e + f*x]^5)/(5*f)
```

Maple [A]
time = 0.05, size = 50, normalized size = 1.04

method	result	siz
derivativedivides	$\frac{a\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)+\frac{b\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$	50
default	$\frac{a\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)+\frac{b\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$	50
risch	$ax+\frac{2i\left(-30a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}-110a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}-20a+3b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+1/5*b*sin(f*x+e)^5/cos(f*x+e)^5)
```

Maxima [A]
time = 0.46, size = 49, normalized size = 1.02

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+15\left(fx+e\right)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

Fricas [A]
time = 3.86, size = 77, normalized size = 1.60

$$\frac{15\,afx\cos\left(fx+e\right)^{5}-\left(\left(20\,a-3\,b\right)\cos\left(fx+e\right)^{4}-\left(5\,a-6\,b\right)\cos\left(fx+e\right)^{2}-3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)*
cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [A]
time = 1.14, size = 54, normalized size = 1.12

$$a\left(\begin{cases} x+\frac{\tan^{3}\left(e+fx\right)}{3f}-\frac{\tan\left(e+fx\right)}{f} & \text{for}\: f\neq 0\\ x\tan^{4}\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases} x\tan^{4}\left(e\right)\sec^{2}\left(e\right) & \text{for}\: f=0\\ \frac{\tan^{5}\left(e+fx\right)}{5f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

```
[Out] a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan(
e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f*x
)**5/(5*f), True))
```

Giac [A]
time = 1.10, size = 45, normalized size = 0.94

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,a\tan\left(fx+e\right)^{3}+15\,(fx+e)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

Mupad [B]
time = 4.68, size = 40, normalized size = 0.83

$$\frac{\frac{b\tan(e+f\,x)^5}{5}+\frac{a\tan(e+f\,x)^3}{3}-a\tan(e+f\,x)+a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
[Out] ((a*tan(e + f*x)^3)/3 - a*tan(e + f*x) + (b*tan(e + f*x)^5)/5 + a*f*x)/f
```

3.319 $\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx$

Optimal. Leaf size=32

$$-ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -a*x+a*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4226, 1816, 209}

$$\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2(a+b(1+x^2))}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \left(a+bx^2-\frac{a}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}-\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+f\right.}{f}$$

$$=-ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.28

$$-\frac{a\text{ArcTan}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*
f)
```

Maple [A]
time = 0.06, size = 41, normalized size = 1.28

method	result	size
derivativedivides	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	41
default	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	41
risch	$-ax-\frac{2i\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(tan(f*x+e)-f*x-e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)
```

Maxima [A]
time = 0.46, size = 36, normalized size = 1.12

$$\frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Fricas [A]
time = 3.59, size = 57, normalized size = 1.78

$$-\frac{3\,afx\cos\left(fx+e\right)^3-\left(\left(3\,a-b\right)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] -1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e))
/(f*cos(f*x + e)^3)
```

Sympy [A]
time = 0.85, size = 42, normalized size = 1.31

$$a\left(\begin{cases}-x+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^2\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x\tan^2\left(e\right)\sec^2\left(e\right) & \text{for}\ f=0\\ \frac{\tan^3\left(e+fx\right)}{3f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
[Out] a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piece
wise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

Giac [A]
time = 0.65, size = 33, normalized size = 1.03

$$\frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Mupad [B]
time = 4.52, size = 29, normalized size = 0.91

$$\frac{\frac{b\tan(e+f\,x)^3}{3}+a\tan(e+f\,x)-a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
[Out] (a*tan(e + f*x) + (b*tan(e + f*x)^3)/3 - a*f*x)/f
```

3.320 $\int (a+b\sec^2(e+fx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^2(e+fx))\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.00, size = 16, normalized size = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
norman	$\frac{ax\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A]
time = 0.26, size = 16, normalized size = 1.07

$$ax+\frac{b\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.
time = 2.97, size = 34, normalized size = 2.27

$$\frac{afx\cos(fx+e)+b\sin(fx+e)}{f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.42, size = 15, normalized size = 1.00

$$ax+\frac{b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Mupad [B]
time = 4.51, size = 17, normalized size = 1.13

$$\frac{b\tan(e+f\,x)+a\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + a*f*x)/f
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=19

$$-ax-\frac{(a+b)\cot(e+fx)}{f}$$

[Out] -a*x-(a+b)*cot(f*x+e)/f

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 209}

$$-\frac{(a+b)\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]

[Out] -(a*x) - ((a + b)*Cot[e + f*x])/f

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^2(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -ax-\frac{(a+b)\cot(e+fx)}{f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 43, normalized size = 2.26

$$
-\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[e + f*x]^2])/f
```

Maple [A]
time = 0.06, size = 33, normalized size = 1.74

method	result	size
derivativedivides	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
default	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
risch	$-ax-\frac{2ia}{f\left(e^{2i(fx+e)}-1\right)}-\frac{2ib}{f\left(e^{2i(fx+e)}-1\right)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

Maxima [A]
time = 0.46, size = 27, normalized size = 1.42

$$
-\frac{(fx+e)a+\frac{a+b}{\tan(fx+e)}}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

Fricas [A]
time = 3.54, size = 37, normalized size = 1.95

$$-\frac{afx\sin(fx+e)+(a+b)\cos(fx+e)}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec^2(e+fx))\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.
time = 0.46, size = 53, normalized size = 2.79

$$-\frac{2(fx+e)a-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a+b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
+ b)/tan(1/2*f*x + 1/2*e))/f
```

Mupad [B]
time = 4.50, size = 19, normalized size = 1.00

$$-a\,x-\frac{\cot(e+f\,x)\,(a+b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

```
[Out] - a*x - (cot(e + f*x)*(a + b))/f
```

3.322 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=33

$$ax+\frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}$$

```
[Out] a*x+a*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4226, 1816, 209}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] a*x + (a*Cot[e + f*x])/f - ((a + b)*Cot[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}-\frac{a}{x^2}+\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f} + \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\right.}{f}$$
$$= ax + \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 51, normalized size = 1.55

$$-\frac{b\cot^3(e+fx)}{3f} - \frac{a\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/3*(b*Cot[e + f*x]^3)/f - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1
/2, -Tan[e + f*x]^2])/(3*f)
```

Maple [A]
time = 0.05, size = 48, normalized size = 1.45

method	result	size
derivativedivides	$\frac{a\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)-\frac{b\left(\cos^3(fx+e)\right)}{3\sin(fx+e)^3}}{f}$	48
default	$\frac{a\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)-\frac{b\left(\cos^3(fx+e)\right)}{3\sin(fx+e)^3}}{f}$	48
risch	$ax+\frac{2i\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+4a+b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^3
)
```

Maxima [A]
time = 0.46, size = 44, normalized size = 1.33

$$\frac{3\,(fx+e)a+\frac{3\,a\tan(fx+e)^2-a-b}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(33) = 66$.
time = 2.93, size = 82, normalized size = 2.48

$$\frac{(4\,a+b)\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)+3\left(afx\cos\left(fx+e\right)^2-afx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] 1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
- a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(31) = 62$.
time = 0.49, size = 111, normalized size = 3.36

$$\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)a-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a-b}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
- 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x
+ 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B]
time = 4.67, size = 35, normalized size = 1.06

$$a\,x-\frac{-a\tan(e+f\,x)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

```
[Out] a*x - (a/3 + b/3 - a*tan(e + f*x)^2)/(f*tan(e + f*x)^3)
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-ax-\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}$$

```
[Out] -a*x-a*cot(f*x+e)/f+1/3*a*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4226, 1816, 209}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - (a*Cot[e + f*x])/f + (a*Cot[e + f*x]^3)/(3*f) - ((a + b)*Cot[e + f
*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^6} - \frac{a}{x^4} + \frac{a}{x^2} - \frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\cot(e+fx)}{f} + \frac{a\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f} - \frac{a\text{Sub}}{}$$
$$= -ax - \frac{a\cot(e+fx)}{f} + \frac{a\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 51, normalized size = 1.00

$$-\frac{b\cot^5(e+fx)}{5f} - \frac{a\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/5*(b*Cot[e + f*x]^5)/f - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3
/2, -Tan[e + f*x]^2])/(5*f)
```

Maple [A]
time = 0.05, size = 63, normalized size = 1.24

method	result	s
derivativedivides	$\frac{a\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)-\frac{b\left(\cos^5(fx+e)\right)}{5\sin(fx+e)^5}}{f}$	6
default	$\frac{a\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)-\frac{b\left(\cos^5(fx+e)\right)}{5\sin(fx+e)^5}}{f}$	6
risch	$-ax-\frac{2i\left(45a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}+23a+3b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x+
e)^5*cos(f*x+e)^5)
```

Maxima [A]
time = 0.46, size = 56, normalized size = 1.10

$$-\frac{15\,(fx+e)a+\frac{15\,a\tan(fx+e)^4-5\,a\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a + 3
*b)/tan(f*x + e)^5)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(50) = 100.
time = 3.40, size = 119, normalized size = 2.33

$$-\frac{(23\,a+3\,b)\cos\left(fx+e\right)^5-35\,a\cos\left(fx+e\right)^3+15\,a\cos\left(fx+e\right)+15\,\left(afx\cos\left(fx+e\right)^4-2\,afx\cos\left(fx+e\right)^2+afx\right)\sin\left(fx+e\right)}{15\,\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x + e
) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x + e)
)/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(47) = 94.
time = 0.48, size = 170, normalized size = 3.33

$$\frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-480\,(fx+e)a+330\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+30\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{330\,a\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4+30\,b\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-35\,a\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-15\,b\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+3\,a+3\,b}{\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1
/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a*t
an(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*
e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b*tan
(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B]
time = 4.89, size = 46, normalized size = 0.90

$$-a\,x-\frac{a\,{\mathrm{tan}\left(e+f\,x\right)}^4-\frac{a\,{\mathrm{tan}\left(e+f\,x\right)}^2}{3}+\frac{a}{5}+\frac{b}{5}}{f\,{\mathrm{tan}\left(e+f\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

```
[Out] - a*x - (a/5 + b/5 - (a*tan(e + f*x)^2)/3 + a*tan(e + f*x)^4)/(f*tan(e + f*
x)^5)
```

3.324 $\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}+\frac{(a-b)b\sec^6(e+fx)}{3f}+\frac{b^2\sec^8(e+fx)}{8f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f-a*(a-b)*sec(f*x+e)^2/f+1/4*(a^2-4*a*b+b^2)*sec(f*x+e)
^4/f+1/3*(a-b)*b*sec(f*x+e)^6/f+1/8*b^2*sec(f*x+e)^8/f
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 457, 90}

$$\frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) - (a*(a - b)*Sec[e + f*x]^2)/f + ((a^2 - 4*a*b
 + b^2)*Sec[e + f*x]^4)/(4*f) + ((a - b)*b*Sec[e + f*x]^6)/(3*f) + (b^2*Sec
[e + f*x]^8)/(8*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)^2}{x^9}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)^2}{x^5}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^5}+\frac{2(a-b)b}{x^4}+\frac{a^2-4ab+b^2}{x^3}-\frac{2a(a-b)}{x^2}+\frac{a^2}{x}\right)dx, x, \cos^2(\right.}{2f}$$

$$= -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{(a^2-4ab+b^2}{4}$$

Mathematica [A]

time = 0.47, size = 126, normalized size = 1.26

$$-\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(24a^2\log(\cos(e+fx))+24a(a-b)\sec^2(e+fx)-6(a^2-4ab+b^2)\sec^4(e+fx)-8(a-b)b\sec^6(e+fx)-3b^2\sec^8(e+fx)\right)}{6f(a+2b+a\cos(2e+2fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -1/6*(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] + 2
4*a*(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a -
b)*b*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(f*(a + 2*b + a*Cos[2*e + 2*f*
x])^2)
```

Maple [A]

time = 0.09, size = 101, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+\frac{ab\left(\sin^6(fx+e)\right)}{3\cos(fx+e)^6}+b^2\left(\frac{\sin^6(fx+e)}{8\cos(fx+e)^8}+\frac{\sin^6(fx+e)}{24\cos(fx+e)^6}\right)}{f}$
default	$\frac{a^2\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+\frac{ab\left(\sin^6(fx+e)\right)}{3\cos(fx+e)^6}+b^2\left(\frac{\sin^6(fx+e)}{8\cos(fx+e)^8}+\frac{\sin^6(fx+e)}{24\cos(fx+e)^6}\right)}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{-4a^2e^{14i(fx+e)}+4ab\,e^{14i(fx+e)}-20a^2e^{12i(fx+e)}+8ab\,e^{12i(fx+e)}+4b^2e^{12i(fx+e)}-44a^2e^{10i(fx+e)}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2-ln(cos(f*x+e)))+1/3*a*b*sin(f*x
+e)^6/cos(f*x+e)^6+b^2*(1/8*sin(f*x+e)^6/cos(f*x+e)^8+1/24*sin(f*x+e)^6/cos
(f*x+e)^6))
```

Maxima [A]
time = 0.26, size = 155, normalized size = 1.55

$$-\frac{12\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\,(a^2-ab)\,\sin(fx+e)^6-6\,(11\,a^2-8\,ab-b^2)\,\sin(fx+e)^4+4\,(15\,a^2-8\,ab-b^2)\,\sin(fx+e)^2-18\,a^2+8\,ab+b^2}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6*
(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x +
e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x
 + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 2.79, size = 105, normalized size = 1.05

$$-\frac{24\,a^2\cos\left(fx+e\right)^8\log\left(-\cos\left(fx+e\right)\right)+24\,(a^2-ab)\cos\left(fx+e\right)^6-6\,(a^2-4\,ab+b^2)\cos\left(fx+e\right)^4-8\,(ab-b^2)\cos\left(fx+e\right)^2-3\,b^2}{24\,f\cos\left(fx+e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^2
- 3*b^2)/(f*cos(f*x + e)^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.
time = 1.61, size = 190, normalized size = 1.90

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\tan^4\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}-\frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{3f}+\frac{ab\sec^2\left(e+fx\right)}{3f}+\frac{b^2\tan^4\left(e+fx\right)\sec^4\left(e+fx\right)}{8f}-\frac{b^2\tan^2\left(e+fx\right)\sec^4\left(e+fx\right)}{12f}+\frac{b^2\sec^4\left(e+fx\right)}{24f} & \text{for}\ f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^5\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f) -
 a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) + b*
*2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e + f*x
)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)**2)*
*2*tan(e)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(94) = 188.
time = 2.11, size = 431, normalized size = 4.31

$$\frac{12\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-12\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{25\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^4+248\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^3+984\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^2+1760\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-512\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-256\,b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+1168\,a^2-1024\,ab+256\,b^2}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/24*(12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 2)) - 12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*
x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (25*a^2*((cos(f
*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^4
+ 248*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1))^3 + 984*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*
x + e) - 1)/(cos(f*x + e) + 1))^2 + 1760*a^2*((cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 512*a*b*((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 256*b^2*
((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1)) + 1168*a^2 - 1024*a*b + 256*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1
) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^4)/f
```

Mupad [B]
time = 4.51, size = 124, normalized size = 1.24

$$\frac{\tan(e+f\,x)^4\left(\frac{(a+b)^2}{4}+\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f}-\frac{\tan(e+f\,x)^2\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f}-\frac{\tan(e+f\,x)^6\left(\frac{b^2}{6}-\frac{b\,(a+b)}{3}\right)}{f}+\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b^2\tan(e+f\,x)^8}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^4*((a + b)^2/4 + b^2/4 - (b*(a + b))/2))/f - (tan(e + f*x)^2*
((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^6*(b^2/6 - (b*(a + b))
/3))/f + (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^8)/(8*f)
```

3.325 $\int (a + b\sec^2(e + fx))^2 \tan^3(e + fx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cos(e+fx))}{f} + \frac{a(a-2b)\sec^2(e+fx)}{2f} + \frac{(2a-b)b\sec^4(e+fx)}{4f} + \frac{b^2\sec^6(e+fx)}{6f}$$

```
[Out] a^2*ln(cos(f*x+e))/f+1/2*a*(a-2*b)*sec(f*x+e)^2/f+1/4*(2*a-b)*b*sec(f*x+e)^
4/f+1/6*b^2*sec(f*x+e)^6/f
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 77}

$$\frac{a^2\log(\cos(e+fx))}{f} + \frac{b(2a-b)\sec^4(e+fx)}{4f} + \frac{a(a-2b)\sec^2(e+fx)}{2f} + \frac{b^2\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (a^2*Log[Cos[e + f*x]])/f + (a*(a - 2*b)*Sec[e + f*x]^2)/(2*f) + ((2*a - b)
*b*Sec[e + f*x]^4)/(4*f) + (b^2*Sec[e + f*x]^6)/(6*f)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^7}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\ &= \frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e}{4f}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 107, normalized size = 1.39

$$\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12a^2\log(\cos(e+fx))+6a(a-2b)\sec^2(e+fx)+3(2a-b)b\sec^4(e+fx)+2b^2\sec^6(e+fx)\right)}{3f(a+2b+a\cos(2e+2fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
[Out] (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(a
 - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x]^
6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

Maple [A]
time = 0.07, size = 89, normalized size = 1.16

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+\frac{ab\left(\sin^4(fx+e)\right)}{2\cos(fx+e)^4}+b^2\left(\frac{\sin^4(fx+e)}{6\cos(fx+e)^6}+\frac{\sin^4(fx+e)}{12\cos(fx+e)^4}\right)}{f}$
default	$\frac{a^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+\frac{ab\left(\sin^4(fx+e)\right)}{2\cos(fx+e)^4}+b^2\left(\frac{\sin^4(fx+e)}{6\cos(fx+e)^6}+\frac{\sin^4(fx+e)}{12\cos(fx+e)^4}\right)}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{2\left(-3a^2e^{10i(fx+e)}+6ab\,e^{10i(fx+e)}-12a^2e^{8i(fx+e)}+12ab\,e^{8i(fx+e)}+6b^2e^{8i(fx+e)}-18a^2e^{6i(fx+e}\right.}{3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(a^2*(1/2*tan(f*x+e)^2+ln(cos(f*x+e)))+1/2*a*b*sin(f*x+e)^4/cos(f*x+e)^4+b^2*(1/6*sin(f*x+e)^4/cos(f*x+e)^6+1/12*sin(f*x+e)^4/cos(f*x+e)^4))

Maxima [A]
time = 0.26, size = 120, normalized size = 1.56

$$\frac{6\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\,(a^2-2\,ab)\sin(fx+e)^4-3\,(4\,a^2-6\,ab-b^2)\sin(fx+e)^2+6\,a^2-6\,ab-b^2}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*(
4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^6
- 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 2.21, size = 84, normalized size = 1.09

$$\frac{12\,a^2\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\,(a^2-2\,ab)\cos\left(fx+e\right)^4+3\,(2\,ab-b^2)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

Sympy [A]
time = 0.79, size = 128, normalized size = 1.66

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{2f}-\frac{ab\sec^2\left(e+fx\right)}{2f}+\frac{b^2\tan^2\left(e+fx\right)\sec^4\left(e+fx\right)}{6f}-\frac{b^2\sec^4\left(e+fx\right)}{12f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f) +
 b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f),
Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(71) = 142.
time = 1.04, size = 385, normalized size = 5.00

$$-\frac{6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{11\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^3+90\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^2+228\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-96\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-48\,b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+184\,a^2-192\,ab+32\,b^2}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")

[Out] -1/12*(6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)) - 6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 228*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 96*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 48*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 184*a^2 - 192*a*b + 32*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f

Mupad [B]
time = 4.54, size = 92, normalized size = 1.19

$$\frac{\tan(e+f\,x)^2\,\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f}-\frac{\tan(e+f\,x)^4\,\left(\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f}-\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b^2\tan(e+f\,x)^6}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)

[Out] (tan(e + f*x)^2*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^4*(b^2/4 - (b*(a + b))/2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^6)/(6*f)

3.326 $\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

```
[Out] -a^2*ln(cos(f*x+e))/f+a*b*sec(f*x+e)^2/f+1/4*b^2*sec(f*x+e)^4/f
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4223, 272, 45}

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) + (a*b*Sec[e + f*x]^2)/f + (b^2*Sec[e + f*x]^4
)/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^5}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 1.71

$$-\frac{(b+a\cos^2(e+fx))^2\left(-b^2-4ab\cos^2(e+fx)+4a^2\cos^4(e+fx)\log(\cos(e+fx))\right)\sec^4(e+fx)}{f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-b^2 - 4*a*b*Cos[e + f*x]^2 + 4*a^2*Cos[e + f*
x]^4*Log[Cos[e + f*x]])*Sec[e + f*x]^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2
))
```

Maple [A]
time = 0.06, size = 41, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{(\sec^4(fx+e))b^2}{4}+(\sec^2(fx+e))ab+a^2\ln(\sec(fx+e))}{f}$	41
default	$\frac{\frac{(\sec^4(fx+e))b^2}{4}+(\sec^2(fx+e))ab+a^2\ln(\sec(fx+e))}{f}$	41
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{4b\left(a\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}-\frac{a^2\ln\left(e^{2i(fx+e)}+1\right)}{f}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/4*sec(f*x+e)^4*b^2+sec(f*x+e)^2*a*b+a^2*ln(sec(f*x+e)))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 1.48

$$-\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin(fx+e)^2-4\,ab-b^2}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)/
(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.07, size = 57, normalized size = 1.19

$$-\frac{4\,a^2\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)-4\,ab\cos\left(fx+e\right)^2-b^2}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2)
/(f*cos(f*x + e)^4)
```

Sympy [A]
time = 0.30, size = 61, normalized size = 1.27

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ab\sec^2\left(e+fx\right)}{f}+\frac{b^2\sec^4\left(e+fx\right)}{4f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b*
*2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(46) = 92.
time = 0.58, size = 334, normalized size = 6.96

$$\frac{2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)^2+12\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-16\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-8\,b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+12\,a^2-32\,ab}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2)) - 2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a^2*((cos(f*x +
 e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 12
*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1)) - 16*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
 1)/(cos(f*x + e) + 1)) - 8*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) + 12*a^2 - 32*a*b)/((cos(f*x + e) + 1)
/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^2)/f
```

Mupad [B]
time = 4.50, size = 61, normalized size = 1.27

$$\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{\tan(e+f\,x)^2\,\left(\frac{b^2}{2}-b\,(a+b)\right)}{f}+\frac{b^2\,\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (tan(e + f*x)^2*(b^2/2 - b*(a + b)))/
f + (b^2*tan(e + f*x)^4)/(4*f)
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

```
[Out] -b*(2*a+b)*ln(cos(f*x+e))/f+(a+b)^2*ln(sin(f*x+e))/f+1/2*b^2*sec(f*x+e)^2/f
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 90}

$$\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cos[e + f*x]])/f) + ((a + b)^2*Log[Sin[e + f*x]])/f + (b
^2*Sec[e + f*x]^2)/(2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^3(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec}{}$$

Mathematica [A]
time = 0.27, size = 84, normalized size = 1.58

$$\frac{2(b^2+2\cos^2(e+fx)(-b(2a+b)\log(\cos(e+fx))+(a+b)^2\log(\sin(e+fx))))(a\cos(e+fx)+b\sec(e+fx))^2}{f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
[Out] (2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*Lo
g[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Cos[
2*(e + f*x)])^2)
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.94

method	result
derivativedivides	$\frac{a^2\ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^2\left(\frac{1}{2\cos(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^2\left(\frac{1}{2\cos(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}+\frac{2b^2e^{2i(fx+e)}}{f(e^{2i(fx+e)}+1)^2}-\frac{2b\ln(e^{2i(fx+e)}+1)a}{f}-\frac{b^2\ln(e^{2i(fx+e)}+1)}{f}+\frac{\ln(e^{2i(fx+e)}-1)a^2}{f}+2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a^2*ln(sin(f*x+e))+2*a*b*ln(tan(f*x+e))+b^2*(1/2/cos(f*x+e)^2+ln(tan(f
*x+e))))
```

Maxima [A]
time = 0.26, size = 67, normalized size = 1.26

$$-\frac{\left(2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(f
*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 2.86, size = 84, normalized size = 1.58

$$-\frac{\left(2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(\cos\left(fx+e\right)^2\right)-\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right)-b^2}{2\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^2
)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(51) = 102.
time = 0.49, size = 247, normalized size = 4.66

$$-\frac{a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+\left(2\,ab+b^2\right)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)-\frac{2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+4\,ab-2\,b^2}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 2)) + (2*a*b + b^2)*log(abs(-(cos(f*x + e) + 1)/(co
```

```
s(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) - (2*a*b*((co
s(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
 + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x
 + e) + 1)) + 4*a*b - 2*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/f
```

Mupad [B]
time = 4.48, size = 58, normalized size = 1.09

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+b^2\right)}{f}-\frac{a^2\,\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f}+\frac{b^2\,{\tan(e+f\,x)}^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x))*(2*a*b + a^2 + b^2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(
2*f) + (b^2*tan(e + f*x)^2)/(2*f)
```

3.328 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=57

$$-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}-\frac{(a^2-b^2)\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*(a+b)^2*csc(f*x+e)^2/f-b^2*ln(cos(f*x+e))/f-(a^2-b^2)*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 457, 90}

$$-\frac{(a^2-b^2)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)^2*Csc[e + f*x]^2)/f - (b^2*Log[Cos[e + f*x]])/f - ((a^2 - b^2
)*Log[Sin[e + f*x]])/f
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}-\frac{(a^2-b^2)\log(\text{si}}{f}$$

Mathematica [A]
time = 0.20, size = 81, normalized size = 1.42

$$-\frac{2\left(b+a\cos^2(e+fx)\right)^2\left((a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))+2(a^2-b^2)\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])
^2)
```

Maple [A]
time = 0.07, size = 64, normalized size = 1.12

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{2(a^2+2ab+b^2)e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{b^2\ln\left(e^{2i(fx+e)}+1\right)}{f}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a^2}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b^2}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-a*b/sin(f*x+e)^2+b^2*(-1/2/sin(
f*x+e)^2+ln(tan(f*x+e))))
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.11

$$-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin(fx+e)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
+ 2*a*b + b^2)/sin(f*x + e)^2)/f
```

Fricas [A]
time = 2.88, size = 105, normalized size = 1.84

$$\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right)}{2\,\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) - (
(a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(f*
cos(f*x + e)^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(55) = 110.
time = 0.53, size = 233, normalized size = 4.09

$$\frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+4\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-4\,b^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) + 2*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e
```

```
) - 1)/(cos(f*x + e) + 1)) + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (
cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*a^2*log(abs(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)) - 4*b^2*log
(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 2)))/f
```

Mupad [B]
time = 4.58, size = 68, normalized size = 1.19

$$\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-b^2\right)}{f}-\frac{\cot(e+f\,x)^2\,\left(\frac{a^2}{2}+a\,b+\frac{b^2}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (log(tan(e + f*x))*(a^2 - b^2))/f - (
cot(e + f*x)^2*(a*b + a^2/2 + b^2/2))/f
```

3.329 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{a(a+b)\csc^2(e+fx)}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}$$

```
[Out] a*(a+b)*csc(f*x+e)^2/f-1/4*(a+b)^2*csc(f*x+e)^4/f+a^2*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 455, 45}

$$\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + b)*Csc[e + f*x]^2)/f - ((a + b)^2*Csc[e + f*x]^4)/(4*f) + (a^2*Log[
Sin[e + f*x]])/f
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)^2}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}-\frac{2a(a+b)}{(-1+x)^2}-\frac{a^2}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a(a+b)\csc^2(e+fx)}{f} - \frac{(a+b)^2\csc^4(e+fx)}{4f} + \frac{a^2\log(\sin(e+}{f}$$

Mathematica [A]
time = 0.27, size = 77, normalized size = 1.51

$$-\frac{\left(b+a\cos^2(e+fx)\right)^2\left(-4a(a+b)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-4a^2\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
+ f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2))
```

Maple [A]
time = 0.07, size = 71, normalized size = 1.39

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)-\frac{ab\left(\cos^4(fx+e)\right)}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)-\frac{ab\left(\cos^4(fx+e)\right)}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{4\left(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}-a^2e^{4i(fx+e)}+b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\ln\left(e^{2i(f}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/2*a*b/sin(f*
x+e)^4*cos(f*x+e)^4-1/4*b^2/sin(f*x+e)^4)
```

Maxima [A]
time = 0.26, size = 64, normalized size = 1.25

$$\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\,(a^2+ab)\sin(fx+e)^2-a^2-2\,ab-b^2}{\sin(fx+e)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*
b - b^2)/sin(f*x + e)^4)/f
```

Fricas [A]
time = 3.73, size = 103, normalized size = 2.02

$$-\frac{4\,(a^2+ab)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\,\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{4\,\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x +
 e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e)^
4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(49) = 98.
time = 0.53, size = 332, normalized size = 6.51

$$\frac{32\,a^2\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-64\,a^2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{\left(a^2+2\,ab+b^2+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*a^2*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a^2*log
(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 12*a^2*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*b^2
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - b^2*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

Mupad [B]
time = 4.61, size = 83, normalized size = 1.63

$$\frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan(e+f\,x)^2\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{f\tan\left(e+f\,x\right)^4}-\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (a^2*log(tan(e + f*x)))/f - ((a*b)/2 + a^2/4 + b^2/4 - tan(e + f*x)^2*(a^2/
2 - b^2/2))/(f*tan(e + f*x)^4) - (a^2*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.330 $\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx$

Optimal. Leaf size=95

$$-a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f-1/3*a^2*tan(f*x+e)^3/f+1/5*a^2*tan(f*x+e)^5/f+1/7*b
*(2*a+b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 209}

$$\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e +
 f*x]^5)/(5*f) + (b*(2*a + b)*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/
(9*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b(1+x^2)\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2-a^2x^2+a^2x^4+b(2a+b)x^6+b^2x^8-\frac{a^2}{1+x^2}\right)\,dx,x\right.}{f}$$

$$= \frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+}{}$$

$$= -a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 275 vs. $2(95) = 190$.

time = 2.16, size = 275, normalized size = 2.89

$$-\frac{4\left(b+a\cos^2(e+fx)\right)^2\sec^9(e+fx)\left(315a^2fx\cos^9(e+fx)-35b^2\sec(e)\sin(fx)-5(18a-19b)b\cos^2(e+fx)\sec(e)\sin(fx)-3(21a^2-90ab+25b^2)\cos^4(e+fx)\sec(e)\sin(fx)+(231a^2-270ab+5b^2)\cos^6(e+fx)\sec(e)\sin(fx)-(483a^2-90ab-10b^2)\cos^8(e+fx)\sec(e)\sin(fx)-35b^2\cos(e+fx)\tan(e)-5(18a-19b)b\cos^3(e+fx)\tan(e)-3(21a^2-90ab+25b^2)\cos^5(e+fx)\tan(e)+(231a^2-270ab+5b^2)\cos^7(e+fx)\tan(e)\right)}{315f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 - 3
5*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] -
3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 - 27
0*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*b^2)
*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a - 19
*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^5*T
an[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(a + 2*b
 + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.08, size = 105, normalized size = 1.11

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\left(\sin^7(fx+e)\right)}{7\cos(fx+e)^7}+b^2\left(\frac{\sin^7(fx+e)}{9\cos(fx+e)^9}+\frac{2\left(\sin^7(fx+e)\right)}{63\cos(fx+e)^7}\right)}{f}$
default	$\frac{a^2\left(\frac{\left(\tan^5(fx+e)\right)}{5}-\frac{\left(\tan^3(fx+e)\right)}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\left(\sin^7(fx+e)\right)}{7\cos(fx+e)^7}+b^2\left(\frac{\sin^7(fx+e)}{9\cos(fx+e)^9}+\frac{2\left(\sin^7(fx+e)\right)}{63\cos(fx+e)^7}\right)}{f}$
risch	$-a^2x-\frac{2i\left(-483a^2+10b^2-5670a^2e^{14i(fx+e)}-16170a^2e^{12i(fx+e)}-1050b^2e^{12i(fx+e)}-11718a^2e^{4i(fx+e)}+90ab+126\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+2/7*a*b*sin(f
*x+e)^7/cos(f*x+e)^7+b^2*(1/9*sin(f*x+e)^7/cos(f*x+e)^9+2/63*sin(f*x+e)^7/c
os(f*x+e)^7))
```

Maxima [A]
time = 0.46, size = 90, normalized size = 0.95

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\,\left(2\,ab+b^2\right)\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-315\,\left(fx+e\right)a^2+315\,a^2\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*tan
(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*x
+ e))/f
```

Fricas [A]
time = 3.51, size = 144, normalized size = 1.52

$$-\frac{315\,a^2fx\cos\left(fx+e\right)^9-\left(\left(483\,a^2-90\,ab-10\,b^2\right)\cos\left(fx+e\right)^8-\left(231\,a^2-270\,ab+5\,b^2\right)\cos\left(fx+e\right)^6+3\,\left(21\,a^2-90\,ab+25\,b^2\right)\cos\left(fx+e\right)^4+5\,\left(18\,ab-19\,b^2\right)\cos\left(fx+e\right)^2+35\,b^2\right)\sin\left(fx+e\right)}{315\,f\cos\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x +
 e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b + 2
5*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\tan^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

Giac [A]
time = 2.65, size = 91, normalized size = 0.96

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+45\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-315\,\left(fx+e\right)a^2+315\,a^2\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 45*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 31
5*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 4.54, size = 126, normalized size = 1.33

$$\frac{\tan(e+f\,x)\,\left({(a+b)}^2+b^2-2\,b\,(a+b)\right)-{\tan(e+f\,x)}^3\,\left(\frac{{(a+b)}^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+{\tan(e+f\,x)}^5\,\left(\frac{{(a+b)}^2}{5}+\frac{b^2}{5}-\frac{2\,b\,(a+b)}{5}\right)-{\tan(e+f\,x)}^7\,\left(\frac{b^2}{7}-\frac{2\,b\,(a+b)}{7}\right)+\frac{b^2\,{\tan(e+f\,x)}^9}{9}-a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*((a + b)^2/3
 + b^2/3 - (2*b*(a + b))/3) + tan(e + f*x)^5*((a + b)^2/5 + b^2/5 - (2*b*(a
 + b))/5) - tan(e + f*x)^7*(b^2/7 - (2*b*(a + b))/7) + (b^2*tan(e + f*x)^9)
/9 - a^2*f*x)/f
```

3.331 $\int (a+b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx$

Optimal. Leaf size=77

$$a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] a^2*x-a^2*tan(f*x+e)/f+1/3*a^2*tan(f*x+e)^3/f+1/5*b*(2*a+b)*tan(f*x+e)^5/f+
1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A]

time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 209}

$$\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}+a^2x+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) + (b*(2*a + b)*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b(1+x^2)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+a^2x^2+b(2a+b)x^4+b^2x^6+\frac{a^2}{1+x^2}\right)\,dx, x, \tan(\right.}{f}$$
$$= -\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+$$
$$= a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+f}{5f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(77) = 154.
time = 1.20, size = 395, normalized size = 5.13

$$\frac{\sec(e)\sec^7(e+fx)\left(3675a^2fx\cos(fx)+3675a^2fx\cos(2e+fx)+2205a^2fx\cos(2e+3fx)+2205a^2fx\cos(4e+3fx)+735a^2fx\cos(4e+5fx)+735a^2fx\cos(6e+5fx)+105a^2fx\cos(6e+7fx)+105a^2fx\cos(8e+7fx)-5320a^2\sin(fx)+1680ab\sin(fx)+840b^2\sin(fx)+4480a^2\sin(2e+fx)-1260ab\sin(2e+fx)+420b^2\sin(2e+fx)-3780a^2\sin(2e+3fx)+924ab\sin(2e+3fx)-168b^2\sin(2e+3fx)+2100a^2\sin(4e+3fx)-840ab\sin(4e+3fx)-420b^2\sin(4e+3fx)-1540a^2\sin(4e+5fx)+168ab\sin(4e+5fx)+84b^2\sin(4e+5fx)+420a^2\sin(6e+5fx)-420ab\sin(6e+5fx)-280a^2\sin(6e+7fx)+84ab\sin(6e+7fx)+12b^2\sin(6e+7fx)\right)}{13440f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] (Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x]
 + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^2*
f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*e +
 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*Sin[f
*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e + f*x]
+ 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*e + 3*
f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*b*Sin[4
*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x] + 168*a*
b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5*f*x] - 4
20*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6*e + 7*f*x
] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

Maple [A]
time = 0.07, size = 94, normalized size = 1.22

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}+b^2\left(\frac{\sin^5(fx+e)}{7\cos(fx+e)^7}+\frac{2\left(\sin^5(fx+e)\right)}{35\cos(fx+e)^5}\right)}{f}$
default	$\frac{a^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}+b^2\left(\frac{\sin^5(fx+e)}{7\cos(fx+e)^7}+\frac{2\left(\sin^5(fx+e)\right)}{35\cos(fx+e)^5}\right)}{f}$

risch	$a^2x + \frac{4i(-105a^2e^{12i(fx+e)}+105ab\,e^{12i(fx+e)}-525a^2e^{10i(fx+e)}+210ab\,e^{10i(fx+e)}+105b^2e^{10i(fx+e)}-1120a^2e^{8i(fx+e)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+2/5*a*b*sin(f*x+e)^5/cos(f*x+e
)^5+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*sin(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A]
time = 0.46, size = 76, normalized size = 0.99

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+b^2\right)\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*tan
(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 3.23, size = 119, normalized size = 1.55

$$\frac{105\,a^2fx\cos\left(fx+e\right)^7-\left(2\left(70\,a^2-21\,ab-3\,b^2\right)\cos\left(fx+e\right)^6-\left(35\,a^2-84\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-6\left(7\,ab-4\,b^2\right)\cos\left(fx+e\right)^2-15\,b^2\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f*x
 + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

Giac [A]
time = 1.38, size = 78, normalized size = 1.01

$$\frac{15\,b^2\tan\left(fx+e\right)^7+42\,ab\tan\left(fx+e\right)^5+21\,b^2\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 4.58, size = 97, normalized size = 1.26

$$\frac{\tan(e+f\,x)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)-\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^5\left(\frac{b^2}{5}-\frac{2\,b\,(a+b)}{5}\right)+\frac{b^2\tan(e+f\,x)^7}{7}+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^3*((a + b)^2/3 + b^2/3 - (2*b*(a + b))/3) - tan(e + f*x)*((a
+ b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^5*(b^2/5 - (2*b*(a + b))/5) + (b
^2*tan(e + f*x)^7)/7 + a^2*f*x)/f
```

3.332 $\int (a+b\sec^2(e+fx))^2 \tan^2(e+fx)\,dx$

Optimal. Leaf size=59

$$-a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -a^2*x+a^2*tan(f*x+e)/f+1/3*b*(2*a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 209}

$$\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f + (b*(2*a + b)*Tan[e + f*x]^3)/(3*f) + (b^2
*Tan[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b(1+x^2)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2+b(2a+b)x^2+b^2x^4-\frac{a^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}-\,a$$

$$= -a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(59) = 118.

time = 0.88, size = 281, normalized size = 4.76

$$-\frac{\sec(e)\sec^5(e+fx)\left(150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)+75a^2fx\cos(4e+3fx)+15a^2fx\cos(4e+5fx)+15a^2fx\cos(6e+5fx)-180a^2\sin(fx)+80ab\sin(fx)-20b^2\sin(fx)+120a^2\sin(2e+fx)-120ab\sin(2e+fx)-60b^2\sin(2e+fx)-120a^2\sin(2e+3fx)+40ab\sin(2e+3fx)+20b^2\sin(2e+3fx)+30a^2\sin(4e+3fx)-60ab\sin(4e+3fx)-30a^2\sin(4e+5fx)+20ab\sin(4e+5fx)+4b^2\sin(4e+5fx)\right)}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -1/480*(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e +
 f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^2*
f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] + 80*
a*b*Sin[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2*e +
 f*x] - 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[2*e +
 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*Sin[4*
e + 3*f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b^2*Sin[
4*e + 5*f*x]))/f
```

Maple [A]

time = 0.06, size = 85, normalized size = 1.44

method	result
derivativedivides	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i\left(-15a^2e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}-60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+30b^2e^{6i(fx+e)}-90a^2e^{4i(fx+e)}+40ab\,e\right.}{15f\left(e^{2i(fx+e)}+1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(tan(f*x+e)-f*x-e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e)^3+b^2*(1/5*sin(
f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A]
time = 0.46, size = 62, normalized size = 1.05

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\,\left(2\,ab+b^2\right)\tan\left(fx+e\right)^3-15\,\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)*
a^2 + 15*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 3.08, size = 91, normalized size = 1.54

$$-\frac{15\,a^2fx\cos\left(fx+e\right)^5-\left(\left(15\,a^2-10\,ab-2\,b^2\right)\cos\left(fx+e\right)^4+\left(10\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)^
4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

Giac [A]
time = 0.81, size = 65, normalized size = 1.10

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+5\,b^2\tan\left(fx+e\right)^3-15\,\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 5*b^2*tan(f*x + e)^3 -
 15*(f*x + e)*a^2 + 15*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 4.66, size = 69, normalized size = 1.17

$$\frac{\tan(e+f\,x)\,\left({\left(a+b\right)}^2+b^2-2\,b\,\left(a+b\right)\right)-{\tan(e+f\,x)}^3\,\left(\frac{b^2}{3}-\frac{2\,b\,\left(a+b\right)}{3}\right)+\frac{b^2\,{\tan(e+f\,x)}^5}{5}-a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*(b^2/3 - (2*
b*(a + b))/3) + (b^2*tan(e + f*x)^5)/5 - a^2*f*x)/f
```

3.333 $\int (a+b\sec^2(e+fx))^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 209}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx\right.}{f}$$
$$=a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(40) = 80.
time = 0.41, size = 106, normalized size = 2.65

$$\frac{4(b+a\cos^2(e+fx))^2\sec^3(e+fx)\left(3a^2fx\cos^3(e+fx)+b^2\sec(e)\sin(fx)+2b(3a+b)\cos^2(e+fx)\sec(e)\sin(fx)+b^2\cos(e+fx)\tan(e)\right)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.00, size = 48, normalized size = 1.20

method	result
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}+3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^2x+3a^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3a^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.26, size = 47, normalized size = 1.18

$$a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Fricas [A]
time = 2.31, size = 62, normalized size = 1.55

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.42, size = 49, normalized size = 1.22

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

Mupad [B]
time = 4.58, size = 42, normalized size = 1.05

$$\frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^2,x)
[Out] ((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=36

$$-a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -a^2*x-(a+b)^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4226, 1816, 209}

$$a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - ((a + b)^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1+x^2)\right)^2}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{f}$$
$$= -a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.
time = 0.79, size = 82, normalized size = 2.28

$$-\frac{4\left(b+a\cos^2(e+fx)\right)^2\sec(e+fx)\left(a^2fx\cos(e+fx)-\left((a+b)^2\cot(e+fx)\csc(e)+b^2\sec(e)\right)\sin(fx)\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)^
2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e + f
*x)])^2)
```

Maple [A]
time = 0.05, size = 66, normalized size = 1.83

method	result	size
derivativedivides	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
default	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
risch	$-a^2x-\frac{2i\left(a^2e^{2i(fx+e)}+2ab\,e^{2i(fx+e)}+a^2+2ab+2b^2\right)}{f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)-
2*cot(f*x+e)))
```

Maxima [A]
time = 0.46, size = 49, normalized size = 1.36

$$-\frac{(fx+e)a^2-b^2\tan(fx+e)+\frac{a^2+2ab+b^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Fricas [A]
time = 2.43, size = 72, normalized size = 2.00

$$-\frac{a^2fx\cos(fx+e)\sin(fx+e)+(a^2+2ab+2b^2)\cos(fx+e)^2-b^2}{f\cos(fx+e)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
[Out] -(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
- b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)^2\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

Giac [A]
time = 0.48, size = 46, normalized size = 1.28

$$-\frac{(fx+e)a^2-b^2\tan(fx+e)+\frac{a^2+2ab+b^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B]
time = 4.64, size = 44, normalized size = 1.22

$$\frac{b^2\tan(e+f\,x)}{f}-a^2\,x-\frac{a^2+2\,a\,b+b^2}{f\tan\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
[Out] (b^2*tan(e + f*x))/f - a^2*x - (2*a*b + a^2 + b^2)/(f*tan(e + f*x))
```

3.335 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$a^2x+\frac{(a^2-b^2)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

[Out] a^2*x+(a^2-b^2)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 209}

$$\frac{(a^2-b^2)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]

[Out] a^2*x + ((a^2 - b^2)*Cot[e + f*x])/f - ((a + b)^2*Cot[e + f*x]^3)/(3*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^2}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^4} + \frac{-a^2+b^2}{x^2} + \frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a^2-b^2)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+x}\right.}{}$$

$$= a^2x + \frac{(a^2-b^2)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(45) = 90.
time = 0.91, size = 160, normalized size = 3.56

$$\frac{\csc(e)\csc^3(e+fx)\left(9a^2fx\cos(fx) - 9a^2fx\cos(2e+fx) - 3a^2fx\cos(2e+3fx) + 3a^2fx\cos(4e+3fx) - 12a^2\sin(fx) + 12b^2\sin(fx) - 12a^2\sin(2e+fx) - 12ab\sin(2e+fx) + 8a^2\sin(2e+3fx) + 4ab\sin(2e+3fx) - 4b^2\sin(2e+3fx)\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*a
^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] + 12
*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Sin[2
*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

Maple [A]
time = 0.05, size = 73, normalized size = 1.62

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\left(\cos^3(fx+e)\right)}{3\sin(fx+e)^3}+b^2\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$	73
default	$\frac{a^2\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\left(\cos^3(fx+e)\right)}{3\sin(fx+e)^3}+b^2\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$	73
risch	$a^2x+\frac{4i\left(3a^2\mathrm{e}^{4i(fx+e)}+3ab\,\mathrm{e}^{4i(fx+e)}-3a^2\mathrm{e}^{2i(fx+e)}+3b^2\mathrm{e}^{2i(fx+e)}+2a^2+ab-b^2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}-1\right)^3}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x+
e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A]
time = 0.46, size = 62, normalized size = 1.38

$$\frac{3\,(fx+e)a^2+\frac{3\,(a^2-b^2)\tan(fx+e)^2-a^2-2\,ab-b^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/t
an(f*x + e)^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(45) = 90.
time = 3.16, size = 104, normalized size = 2.31

$$\frac{2\,(2\,a^2+ab-b^2)\cos(fx+e)^3-3\,(a^2-b^2)\cos(fx+e)+3\,\left(a^2fx\cos(fx+e)^2-a^2fx\right)\sin(fx+e)}{3\,\left(f\cos(fx+e)^2-f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3*
(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*si
n(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^2(e+fx)\right)^2\cot^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(43) = 86.
time = 0.47, size = 176, normalized size = 3.91

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)a^2-15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2-2\,ab-b^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(1
/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*b*
tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x + 1
/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 - a^2
 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B]
time = 4.61, size = 53, normalized size = 1.18

$$a^2\,x-\frac{\frac{2\,a\,b}{3}-{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(a^2-b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\,{\mathrm{tan}\left(e+f\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*a*b)/3 - tan(e + f*x)^2*(a^2 - b^2) + a^2/3 + b^2/3)/(f*tan(e +
 f*x)^3)
```

3.336 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=65

$$-a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

```
[Out] -a^2*x-a^2*cot(f*x+e)/f+1/3*(a^2-b^2)*cot(f*x+e)^3/f-1/5*(a+b)^2*cot(f*x+e)
^5/f
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 209}

$$\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[e + f*x])/f + ((a^2 - b^2)*Cot[e + f*x]^3)/(3*f) - ((a
+ b)^2*Cot[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1+x^2)\right)^2}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{-a^2+b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+f}{5f}$$
$$= -a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5}{5f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 256 vs. $2(65) = 130$.
time = 1.10, size = 256, normalized size = 3.94

$$\frac{\csc(e)\csc^5(e+fx)\left(-150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)-75a^2fx\cos(4e+3fx)-15a^2fx\cos(4e+5fx)+15a^2fx\cos(6e+5fx)+280a^2\sin(fx)+120ab\sin(fx)+20b^2\sin(fx)+180a^2\sin(2e+fx)-60b^2\sin(2e+fx)-140a^2\sin(2e+3fx)+20b^2\sin(2e+3fx)-90a^2\sin(4e+3fx)-60ab\sin(4e+3fx)+46a^2\sin(4e+5fx)+12ab\sin(4e+5fx)-4b^2\sin(4e+5fx)\right)}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
+ 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*Co
s[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*b*S
in[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f*x]
- 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e + 3*f
*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[4*e +
5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)
```

Maple [A]
time = 0.06, size = 107, normalized size = 1.65

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\left(\cos^5(fx+e)\right)}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos^3(fx+e)}{5\sin(fx+e)^5}-\frac{2\left(\cos^3(fx+e)\right)}{15\sin(fx+e)^3}\right)}{f}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\left(\cos^5(fx+e)\right)}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos^3(fx+e)}{5\sin(fx+e)^5}-\frac{2\left(\cos^3(fx+e)\right)}{15\sin(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i\left(45a^2e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}-90a^2e^{6i(fx+e)}+30b^2e^{6i(fx+e)}+140a^2e^{4i(fx+e)}+60ab\,e^{4i(fx+e)}+10b^2e^{4i}\right.}{15f\left(e^{2i(fx+e)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin(
f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15/sin(f*x+e)^3
*cos(f*x+e)^3))
```

Maxima [A]
time = 0.46, size = 76, normalized size = 1.17

$$-\frac{15\,(fx+e)a^2+\frac{15\,a^2\tan(fx+e)^4-5\,(a^2-b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(64) = 128.
time = 3.49, size = 145, normalized size = 2.23

$$-\frac{(23\,a^2+6\,ab-2\,b^2)\cos(fx+e)^5-5\,(7\,a^2-b^2)\cos(fx+e)^3+15\,a^2\cos(fx+e)+15\,\left(a^2fx\cos(fx+e)^4-2\,a^2fx\cos(fx+e)^2+a^2fx\right)\sin(fx+e)}{15\,\left(f\cos(fx+e)^4-2\,f\cos(fx+e)^2+f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x +
e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f*x
 + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 +
f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(61) = 122.
time = 0.52, size = 273, normalized size = 4.20

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+5\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-480\,(fx+e)a^2+330\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+60\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-30\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{330\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+60\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-30\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+5\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*
tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f*x
 + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2*ta
n(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x + 1/2
*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4 - 30*
b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*tan(1/2
*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan
(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B]
time = 4.84, size = 68, normalized size = 1.05

$$-a^2\,x-\frac{\frac{2\,a\,b}{5}+\frac{a^2}{5}+\frac{b^2}{5}-{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)+a^2\,{\mathrm{tan}\left(e+f\,x\right)}^4}{f\,{\mathrm{tan}\left(e+f\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] - a^2*x - ((2*a*b)/5 + a^2/5 + b^2/5 - tan(e + f*x)^2*(a^2/3 - b^2/3) + a^2
*tan(e + f*x)^4)/(f*tan(e + f*x)^5)
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{(a+2b)\log(\cos(e+fx))}{b^2f}-\frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f}+\frac{\sec^2(e+fx)}{2bf}$$

```
[Out] (a+2*b)*ln(cos(f*x+e))/b^2/f-1/2*(a+b)^2*ln(b+a*cos(f*x+e)^2)/a/b^2/f+1/2*s
ec(f*x+e)^2/b/f
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$-\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f}+\frac{(a+2b)\log(\cos(e+fx))}{b^2f}+\frac{\sec^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*Log[Cos[e + f*x]])/(b^2*f) - ((a + b)^2*Log[b + a*Cos[e + f*x]^2
])/(2*a*b^2*f) + Sec[e + f*x]^2/(2*b*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{(a+2b)\log(\cos(e+fx))}{b^2f}-\frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f}+\frac{\sec^2(e+fx)}{2bf}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 99, normalized size = 1.43

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(2a(a+2b)\log(\cos(e+fx))-(a+b)^2\log\left(a+b-a\sin^2(e+fx)\right)+ab\sec^2(e+fx)\right)}{4ab^2f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e + f
*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*a*
b^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.08, size = 67, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2b^2a}+\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}}{f}$
default	$\frac{-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2b^2a}+\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}}{f}$
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,\mathrm{e}^{2i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)a}{b^2f}+\frac{2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{bf}-\frac{a\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right.}{2b^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/2*(a^2+2*a*b+b^2)/b^2/a*ln(b+a*cos(f*x+e)^2)+(a+2*b)/b^2*ln(cos(f*x
+e))+1/2/b/cos(f*x+e)^2)
```

Maxima [A]
time = 0.26, size = 84, normalized size = 1.22

$$\frac{\frac{(a+2\,b)\log\left(\sin(fx+e)^2-1\right)}{b^2}-\frac{1}{b\sin(fx+e)^2-b}-\frac{(a^2+2\,ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{ab^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a^
2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

Fricas [A]
time = 3.26, size = 89, normalized size = 1.29

$$-\frac{(a^2+2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,(a^2+2\,ab)\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-ab}{2\,ab^2f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^2
 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(65) = 130.
time = 1.75, size = 373, normalized size = 5.41

$$-\frac{\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\,a+2\,b\right|\right)}{a^2b^2+ab^3}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a}-\frac{(a+2\,b)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{b^2}+\frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,a+8\,b}{b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) +
```

```
1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b)
)/(a^2*b^2 + a*b^3) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/a - (a + 2*b)*log(abs(-(cos(f*x + e
) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2
 + (a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x
+ e) + 1)) + 2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1
)/(cos(f*x + e) + 1)) + 2*a + 8*b)/(b^2*((cos(f*x + e) + 1)/(cos(f*x + e) -
 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)))/f
```

Mupad [B]
time = 4.61, size = 103, normalized size = 1.49

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}-\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{b\,f}-\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f}+\frac{\tan(e+f\,x)^2}{2\,b\,f}-\frac{a\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - log(a + b + b*tan(e + f*x)^2)/(b*f) - log
(a + b + b*tan(e + f*x)^2)/(2*a*f) + tan(e + f*x)^2/(2*b*f) - (a*log(a + b
+ b*tan(e + f*x)^2))/(2*b^2*f)
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$-\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

```
[Out] -ln(cos(f*x+e))/b/f+1/2*(a+b)*ln(b+a*cos(f*x+e)^2)/a/b/f
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 78}

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/(b*f)) + ((a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 41, normalized size = 0.91

$$
\frac{-2a\log(\cos(e+fx))+(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
[Out] (-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{(a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
default	$\frac{\frac{(a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
risch	$-\frac{ix}{a}-\frac{2ie}{af}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2*(a+b)/b/a*ln(b+a*cos(f*x+e)^2)-1/b*ln(cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 52, normalized size = 1.16

$$\frac{\frac{(a+b)\log\left(a\sin(fx+e)^2-a-b\right)}{ab}-\frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)/
b)/f
```

Fricas [A]
time = 3.10, size = 43, normalized size = 0.96

$$\frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(43) = 86.
time = 0.77, size = 222, normalized size = 4.93

$$\frac{\frac{(a^2+2\,ab+b^2)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\,a+2\,b\right|\right)}{a^2b+ab^2}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(cos(f*x +
```

```
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^2*b + a*
b^2) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 2))/a - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)
 - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b)/f
```

Mupad [B]
time = 4.49, size = 64, normalized size = 1.42

$$\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f}+\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*a*f) + log(a + b + b*tan(e + f*x)^2)/(2*b*
f) - log(tan(e + f*x)^2 + 1)/(2*a*f)
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

```
[Out] -1/2*ln(b+a*cos(f*x+e)^2)/a/f
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4223, 266}

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*Log[b + a*Cos[e + f*x]^2]/(a*f)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 26, normalized size = 1.13

$$-\frac{\log(a+2b+a\cos(2(e+fx)))}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
[Out] -1/2*Log[a + 2*b + a*Cos[2*(e + f*x)]]/(a*f)
```

Maple [A]
time = 0.04, size = 35, normalized size = 1.52

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
default	$\frac{-\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1/2/a*ln(a+b*sec(f*x+e)^2)+1/a*ln(sec(f*x+e)))
```

Maxima [A]
time = 0.26, size = 27, normalized size = 1.17

$$-\frac{\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

Fricas [A]
time = 3.07, size = 22, normalized size = 0.96

$$-\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(19) = 38$.
time = 7.24, size = 117, normalized size = 5.09

$$\begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^2(e)} & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x \tan(e)}{a + b \sec^2(e)} & \text{for } f = 0 \\ -\frac{1}{2bf \sec^2(e + fx)} & \text{for } a = 0 \\ \frac{\log(\tan^2(e + fx) + 1)}{2af} & \text{for } b = 0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}} + \sec(e + fx)\right)}{2af} - \frac{\log\left(\sqrt{-\frac{a}{b}} + \sec(e + fx)\right)}{2af} + \frac{\log(\tan^2(e + fx) + 1)}{2af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x*tan(
e)/(a + b*sec(e)**2), Eq(f, 0)), (-1/(2*b*f*sec(e + f*x)**2), Eq(a, 0)), (l
og(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (-log(-sqrt(-a/b) + sec(e + f*x
))/(2*a*f) - log(sqrt(-a/b) + sec(e + f*x))/(2*a*f) + log(tan(e + f*x)**2 +
 1)/(2*a*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(21) = 42$.
time = 0.70, size = 129, normalized size = 5.61

$$-\frac{\frac{\log\left(a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a} - \frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a - 2*log(abs(-(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B]
time = 4.56, size = 63, normalized size = 2.74

$$\frac{\operatorname{atanh}\left(\frac{a}{2\left(\frac{3a}{2} + 2b + \frac{a\cos(2e+2fx)}{2}\right)} - \frac{a\cos(2e+2fx)}{2\left(\frac{3a}{2} + 2b + \frac{a\cos(2e+2fx)}{2}\right)}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atanh(a/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)) - (a*cos(2*e + 2*f*x))
/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)))/(a*f)
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f}+\frac{\log(\sin(e+fx))}{(a+b)f}$$

```
[Out] 1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+ln(sin(f*x+e))/(a+b)/f
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 78}

$$\frac{\log(\sin(e+fx))}{f(a+b)}+\frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)*f) + Log[Sin[e + f*x]]/((a + b)*
f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f} + \frac{\log(\sin(e+fx))}{(a+b)f}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 43, normalized size = 0.93

$$\frac{2a\log(\sin(e+fx)) + b\log\left(a+b-a\sin^2(e+fx)\right)}{2a^2f+2abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b*
f)
```

Maple [A]
time = 0.10, size = 68, normalized size = 1.48

method	result	siz
derivativedivides	$\frac{\frac{b\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)a}+\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$	68
default	$\frac{\frac{b\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)a}+\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$	68
risch	$\frac{ix}{a}-\frac{2ix}{a+b}-\frac{2ie}{f(a+b)}-\frac{2ibx}{a(a+b)}-\frac{2ibe}{af(a+b)}+\frac{\ln\left(e^{2i(fx+e)}-1\right)}{f(a+b)}+\frac{b\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af(a+b)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a+b)*b/a*ln(b+a*cos(f*x+e)^2)+1/(2*a+2*b)*ln(cos(f*x+e)-1)+1/(2*a
+2*b)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 52, normalized size = 1.13

$$\frac{\frac{b \log\left(a \sin(fx+e)^2 - a - b\right)}{a^2+ab} + \frac{\log\left(\sin(fx+e)^2\right)}{a+b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a +
 b))/f
```

Fricas [A]
time = 3.91, size = 44, normalized size = 0.96

$$\frac{b \log\left(a \cos\left(fx+e\right)^2 + b\right) + 2\,a \log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\,(a^2+ab)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(44) = 88.
time = 0.49, size = 166, normalized size = 3.61

$$\frac{\frac{b \log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2+ab} + \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b} - \frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2 + a*b) + log(abs(-cos(
f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a + b) - 2*log(abs(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B]
time = 4.70, size = 65, normalized size = 1.41

$$\frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a+b\right)}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,a\,f}+\frac{b\,\ln\left(b\,{\tan(e+f\,x)}^2+a+b\right)}{2\,f\,\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x))/(f*(a + b)) - log(tan(e + f*x)^2 + 1)/(2*a*f) + (b*log(a
+ b + b*tan(e + f*x)^2))/(2*f*(a*b + a^2))
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\csc^2(e+fx)}{2(a+b)f}-\frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f}-\frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

```
[Out] -1/2*csc(f*x+e)^2/(a+b)/f-1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-(a+2*b)*
ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$-\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2}-\frac{\csc^2(e+fx)}{2f(a+b)}-\frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*Csc[e + f*x]^2/((a + b)*f) - (b^2*Log[b + a*Cos[e + f*x]^2])/(2*a*(a +
 b)^2*f) - ((a + 2*b)*Log[Sin[e + f*x]])/((a + b)^2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\csc^2(e+fx)}{2(a+b)f}-\frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f}-\frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 100, normalized size = 1.35

$$
-\frac{(a+2b+a\cos(2(e+fx)))\left(a(a+b)\csc^2(e+fx)+2a(a+2b)\log(\sin(e+fx))+b^2\log\left(a+b-a\sin^2(e+fx)\right)\right)\sec^2(e+fx)}{4a(a+b)^2f\left(a+b\sec^2(e+fx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a + 2
*b)*Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2)/
(a*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.09, size = 119, normalized size = 1.61

method	result
derivativedivides	$\frac{-\frac{b^2\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(-a-2b)\ln(\cos(fx+e)-1)}{2(a+b)^2}-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx}{2(a+b)^2}}{f}$
default	$\frac{-\frac{b^2\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(-a-2b)\ln(\cos(fx+e)-1)}{2(a+b)^2}-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx}{2(a+b)^2}}{f}$
risch	$-\frac{ix}{a}+\frac{2iax}{a^2+2ab+b^2}+\frac{2iae}{f(a^2+2ab+b^2)}+\frac{4ibx}{a^2+2ab+b^2}+\frac{4ibe}{f(a^2+2ab+b^2)}+\frac{2ib^2x}{a(a^2+2ab+b^2)}+\frac{2ib^2e}{af(a^2+2ab+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*b^2/(a+b)^2/a*ln(b+a*cos(f*x+e)^2)+1/(4*a+4*b)/(cos(f*x+e)-1)+1/2
*(-a-2*b)/(a+b)^2*ln(cos(f*x+e)-1)-1/(4*a+4*b)/(1+cos(f*x+e))+1/2*(-a-2*b)/
(a+b)^2*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 90, normalized size = 1.22

$$-\frac{\frac{b^{2}\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{3}+2\,a^{2}b+ab^{2}}+\frac{(a+2\,b)\log\left(\sin\left(fx+e\right)^{2}\right)}{a^{2}+2\,ab+b^{2}}+\frac{1}{(a+b)\sin\left(fx+e\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b)
*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

Fricas [A]
time = 3.70, size = 131, normalized size = 1.77

$$\frac{a^{2}+ab-\left(b^{2}\cos\left(fx+e\right)^{2}-b^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)-2\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}-a^{2}-2\,ab\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)}{2\left(\left(a^{3}+2\,a^{2}b+ab^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{3}+2\,a^{2}b+ab^{2}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*(
(a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3 +
2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{3}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(70) = 140.
time = 0.51, size = 294, normalized size = 3.97

$$-\frac{\frac{4\,b^{2}\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)}{a^{3}+2\,a^{2}b+ab^{2}}+\frac{4\,(a+2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^{2}+2\,ab+b^{2}}-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{(a^{2}+2\,ab+b^{2})(\cos(fx+e)-1)}-\frac{8\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}-\frac{\cos(fx+e)-1}{(a+b)(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^3 + 2*a^2*b + a*b^2
) + 4*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2 + 2*
a*b + b^2) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^2 + 2*a*b + b^2)*(
cos(f*x + e) - 1)) - 8*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))
/a - (cos(f*x + e) - 1)/((a + b)*(cos(f*x + e) + 1)))/f
```

Mupad [B]
time = 4.94, size = 98, normalized size = 1.32

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}-\frac{\cot(e+f\,x)^2}{2\,f\,\left(a+b\right)}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+2\,b\right)}{f\,\left(a^2+2\,a\,b+b^2\right)}-\frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f\,\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a*f) - cot(e + f*x)^2/(2*f*(a + b)) - (log(tan(e
 + f*x))*(a + 2*b))/(f*(2*a*b + a^2 + b^2)) - (b^2*log(a + b + b*tan(e + f*
x)^2))/(2*a*f*(a + b)^2)
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f}-\frac{\csc^4(e+fx)}{4(a+b)f}+\frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f}+\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3f}$$

```
[Out] 1/2*(2*a+3*b)*csc(f*x+e)^2/(a+b)^2/f-1/4*csc(f*x+e)^4/(a+b)/f+1/2*b^3*ln(b+
a*cos(f*x+e)^2)/a/(a+b)^3/f+(a^2+3*a*b+3*b^2)*ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3}+\frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3}-\frac{\csc^4(e+fx)}{4f(a+b)}+\frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + 3*b)*Csc[e + f*x]^2)/(2*(a + b)^2*f) - Csc[e + f*x]^4/(4*(a + b)*f)
 + (b^3*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)^3*f) + ((a^2 + 3*a*b + 3*b^
2)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^3(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3}+\frac{-2a-3b}{(a+b)^2(-1+x)^2}+\frac{-a^2-3ab-3b^2}{(a+b)^3(-1+x)}-\frac{b^3}{(a+b)^3(b+ax)}\right)\,dx, x, \text{co}\right.}{2f}$$

$$= \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f}-\frac{\csc^4(e+fx)}{4(a+b)f}+\frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f}+\frac{(a^2+3a}{}$$

Mathematica [A]
time = 0.66, size = 138, normalized size = 1.28

$$\frac{(a+2b+a\cos(2e+2fx))\left(\frac{2(2a+3b)\csc^2(e+fx)}{(a+b)^2}-\frac{\csc^4(e+fx)}{a+b}+\frac{4(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3}+\frac{2b^3\log(a+b-a\sin^2(e+fx))}{a(a+b)^3}\right)\sec^2(e+fx)}{8f\,(a+b\sec^2(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2 -
 Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a +
b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]^2)
/(8*f*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.11, size = 180, normalized size = 1.67

method	result
derivativedivides	$\frac{\frac{b^3\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(\cos(fx+e)-1)^2}-\frac{7a+11b}{16(a+b)^2(\cos(fx+e)-1)}+\frac{\left(a^2+3ab+3b^2\right)\ln(\cos(fx+e)-1)}{2(a+b)^3}-\frac{1}{2(8a+8b)(1+\cdots}}{f}$
default	$\frac{\frac{b^3\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(\cos(fx+e)-1)^2}-\frac{7a+11b}{16(a+b)^2(\cos(fx+e)-1)}+\frac{\left(a^2+3ab+3b^2\right)\ln(\cos(fx+e)-1)}{2(a+b)^3}-\frac{1}{2(8a+8b)(1+\cdots}}{f}$
risch	$\frac{ix}{a}-\frac{2ia^2x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2ia^2e}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6iabx}{a^3+3a^2b+3a\,b^2+b^3}-\frac{6iabe}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6}{a^3+3a^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b^3/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)-1/2/(8*a+8*b)/(cos(f*x+e)-1)^2-
1/16*(7*a+11*b)/(a+b)^2/(cos(f*x+e)-1)+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(cos
(f*x+e)-1)-1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-7*a-11*b)/(a+b)^2/(1+cos(f
*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 149, normalized size = 1.38

$$\frac{\frac{2\,b^3\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^4+3\,a^3b+3\,a^2b^2+ab^3}+\frac{2\,\left(a^2+3\,ab+3\,b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,\left(2\,a+3\,b\right)\sin\left(fx+e\right)^2-a-b}{\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f*x
+ e)^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(106) = 212$.
time = 4.21, size = 274, normalized size = 2.54

$$\frac{3\,a^3+8\,a^2b+5\,ab^2-2\,\left(2\,a^3+5\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2+2\,\left(b^3\cos\left(fx+e\right)^4-2\,b^3\cos\left(fx+e\right)^2+b^3\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+4\,\left(\left(a^3+3\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^4+a^3+3\,a^2b+3\,ab^2-2\,\left(a^3+3\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{4\,\left(\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-2\,\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)
^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x + e)
^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a
*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/(
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(102) = 204.
time = 0.52, size = 513, normalized size = 4.75

$$\frac{\frac{32\,b^3\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+3\,a^3b+3\,a^2b^2+ab^3}+\frac{32\,(a^2+3\,ab+3\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{20\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2+2\,ab+b^2}-\frac{\left(a^2+2\,ab+b^2+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{32\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{20\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{144\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{144\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(a^3+3\,a^2b+3\,ab^2+b^3)(\cos(fx+e)-1)^2}-\frac{64\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(32*b^3*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3) + 32*(a^2 + 3*a*b + 3*b^2)*log(abs(-cos(f*x + e) + 1)/abs(co
s(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (12*a*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 20*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2)/(a^2 + 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 20*b^
2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b
^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2) - 64*log(abs(-(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B]
time = 4.91, size = 160, normalized size = 1.48

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+3\,a\,b+3\,b^2\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}-\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(\frac{b}{2\,(a+b)^2}+\frac{1}{2\,(a+b)}+\frac{b^2}{2\,(a+b)^3}-\frac{1}{2\,a}\right)}{f}-\frac{\cot(e+f\,x)^4\,\left(\frac{1}{4\,(a+b)}-\frac{\tan(e+f\,x)^2\,(a+2\,b)}{2\,(a+b)^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(3*a*b + a^2 + 3*b^2))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3
)) - log(tan(e + f*x)^2 + 1)/(2*a*f) - (log(a + b + b*tan(e + f*x)^2)*(b/(2
*(a + b)^2) + 1/(2*(a + b)) + b^2/(2*(a + b)^3) - 1/(2*a)))/f - (cot(e + f*
x)^4*(1/(4*(a + b)) - (tan(e + f*x)^2*(a + 2*b))/(2*(a + b)^2)))/f
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$-\frac{x}{a}+\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -x/a+(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(5/2)/f-(a+2*b)
*tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.19, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 490, 596, 536, 209, 211}

$$\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + ((a + b)^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(5
/2)*f) - ((a + 2*b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan^3(e+fx)}{3bf} - \frac{\mathrm{Subst}\left(\int \frac{x^2(3(a+b)+3(a+2b)x^2)}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3bf} \\
&= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\mathrm{Subst}\left(\int \frac{3(a+b)(a+2b)+3(a^2+3ab+3b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx,\right.}{3b^2f} \\
&= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a}{} \\
&= -\frac{x}{a} + \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f} - \frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.05, size = 229, normalized size = 2.76

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{3x}{a}-\frac{3(a+b)^{5/2}\mathrm{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{ab^2f\sqrt{b(\cos(e)-i\sin(e))^4}}-\frac{(3a+7b)\sec(e)\sec(e+fx)\sin(fx)}{b^2f}+\frac{\sec(e)\sec^3(e+fx)\sin(fx)}{bf}+\frac{\sec^2(e+fx)\tan(e)}{bf}\right)}{6(a+b\sec^2(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2)
*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*
e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin
[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*Sec[e
 + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (Sec[e
 + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

Maple [A]
time = 0.13, size = 101, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{b^2a\sqrt{(a+b)\,b}}}{f}$

default	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{b^2a\sqrt{(a+b)\,b}}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(3a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+12b\,e^{2i(fx+e)}+3a+7b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-(a+b)\,b}\,a\ln\left(e^{2i(fx+e)}+\frac{2}{-}\right.}{2b^3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+2*b*tan(f*x+e))-1/a*arctan(ta
n(f*x+e))+1/b^2*(a^3+3*a^2*b+3*a*b^2+b^3)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*
x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.46, size = 99, normalized size = 1.19

$$-\frac{\frac{3\,(fx+e)}{a}-\frac{b\tan(fx+e)^3-3\,(a+2\,b)\tan(fx+e)}{b^2}-\frac{3\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,ab^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(f*x + e)/a - (b*tan(f*x + e)^3 - 3*(a + 2*b)*tan(f*x + e))/b^2 - 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sq
rt((a + b)*b)*a*b^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(76) = 152.
time = 3.16, size = 393, normalized size = 4.73

$$\left[-\frac{12\,b^2fx\cos(fx+e)^3-3\,(a^2+2\,ab+b^2)\sqrt{-\frac{a+b}{b}}\cos(fx+e)^3\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+4\,\left((3\,a^2+7\,ab)\cos(fx+e)^2-ab\right)\sin(fx+e)}{12\,ab^2f\cos(fx+e)^3},\ -\frac{6\,b^2fx\cos(fx+e)^3+3\,(a^2+2\,ab+b^2)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3+2\,\left((3\,a^2+7\,ab)\cos(fx+e)^2-ab\right)\sin(fx+e)}{6\,ab^2f\cos(fx+e)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)*
cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt
(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
```

```
+ b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*
cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^2)*sqr
t((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a
+ b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b)*cos(f*
x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 2.20, size = 126, normalized size = 1.52

$$-\frac{\frac{3\,(fx+e)}{a}-\frac{3\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,ab^2}-\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)-6\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(f*x + e)/a - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((f*x + e)
/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2
)*a*b^2) - (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x + e) - 6*b^2*tan(f*x + e))/b
^3)/f
```

Mupad [B]
time = 4.92, size = 1109, normalized size = 13.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) - atan((40*a^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b
^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a^3*tan(e + f*x))/(30*a
*b^2 + 40*a^2*b + 30*a^3 + 10*b^3 + (12*a^4)/b + (2*a^5)/b^2) + (12*a^4*tan
(e + f*x))/(30*a*b^3 + 30*a^3*b + 12*a^4 + 10*b^4 + 40*a^2*b^2 + (2*a^5)/b)
```

```
 + (2*a^5*tan(e + f*x))/(30*a*b^4 + 12*a^4*b + 2*a^5 + 10*b^5 + 40*a^2*b^3
+ 30*a^3*b^2) + (10*b^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b^2 + (30*a^3)/
b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a*b*tan(e + f*x))/(30*a*b + 40*a^2 +
10*b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3))/(a*f) - (tan(e + f*x)*(a
 + 2*b))/(b^2*f) - (atan((((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5
 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-
b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f
*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i
)/(2*a*b^5) + ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b +
 a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^
5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b
^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5))
/((2*(12*a*b^4 + 6*a^4*b + a^5 + 3*b^5 + 19*a^2*b^3 + 15*a^3*b^2))/b^3 - ((
-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 1
5*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-b^5*(a + b)^5)^(1/2)*((8*a^2
*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*
(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5) + ((-b^5*(a + b)^5)
^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*
a^3*b^3 + 15*a^4*b^2))/b^3 - ((-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b
^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5
)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*b^
5*f)
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{(a+b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/a-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(3/2)/f+tan(f*x+
e)/b/f
```

Rubi [A]
time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 490, 536, 209, 211}

$$-\frac{(a+b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{x}{a}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(3/2)
*f) + Tan[e + f*x]/(b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx,\right.}{abf} \\
&= \frac{x}{a} - \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.24, size = 206, normalized size = 3.49

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left((a+b)^2\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\sqrt{b(i\cos(e)+\sin(e))^4}(bfx+a\sec(e)\sec(e+fx)\sin(fx))\right)}{2ab\sqrt{a+b}f(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x])))
/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.10, size = 72, normalized size = 1.22

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{ab\sqrt{(a+b)\,b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{ab\sqrt{(a+b)\,b}}}{f}$
risch	$\frac{x}{a}+\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{-(a+b)\,b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)\,b}+a+2b}{a}\right)}{2b^2f}+\frac{\sqrt{-(a+b)\,b}\,\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b*tan(f*x+e)+1/a*arctan(tan(f*x+e))+(-a^2-2*a*b-b^2)/a/b/((a+b)*b)^(
1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.47, size = 69, normalized size = 1.17

$$\frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*x + e)/s
qrt((a + b)*b))/(sqrt((a + b)*b)*a*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.
time = 3.02, size = 315, normalized size = 5.34

$$\left[\frac{4\,bfx\cos\left(fx+e\right)+(a+b)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)+4\,a\sin\left(fx+e\right)}{4\,abf\cos\left(fx+e\right)}, \frac{2\,bfx\cos\left(fx+e\right)+(a+b)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)\cos\left(fx+e\right)+2\,a\sin\left(fx+e\right)}{2\,abf\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a^
2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((
a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(f*x
+ e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((a + b
)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos(f*x +
e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 1.00, size = 87, normalized size = 1.47

$$\frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{ab+b^2}\,ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a^2 + 2*a*b + b^2)/(sqrt(a*b + b^2)*a*
b))/f
```

Mupad [B]
time = 4.68, size = 410, normalized size = 6.95

$$\frac{\operatorname{atan}\left(\frac{8\,a^2\,\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{2\,a^3\,\tan(e+f\,x)}{2\,a^3+8\,a^2\,b+12\,a\,b^2+6\,b^3}+\frac{6\,b^2\,\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}+\frac{12\,a\,b\tan(e+f\,x)}{12\,a\,b+8\,a^2+6\,b^2+\frac{2\,a^3}{b}}\right)}{a\,f}+\frac{\tan(e+f\,x)}{b\,f}+\frac{\operatorname{atanh}\left(\frac{6\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}}{18\,a\,b^2+20\,a^2\,b+10\,a^3+6\,b^3+\frac{2\,a^4}{b}}+\frac{6\,a\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}}{2\,a^4+10\,a^3\,b+20\,a^2\,b^2+18\,a\,b^3+6\,b^4}+\frac{2\,a^2\,\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}}{2\,a^4\,b+10\,a^3\,b^2+20\,a^2\,b^3+18\,a\,b^4+6\,b^5}\right)\,\sqrt{-b^3\,(a+b)^3}}{a\,b^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((8*a^2*tan(e + f*x))/(12*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (2*a^3*tan
(e + f*x))/(12*a*b^2 + 8*a^2*b + 2*a^3 + 6*b^3) + (6*b^2*tan(e + f*x))/(12*
a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (12*a*b*tan(e + f*x))/(12*a*b + 8*a^2 +
6*b^2 + (2*a^3)/b))/(a*f) + tan(e + f*x)/(b*f) + (atanh((6*tan(e + f*x)*(-
3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^2 + 20*a^2*b + 10*a^3 +
 6*b^3 + (2*a^4)/b) + (6*a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*
b^3)^(1/2))/(18*a*b^3 + 10*a^3*b + 2*a^4 + 6*b^4 + 20*a^2*b^2) + (2*a^2*tan
(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^4 + 2*a^4*
b + 6*b^5 + 20*a^2*b^3 + 10*a^3*b^2))*(-b^3*(a + b)^3)^(1/2))/(a*b^3*f)
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$-\frac{x}{a}+\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}$$

```
[Out] -x/a+arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a/f/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4226, 2000, 492, 209, 211}

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b
]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{af} \\
&= -\frac{x}{a} + \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,f}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 184, normalized size = 4.00

$$
-\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}\,fx\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))\right)}{2a\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b
*(Cos[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(a*Sqrt[a + b]*f*(a + b*Sec[e + f
*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.10, size = 48, normalized size = 1.04

method	result
derivativedivides	$\dfrac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a\sqrt{(a+b)\,b}}}{f}$
default	$\dfrac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{a\sqrt{(a+b)\,b}}}{f}$
risch	$-\frac{x}{a}-\frac{\sqrt{-(a+b)\,b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)\,b}+a+2b}{a}\right)}{2bfa}+\frac{\sqrt{-(a+b)\,b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{}}{}\right.}{2bfa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a*arctan(tan(f*x+e))+(a+b)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((
a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.46, size = 47, normalized size = 1.02

$$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] ((a + b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x
+ e)/a)/f
```

Fricas [A]
time = 2.66, size = 236, normalized size = 5.13

$$\left[-\frac{4\,fx-\sqrt{-\frac{a+b}{b}}\,\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\,\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,af},\,-\frac{2\,fx+\sqrt{\frac{a+b}{b}}\,\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*c
os(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*
x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.57, size = 66, normalized size = 1.43

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+b)}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*(a + b)/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B]
time = 4.72, size = 126, normalized size = 2.74

$$-\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{a\,f}-\frac{\operatorname{atanh}\left(\frac{2\,a\,b^2\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{2\,a^2\,b^2+2\,a\,b^3}\right)\,\sqrt{-b\,(a+b)}}{a\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] - atan((2*a*b^2*tan(e + f*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(e + f*x))/
(2*a*b^2 + 2*a^2*b))/(a*f) - (atanh((2*a*b^2*tan(e + f*x)*(- a*b - b^2)^(1/
2))/(2*a*b^3 + 2*a^2*b^2))*(-b*(a + b))^(1/2))/(a*b*f)
```

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{a}+\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

[Out] x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4212, 3260, 211}

$$\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}$$
$$= \frac{x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx, x, \cot(e+fx)\right)}{af}$$
$$= \frac{x}{a} + \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 182, normalized size = 4.04

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}\,fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))\right)}{2a\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.00, size = 46, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(\ldots}}{}\right.}{2(a+b)fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*arctan(tan(f*x+e))-1/a*b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2)))
```

Maxima [A]
time = 0.46, size = 46, normalized size = 1.02

$$-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)/
a)/f
```

Fricas [A]
time = 3.45, size = 241, normalized size = 5.36

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(fx+e)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos(fx+e)^3-\left(ab+b^2\right)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{4\,af},\ \frac{2\,fx+\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a+b\sec^2{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.41, size = 65, normalized size = 1.44

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}\,a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B]
time = 4.86, size = 460, normalized size = 10.22

$$\frac{x}{a}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+ba}+\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+ba}}{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{a^2+ba}-\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b\,(a+b)}}{a^2+ba}}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{f\left(a^2+ba\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2),x)
```

```
[Out] x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(
a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) + ((
2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(
a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a
*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^3
+ 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a
*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x) + ((2*a^
2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b
+ a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^
2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$-\frac{x}{a}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}$$

```
[Out] -x/a+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(3/2)/f-cot(f*x
+e)/(a+b)/f
```

Rubi [A]
time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4226, 2000, 491, 536, 209, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(3
/2)*f) - Cot[e + f*x]/((a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-2b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\
&= -\frac{\cot(e+fx)}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan\right.}{a(a+b)f}\\
&= -\frac{x}{a} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.51, size = 204, normalized size = 3.29

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b^2\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}((a+b)fx-a\csc(e)\csc(e+fx)\sin(fx))\right)}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*(
Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Sin[f*
x])))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^
4])
```

Maple [A]
time = 0.15, size = 68, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}}{f}$
risch	$-\frac{x}{a}-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^2fa}+\frac{\sqrt{-(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a+b)/tan(f*x+e)-1/a*arctan(tan(f*x+e))+1/(a+b)*b^2/a/((a+b)*b)^(1/
2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.46, size = 69, normalized size = 1.11

$$\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+ab)\sqrt{(a+b)b}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + a*b)*sqrt((a + b)*b)) -
 (f*x + e)/a - 1/((a + b)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.
time = 2.84, size = 328, normalized size = 5.29

$$\left[-\frac{4\,(a+b)fx\sin\left(fx+e\right)-b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin\left(fx+e\right)+4\,a\cos\left(fx+e\right)}{4\,(a^2+ab)f\sin\left(fx+e\right)}, -\frac{2\,(a+b)fx\sin\left(fx+e\right)+b\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)+2\,a\cos\left(fx+e\right)}{2\,(a^2+ab)f\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x +
e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f*x*si
n(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(f*x + e
))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.47, size = 87, normalized size = 1.40

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x +
 e)))/f
```

Mupad [B]
time = 6.25, size = 637, normalized size = 10.27

$$-\frac{a\,b^2+2\,a^2\,b+a^3+a^3\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))+b^3\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))+3\,a\,b^2\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))+3\,a^2\,b\tan(e+f\,x)\,\mathrm{atan}(\tan(e+f\,x))-\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,{\left(-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6\right)}^{3/2}\,1{}\mathrm{i}+b\tan(e+f\,x)\,{\left(-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6\right)}^{3/2}\,2{}\mathrm{i}+b^7\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,2{}\mathrm{i}+a\,b^6\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,10{}\mathrm{i}+a^6\,b\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,1{}\mathrm{i}+a^2\,b^5\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,21{}\mathrm{i}+a^3\,b^4\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,24{}\mathrm{i}+a^4\,b^3\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,16{}\mathrm{i}+a^5\,b^2\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,6{}\mathrm{i}}{a^8\,b^2+8\,a^7\,b^3+28\,a^6\,b^4+55\,a^5\,b^5+65\,a^4\,b^6+46\,a^3\,b^7+18\,a^2\,b^8+3\,a\,b^9}\right)\tan(e+f\,x)\,\sqrt{-a^3\,b^3-3\,a^2\,b^4-3\,a\,b^5-b^6}\,1{}\mathrm{i}}{f\tan(e+f\,x)\,a^4+3\,f\tan(e+f\,x)\,a^3\,b+3\,f\tan(e+f\,x)\,a^2\,b^2+f\tan(e+f\,x)\,a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

```
[Out] -(a*b^2 + 2*a^2*b + a^3 - atan((a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4
 - a^3*b^3)^(3/2)*1i + b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^
3)^(3/2)*2i + b^7*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2
)*2i + a*b^6*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*10i
 + a^6*b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i + a^
2*b^5*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*21i + a^3*
b^4*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*24i + a^4*b^
3*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*16i + a^5*b^2*
tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*6i)/(3*a*b^9 + 1
8*a^2*b^8 + 46*a^3*b^7 + 65*a^4*b^6 + 55*a^5*b^5 + 28*a^6*b^4 + 8*a^7*b^3 +
 a^8*b^2))*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i +
a^3*tan(e + f*x)*atan(tan(e + f*x)) + b^3*tan(e + f*x)*atan(tan(e + f*x)) +
 3*a*b^2*tan(e + f*x)*atan(tan(e + f*x)) + 3*a^2*b*tan(e + f*x)*atan(tan(e
+ f*x)))/(a^4*f*tan(e + f*x) + a*b^3*f*tan(e + f*x) + 3*a^3*b*f*tan(e + f*x
) + 3*a^2*b^2*f*tan(e + f*x))
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{x}{a}-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f}+\frac{(a+2b)\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}$$

```
[Out] x/a-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(5/2)/f+(a+2*b)*
cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

Rubi [A]
time = 0.17, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 491, 597, 536, 209, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)
*f) + ((a + 2*b)*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3(a+2b)-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)-3b(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \text{ta}\right.}{3(a+b)^2f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{b^3\text{Sul}}{}$$

$$= \frac{x}{a} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.80, size = 390, normalized size = 4.53

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3b^3\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))+\frac{1}{8}\sqrt{a+b}\csc(e)\csc^3(e+fx)\sqrt{b(\cos(e)-i\sin(e))^4}\,(9(a+b)^2fx\cos(fx)-9(a+b)^2fx\cos(2e+fx)-3a^2fx\cos(2e+3fx)-6abfx\cos(2e+3fx)-3b^2fx\cos(2e+3fx)+3a^2fx\cos(4e+3fx)+6abfx\cos(4e+3fx)+3b^2fx\cos(4e+3fx)-12a^2\sin(fx)-24ab\sin(fx)-12a^2\sin(2e+fx)-18ab\sin(2e+fx)+8a^2\sin(2e+3fx)+14ab\sin(2e+3fx))\right)}{6a(a+b)^{5/2}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Cos[f*
x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a*b*f*
x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x
] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*
x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x] + 8*a^
2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A]
time = 0.14, size = 90, normalized size = 1.05

method	result

derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^2a\sqrt{(a+b)\,b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{(a+b)^2a\sqrt{(a+b)\,b}}}{f}$
risch	$\frac{x}{a}+\frac{2i(6a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-12b\,e^{2i(fx+e)}+4a+7b)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)\,b}\;b^2\ln\left(e^{2i(fx+e)}+\frac{2i}{}\right.}{2(a+b)^3fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a*arctan(tan(f*x+e))-1/3/(a+b)/tan(f*x+e)^3-(-a-2*b)/(a+b)^2/tan(f*x
+e)-1/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.46, size = 110, normalized size = 1.28

$$-\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)b}}-\frac{3\,(fx+e)}{a}-\frac{3\,(a+2\,b)\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 2*a^2*b + a*b^2)
*sqrt((a + b)*b)) - 3*(f*x + e)/a - (3*(a + 2*b)*tan(f*x + e)^2 - a - b)/((
a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(79) = 158$.
time = 3.53, size = 559, normalized size = 6.50

$$\left[\frac{4\,(4\,a^2+7\,ab)\cos(fx+e)^3+3\,(b^2\cos(fx+e)^2-b^2)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)-12\,(a^2+2\,ab)\cos(fx+e)+12\,\left((a^2+2\,ab+b^2)fx\cos(fx+e)^2-(a^2+2\,ab+b^2)fx\right)\sin(fx+e)}{12\,\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2-(a^3+2\,a^2b+ab^2)f\right)\sin(fx+e)},\ \frac{2\,(4\,a^2+7\,ab)\cos(fx+e)^3+3\,(b^2\cos(fx+e)^2-b^2)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-6\,(a^2+2\,ab)\cos(fx+e)+6\,\left((a^2+2\,ab+b^2)fx\cos(fx+e)^2-(a^2+2\,ab+b^2)fx\right)\sin(fx+e)}{6\,\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2-(a^3+2\,a^2b+ab^2)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
```

```
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) + 12*((
a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(f*x +
e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)
^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/
(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2*a*b)*cos(
f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*
f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^
2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.50, size = 134, normalized size = 1.56

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{(a^3+2\,a^2b+ab^2)\sqrt{ab+b^2}}-\frac{3\,(fx+e)}{a}-\frac{3\,a\tan(fx+e)^2+6\,b\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a -
 (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan
(f*x + e)^3))/f
```

Mupad [B]
time = 8.97, size = 2644, normalized size = 30.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

```
[Out] atan((10*b^12*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b
^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 +
22*a^9*b^3 + 2*a^10*b^2) + (80*a*b^11*tan(e + f*x))/(80*a*b^11 + 10*b^12 +
290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*
a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (290*a^2*b^10*tan(e + f*
x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a
^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2)
 + (630*a^3*b^9*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3
*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4
+ 22*a^9*b^3 + 2*a^10*b^2) + (912*a^4*b^8*tan(e + f*x))/(80*a*b^11 + 10*b^1
2 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 +
330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (922*a^5*b^7*tan(e +
 f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 92
2*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b
^2) + (660*a^6*b^6*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*
a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b
^4 + 22*a^9*b^3 + 2*a^10*b^2) + (330*a^7*b^5*tan(e + f*x))/(80*a*b^11 + 10*
b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6
 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (110*a^8*b^4*tan(
e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 +
 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^1
0*b^2) + (22*a^9*b^3*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 63
0*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8
*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (2*a^10*b^2*tan(e + f*x))/(80*a*b^11 + 10
*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^
6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2))/(a*f) - (1/(3*(a
+ b)) - (tan(e + f*x)^2*(a + 2*b))/(a + b)^2)/(f*tan(e + f*x)^3) - (atan(((
(-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 +
280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a
^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12
+ 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 64
4*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 - (tan(e
+ f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 +
2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 +
 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5))
)/(2*a*(a + b)^5))*1i)/(a*(a + b)^5) + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*
x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^
5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/
2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*
a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*
b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a
^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712
*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*
a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5))*1i)/(a*(a + b)^5
))/(26*a*b^11 + 4*b^12 + 72*a^2*b^10 + 110*a^3*b^9 + 100*a^4*b^8 + 54*a^5*b
```

^7 + 16*a^6*b^6 + 2*a^7*b^5 + ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 - ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 - (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5)))/(a*(a + b)^5) - ((-b^5*(a + b)^5)^(1/2)*((tan(e + f*x)*(32*a*b^12 + 4*b^13 + 120*a^2*b^11 + 280*a^3*b^10 + 450*a^4*b^9 + 516*a^5*b^8 + 422*a^6*b^7 + 240*a^7*b^6 + 90*a^8*b^5 + 20*a^9*b^4 + 2*a^10*b^3))/2 + ((-b^5*(a + b)^5)^(1/2)*(6*a^2*b^12 + 54*a^3*b^11 + 218*a^4*b^10 + 520*a^5*b^9 + 812*a^6*b^8 + 868*a^7*b^7 + 644*a^8*b^6 + 328*a^9*b^5 + 110*a^10*b^4 + 22*a^11*b^3 + 2*a^12*b^2 + (tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(16*a^2*b^13 + 168*a^3*b^12 + 800*a^4*b^11 + 2280*a^5*b^10 + 4320*a^6*b^9 + 5712*a^7*b^8 + 5376*a^8*b^7 + 3600*a^9*b^6 + 1680*a^10*b^5 + 520*a^11*b^4 + 96*a^12*b^3 + 8*a^13*b^2))/(4*a*(a + b)^5)))/(2*a*(a + b)^5)))/(a*(a + b)^5)))*(-b^5*(a + b)^5)^(1/2)*1i)/(a*f*(a + b)^5)

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=120

$$-\frac{x}{a}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f}-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f}+\frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f}-\frac{\cot^5(e+fx)}{5(a+b)f}$$

```
[Out] -x/a+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(7/2)/f-(a^2+3*
a*b+3*b^2)*cot(f*x+e)/(a+b)^3/f+1/3*(a+2*b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(
f*x+e)^5/(a+b)/f
```

Rubi [A]
time = 0.24, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 491, 597, 536, 209, 211}

$$-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(7
/2)*f) - ((a^2 + 3*a*b + 3*b^2)*Cot[e + f*x])/((a + b)^3*f) + ((a + 2*b)*Co
t[e + f*x]^3)/(3*(a + b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5(a+2b)-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{-15(a^2+3ab+3b^2)-15b(a+2b)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, \right.}{15(a+b)^2f}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} + \text{Subs}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \text{Subs}$$

$$= -\frac{x}{a} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot}{3(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.05, size = 671, normalized size = 5.59

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)^3*f
*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e + 3*f*x
] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] + 75*b^
3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x*Cos[4*
e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e + 3*f*x] -
 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*a*b^2*f*x
*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[6*e + 5*f*
x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*x] + 15*b^3
*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] + 680*a*b^2*S
in[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] + 480*a*b^2*Sin
```

[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e + 3*f*x] - 400*a *b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*Sin[4*e + 3*f*x] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] + 132*a^2*b*Sin[4* e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2))

Maple [A]
time = 0.17, size = 118, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 a\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3 a\sqrt{(a+b)b}}}{f}$
risch	$-\frac{x}{a}-\frac{2i(45a^2e^{8i(fx+e)}+120ab\,e^{8i(fx+e)}+90b^2e^{8i(fx+e)}-90a^2e^{6i(fx+e)}-270ab\,e^{6i(fx+e)}-240b^2e^{6i(fx+e)}+140a^2e^{4i(}}{15f(a+b)^3(e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a*arctan(tan(f*x+e))-1/5/(a+b)/tan(f*x+e)^5-1/3*(-a-2*b)/(a+b)^2/ta
n(f*x+e)^3-(a^2+3*a*b+3*b^2)/(a+b)^3/tan(f*x+e)+1/(a+b)^3*b^4/a/((a+b)*b)^(
1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.47, size = 166, normalized size = 1.38

$$\frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{(a+b)b}}-\frac{15\,(fx+e)}{a}-\frac{15\,(a^2+3\,ab+3\,b^2)\tan(fx+e)^4-5\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*sqrt((a + b)*b)) - 15*(f*x + e)/a - (15*(a^2 + 3*a*b + 3*b^2)
*tan(f*x + e)^4 - 5*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b +
3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(112) = 224$.
time = 3.20, size = 867, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a^
2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e
)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(
3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (
a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*b + 3*
a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^
4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*
cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(23*a^3 + 66*
a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*b^2)*cos(f*x
 + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*sqrt(b/(a +
b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)
+ 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x)
*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e + fx)}{a + b\sec^2(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A]
time = 0.52, size = 212, normalized size = 1.77

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab+b^2}} - \frac{15(fx+e)}{a} - \frac{15a^2\tan(fx+e)^4+45ab\tan(fx+e)^4+45b^2\tan(fx+e)^4-5a^2\tan(fx+e)^2-15ab\tan(fx+e)^2-10b^2\tan(fx+e)^2+3a^2+6ab+3b^2}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) - 15
*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*tan(
f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(f*x
+ e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x +
e)^5))/f
```

Mupad [B]
time = 10.17, size = 2500, normalized size = 20.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

```
[Out] (atan(((((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15 +
 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*a^
8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 + 210
*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b
^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14168*a
^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^11*b^8
 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*a^16*b
^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 + 248*a
^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*b^13 +
 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b^9 + 8
8088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 1080*a^16
*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))*(-b^7
*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7) + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 +
 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b
^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 273
0*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 + (
(-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5
*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 +
22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14
*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a + b
)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25
480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*
a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*
b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a +
 b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7))/(60*a*b
^16 + 6*b^17 + 272*a^2*b^15 + 738*a^3*b^14 + 1332*a^4*b^13 + 1680*a^5*b^12
+ 1512*a^6*b^11 + 972*a^7*b^10 + 438*a^8*b^9 + 132*a^9*b^8 + 24*a^10*b^7 +
```

```
2*a^11*b^6 + (((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*
b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12
888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6
 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8
*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 1
4168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^
11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*
a^16*b^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18 +
 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a^7*
b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^11*b
^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5 + 108
0*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a + b)^7))
*(-b^7*(a + b)^7)^(1/2))/(a*(a + b)^7) - (((tan(e + f*x)*(48*a*b^17 + 4*b^1
8 + 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^
6*b^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 +
2730*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2
+ ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*
a^5*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10
 + 22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a
^14*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (tan(e + f*x)*(-b^7*(a
+ b)^7)^(1/2)*(16*a^2*b^18 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 +
 25480*a^6*b^14 + 58968*a^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 1544
40*a^10*b^10 + 131560*a^11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^
14*b^6 + 5320*a^15*b^5 + 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(
a + b)^7)))/(2*a*(a + b)^7))*(-b^7*(a + b)^7)^(1/2))/(a*(a + b)^7)))*(-b^7*
(a + b)^7)^(1/2)*1i)/(a*f*(a + b)^7) - (1/(5*(a + b)) + (tan(e + f*x)^4*(3*
a*b + a^2 + 3*b^2))/(a + b)^3 - (tan(e + f*x)^2*(a + 2*b))/(3*(a + b)^2))/(
f*tan(e + f*x)^5) - atan((14*b^17*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952
*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11
 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^
11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (952*a^
2*b^15*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 +
 8484*a^4*b^13 + 15848*a^5*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a
^8*b^9 + 16016*a^9*b^8 + 8736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 24
0*a^13*b^4 + 32*a^14*b^3 + 2*a^15*b^2) + (3388*a^3*b^14*tan(e + f*x))/(168*
a*b^16 + 14*b^17 + 952*a^2*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a^5
*b^12 + 22808*a^6*b^11 + 25722*a^7*b^10 + 22878*a^8*b^9 + 16016*a^9*b^8 + 8
736*a^10*b^7 + 3640*a^11*b^6 + 1120*a^12*b^5 + 240*a^13*b^4 + 32*a^14*b^3 +
 2*a^15*b^2) + (8484*a^4*b^13*tan(e + f*x))/(168*a*b^16 + 14*b^17 + 952*a^2
*b^15 + 3388*a^3*b^14 + 8484*a^4*b^13 + 15848*a...
```

3.350 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)}-\frac{\log(\cos(e+fx))}{b^2f}-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}$$

```
[Out] -1/2*(a+b)^2/a^2/b/f/(b+a*cos(f*x+e)^2)-ln(cos(f*x+e))/b^2/f-1/2*(1/a^2-1/b
^2)*ln(b+a*cos(f*x+e)^2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\log(\cos(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a + b)^2/(a^2*b*f*(b + a*Cos[e + f*x]^2)) - Log[Cos[e + f*x]]/(b^2*f)
 - ((a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/(2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)^2}{2a^2bf\,(b+a\cos^2(e+fx))} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e\right.}{2f}$$

Mathematica [A]
time = 0.49, size = 109, normalized size = 1.42

$$-\frac{(a+2b+a\cos(2e+2fx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cos^2(e+fx))}+\frac{2\log(\cos(e+fx))}{b^2}+\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)\right)\sec^4(e+fx)}{8f\,(a+b\sec^2(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x
]^2)) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x
]^2])*Sec[e + f*x]^4)/(f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.09, size = 72, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{(a+b)\left(-\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(a-b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
default	$\frac{\frac{(a+b)\left(-\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(a-b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2(a^2+2ab+b^2)e^{2i(fx+e)}}{a^2bf\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{b^2f}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx\right.}{a}}{2b^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/2/b^2*(a+b)*(-(a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(a-b)/a^2*ln(b+a*cos(f*x+e)^2))-1/b^2*ln(cos(f*x+e)))

Maxima [A]
time = 0.26, size = 101, normalized size = 1.31

$$\frac{\frac{a^2+2\,ab+b^2}{a^3b\sin\left(fx+e\right)^2-a^3b-a^2b^2}-\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}+\frac{\left(a^2-b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(sin(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2))/f

Fricas [A]
time = 3.41, size = 123, normalized size = 1.60

$$-\frac{a^2b+2\,ab^2+b^3-\left(a^2b-b^3+\left(a^3-ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,\left(a^3\cos\left(fx+e\right)^2+a^2b\right)\log\left(-\cos\left(fx+e\right)\right)}{2\,\left(a^3b^2f\cos\left(fx+e\right)^2+a^2b^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] -1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x + e)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)

[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(73) = 146.
time = 1.81, size = 532, normalized size = 6.91

$$\frac{\frac{\left(a^3+a^2b-ab^2-b^3\right)\log\left(\left|-a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-b\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-2\,a+2\,b\right|\right)}{a^3b^2+a^2b^3}+\frac{\log\left(\left|-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right|\right)}{a^2}-\frac{\log\left(\left|-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right|\right)}{b^2}-\frac{a^3\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+a^2b\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-ab^2\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-b^3\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+2\,a^3-6\,a^2b-6\,ab^2+2\,b^3}{\left(a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+b\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+2\,a-2\,b\right)a^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + a^2*b - a*b^2 - b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x +
e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^
3*b^2 + a^2*b^3) + log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + 2))/a^2 - log(abs(-(cos(f*x + e) + 1)/(cos
(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2 - (a^3*((c
os(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
) + a^2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1)) - a*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
e) - 1)/(cos(f*x + e) + 1)) - b^3*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) +
(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a^3 - 6*a^2*b - 6*a*b^2 + 2*b^3)
/((a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1)) + b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) + 2*a - 2*b)*a^2*b^2))/f
```

Mupad [B]
time = 4.65, size = 170, normalized size = 2.21

$$\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f}-\frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a^2\,f}+\frac{a^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}+\frac{b^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}+\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f}+\frac{a\,b}{f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(a + b + b*tan(e + f*x)^2)/(2*b^2*f) - log(a + b + b*tan(e + f*x)^2)/(2*
a^2*f) + a^2/(2*f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + b^2/(2*f*(a*b
^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + log(tan(e + f*x)^2 + 1)/(2*a^2*f) +
 (a*b)/(f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2))
```

3.351 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

```
[Out] 1/2*(a+b)/a^2/f/(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 455, 45}

$$\frac{a+b}{2a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*f*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^2
*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2} - \frac{1}{a(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a+b}{2a^2f\,(b+a\cos^2(e+fx))} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

Mathematica [A]
time = 0.77, size = 81, normalized size = 1.59

$$\frac{2(a+b)+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*x)
]))
```

Maple [A]
time = 0.07, size = 50, normalized size = 0.98

method	result	size
derivativedivides	$\frac{-\frac{-a-b}{2a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^2}}{f}$	50
default	$\frac{-\frac{-a-b}{2a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^2}}{f}$	50
risch	$-\frac{ix}{a^2}-\frac{2ie}{a^2f}+\frac{2(a+b)\mathrm{e}^{2i(fx+e)}}{a^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}+\frac{\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(-a-b)/a^2/(b+a*cos(f*x+e)^2)+1/2/a^2*ln(b+a*cos(f*x+e)^2))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 1.20

$$-\frac{\frac{a+b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a
 - b)/a^2)/f
```

Fricas [A]
time = 4.04, size = 56, normalized size = 1.10

$$\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(f
*x + e)^2 + a^2*b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(47) = 94.
time = 0.84, size = 314, normalized size = 6.16

$$\frac{\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}-\frac{a+b+\frac{6\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 2*log(abs(-(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 1))/a^2 - (a + b + 6*a*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^2))/f
```

Mupad [B]
time = 4.59, size = 97, normalized size = 1.90

$$-\frac{\mathrm{atanh}\left(\frac{4\,b^2\,\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan(e+f\,x)^2+\frac{8\,b^3\,\tan(e+f\,x)^2}{a}}\right)}{a^2\,f}-\frac{a+b}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] - atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 +
(8*b^3*tan(e + f*x)^2)/a))/(a^2*f) - (a + b)/(2*a*b*f*(a + b + b*tan(e + f*
x)^2))
```

3.352 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=49

$$-\frac{b}{2a^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

```
[Out] -1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 272, 45}

$$-\frac{b}{2a^2f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*b/(a^2*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^2*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{b}{2a^2f\,(b+a\cos^2(e+fx))} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

Mathematica [A]
time = 0.52, size = 79, normalized size = 1.61

$$-\frac{2b+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x)
]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [A]
time = 0.07, size = 61, normalized size = 1.24

method	result	size
derivativedivides	$\frac{-\frac{b\left(-\frac{a}{b\left(a+b\left(\sec^2(fx+e)\right)\right)}+\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{b}\right)}{2a^2}+\frac{\ln(\sec(fx+e))}{a^2}}{f}$	61
default	$\frac{-\frac{b\left(-\frac{a}{b\left(a+b\left(\sec^2(fx+e)\right)\right)}+\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{b}\right)}{2a^2}+\frac{\ln(\sec(fx+e))}{a^2}}{f}$	61
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2b\,e^{2i(fx+e)}}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/a^2*b*(-a/b/(a+b*sec(f*x+e)^2)+1/b*ln(a+b*sec(f*x+e)^2))+1/a^2*ln
(sec(f*x+e)))
```

Maxima [A]
time = 0.26, size = 59, normalized size = 1.20

$$\frac{\frac{b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/a
^2)/f
```

Fricas [A]
time = 4.20, size = 55, normalized size = 1.12

$$-\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
+ e)^2 + a^2*b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(45) = 90.
time = 0.57, size = 384, normalized size = 7.84

$$\frac{\frac{a^2+2\,ab+b^2+\frac{2\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{2\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3+a^2b)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}-\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2}+\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e) - 1)/(cos(f*
```

```
x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2)/((a^3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - log(a + b
+ 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 + 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 1))/a^2)/f
```

Mupad [B]
time = 4.48, size = 90, normalized size = 1.84

$$\frac{\operatorname{atanh}\left(\frac{4\,b^2\,\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan(e+f\,x)^2+\frac{8\,b^3\,\tan(e+f\,x)^2}{a}}\right)}{a^2\,f}+\frac{1}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8
*b^3*tan(e + f*x)^2)/a))/(a^2*f) + 1/(2*a*f*(a + b + b*tan(e + f*x)^2))
```

3.353 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)}+\frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f}+\frac{\log(\sin(e+fx))}{(a+b)^2f}$$

```
[Out] 1/2*b^2/a^2/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*b*(2*a+b)*ln(b+a*cos(f*x+e)^2)/a
^2/(a+b)^2/f+ln(sin(f*x+e))/(a+b)^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 90}

$$\frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)}+\frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2}+\frac{\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*f*(b + a*Cos[e + f*x]^2)) + (b*(2*a + b)*Log[b + a*Cos[e
 + f*x]^2])/(2*a^2*(a + b)^2*f) + Log[Sin[e + f*x]]/((a + b)^2*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b^2}{a(a+b)(b+ax)^2} - \frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{b^2}{2a^2(a+b)f\,(b+a\cos^2(e+fx))} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(s}{(}$$

Mathematica [A]
time = 0.38, size = 112, normalized size = 1.35

$$\frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(2\log(\sin(e+fx)) + \frac{b(2a+b)\log(a+b-a\sin^2(e+fx))}{a^2} + \frac{b^2(a+b)}{a^2(a+b-a\sin^2(e+fx))}\right)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b*
(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b -
a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.14, size = 91, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(2a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(\cos(fx+e)-1)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(2a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(\cos(fx+e)-1)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$\frac{ix}{a^2}-\frac{2ix}{a^2+2ab+b^2}-\frac{2ie}{f(a^2+2ab+b^2)}-\frac{4ibx}{a(a^2+2ab+b^2)}-\frac{4ibe}{af(a^2+2ab+b^2)}-\frac{2ib^2x}{a^2(a^2+2ab+b^2)}-\frac{2ib^2e}{a^2f(a^2+2ab+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b/(a+b)^2*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(2*a+b)/a^2*ln(b+a*cos(f
*x+e)^2))+1/2/(a+b)^2*ln(cos(f*x+e)-1)+1/2/(a+b)^2*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 120, normalized size = 1.45

$$\frac{\frac{b^2}{a^4+2\,a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2}+\frac{(2\,ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+2\,a^3b+a^2b^2}+\frac{\log\left(\sin(fx+e)^2\right)}{a^2+2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
+ b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(f*
x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

Fricas [A]
time = 2.94, size = 143, normalized size = 1.72

$$\frac{ab^2+b^3+\left(2\,ab^2+b^3+\left(2\,a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,\left(a^3\cos\left(fx+e\right)^2+a^2b\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\,\left(\left(a^5+2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(a
*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e))
)/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^
3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(79) = 158.
time = 0.50, size = 396, normalized size = 4.77

$$\frac{\frac{(2\,ab+b^2)\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+2\,a^3b+a^2b^2}+\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2\,ab+b^2}-\frac{2\,ab+b^2+\frac{4\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3+a^2b)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/2*((2*a*b + b^2)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 2*a^3*b + a^2*b^2) + log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) - (2*a*b + b^2 + 4*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^2)/f

Mupad [B]
time = 4.85, size = 106, normalized size = 1.28

$$\frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,a^2\,f}-\frac{b}{2\,a\,f\,\left(a+b\right)\,\left(b\,{\tan\left(e+f\,x\right)}^2+a+b\right)}+\frac{b\,\ln\left(b\,{\tan(e+f\,x)}^2+a+b\right)\,\left(2\,a+b\right)}{2\,a^2\,f\,{\left(a+b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)

[Out] log(tan(e + f*x))/(f*(2*a*b + a^2 + b^2)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) - b/(2*a*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + (b*log(a + b + b*tan(e + f*x)^2)*(2*a + b))/(2*a^2*f*(a + b)^2)

3.354 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=111

$$-\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\csc^2(e+fx)}{2(a+b)^2f}-\frac{b^2(3a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^3f}-\frac{(a+3b)\log(\sin(e+$$

```
[Out] -1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^2/f-1/2*b^
2*(3*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^3/f-(a+3*b)*ln(sin(f*x+e))/(a+b)^3
/f
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$-\frac{b^3}{2a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^3}-\frac{\csc^2(e+fx)}{2f(a+b)^2}-\frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/2*b^3/(a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a +
b)^2*f) - (b^2*(3*a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a^2*(a + b)^3*f) - (
(a + 3*b)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2} + \frac{a+3b}{(a+b)^3(-1+x)} - \frac{b^3}{a(a+b)^2(b+ax)^2} + \frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right)\,dx, x,}{2f}$$
$$= -\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+fx)}{2(a+b)^2f} - \frac{b^2(3a+b)\log\left(b+a\cos\right.}{2a^2(a+b)^3}$$

Mathematica [A]
time = 1.47, size = 130, normalized size = 1.17

$$-\frac{(a+2b+a\cos(2(e+fx)))^2\left((a+b)\csc^2(e+fx)+2(a+3b)\log(\sin(e+fx))+\frac{b^2\left(\frac{2b(a+b)}{a+2b+a\cos(2(e+fx))}+(3a+b)\log\left(a+b-a\sin^2(e+fx)\right)\right)}{a^2}\right)\sec^4(e+fx)}{8(a+b)^3f\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*b
)*Log[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) +
(3*a + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/((a + b)^3*f
*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.13, size = 141, normalized size = 1.27

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(3a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^3}+\frac{1}{4(a+b)^2(\cos(fx+e)-1)}+\frac{(-a-3b)\ln(\cos(fx+e)-1)}{2(a+b)^3}-\frac{}{4(a+b)^2(1}}{f}$
default	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(3a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^3}+\frac{1}{4(a+b)^2(\cos(fx+e)-1)}+\frac{(-a-3b)\ln(\cos(fx+e)-1)}{2(a+b)^3}-\frac{}{4(a+b)^2(1}}{f}$

risch	$-\frac{ix}{a^2}+\frac{2iax}{a^3+3a^2b+3a\,b^2+b^3}+\frac{2iae}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6ibx}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6ibe}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{}{a(a^3+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(a+b)^3*b^2*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(3*a+b)/a^2*ln(b+a*co
s(f*x+e)^2))+1/4/(a+b)^2/(cos(f*x+e)-1)+1/2*(-a-3*b)/(a+b)^3*ln(cos(f*x+e)-
1)-1/4/(a+b)^2/(1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 197, normalized size = 1.77

$$-\frac{\frac{(3\,ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{(a+3\,b)\log\left(\sin(fx+e)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{a^3+a^2b-(a^3-b^3)\sin(fx+e)^2}{(a^5+2\,a^4b+a^3b^2)\sin(fx+e)^4-(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sin(fx+e)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b^2)
*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(109) = 218.
time = 4.39, size = 321, normalized size = 2.89

$$\frac{a^3b+a^2b^2+ab^3+b^4+(a^4+a^3b-ab^3-b^4)\cos(fx+e)^2-\left((3\,a^2b^2+ab^3)\cos(fx+e)^4-3\,ab^3-b^4-(3\,a^2b^2-2\,ab^3-b^4)\cos(fx+e)^2\right)\log\left(a\cos(fx+e)^2+b\right)-2\left((a^4+3\,a^3b)\cos(fx+e)^4-a^3b-3\,a^2b^2-(a^4+2\,a^3b-3\,a^2b^2)\cos(fx+e)^2\right)\log\left(\frac{1}{2}\sin(fx+e)\right)}{2\left((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cos(fx+e)^4-(a^6+2\,a^5b-2\,a^3b^3-a^2b^4)f\cos(fx+e)^2-(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 - 2
*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^3*b
)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(f*x
+ e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. 2(105) = 210.
time = 0.59, size = 811, normalized size = 7.31

$$-\frac{\frac{12\,(3\,ab^2+b^3)\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{12\,(a+3\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a^4+6\,a^3b+3\,a^2b^2+\frac{10\,a^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{16\,a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{30\,a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{32\,ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{11\,a^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{22\,a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{27\,a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{16\,ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{16\,b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{4\,a^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{16\,a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{36\,a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{32\,ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{8\,b^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\left(\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}-\frac{3\,(\cos(fx+e)-1)}{(a^2+2\,ab+b^2)(\cos(fx+e)+1)}-\frac{24\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3) + 12*(a + 3*b)*log(abs(-cos(f*x + e) + 1)/abs
(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^4 + 6*a^3*b + 3*
a^2*b^2 + 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 30*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b^4*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + 22*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 16*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 4*a^4*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*a^3*b*(cos(f*x + e) - 1)^3/(cos(
f*x + e) + 1)^3 + 36*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 32
*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*b^4*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2
*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b +
 b^2)*(cos(f*x + e) + 1)) - 24*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + 1))/a^2)/f
```

Mupad [B]
time = 5.05, size = 160, normalized size = 1.44

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f}-\frac{\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^2\,(a\,b-b^2)}{2\,a\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+(a+b)\,\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan(e+f\,x)\right)\,(a+3\,b)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a+b)}{2\,a^2\,f\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*a^2*f) - (1/(2*(a + b)) + (tan(e + f*x)^2*(a*b -
 b^2))/(2*a*(a + b)^2))/(f*(tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)) - (
log(tan(e + f*x))*(a + 3*b))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^2*log
(a + b + b*tan(e + f*x)^2)*(3*a + b))/(2*a^2*f*(a + b)^3)
```

3.355 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=140

$$\frac{b^4}{2a^2(a+b)^3f\left(b+a\cos^2(e+fx)\right)}+\frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f}-\frac{\csc^4(e+fx)}{4(a+b)^2f}+\frac{b^3(4a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^4f}$$

```
[Out] 1/2*b^4/a^2/(a+b)^3/f/(b+a*cos(f*x+e)^2)+(a+2*b)*csc(f*x+e)^2/(a+b)^3/f-1/4
*csc(f*x+e)^4/(a+b)^2/f+1/2*b^3*(4*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^4/f+
(a^2+4*a*b+6*b^2)*ln(sin(f*x+e))/(a+b)^4/f
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{b^4}{2a^2f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4}+\frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{f(a+b)^4}-\frac{\csc^4(e+fx)}{4f(a+b)^2}+\frac{(a+2b)\csc^2(e+fx)}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^4/(2*a^2*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) + ((a + 2*b)*Csc[e + f*x]^2)
/((a + b)^3*f) - Csc[e + f*x]^4/(4*(a + b)^2*f) + (b^3*(4*a + b)*Log[b + a*
Cos[e + f*x]^2])/(2*a^2*(a + b)^4*f) + ((a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f
*x]])/((a + b)^4*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^3(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^3(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)^3} - \frac{2(a+2b)}{(a+b)^3(-1+x)^2} + \frac{-a^2-4ab-6b^2}{(a+b)^4(-1+x)} + \frac{b^4}{a(a+b)^3(b+ax)^2} - \frac{b^3}{a(a+\cdots}\right.\right.}{2f}$$
$$= \frac{b^4}{2a^2(a+b)^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f} - \frac{\csc^4(e+fx)}{4(a+b)^2f} + \frac{b\cdots}{\cdots}$$

Mathematica [A]
time = 2.03, size = 162, normalized size = 1.16

$$\frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(4(a+b)(a+2b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2+4ab+6b^2)\log(\sin(e+fx))+\frac{2b^3(4a+b)\log(a+b-a\sin^2(e+fx))}{a^2}+\frac{2b^4(a+b)}{a^2(a+b-a\sin^2(e+fx))}\right)}{16(a+b)^4f\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[e
 + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e +
f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a + b
))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*x]^2
)^2)
```

Maple [A]
time = 0.15, size = 201, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(4a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(\cos(fx+e)-1)^2}-\frac{7a+15b}{16(a+b)^3(\cos(fx+e)-1)}+\frac{\left(a^2+4ab+6b^2\cdots\right.}{2(\cdots}}{f}$
default	$\frac{\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{(4a+b)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(\cos(fx+e)-1)^2}-\frac{7a+15b}{16(a+b)^3(\cos(fx+e)-1)}+\frac{\left(a^2+4ab+6b^2\cdots\right.}{2(\cdots}}{f}$

risch	$-\frac{12ib^2x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}-\frac{8ib^3x}{a(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{12ib^2e}{f(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{2}{a^2(a^4+4a^3b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b^3/(a+b)^4*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(4*a+b)/a^2*ln(b+a*cos
(f*x+e)^2))-1/16/(a+b)^2/(cos(f*x+e)-1)^2-1/16*(7*a+15*b)/(a+b)^3/(cos(f*x+
e)-1)+1/2*(a^2+4*a*b+6*b^2)/(a+b)^4*ln(cos(f*x+e)-1)-1/16/(a+b)^2/(1+cos(f*
x+e))^2-1/16*(-7*a-15*b)/(a+b)^3/(1+cos(f*x+e))+1/2*(a^2+4*a*b+6*b^2)/(a+b)
^4*ln(1+cos(f*x+e)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(139) = 278$.
time = 0.27, size = 285, normalized size = 2.04

$$\frac{\frac{2\,(4\,ab^3+b^4)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4}+\frac{2\,(a^2+4\,ab+6\,b^2)\log\left(\sin\left(fx+e\right)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,(2\,a^4+4\,a^3b-b^4)\sin\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-(5\,a^4+13\,a^3b+8\,a^2b^2)\sin\left(fx+e\right)^2}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sin\left(fx+e\right)^6-(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)/(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*sin
(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*sin(
f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(139) = 278$.
time = 4.84, size = 570, normalized size = 4.07

$$\frac{3\,a^4b+10\,a^3b^2+7\,a^2b^3+2\,ab^4+2\,b^5-2\,(2\,a^5+6\,a^4b+4\,a^3b^2-ab^4-b^5)\cos\left(fx+e\right)^4+(3\,a^5+6\,a^4b-5\,a^3b^2-8\,a^2b^3-4\,ab^4-4\,b^5)\cos\left(fx+e\right)^2+2\,\left((4\,a^2b^3+ab^4)\cos\left(fx+e\right)^6+4\,ab^4+b^5-(8\,a^2b^3-2\,ab^4-b^5)\cos\left(fx+e\right)^4+(4\,a^2b^3-7\,ab^4-2\,b^5)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+4\,\left((a^5+4\,a^4b+6\,a^3b^2)\cos\left(fx+e\right)^6+a^4b+4\,a^3b^2+6\,a^2b^3-(2\,a^5+7\,a^4b+8\,a^3b^2-6\,a^2b^3)\cos\left(fx+e\right)^4+(a^5+2\,a^4b-2\,a^3b^2-12\,a^2b^3)\cos\left(fx+e\right)^2\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)}{4\,\left((a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)f\cos\left(fx+e\right)^6-(2\,a^7+7\,a^6b+8\,a^5b^2+2\,a^4b^3-2\,a^3b^4-a^2b^5)f\cos\left(fx+e\right)^4+(a^7+2\,a^6b-2\,a^5b^2-8\,a^4b^3-7\,a^3b^4-2\,a^2b^5)f\cos\left(fx+e\right)^2+(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4*
b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^2
- 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*cos(
f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (
4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*
((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3
```

```
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 + 2*a^4*
b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^7 +
4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^
6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7
+ 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(134) = 268.
time = 0.62, size = 913, normalized size = 6.52

$$\frac{\frac{32\,(4ab^3+b^4)\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4}+\frac{32\,(a^2+4ab+6b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{\frac{12a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{40ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{2ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{\left(a^2+2ab+b^2+\frac{12a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{40ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{192ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{288b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(fx+e)-1)^2}-\frac{32\left(4a^2b^3+5ab^4+b^5+\frac{8a^2b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2ab^4(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b^5(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4a^2b^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{5ab^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^5(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}-\frac{64\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*(4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 4*
a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4) + 32*(a^2 + 4*a*b + 6*b^2)*log(abs
(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4) - (12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 192*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2 + 288*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e
) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)^2)
 - 32*(4*a^2*b^3 + 5*a*b^4 + b^5 + 8*a^2*b^3*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - 2*a*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^5*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 4*a^2*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 5*a*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^5*(cos(f*x +
```

```
e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a
^2*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 64*log(abs(-(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 1))/a^2)/f
```

Mupad [B]
time = 6.09, size = 206, normalized size = 1.47

$$\frac{\frac{\tan(e+f\,x)^2\,(2\,a+5\,b)}{4\,(a+b)^2}-\frac{1}{4\,(a+b)}+\frac{\tan(e+f\,x)^4\,(a^2\,b+3\,a\,b^2-b^3)}{2\,a\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+(a+b)\,\tan\left(e+f\,x\right)^4\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f}+\frac{\ln\left(\tan(e+f\,x)\right)\,(a^2+4\,a\,b+6\,b^2)}{f\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}+\frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(4\,a+b)}{2\,a^2\,f\,(a+b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(2*a + 5*b))/(4*(a + b)^2) - 1/(4*(a + b)) + (tan(e + f*x)
^4*(3*a*b^2 + a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^4*(a + b) + b
*tan(e + f*x)^6)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) + (log(tan(e + f*x))*
(4*a*b + a^2 + 6*b^2))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (b
^3*log(a + b + b*tan(e + f*x)^2)*(4*a + b))/(2*a^2*f*(a + b)^4)
```

3.356 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{x}{a^2}-\frac{(3a-2b)(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2-1/2*(3*a-2*b)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2
/b^(5/2)/f+1/2*(3*a+b)*tan(f*x+e)/a/b^2/f-1/2*(a+b)*tan(f*x+e)^3/a/b/f/(a+b
+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 481, 596, 536, 209, 211}

$$-\frac{(3a-2b)(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}-\frac{x}{a^2}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) - ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(2*a^2*b^(5/2)*f) + ((3*a + b)*Tan[e + f*x])/(2*a*b^2*f) - ((a + b)*
Tan[e + f*x]^3)/(2*a*b*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+b)x^2)}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx\right.}{2abf} \\
&= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{(a+b)(3a+b)+(3}{(1+x^2)(a}\right.}{} \\
&= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \text{ta}\right.}{a^2f} \\
&= -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{}{2a}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.09, size = 286, normalized size = 2.40

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2}+\frac{(3a-2b)(a+b)^{3/2}\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{a^2b^2f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{2(a+2b+a\cos(2(e+fx)))\sec(e)\sec(e+fx)\sin(fx)}{b^2f}-\frac{(a+b)^2((a+2b)\sin(2e)-a\sin(2fx))}{a^2b^2f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8\left(a+b\sec^2(e+fx)\right)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*
Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[
2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*b)*Sin
[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8
*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.14, size = 125, normalized size = 1.05

method	result

derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{a+b+b\left(\tan^2(fx+e)\right)}+\frac{\left(3a^3+4a^2b-a\,b^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2\sqrt{(a+b)\,b}}}{a^2b^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{a+b+b\left(\tan^2(fx+e)\right)}+\frac{\left(3a^3+4a^2b-a\,b^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2\sqrt{(a+b)\,b}}}{a^2b^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i(3a^3e^{4i(fx+e)}+4a^2b\,e^{4i(fx+e)}+5a\,b^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}+6a^3e^{2i(fx+e)}+14a^2b\,e^{2i(fx+e)}+6a\,b^2e^{2i(fx+e}}{a^2b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b^2*tan(f*x+e)-1/a^2*arctan(tan(f*x+e))-1/a^2/b^2*((-1/2*a^3-a^2*b-1
/2*a*b^2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a^3+4*a^2*b-a*b^2-2*b^3)/(
(a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.46, size = 132, normalized size = 1.11

$$\frac{\frac{(a^2+2\,ab+b^2)\tan(fx+e)}{ab^3\tan(fx+e)^2+a^2b^2+ab^3}-\frac{2\,(fx+e)}{a^2}+\frac{2\,\tan(fx+e)}{b^2}-\frac{(3\,a^3+4\,a^2b-ab^2-2\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*tan(f*x + e)/(a*b^3*tan(f*x + e)^2 + a^2*b^2 + a*b
^3) - 2*(f*x + e)/a^2 + 2*tan(f*x + e)/b^2 - (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2*b^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(109) = 218.
time = 3.49, size = 540, normalized size = 4.54

$$\left[-\frac{8\,ab^2fx\cos(fx+e)^3+8\,b^3fx\cos(fx+e)+\left((3\,a^3+a^2b-2\,ab^2)\cos(fx+e)^3+(3\,a^2b+ab^2-2\,b^3)\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)-4\left(2\,a^2b+(3\,a^3+2\,a^2b+ab^2)\cos(fx+e)^2\right)\sin(fx+e)}{8\left(a^3b^2f\cos(fx+e)^3+a^2b^3f\cos(fx+e)\right)},\ -\frac{4\,ab^2fx\cos(fx+e)^3+4\,b^3fx\cos(fx+e)-\left((3\,a^3+a^2b-2\,ab^2)\cos(fx+e)^3+(3\,a^2b+ab^2-2\,b^3)\cos(fx+e)\right)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)-2\left(2\,a^2b+(3\,a^3+2\,a^2b+ab^2)\cos(fx+e)^2\right)\sin(fx+e)}{4\left(a^3b^2f\cos(fx+e)^3+a^2b^3f\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2*
b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(
-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*c
os(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(
a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x + e)
)/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2*f*x*co
s(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*cos(f*x
+ e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f
*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x
+ e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 2.36, size = 155, normalized size = 1.30

$$-\frac{\frac{2\,(fx+e)}{a^2}-\frac{2\,\tan(fx+e)}{b^2}+\frac{\left(3\,a^3+4\,a^2b-ab^2-2\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^2b^2}-\frac{a^2\tan(fx+e)+2\,ab\tan(fx+e)+b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/a^2 - 2*tan(f*x + e)/b^2 + (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^2*b^2) - (a^2*tan(f*x + e) + 2*a*b*tan(f*x + e)
+ b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b^2))/f
```

Mupad [B]
time = 4.99, size = 765, normalized size = 6.43

$$\frac{\tan(e+f\,x)}{b^2\,f}-\frac{\mathrm{atan}\left(\frac{5\,\tan(e+f\,x)}{\frac{12\,a}{b}-\frac{10\,b}{a}-\frac{15\,b^2}{2\,a^2}+\frac{9\,a^2}{2\,b^2}+5}-\frac{10\,\tan(e+f\,x)}{\frac{5\,a}{b}-\frac{15\,b}{2\,a}+\frac{12\,a^2}{b^2}+\frac{9\,a^3}{2\,b^3}-10}+\frac{12\,a\,\tan(e+f\,x)}{12\,a+5\,b-\frac{10\,b^2}{a}+\frac{9\,a^2}{2\,b}-\frac{15\,b^3}{2\,a^2}}-\frac{15\,b\,\tan(e+f\,x)}{2\left(\frac{5\,a^2}{b}-\frac{15\,b}{2}-10\,a+\frac{12\,a^3}{b^2}+\frac{9\,a^4}{2\,b^3}\right)}+\frac{9\,a^2\,\tan(e+f\,x)}{2\left(12\,a\,b+\frac{9\,a^2}{2}+5\,b^2-\frac{10\,b^3}{a}-\frac{15\,b^4}{2\,a^2}\right)}\right)}{a^2\,f}+\frac{\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,f\,\left(b^3\,\tan(e+f\,x)^2+b^3+a\,b^2\right)}-\frac{\mathrm{atan}\left(-\frac{\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,35{}\mathrm{i}}{4\left(9\,a^3\,b-\frac{85\,a\,b^3}{4}+\frac{81\,a^4}{4}+\frac{25\,b^4}{4}-\frac{49\,a^2\,b^2}{2}+\frac{15\,b^5}{2\,a}+\frac{27\,a^5}{4\,b}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,15{}\mathrm{i}}{2\left(\frac{25\,a\,b^3}{4}-\frac{49\,a^3\,b}{2}+9\,a^4+\frac{15\,b^4}{2}-\frac{85\,a^2\,b^2}{4}+\frac{81\,a^5}{4\,b}+\frac{27\,a^6}{4\,b^2}\right)}-\frac{\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,45{}\mathrm{i}}{4\left(\frac{81\,a^3\,b}{4}-\frac{49\,a\,b^3}{2}+\frac{27\,a^4}{4}-\frac{85\,b^4}{4}+9\,a^2\,b^2+\frac{25\,b^5}{4\,a}+\frac{15\,b^6}{2\,a^2}\right)}+\frac{a^2\,\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,27{}\mathrm{i}}{4\left(9\,a^2\,b^4-\frac{85\,b^6}{4}-\frac{49\,a\,b^5}{2}+\frac{81\,a^3\,b^3}{4}+\frac{27\,a^4\,b^2}{4}+\frac{25\,b^7}{4\,a}+\frac{15\,b^8}{2\,a^2}\right)}+\frac{a\,\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,27{}\mathrm{i}}{4\left(\frac{27\,a^4\,b}{4}-\frac{49\,a\,b^4}{2}-\frac{85\,b^5}{4}+9\,a^2\,b^3+\frac{81\,a^3\,b^2}{4}+\frac{25\,b^6}{4\,a}+\frac{15\,b^7}{2\,a^2}\right)}\right)\,\sqrt{-b^5\,{\left(a+b\right)}^3}\,\left(3\,a-2\,b\right)\,1{}\mathrm{i}}{2\,a^2\,b^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(b^2*f) - atan((5*tan(e + f*x))/((12*a)/b - (10*b)/a - (15*b^2
)/(2*a^2) + (9*a^2)/(2*b^2) + 5) - (10*tan(e + f*x))/((5*a)/b - (15*b)/(2*a
) + (12*a^2)/b^2 + (9*a^3)/(2*b^3) - 10) + (12*a*tan(e + f*x))/(12*a + 5*b
- (10*b^2)/a + (9*a^2)/(2*b) - (15*b^3)/(2*a^2)) - (15*b*tan(e + f*x))/(2*(
(5*a^2)/b - (15*b)/2 - 10*a + (12*a^3)/b^2 + (9*a^4)/(2*b^3))) + (9*a^2*tan
(e + f*x))/(2*(12*a*b + (9*a^2)/2 + 5*b^2 - (10*b^3)/a - (15*b^4)/(2*a^2)))
)/(a^2*f) + (tan(e + f*x)*(2*a*b + a^2 + b^2))/(2*a*f*(a*b^2 + b^3 + b^3*ta
n(e + f*x)^2)) - (atan((tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5
)^(1/2)*15i)/(2*((25*a*b^3)/4 - (49*a^3*b)/2 + 9*a^4 + (15*b^4)/2 - (85*a^2
*b^2)/4 + (81*a^5)/(4*b) + (27*a^6)/(4*b^2))) - (tan(e + f*x)*(- 3*a*b^7 -
b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*35i)/(4*(9*a^3*b - (85*a*b^3)/4 + (81*a^4)
/4 + (25*b^4)/4 - (49*a^2*b^2)/2 + (15*b^5)/(2*a) + (27*a^5)/(4*b))) - (tan
(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*45i)/(4*((81*a^3*b)
/4 - (49*a*b^3)/2 + (27*a^4)/4 - (85*b^4)/4 + 9*a^2*b^2 + (25*b^5)/(4*a) +
(15*b^6)/(2*a^2))) + (a^2*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b
^5)^(1/2)*27i)/(4*(9*a^2*b^4 - (85*b^6)/4 - (49*a*b^5)/2 + (81*a^3*b^3)/4 +
 (27*a^4*b^2)/4 + (25*b^7)/(4*a) + (15*b^8)/(2*a^2))) + (a*tan(e + f*x)*(-
3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*((27*a^4*b)/4 - (49*a*b^
4)/2 - (85*b^5)/4 + 9*a^2*b^3 + (81*a^3*b^2)/4 + (25*b^6)/(4*a) + (15*b^7)/
(2*a^2))))*(-b^5*(a + b)^3)^(1/2)*(3*a - 2*b)*1i)/(2*a^2*b^5*f)
```

3.357 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal. Leaf size=90

$$\frac{x}{a^2}+\frac{(a-2b)\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^2+1/2*(a-2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/b^
(3/2)/f-1/2*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 481, 536, 209, 211}

$$\frac{(a-2b)\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}+\frac{x}{a^2}-\frac{(a+b)\tan(e+fx)}{2abf\,(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(2*a^2*b^(3/2)*f) - ((a + b)*Tan[e + f*x])/(2*a*b*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right.}{2abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{((a-}{}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\,(a+b+b\tan^2(e+fx)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.78, size = 249, normalized size = 2.77

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{(-a^2+ab+2b^2)\mathrm{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{b\sqrt{a+b}\,f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{(a+b)((a+2b)\sin(2e)-a\sin(2fx))}{bf(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-
((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(b*Sq
rt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*S
ec[e + f*x]^2)^2)
```

Maple [A]
time = 0.12, size = 85, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2b\sqrt{(a+b)\,b}}\right)}{a^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right)}{2b\sqrt{(a+b)\,b}}\right)}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(a^2\mathrm{e}^{2i(fx+e)}+3ab\,\mathrm{e}^{2i(fx+e)}+2b^2\mathrm{e}^{2i(fx+e)}+a^2+ab\right)}{a^2bf\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}+\frac{\sqrt{-(a+b)\,b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)\,b}}{a}\right.}{4b^2fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*arctan(tan(f*x+e))+(a+b)/a^2*(-1/2*a/b*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)+1/2*(a-2*b)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]

time = 0.47, size = 100, normalized size = 1.11

$$-\frac{\frac{(a+b)\tan(fx+e)}{ab^2\tan(fx+e)^2+a^2b+ab^2}-\frac{2(fx+e)}{a^2}-\frac{(a^2-ab-2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)*tan(f*x + e)/(a*b^2*tan(f*x + e)^2 + a^2*b + a*b^2) - 2*(f*x
+ e)/a^2 - (a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt
((a + b)*b)*a^2*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(81) = 162.
time = 3.41, size = 413, normalized size = 4.59

$$\left[\frac{8\,abfx\cos(fx+e)^2+8\,b^2fx-4\,(a^2+ab)\cos(fx+e)\sin(fx+e)-((a^2-2\,ab)\cos(fx+e)^2+ab-2\,b^2)\sqrt{-\frac{a+b}{b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)}{8\left(a^3bf\cos(fx+e)^2+a^2b^2f\right)},\ \frac{4\,abfx\cos(fx+e)^2+4\,b^2fx-2\,(a^2+ab)\cos(fx+e)\sin(fx+e)-((a^2-2\,ab)\cos(fx+e)^2+ab-2\,b^2)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)}{4\left(a^3bf\cos(fx+e)^2+a^2b^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*sin
(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^3*b
*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2*f*x -
 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 +
a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x + e)^2 +
 a^2*b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 1.12, size = 120, normalized size = 1.33

$$\frac{\frac{2\,(fx+e)}{a^2}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-ab-2\,b^2)}{\sqrt{ab+b^2}\,a^2b}-\frac{a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)/a^2 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - a*b - 2*b^2)/(sqrt(a*b + b^2)*a^2*b) - (a
*tan(f*x + e) + b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b))/f
```

Mupad [B]
time = 4.78, size = 285, normalized size = 3.17

$$\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)}{\frac{3\,b}{2\,a}-\frac{a}{2\,b}+1}-\frac{\tan(e+f\,x)}{2\,\left(\frac{b}{a}+\frac{3\,b^2}{2\,a^2}-\frac{1}{2}\right)}+\frac{3\,b\tan(e+f\,x)}{2\,\left(a+\frac{3\,b}{2}-\frac{a^2}{2\,b}\right)}\right)}{a^2\,f}-\frac{\tan(e+f\,x)\,\left(a+b\right)}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}-\frac{\mathrm{atanh}\left(\frac{3\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{2\,\left(\frac{a\,b}{4}-a^2+\frac{3\,b^2}{2}+\frac{a^3}{4\,b}\right)}-\frac{5\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\,\left(\frac{a^2}{4}-a\,b+\frac{b^2}{4}+\frac{3\,b^3}{2\,a}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\,\left(\frac{a\,b}{4}-b^2+\frac{b^3}{4\,a}+\frac{3\,b^4}{2\,a^2}\right)}\right)\,\sqrt{-b^3\,\left(a+b\right)}\,\left(a-2\,b\right)}{2\,a^2\,b^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan(tan(e + f*x)/((3*b)/(2*a) - a/(2*b) + 1) - tan(e + f*x)/(2*(b/a + (3*b
^2)/(2*a^2) - 1/2)) + (3*b*tan(e + f*x))/(2*(a + (3*b)/2 - a^2/(2*b))))/(a^
2*f) - (tan(e + f*x)*(a + b))/(2*a*b*f*(a + b + b*tan(e + f*x)^2)) - (atanh
((3*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(2*((a*b)/4 - a^2 + (3*b^2)/2 + a^3
/(4*b))) - (5*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*(a^2/4 - a*b + b^2/4 +
 (3*b^3)/(2*a))) + (tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*((a*b)/4 - b^2 +
 b^3/(4*a) + (3*b^4)/(2*a^2))))*(-b^3*(a + b))^(1/2)*(a - 2*b))/(2*a^2*b^3*
f)
```

3.358 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2} dx$

Optimal. Leaf size=85

$$-\frac{x}{a^2}+\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f}+\frac{\tan(e+fx)}{2af(a+b+b\tan^2(e+fx))}$$

```
[Out] -x/a^2+1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/f/b^(1/2)/(a+
b)^(1/2)+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 482, 536, 209, 211}

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}f\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(e+fx)}{2af(a+b\tan^2(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^2*Sq
rt[b]*Sqrt[a + b]*f) + Tan[e + f*x]/(2*a*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)}{2af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}\\
&= \frac{\tan(e+fx)}{2af\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(a+2b)\text{Su}}{}\\
&= -\frac{x}{a^2} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,\sqrt{a+b}\,f} + \frac{\tan(e+fx)}{2af\,(a+b+b\tan^2(e+fx))}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 8.48, size = 346, normalized size = 4.07

$$\frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\operatorname{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{\left(a^2+8ab+8b^2\right)((a+2b)\sin(2e)-a\sin(2fx))}{b(a+b)f(a+2b+a\cos(2(e+fx)))(\cos(e)-\sin(e))(\cos(e)+\sin(e))}}{a^2}+\frac{\frac{(a+2b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}}{b^{3/2}f}\right)}{64\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2*
b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a +
b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a
^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)
 - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(
b^(3/2)*f)))/(64*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]

time = 0.13, size = 77, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2iba+2ib^2-a\sqrt{-ab-b^2}-2b\sqrt{-ab}}{a\sqrt{-ab-b^2}}\right.}{4\sqrt{-ab-b^2}\,fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*arctan(tan(f*x+e))+1/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)
+1/2*(a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Maxima [A]

time = 0.46, size = 79, normalized size = 0.93

$$\frac{\frac{\tan(fx+e)}{ab\tan(fx+e)^2+a^2+ab}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}-\frac{2(fx+e)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(f*x + e)/(a*b*tan(f*x + e)^2 + a^2 + a*b) + (a + 2*b)*arctan(b*tan
(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) - 2*(f*x + e)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(76) = 152$.
time = 3.82, size = 478, normalized size = 5.62

$$\left[-\frac{8(a^2b+ab^2)fx\cos(fx+e)^2+8(ab^2+b^3)fx-4(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+((a^2+2ab)\cos(fx+e)^2+ab+2b^2)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4((a+2b)\cos(fx+e)^3-b\cos(fx+e))\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)}{8((a^4b+a^3b^2)f\cos(fx+e)^2+(a^3b^2+a^2b^3)f)}, -\frac{4(a^2b+ab^2)fx\cos(fx+e)^2+4(ab^2+b^3)fx-2(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+((a^2+2ab)\cos(fx+e)^2+ab+2b^2)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)}{4((a^4b+a^3b^2)f\cos(fx+e)^2+(a^3b^2+a^2b^3)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2*
b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
+ 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))
*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)
, -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x - 2*(a^2
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
 + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2
+ (a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.69, size = 94, normalized size = 1.11

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{\sqrt{ab+b^2}\,a^2}-\frac{2\,(fx+e)}{a^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(sqrt(a*b + b^2)*a^2) - 2*(f*x + e)/a^2 + tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)*a))/f
```

Mupad [B]
time = 5.00, size = 711, normalized size = 8.36

$$\frac{\tan(e+fx)}{2af\left(b\tan(e+fx)^2+a+b\right)}-\frac{x}{a^2}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\tan(e+fx)\left(a^2b+4ab^2+8b^3\right)}{2a^2}-\frac{\left(2ab^2-\frac{\tan(e+fx)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}\,1i}{4\left(a^3b+a^2b^2\right)}+\frac{\left(\frac{\tan(e+fx)\left(a^2b+4ab^2+8b^3\right)}{2a^2}+\frac{\left(2ab^2+\frac{\tan(e+fx)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}\,1i}{4\left(a^3b+a^2b^2\right)}}{\frac{b^2+\frac{ab}{2}}{a^3}-\frac{\left(\frac{\tan(e+fx)\left(a^2b+4ab^2+8b^3\right)}{2a^2}-\frac{\left(2ab^2-\frac{\tan(e+fx)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}+\frac{\left(\frac{\tan(e+fx)\left(a^2b+4ab^2+8b^3\right)}{2a^2}+\frac{\left(2ab^2+\frac{\tan(e+fx)\left(16a^5b^2+32a^4b^3\right)(a+2b)\sqrt{-b(a+b)}}{8a^2\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}\right)(a+2b)\sqrt{-b(a+b)}}{4\left(a^3b+a^2b^2\right)}}\right)(a+2b)\sqrt{-b(a+b)}\,1i}{2f\left(a^3b+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(2*a*f*(a + b + b*tan(e + f*x)^2)) - x/a^2 - (atan(((((tan(e +
 f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^
4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2))
)*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b
))^(1/2)*1i)/(4*(a^3*b + a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b
^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b
)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/
2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2)*1i)/(4*(a^3*b + a^2
*b^2)))/(((a*b)/2 + b^2)/a^3 - (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(
2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*
(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4
*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)) +
(((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f
*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b +
 a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*
(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2))))*(a + 2*b)*(-b*(a + b))^(1/2)*1i
)/(2*f*(a^3*b + a^2*b^2))
```

3.359 $\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{x}{a^2}-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b
)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4213, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e \cdots\right)}{2a(a+b)f \cdots}\\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f}-\cdots\\
&= \frac{x}{a^2}-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f}-\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+\cdots\right)}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.11, size = 240, normalized size = 2.61

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\,\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
```

```
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A]
time = 0.00, size = 90, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)\,b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)}}{a}\right.}{4(a+b)^2fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*b*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a
+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

Maxima [A]
time = 0.46, size = 110, normalized size = 1.20

$$-\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)^
2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b)*
sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(83) = 166$.

time = 3.65, size = 455, normalized size = 4.95

$$\left[\frac{8\,(a^2+ab)fx\cos\left(fx+e\right)^2 - 4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 8\,(ab+b^2)fx + \left((3\,a^2+2\,ab)\cos\left(fx+e\right)^2 + 3\,ab + 2\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos(fx+e)^2 + 4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3 - (ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e) + b^2}{a^2\cos(fx+e)^4 + 2\,ab\cos(fx+e)^2 + b^2}\right)}{8\left((a^4+a^3b)f\cos\left(fx+e\right)^2 + (a^3b+a^2b^2)f\right)}, \frac{4\,(a^2+ab)fx\cos\left(fx+e\right)^2 - 2\,ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 4\,(ab+b^2)fx + \left((3\,a^2+2\,ab)\cos\left(fx+e\right)^2 + 3\,ab + 2\,b^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{((a+2\,b)\cos(fx+e)^2 - b)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)}{4\left((a^4+a^3b)f\cos\left(fx+e\right)^2 + (a^3b+a^2b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A]

time = 0.42, size = 114, normalized size = 1.24

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B]
time = 6.39, size = 2056, normalized size = 22.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^
2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8
*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b^5
+ 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b
^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b
^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2
)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b +
a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^
4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3
 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) + (tan(e + f
*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2
+ (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2))
 + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^
2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5
 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*a*b^3)/2 + b^4)
/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*x)*(20*a*b^4 + 8
*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(a + b)^3)^(1/2)*(
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (tan(e + f*
x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 +
 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3
*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)
^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((ta
n(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
 ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5
+ a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 8
0*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b
 + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b
^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((tan(e
 + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((
-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a
^3*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a
^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b +
a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a
^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3
*a^3*b^2)) + (((tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b +
 a^4 + a^2*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^
```

```
2)/(2*a^4*b + a^5 + a^3*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*
b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 +
 a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b
+ a^5 + a^2*b^3 + 3*a^3*b^2)))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(2*f
*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b)*(
a + b + b*tan(e + f*x)^2))
```

3.360 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{x}{a^2}+\frac{b^{3/2}(5a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f}-\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f}-\frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

```
[Out] -x/a^2+1/2*b^(3/2)*(5*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(5/2)/f-1/2*(2*a-b)*cot(f*x+e)/a/(a+b)^2/f-1/2*b*cot(f*x+e)/a/(a+b)/f/(a
+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 483, 597, 536, 209, 211}

$$\frac{b^{3/2}(5a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}}-\frac{x}{a^2}-\frac{(2a-b)\cot(e+fx)}{2af(a+b)^2}-\frac{b\cot(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(3/2)*(5*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(5/2)*f) - ((2*a - b)*Cot[e + f*x])/(2*a*(a + b)^2*f) - (b*
Cot[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e\right.}{2a(a+b)f} \\
&= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{2a^2+}{(1}\right.}{} \\
&= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{} \\
&= -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{}{2a(a+b}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.27, size = 288, normalized size = 2.38

$$
\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2} - \frac{b^2(5a+2b)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))(\cos(2e)-i\sin(2e))}{a^2(a+b)^{5/2}f\sqrt{b(\cos(e)-i\sin(e))^4}} + \frac{2(a+2b+a\cos(2(e+fx)))\csc(e)\csc(e+fx)\sin(fx)}{(a+b)^2f} + \frac{b^2(-((a+2b)\sin(2e))+a\sin(2fx))}{a^2(a+b)^2f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{8\left(a+b\sec^2(e+fx)\right)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(a^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[
e]))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A]
time = 0.17, size = 102, normalized size = 0.84

method	result

derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2a^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(-2a^3\mathrm{e}^{4i(fx+e)}+a\,b^2\mathrm{e}^{4i(fx+e)}+2b^3\mathrm{e}^{4i(fx+e)}-4a^3\mathrm{e}^{2i(fx+e)}-8a^2b\,\mathrm{e}^{2i(fx+e)}-2b^3\mathrm{e}^{2i(fx+e)}-2a^3-a\,b^2\right)}{a^2(a+b)^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)\left(\mathrm{e}^{2i(fx+e)}-1\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*arctan(tan(f*x+e))-1/(a+b)^2/tan(f*x+e)+b^2/(a+b)^2/a^2*(1/2*a*
tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(
f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.46, size = 168, normalized size = 1.39

$$\frac{\frac{(5\,ab^2+2\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)b}}-\frac{(2\,ab-b^2)\tan(fx+e)^2+2\,a^2+2\,ab}{(a^3b+2\,a^2b^2+ab^3)\tan(fx+e)^3+(a^4+3\,a^3b+3\,a^2b^2+ab^3)\tan(fx+e)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt((a + b)*b)) - ((2*a*b - b^2)*tan(f*x + e)^2 + 2*a^2 + 2*
a*b)/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(f*x + e)^3 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*tan(f*x + e)) - 2*(f*x + e)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(111) = 222.
time = 4.84, size = 630, normalized size = 5.21

$$\left[-\frac{4\,(2\,a^3+ab^2)\cos(fx+e)^3-(5\,ab^2+2\,b^3+(5\,a^2b+2\,ab^2)\cos(fx+e)^2)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)\sin(fx+e)+4\,(2\,a^2b-ab^2)\cos(fx+e)+8\,((a^3+2\,a^2b+ab^2)fx\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)fx)\sin(fx+e)}{8\,((a^5+2\,a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2\,a^3b^2+a^2b^3)f)\sin(fx+e)},\ -\frac{2\,(2\,a^3+ab^2)\cos(fx+e)^3+(5\,ab^2+2\,b^3+(5\,a^2b+2\,ab^2)\cos(fx+e)^2)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{((a+2\,b)\cos(fx+e)^2-b)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+2\,(2\,a^2b-ab^2)\cos(fx+e)+4\,((a^3+2\,a^2b+ab^2)fx\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)fx)\sin(fx+e)}{4\,((a^5+2\,a^4b+a^3b^2)f\cos(fx+e)^2+(a^4b+2\,a^3b^2+a^2b^3)f)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a
*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
 + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a^2*b
- a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^
2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a^3
+ a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*cos(f*x +
e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b - a*b^2)*cos(
f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.52, size = 175, normalized size = 1.45

$$\frac{\frac{(5ab^2+2b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+2a^3b+a^2b^2)\sqrt{ab+b^2}}-\frac{2ab\tan(fx+e)^2-b^2\tan(fx+e)^2+2a^2+2ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^3+2a^2b+ab^2)}-\frac{2(fx+e)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) - (2
*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x + e)^
3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x + e)
/a^2)/f
```

Mupad [B]
time = 9.30, size = 2500, normalized size = 20.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
[Out] (atan((((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160
*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6
+ 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*
a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 2956
8*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b
^4 + 1408*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5
*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10
+ 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 5
3760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^
7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)
*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*
a^5*b^2)) + ((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 +
16160*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10
*b^6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2
)*(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 +
 29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a
^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/
2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*
b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^
6 + 53760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*
a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b
 + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^
(1/2)*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3
+ 10*a^5*b^2)))/(80*a^5*b^9 - 208*a^3*b^11 - 416*a^4*b^10 - 32*a^2*b^12 + 1
600*a^6*b^8 + 2768*a^7*b^7 + 2272*a^8*b^6 + 944*a^9*b^5 + 160*a^10*b^4 + ((
tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^10
 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*a^
11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*
(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*b^
8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 1408
*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)
*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 138240*
a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^15
*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7 +
a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2*b
^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a + 2
```

b))/(4(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) -
((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 16160*a^6*b^
10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^6 + 3280*
a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*(a + b)^5)^(1/2)*(5*a + 2*b
)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 + 29568*a^10*
b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a^14*b^4 + 14
08*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*
b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^10*b^10 + 13824
0*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14*b^6 + 53760*a^
15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4*(5*a^6*b + a^7
+ a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*a^6*b + a^7 + a^2
*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a + b)^5)^(1/2)*(5*a +
2*b))/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))
)*(-b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*1i)/(2*f*(5*a^6*b + a^7 + a^2*b^5 + 5*
a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)) - atan((240*a^3*b^11*tan(e + f*x))/(240
*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 +
18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2)
+ (2080*a^4*b^10*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^
9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^
10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (7760*a^5*b^9*tan(e + f*x))/(240*a^3
*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 1840
0*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (
16384*a^6*b^8*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 +
16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b
^4 + 704*a^11*b^3 + 64*a^12*b^2) + (21584*a^7*b^7*tan(e + f*x))/(240*a^3*b^
11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 16384*a^6*b^8 + 21584*a^7*b^7 + 18400*a
^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4 + 704*a^11*b^3 + 64*a^12*b^2) + (184
00*a^8*b^6*tan(e + f*x))/(240*a^3*b^11 + 2080*a^4*b^10 + 7760*a^5*b^9 + 163
84*a^6*b^8 + 21584*a^7*b^7 + 18400*a^8*b^6 + 10160*a^9*b^5 + 3520*a^10*b^4
+ 704*a^11*b^3 + 64*a^12*b^2) + (10160*a^9*b^5*...

3.361 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{x}{a^2}-\frac{b^{5/2}(7a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f}+\frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f}-\frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f}-\frac{}{2a(a+}$$

```
[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+b
)^(7/2)/f+1/2*(2*a^2+6*a*b-b^2)*cot(f*x+e)/a/(a+b)^3/f-1/6*(2*a-3*b)*cot(f*
x+e)^3/a/(a+b)^2/f-1/2*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 483, 597, 536, 209, 211}

$$-\frac{b^{5/2}(7a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}}+\frac{(2a^2+6ab-b^2)\cot(e+fx)}{2af(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2}-\frac{b\cot^3(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(7/2)*f) + ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(2*a*(a + b)^3
*f) - ((2*a - 3*b)*Cot[e + f*x]^3)/(6*a*(a + b)^2*f) - (b*Cot[e + f*x]^3)/(
2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-3b-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(\right.}{2a(a+b)f}$$
$$= -\frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \right.}{}$$
$$= \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e}{2a(a+b)f\,(a+b+}$$
$$= \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e}{2a(a+b)f\,(a+b+}$$
$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.07, size = 1896, normalized size = 11.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((7*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*((b^3*ArcTan[S
ec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*
Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2
*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(8*a^2*Sqrt[a + b]*f*Sqrt[b*Cos[
4*e] - I*b*Sin[4*e]]) - ((I/8)*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]
*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]
*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3
*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e +
 f*x]^3*Sec[2*e]*Sec[e + f*x]^4*(-6*a^4*f*x*Cos[f*x] - 54*a^3*b*f*x*Cos[f*x
] - 126*a^2*b^2*f*x*Cos[f*x] - 114*a*b^3*f*x*Cos[f*x] - 36*b^4*f*x*Cos[f*x]
 + 3*a^4*f*x*Cos[3*f*x] - 3*a^3*b*f*x*Cos[3*f*x] - 27*a^2*b^2*f*x*Cos[3*f*x
] - 33*a*b^3*f*x*Cos[3*f*x] - 12*b^4*f*x*Cos[3*f*x] + 6*a^4*f*x*Cos[2*e - f
```

```
*x] + 54*a^3*b*f*x*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e - f*x] + 114*a*
b^3*f*x*Cos[2*e - f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f*x*Cos[2*e + f*
x] + 54*a^3*b*f*x*Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e + f*x] + 114*a*b
^3*f*x*Cos[2*e + f*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*f*x*Cos[4*e + f*x
] - 54*a^3*b*f*x*Cos[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e + f*x] - 114*a*b^
3*f*x*Cos[4*e + f*x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4*f*x*Cos[2*e + 3*f*
x] + 3*a^3*b*f*x*Cos[2*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[2*e + 3*f*x] + 33*a*
b^3*f*x*Cos[2*e + 3*f*x] + 12*b^4*f*x*Cos[2*e + 3*f*x] + 3*a^4*f*x*Cos[4*e
+ 3*f*x] - 3*a^3*b*f*x*Cos[4*e + 3*f*x] - 27*a^2*b^2*f*x*Cos[4*e + 3*f*x] -
 33*a*b^3*f*x*Cos[4*e + 3*f*x] - 12*b^4*f*x*Cos[4*e + 3*f*x] - 3*a^4*f*x*Co
s[6*e + 3*f*x] + 3*a^3*b*f*x*Cos[6*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[6*e + 3*
f*x] + 33*a*b^3*f*x*Cos[6*e + 3*f*x] + 12*b^4*f*x*Cos[6*e + 3*f*x] - 3*a^4*
f*x*Cos[2*e + 5*f*x] - 9*a^3*b*f*x*Cos[2*e + 5*f*x] - 9*a^2*b^2*f*x*Cos[2*e
 + 5*f*x] - 3*a*b^3*f*x*Cos[2*e + 5*f*x] + 3*a^4*f*x*Cos[4*e + 5*f*x] + 9*a
^3*b*f*x*Cos[4*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[4*e + 5*f*x] + 3*a*b^3*f*x*Co
s[4*e + 5*f*x] - 3*a^4*f*x*Cos[6*e + 5*f*x] - 9*a^3*b*f*x*Cos[6*e + 5*f*x]
- 9*a^2*b^2*f*x*Cos[6*e + 5*f*x] - 3*a*b^3*f*x*Cos[6*e + 5*f*x] + 3*a^4*f*x
*Cos[8*e + 5*f*x] + 9*a^3*b*f*x*Cos[8*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[8*e +
5*f*x] + 3*a*b^3*f*x*Cos[8*e + 5*f*x] - 12*a^4*Sin[f*x] - 60*a^3*b*Sin[f*x]
 - 96*a^2*b^2*Sin[f*x] + 18*b^4*Sin[f*x] + 4*a^4*Sin[3*f*x] + 36*a^3*b*Sin[
3*f*x] + 80*a^2*b^2*Sin[3*f*x] - 6*a*b^3*Sin[3*f*x] + 6*b^4*Sin[3*f*x] + 4*
a^4*Sin[2*e - f*x] + 76*a^3*b*Sin[2*e - f*x] + 144*a^2*b^2*Sin[2*e - f*x] +
 18*b^4*Sin[2*e - f*x] - 4*a^4*Sin[2*e + f*x] - 76*a^3*b*Sin[2*e + f*x] - 1
44*a^2*b^2*Sin[2*e + f*x] + 6*a*b^3*Sin[2*e + f*x] + 18*b^4*Sin[2*e + f*x]
- 12*a^4*Sin[4*e + f*x] - 60*a^3*b*Sin[4*e + f*x] - 96*a^2*b^2*Sin[4*e + f*
x] - 6*a*b^3*Sin[4*e + f*x] - 18*b^4*Sin[4*e + f*x] - 12*a^4*Sin[2*e + 3*f*
x] - 24*a^3*b*Sin[2*e + 3*f*x] + 6*a*b^3*Sin[2*e + 3*f*x] - 6*b^4*Sin[2*e +
 3*f*x] + 4*a^4*Sin[4*e + 3*f*x] + 36*a^3*b*Sin[4*e + 3*f*x] + 80*a^2*b^2*S
in[4*e + 3*f*x] - 3*a*b^3*Sin[4*e + 3*f*x] - 6*b^4*Sin[4*e + 3*f*x] - 12*a^
4*Sin[6*e + 3*f*x] - 24*a^3*b*Sin[6*e + 3*f*x] + 3*a*b^3*Sin[6*e + 3*f*x] +
 6*b^4*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] + 20*a^3*b*Sin[2*e + 5*f*x
] + 3*a*b^3*Sin[2*e + 5*f*x] - 3*a*b^3*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5
*f*x] + 20*a^3*b*Sin[6*e + 5*f*x]))/(384*a^2*(a + b)^3*f*(a + b*Sec[e + f*x
]^2)^2)
```

Maple [A]
time = 0.21, size = 124, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^3}}{f}$

default	$\dfrac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)\,b}}\right.}{2\sqrt{(a+b)\,b}}\right.}{a^2(a+b)^3}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(-12a^4\mathrm{e}^{8i(fx+e)}-24a^3b\,\mathrm{e}^{8i(fx+e)}+3a\,b^3\mathrm{e}^{8i(fx+e)}+6b^4\mathrm{e}^{8i(fx+e)}-12a^4\mathrm{e}^{6i(fx+e)}-60a^3b\,\mathrm{e}^{6i(fx+e)}-96a^2b^2\mathrm{e}^{6i(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^2*arctan(tan(f*x+e))-1/3/(a+b)^2/tan(f*x+e)^3-(-a-3*b)/(a+b)^3/tan
(f*x+e)-b^3/a^2/(a+b)^3*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+2*b
)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.47, size = 241, normalized size = 1.51

$$-\frac{\frac{3\left(7ab^3+2b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\sqrt{(a+b)b}}-\frac{3\left(2a^2b+6ab^2-b^3\right)\tan(fx+e)^4-2a^3-4a^2b-2ab^2+2\left(3a^3+11a^2b+8ab^2\right)\tan(fx+e)^2}{\left(a^4b+3a^3b^2+3a^2b^3+ab^4\right)\tan(fx+e)^5+\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)\tan(fx+e)^3}-\frac{6(fx+e)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(7*a*b^3 + 2*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)) - (3*(2*a^2*b + 6*a*b^2 - b^3
)*tan(f*x + e)^4 - 2*a^3 - 4*a^2*b - 2*a*b^2 + 2*(3*a^3 + 11*a^2*b + 8*a*b^
2)*tan(f*x + e)^2)/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(f*x + e)^5
+ (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(f*x + e)^3) - 6*(f*x
+ e)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(149) = 298$.
time = 3.57, size = 1013, normalized size = 6.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b -
 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x +
```

```
e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(
f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2*(8*a^4 + 20
*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2*b^2 + 3*a*b^
3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 7*a*b^3 - 2*b
^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x +
 e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f*x + e) + 12*((a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*
b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f*x)*si
n(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3
*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.57, size = 212, normalized size = 1.32

$$-\frac{\frac{3\,b^3\tan(fx+e)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\left(b\tan(fx+e)^2+a+b\right)}+\frac{3\,(7\,ab^3+2\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab+b^2}}-\frac{6\,(fx+e)}{a^2}-\frac{2\left(3\,a\tan(fx+e)^2+9\,b\tan(fx+e)^2-a-b\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*tan(f
*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B]
time = 10.46, size = 2500, normalized size = 15.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(3*a + 8*b))/(3*(a + b)^2) - 1/(3*(a + b)) + (tan(e + f*x)
^4*(6*a*b^2 + 2*a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) +
 b*tan(e + f*x)^5)) + atan((560*a^3*b^16*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (7280*a^4*b^15*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (4
2560*a^5*b^14*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14
+ 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6
99840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3584
0*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (149184*a^6*b^1
3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6
*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b
^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 +
 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (351904*a^7*b^12*tan(e + f*
x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3519
04*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*
a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b
^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (593440*a^8*b^11*tan(e + f*x))/(560*a^3
*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12
+ 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 2
78768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^
16*b^3 + 64*a^17*b^2) + (741120*a^9*b^10*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (699840*a^10*b^9*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 +
42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 7411
20*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*
a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) +
(505008*a^11*b^8*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^
14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10
+ 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 3
5840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (278768*a^12
```

```
*b^7*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*
a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^1
0*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^
5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (116480*a^13*b^6*tan(e +
 f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 3
51904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 5050
08*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^1
5*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (35840*a^14*b^5*tan(e + f*x))/(560*a
^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^1
2 + 593440*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 +
 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*
a^16*b^3 + 64*a^17*b^2) + (7680*a^15*b^4*tan(e + f*x))/(560*a^3*b^16 + 7280
*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8
*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b
^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*
a^17*b^2) + (1024*a^16*b^3*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42
560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120
*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (6
4*a^17*b^2*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 1
49184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 6998
40*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 + 35840*a
^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2))/(a^2*f) - (atan((((
tan(e + f*x)*(128*a^3*b^18 + 1984*a^4*b^17 + 13840*a^5*b^16 + 57680*a^6*b^1
5 + 161280*a^7*b^14 + 322560*a^8*b^13 + 480928*a^9*b^12 + 550560*a^10*b^11
+ 494400*a^11*b^10 + 352640*a^12*b^9 + 199696*a^13*b^8 + 88144*a^14*b^7 + 2
9120*a^15*b^6 + 6720*a^16*b^5 + 960*a^17*b^4 + 64*a^18*b^3) - ((-b^5*(a + b
)^7)^(1/2)*(7*a + 2*b)*(64*a^6*b^17 + 1536*a^7*b^16 + 13952*a^8*b^15 + 7104
0*a^9*b^14 + 235968*a^10*b^13 + 551936*a^11*b^12 + 948992*a^12*b^11 + 12291
84*a^13*b^10 + 1214400*a^14*b^9 + 918016*a^15*b...
```

3.362 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=207

$$-\frac{x}{a^2}+\frac{b^{7/2}(9a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f}-\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f}+\frac{(2a^2+6ab-3b^2)\cot}{6a(a+b)^3f}$$

```
[Out] -x/a^2+1/2*b^(7/2)*(9*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(9/2)/f-1/2*(2*a^3+8*a^2*b+12*a*b^2-b^3)*cot(f*x+e)/a/(a+b)^4/f+1/6*(2*a
^2+6*a*b-3*b^2)*cot(f*x+e)^3/a/(a+b)^3/f-1/10*(2*a-5*b)*cot(f*x+e)^5/a/(a+b
)^2/f-1/2*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.30, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 483, 597, 536, 209, 211}

$$\frac{b^{7/2}(9a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}}+\frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6af(a+b)^3}-\frac{x}{a^2}-\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2af(a+b)^4}-\frac{(2a-5b)\cot^5(e+fx)}{10af(a+b)^2}-\frac{b\cot^5(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(9/2)*f) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x]
)/(2*a*(a + b)^4*f) + ((2*a^2 + 6*a*b - 3*b^2)*Cot[e + f*x]^3)/(6*a*(a + b)
^3*f) - ((2*a - 5*b)*Cot[e + f*x]^5)/(10*a*(a + b)^2*f) - (b*Cot[e + f*x]^5
)/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-5b-7bx^2}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(\right.}{2a(a+b)f}$$

$$= -\frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))} - \text{Subst}\left(\int 5\right.$$

$$= \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(}{2a(a+b)f\,(a+b}$$

$$= -\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} -$$

$$= -\frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} -$$

$$= -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(}{2a(a+b)^4f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.49, size = 3028, normalized size = 14.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-1/8*(b^4*Arc
Tan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((
I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]
) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*C
os[4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a +
 b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b
*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x
])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b
)^4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[
e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4*b*f*x*Cos
```

[f*x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] + 2475*a*b^4*
f*x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 240*a^4*b*f*x
*Cos[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*Cos[3*f*x] + 1
185*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*x*Cos[2*e - f*
x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e - f*x] - 3900*
a^2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x] - 600*b^5*f*x*Co
s[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*Cos[2*e + f*x] - 2
850*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e + f*x] - 2475*a*b
^4*f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*a^5*f*x*Cos[4*e + f
*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*x*Cos[4*e + f*x] + 3900
*a^2*b^3*f*x*Cos[4*e + f*x] + 2475*a*b^4*f*x*Cos[4*e + f*x] + 600*b^5*f*x*C
os[4*e + f*x] + 15*a^5*f*x*Cos[2*e + 3*f*x] - 240*a^4*b*f*x*Cos[2*e + 3*f*x
] - 1110*a^3*b^2*f*x*Cos[2*e + 3*f*x] - 1740*a^2*b^3*f*x*Cos[2*e + 3*f*x] -
 1185*a*b^4*f*x*Cos[2*e + 3*f*x] - 300*b^5*f*x*Cos[2*e + 3*f*x] - 15*a^5*f*
x*Cos[4*e + 3*f*x] + 240*a^4*b*f*x*Cos[4*e + 3*f*x] + 1110*a^3*b^2*f*x*Cos[
4*e + 3*f*x] + 1740*a^2*b^3*f*x*Cos[4*e + 3*f*x] + 1185*a*b^4*f*x*Cos[4*e +
 3*f*x] + 300*b^5*f*x*Cos[4*e + 3*f*x] + 15*a^5*f*x*Cos[6*e + 3*f*x] - 240*
a^4*b*f*x*Cos[6*e + 3*f*x] - 1110*a^3*b^2*f*x*Cos[6*e + 3*f*x] - 1740*a^2*b
^3*f*x*Cos[6*e + 3*f*x] - 1185*a*b^4*f*x*Cos[6*e + 3*f*x] - 300*b^5*f*x*Cos
[6*e + 3*f*x] + 45*a^5*f*x*Cos[2*e + 5*f*x] + 120*a^4*b*f*x*Cos[2*e + 5*f*x
] + 30*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[2*e + 5*f*x] - 19
5*a*b^4*f*x*Cos[2*e + 5*f*x] - 60*b^5*f*x*Cos[2*e + 5*f*x] - 45*a^5*f*x*Cos
[4*e + 5*f*x] - 120*a^4*b*f*x*Cos[4*e + 5*f*x] - 30*a^3*b^2*f*x*Cos[4*e + 5
*f*x] + 180*a^2*b^3*f*x*Cos[4*e + 5*f*x] + 195*a*b^4*f*x*Cos[4*e + 5*f*x] +
 60*b^5*f*x*Cos[4*e + 5*f*x] + 45*a^5*f*x*Cos[6*e + 5*f*x] + 120*a^4*b*f*x*
Cos[6*e + 5*f*x] + 30*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[6*
e + 5*f*x] - 195*a*b^4*f*x*Cos[6*e + 5*f*x] - 60*b^5*f*x*Cos[6*e + 5*f*x] -
 45*a^5*f*x*Cos[8*e + 5*f*x] - 120*a^4*b*f*x*Cos[8*e + 5*f*x] - 30*a^3*b^2*
f*x*Cos[8*e + 5*f*x] + 180*a^2*b^3*f*x*Cos[8*e + 5*f*x] + 195*a*b^4*f*x*Cos
[8*e + 5*f*x] + 60*b^5*f*x*Cos[8*e + 5*f*x] - 15*a^5*f*x*Cos[4*e + 7*f*x] -
 60*a^4*b*f*x*Cos[4*e + 7*f*x] - 90*a^3*b^2*f*x*Cos[4*e + 7*f*x] - 60*a^2*b
^3*f*x*Cos[4*e + 7*f*x] - 15*a*b^4*f*x*Cos[4*e + 7*f*x] + 15*a^5*f*x*Cos[6*
e + 7*f*x] + 60*a^4*b*f*x*Cos[6*e + 7*f*x] + 90*a^3*b^2*f*x*Cos[6*e + 7*f*x
] + 60*a^2*b^3*f*x*Cos[6*e + 7*f*x] + 15*a*b^4*f*x*Cos[6*e + 7*f*x] - 15*a^
5*f*x*Cos[8*e + 7*f*x] - 60*a^4*b*f*x*Cos[8*e + 7*f*x] - 90*a^3*b^2*f*x*Cos
[8*e + 7*f*x] - 60*a^2*b^3*f*x*Cos[8*e + 7*f*x] - 15*a*b^4*f*x*Cos[8*e + 7*
f*x] + 15*a^5*f*x*Cos[10*e + 7*f*x] + 60*a^4*b*f*x*Cos[10*e + 7*f*x] + 90*a
^3*b^2*f*x*Cos[10*e + 7*f*x] + 60*a^2*b^3*f*x*Cos[10*e + 7*f*x] + 15*a*b^4*
f*x*Cos[10*e + 7*f*x] - 10*a^5*Sin[f*x] + 860*a^4*b*Sin[f*x] + 3120*a^3*b^2
*Sin[f*x] + 3600*a^2*b^3*Sin[f*x] - 300*b^5*Sin[f*x] + 46*a^5*Sin[3*f*x] -
508*a^4*b*Sin[3*f*x] - 2324*a^3*b^2*Sin[3*f*x] - 3120*a^2*b^3*Sin[3*f*x] +
75*a*b^4*Sin[3*f*x] - 150*b^5*Sin[3*f*x] - 240*a^5*Sin[2*e - f*x] - 1840*a^
4*b*Sin[2*e - f*x] - 4840*a^3*b^2*Sin[2*e - f*x] - 5040*a^2*b^3*Sin[2*e - f
*x] - 300*b^5*Sin[2*e - f*x] + 240*a^5*Sin[2*e + f*x] + 1840*a^4*b*Sin[2*e
+ f*x] + 4840*a^3*b^2*Sin[2*e + f*x] + 5040*a^2*b^3*Sin[2*e + f*x] - 75*a*b

```
^4*Sin[2*e + f*x] - 300*b^5*Sin[2*e + f*x] - 10*a^5*Sin[4*e + f*x] + 860*a^
4*b*Sin[4*e + f*x] + 3120*a^3*b^2*Sin[4*e + f*x] + 3600*a^2*b^3*Sin[4*e + f
*x] + 75*a*b^4*Sin[4*e + f*x] + 300*b^5*Sin[4*e + f*x] - 240*a^4*b*Sin[2*e
+ 3*f*x] - 900*a^3*b^2*Sin[2*e + 3*f*x] - 1200*a^2*b^3*Sin[2*e + 3*f*x] - 7
5*a*b^4*Sin[2*e + 3*f*x] + 150*b^5*Sin[2*e + 3*f*x] + 46*a^5*Sin[4*e + 3*f*
x] - 508*a^4*b*Sin[4*e + 3*f*x] - 2324*a^3*b^2*Sin[4*e + 3*f*x] - 3120*a^2*
b^3*Sin[4*e + 3*f*x] + 60*a*b^4*Sin[4*e + 3*f*x] + 150*b^5*Sin[4*e + 3*f*x]
 - 240*a^4*b*Sin[6*e + 3*f*x] - 900*a^3*b^2*Sin[6*e + 3*f*x] - 1200*a^2*b^3
*Sin[6*e + 3*f*x] - 60*a*b^4*Sin[6*e + 3*f*x] - 150*b^5*Sin[6*e + 3*f*x] -
48*a^5*Sin[2*e + 5*f*x] - 32*a^4*b*Sin[2*e + 5*f*x] + 340*a^3*b^2*Sin[2*e +
 5*f*x] + 864*a^2*b^3*Sin[2*e + 5*f*x] - 60*a*b^4*Sin[2*e + 5*f*x] + 30*b^5
*Sin[2*e + 5*f*x] - 90*a^5*Sin[4*e + 5*f*x] - 3...
```

Maple [A]

time = 0.25, size = 152, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-\frac{a^2+4ab+6b^2}{(a+b)^4\tan(fx+e)}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(9a+2b)\text{ ar}}{2}\right.}{(a+b)^4a^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-\frac{a^2+4ab+6b^2}{(a+b)^4\tan(fx+e)}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\left(\tan^2(fx+e)\right)}+\frac{(9a+2b)\text{ ar}}{2}\right.}{(a+b)^4a^2}}{f}$
risch	$-\frac{x}{a^2}-\frac{i\left(-60a\,b^4\mathrm{e}^{2i(fx+e)}+300a^4b\,\mathrm{e}^{12i(fx+e)}+300a^3b^2\mathrm{e}^{12i(fx+e)}-15a\,b^4\mathrm{e}^{12i(fx+e)}+240a^4b\,\mathrm{e}^{10i(fx+e)}+1840a^4b\,\mathrm{e}^{6}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*arctan(tan(f*x+e))-1/5/(a+b)^2/tan(f*x+e)^5-1/3*(-a-3*b)/(a+b)^
3/tan(f*x+e)^3-(a^2+4*a*b+6*b^2)/(a+b)^4/tan(f*x+e)+b^4/(a+b)^4/a^2*(1/2*a*
tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(
f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]

time = 0.48, size = 326, normalized size = 1.57

$$\frac{\frac{15\,(9\,ab^4+2\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{(a+b)b}}-\frac{15\,(2\,a^3b+8\,a^2b^2+12\,ab^3-b^4)\tan(fx+e)^6+10\,(3\,a^4+14\,a^3b+26\,a^2b^2+15\,ab^3)\tan(fx+e)^4+6\,a^4+18\,a^3b+18\,a^2b^2+6\,ab^3-2\,(5\,a^4+22\,a^3b+29\,a^2b^2+12\,ab^3)\tan(fx+e)^2}{(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5)\tan(fx+e)^7+(a^6+5\,a^5b+10\,a^4b^2+10\,a^3b^3+5\,a^2b^4+ab^5)\tan(fx+e)^5}-\frac{30\,(fx+e)}{a^2}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(9*a*b^4 + 2*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 + 4
*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt((a + b)*b)) - (15*(2*a^3*b +
 8*a^2*b^2 + 12*a*b^3 - b^4)*tan(f*x + e)^6 + 10*(3*a^4 + 14*a^3*b + 26*a^2
*b^2 + 15*a*b^3)*tan(f*x + e)^4 + 6*a^4 + 18*a^3*b + 18*a^2*b^2 + 6*a*b^3 -
 2*(5*a^4 + 22*a^3*b + 29*a^2*b^2 + 12*a*b^3)*tan(f*x + e)^2)/((a^5*b + 4*a
^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*tan(f*x + e)^7 + (a^6 + 5*a^5*b + 1
0*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*tan(f*x + e)^5) - 30*(f*x + e)/
a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(195) = 390$.
time = 3.40, size = 1547, normalized size = 7.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 4
*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5
 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3
 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3
 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x
 + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*
b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8*a^3*b^2
+ 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*
b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3
+ 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^
3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 -
2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*
b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*
b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46*a^5 + 172*a^4*b +
 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 234*a^4*b + 218*a^3*b
^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*a^5 + 10*a^4*b - 20*a^3
*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3 + 15*((9*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)
^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
```

```
+ e)))*sin(f*x + e) + 30*(2*a^4*b + 8*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x
 + e) + 60*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e
)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos
(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x
+ e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
- (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x
 + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f
*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*
sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 298, normalized size = 1.44

$$\frac{\frac{15\,b^{4}\tan\left(fx+e\right)}{\left(a^{5}+4\,a^{4}b+6\,a^{3}b^{2}+4\,a^{2}b^{3}+ab^{4}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)}+\frac{15\,\left(9\,ab^{4}+2\,b^{5}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}\right)\sqrt{ab+b^{2}}}-\frac{30\,\left(fx+e\right)}{a^{2}}-\frac{2\,\left(15\,a^{2}\tan\left(fx+e\right)^{4}+60\,ab\tan\left(fx+e\right)^{4}+90\,b^{2}\tan\left(fx+e\right)^{4}-5\,a^{2}\tan\left(fx+e\right)^{2}-20\,ab\tan\left(fx+e\right)^{2}-15\,b^{2}\tan\left(fx+e\right)^{2}+3\,a^{2}+6\,ab+3\,b^{2}\right)}{\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\tan\left(fx+e\right)^{5}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi +
 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*(15*
a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5*a^2*
tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^2 + 6*
a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^5))/
f
```

Mupad [**B**]
time = 10.86, size = 2500, normalized size = 12.08

$$\text{Too large to display}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

[Out] atan(((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 9
97248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^1
5 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112
*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 25
95328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*
a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 10
2400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 +
23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^
16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 +
71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*
b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 +
256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3*b^23 + 2624
*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^
8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 1207507
2*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 +
4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 +
72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2) -
(((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*
a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28
745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*
b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*
a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b
^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a
^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 238141
44*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^1
4 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938
560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 +
1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a
^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b
^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18
+ 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12
*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 49774
08*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960
*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))/(2*a^2))/(2752*a
^4*b^20 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^1
9 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^1
3*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 348
74112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8
+ 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 +
2688*a^25*b^3 + 128*a^26*b^2 - (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22
+ 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^
18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182438
40*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b
^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*
a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*

b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) - tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 310080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23*b^3))*1i)/(2*a^2) - 32*a^2*b^22 - 144*a^3*b^21 - ((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8 + 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 + 2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^22 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 118243840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 6449664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632*a^27*b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154592*a^15*b^11 + 4977408*a^16*...

3.363 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=78

$$\frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}-\frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

```
[Out] 1/4*(a+b)^2/a^3/f/(b+a*cos(f*x+e)^2)^2+(-a-b)/a^3/f/(b+a*cos(f*x+e)^2)-1/2*
ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 455, 45}

$$\frac{(a+b)^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{a+b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (a + b)^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - (a + b)/(a^3*f*(b + a*Cos[e
+ f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+\right.}{2a^3f}
\end{aligned}$$

Mathematica [A]
time = 2.37, size = 136, normalized size = 1.74

$$-\frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+fx)))+2a\cos(2(e+fx))(2(a+b)+(a+2b)\log(a+2b+a\cos(2(e+fx))))}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)
]] + a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(
e + f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f
*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.09, size = 80, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{-2a-2b}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{a^2+2ab+b^2}{4a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}-\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^3}}{f}$
default	$\frac{\frac{-2a-2b}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)}+\frac{a^2+2ab+b^2}{4a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}-\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^3}}{f}$
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3f}-\frac{4\left(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}+a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+3b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/2*(-2*a-2*b)/a^3/(b+a*cos(f*x+e)^2)+1/4*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2-1/2/a^3*ln(b+a*cos(f*x+e)^2))

Maxima [A]
time = 0.26, size = 116, normalized size = 1.49

$$\frac{\frac{4\,(a^2+ab)\sin(fx+e)^2-3\,a^2-6\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] 1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2 - a - b)/a^3)/f

Fricas [A]
time = 3.35, size = 122, normalized size = 1.56

$$-\frac{4\,(a^2+ab)\cos\left(fx+e\right)^2-a^2+2\,ab+3\,b^2+2\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)}{4\,\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. 2(66) = 132.
time = 83.21, size = 921, normalized size = 11.81

$$\begin{cases} \frac{\tilde{\infty} x\tan^5(e)}{\sec^6(e)} & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}}{a^3} & \text{for } b=0 \\ \frac{-\frac{\tan^4(e+fx)}{2f\sec^6(e+fx)}-\frac{\tan^2(e+fx)}{2f\sec^6(e+fx)}-\frac{1}{6f\sec^6(e+fx)}}{b^3} & \text{for } a=0 \\ \frac{x\tan^5(e)}{(a+b\sec^2(e))^3} & \text{for } f=0 \\ -\frac{2a^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2a^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2a^2\log\left(\tan^2(e+fx)+1\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{a^2\tan^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2a^2\tan^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{4ab\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{4ab\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{4ab\log\left(\tan^2(e+fx)+1\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2ab\tan^2(e+fx)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2b^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2b^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2b^2\log\left(\tan^2(e+fx)+1\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)

[Out] Piecewise((zoo*x*tan(e)**5/sec(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), ((-tan(e + f*x)**4/(2*f*sec(e + f*x)**6) - tan(e + f*x)**2/(2*f*sec(e + f*x)**6) - 1/(6*f*sec(e + f*x)**6))/b**3, Eq(a, 0)), (x*tan(e)**5/(a + b*sec(e)**2)**3, Eq(f, 0)), (-2*a**2*log(-sqrt(-a/b) + sec(e + f*x))/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*a**2*log(sqrt(-a/b) + sec(e + f*x))/(4*a**5*f + 8*a**4*b*f*sec(e +

```
f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 2*a**2*log(tan(e + f*x)**2 + 1)/
(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + a
**2*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*
sec(e + f*x)**4) - 2*a**2*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*
x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 4*a*b*log(-sqrt(-a/b) + sec(e + f*
x))*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*
sec(e + f*x)**4) - 4*a*b*log(sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**2/(4*
a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 4*a*
b*log(tan(e + f*x)**2 + 1)*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f
*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*a*b*tan(e + f*x)**2*sec(e + f*x
)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4
) - 2*b**2*log(-sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**4/(4*a**5*f + 8*a*
*4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*b**2*log(sqrt(-
a/b) + sec(e + f*x))*sec(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2
 + 4*a**3*b**2*f*sec(e + f*x)**4) + 2*b**2*log(tan(e + f*x)**2 + 1)*sec(e +
 f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x
)**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(74) = 148$.

time = 2.05, size = 531, normalized size = 6.81

$$-\frac{\frac{2\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3}-\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}-\frac{3a^2+6ab+3b^2+\frac{20a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{50a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{28ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{18b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{20a^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{8ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{12b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{6ab(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{3b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2a^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 - 4*log(abs(-(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1))/a^3 - (3*a^2 + 6*a*b + 3*b^2 + 20*a^2*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 12*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 50*a^2*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 - 28*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^2*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 - 12*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^2*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 + 6*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e
) + 1)^4 + 3*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a + b + 2*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2)^2*a^3))/f
```

Mupad [B]

time = 4.60, size = 166, normalized size = 2.13

$$\frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}+\frac{\frac{-a^3+3\,a\,b^2+2\,b^3}{4\,a^2\,b^2}-\frac{\tan(e+f\,x)^2\,(a^2-b^2)}{2\,a^2\,b}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8
*b^3*tan(e + f*x)^2)/a))/(a^3*f) + ((3*a*b^2 - a^3 + 2*b^3)/(4*a^2*b^2) - (
tan(e + f*x)^2*(a^2 - b^2))/(2*a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)
^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.364 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{a+2b}{2a^3f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

```
[Out] -1/4*b*(a+b)/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*(a+2*b)/a^3/f/(b+a*cos(f*x+e)^2
)+1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 78}

$$-\frac{b(a+b)}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}+\frac{a+2b}{2a^3f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*(b*(a + b))/(a^3*f*(b + a*Cos[e + f*x]^2)^2) + (a + 2*b)/(2*a^3*f*(b +
 a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3}+\frac{a+2b}{a^2(b+ax)^2}-\frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{b(a+b)}{4a^3f\,(b+a\cos^2(e+fx))^2}+\frac{a+2b}{2a^3f\,(b+a\cos^2(e+fx))}+\frac{\log\left(b+a\cos^2(e+\right.}{2a^3f}\end{aligned}$$

Mathematica [A]

time = 1.16, size = 131, normalized size = 1.62

$$\frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+fx)))+2a(a+2b)\cos(2(e+fx))(1+\log(a+2b+a\cos(2(e+fx))))}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*Co
s[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

Maple [A]

time = 0.09, size = 73, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{-a-2b}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)}-\frac{(a+b)b}{4a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^3}}{f}$
default	$\frac{-\frac{-a-2b}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)}-\frac{(a+b)b}{4a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{2a^3}}{f}$
risch	$-\frac{ix}{a^3}-\frac{2ie}{a^3f}+\frac{2a^2e^{6i(fx+e)}+4ab\,e^{6i(fx+e)}+4a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+12b^2e^{4i(fx+e)}+2a^2e^{2i(fx+e)}+4ab\,e^{2i(fx+}}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/2*(-a-2*b)/a^3/(b+a*cos(f*x+e)^2)-1/4*(a+b)*b/a^3/(b+a*cos(f*x+e)^2
)^2+1/2/a^3*ln(b+a*cos(f*x+e)^2))

Maxima [A]
time = 0.26, size = 117, normalized size = 1.44

$$-\frac{\frac{2\,(a^2+2\,ab)\,\sin(fx+e)^2-2\,a^2-5\,ab-3\,b^2}{a^5\,\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\,\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")

[Out] -1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*x
 + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log
(a*sin(f*x + e)^2 - a - b)/a^3)/f

Fricas [A]
time = 3.18, size = 117, normalized size = 1.44

$$\frac{2\,(a^2+2\,ab)\cos\left(fx+e\right)^2+ab+3\,b^2+2\,\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)}{4\,\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")

[Out] 1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 933 vs. 2(66) = 132.
time = 83.00, size = 933, normalized size = 11.52

$$\begin{cases} \frac{\tilde{\infty}x\tan^3(e)}{\sec^6(e)} & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{-\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{\tan^2(e+fx)}{2f}}{a^3} & \text{for } b=0 \\ \frac{-\frac{\tan^2(e+fx)}{4f\sec^6(e+fx)}-\frac{1}{12f\sec^6(e+fx)}}{b^3} & \text{for } a=0 \\ \frac{x\tan^3(e)}{(a+b\sec^2(e))^3} & \text{for } f=0 \\ \frac{2a^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2a^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2a^2\log\left(\tan^2(e+fx)+1\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2a^2\tan^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{a^2}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{4ab\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{4ab\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{4ab\log\left(\tan^2(e+fx)+1\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{ab\tan^2(e+fx)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{ab\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2b^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}+\frac{2b^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)}-\frac{2b^2\log\left(\tan^2(e+fx)+1\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)

[Out] Piecewise((zoo*x*tan(e)**3/sec(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-l
og(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), ((-

```
tan(e + f*x)**2/(4*f*sec(e + f*x)**6) - 1/(12*f*sec(e + f*x)**6))/b**3, Eq(
a, 0)), (x*tan(e)**3/(a + b*sec(e)**2)**3, Eq(f, 0)), (2*a**2*log(-sqrt(-a/
b) + sec(e + f*x))/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*s
ec(e + f*x)**4) + 2*a**2*log(sqrt(-a/b) + sec(e + f*x))/(4*a**5*f + 8*a**4*
b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*a**2*log(tan(e + f
*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e +
f*x)**4) + 2*a**2*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 +
4*a**3*b**2*f*sec(e + f*x)**4) - a**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**
2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 4*a*b*log(-sqrt(-a/b) + sec(e + f*x))*
sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(
e + f*x)**4) + 4*a*b*log(sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**2/(4*a**5
*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 4*a*b*lo
g(tan(e + f*x)**2 + 1)*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)*
*2 + 4*a**3*b**2*f*sec(e + f*x)**4) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(
4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - a*
b*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*se
c(e + f*x)**4) + 2*b**2*log(-sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**4/(4*
a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 2*b*
*2*log(sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*se
c(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*b**2*log(tan(e + f*x)**2
 + 1)*sec(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*
f*sec(e + f*x)**4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(75) = 150$.

time = 1.04, size = 656, normalized size = 8.10

$$-\frac{\frac{3a^3+9a^2b+9ab^2+3b^3+\frac{20a^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{34a^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{22a^2b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{10ab^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{18b^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{20a^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{28a^2b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{4ab^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{12b^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{9a^2b(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{9ab^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{3b^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^4+a^3b)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}-\frac{2\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3}+\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*((3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 20*a^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 28*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*a*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*b^3*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 34*a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 22*a^2*b*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 10*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 18*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^3*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 28*a^2*b*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 - 4*a*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12
*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^3*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4 + 9*a^2*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
 9*a*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^3*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4)/((a^4 + a^3*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
```

```
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 + 4*log(abs(-(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B]
time = 4.45, size = 153, normalized size = 1.89

$$-\frac{\frac{a^2+3\,a\,b+2\,b^2}{4\,a^2\,b}+\frac{b\,\tan\left(e+f\,x\right)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^4\right)}-\frac{\mathrm{atanh}\left(\frac{4\,b^2\,\tan\left(e+f\,x\right)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan\left(e+f\,x\right)^2+\frac{8\,b^3\,\tan\left(e+f\,x\right)^2}{a}}\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - ((3*a*b + a^2 + 2*b^2)/(4*a^2*b) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b
+ a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - atanh
((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*
tan(e + f*x)^2)/a))/(a^3*f)
```

3.365 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=74

$$\frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}-\frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

```
[Out] 1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2-b/a^3/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 272, 45}

$$\frac{b^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - b/(a^3*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{b^2}{4a^3f\,(b+a\cos^2(e+fx))^2} - \frac{b}{a^3f\,(b+a\cos^2(e+fx))} - \frac{\log\left(b+a\cos^2(e+\right.}{2a^3f}$$

Mathematica [A]
time = 1.57, size = 129, normalized size = 1.74

$$-\frac{2b(2a+3b)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+fx)))+2a\cos(2(e+fx))(2b+(a+2b)\log(a+2b+a\cos(2(e+fx))))}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^2
*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*x)
]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f*(a + 2*b + a*
Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.05, size = 83, normalized size = 1.12

method	result	si
derivativedivides	$\frac{-\frac{b\left(-\frac{a^2}{2b\left(a+b\left(\sec^2(fx+e)\right)\right)^2}-\frac{a}{b\left(a+b\left(\sec^2(fx+e)\right)\right)}+\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{b}\right)}{2a^3}+\frac{\ln(\sec(fx+e))}{a^3}}{f}$	8
default	$\frac{-\frac{b\left(-\frac{a^2}{2b\left(a+b\left(\sec^2(fx+e)\right)\right)^2}-\frac{a}{b\left(a+b\left(\sec^2(fx+e)\right)\right)}+\frac{\ln\left(a+b\left(\sec^2(fx+e)\right)\right)}{b}\right)}{2a^3}+\frac{\ln(\sec(fx+e))}{a^3}}{f}$	8
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3f}-\frac{4b\left(a\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right)}{a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^3f}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*b/a^3*(-1/2*a^2/b/(a+b*sec(f*x+e)^2)^2-a/b/(a+b*sec(f*x+e)^2)+1/b
*ln(a+b*sec(f*x+e)^2))+1/a^3*ln(sec(f*x+e)))
```

Maxima [A]
time = 0.26, size = 106, normalized size = 1.43

$$\frac{\frac{4\,ab\sin(fx+e)^2-4\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*a
^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2 -
 a - b)/a^3)/f
```

Fricas [A]
time = 3.88, size = 108, normalized size = 1.46

$$-\frac{4\,ab\cos\left(fx+e\right)^{2}+3\,b^{2}+2\,\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)}{4\,\left(a^{5}f\cos\left(fx+e\right)^{4}+2\,a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*
cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(61) = 122.
time = 82.86, size = 782, normalized size = 10.57

$$\begin{cases} \frac{\tilde{\infty} x\tan(e)}{\sec^6(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2a^3f} & \text{for } b=0 \\ -\frac{1}{6b^3f\sec^6(e+fx)} & \text{for } a=0 \\ \frac{x\tan(e)}{(a+b\sec^2(e))^3} & \text{for } f=0 \\ -\frac{2a^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} - \frac{2a^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} + \frac{2a^2\log(\tan^2(e+fx)+1)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} + \frac{3a^2}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} - \frac{4ab\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} - \frac{4ab\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} + \frac{4ab\log(\tan^2(e+fx)+1)\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} + \frac{2ab\sec^2(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} - \frac{2b^2\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} - \frac{2b^2\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} + \frac{2b^2\log(\tan^2(e+fx)+1)\sec^4(e+fx)}{4a^5f+8a^4bf\sec^2(e+fx)+4a^3b^2f\sec^4(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(ta
n(e + f*x)**2 + 1)/(2*a**3*f), Eq(b, 0)), (-1/(6*b**3*f*sec(e + f*x)**6), E
q(a, 0)), (x*tan(e)/(a + b*sec(e)**2)**3, Eq(f, 0)), (-2*a**2*log(-sqrt(-a/
b) + sec(e + f*x))/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*s
ec(e + f*x)**4) - 2*a**2*log(sqrt(-a/b) + sec(e + f*x))/(4*a**5*f + 8*a**4*
```

```
b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 2*a**2*log(tan(e + f
*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e +
f*x)**4) + 3*a**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*se
c(e + f*x)**4) - 4*a*b*log(-sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**2/(4*a
**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 4*a*b
*log(sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(
e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 4*a*b*log(tan(e + f*x)**2 +
1)*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*s
ec(e + f*x)**4) + 2*a*b*sec(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)
**2 + 4*a**3*b**2*f*sec(e + f*x)**4) - 2*b**2*log(-sqrt(-a/b) + sec(e + f*x
))*sec(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*s
ec(e + f*x)**4) - 2*b**2*log(sqrt(-a/b) + sec(e + f*x))*sec(e + f*x)**4/(4*
a**5*f + 8*a**4*b*f*sec(e + f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4) + 2*b*
*2*log(tan(e + f*x)**2 + 1)*sec(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*sec(e +
f*x)**2 + 4*a**3*b**2*f*sec(e + f*x)**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(70) = 140$.

time = 0.71, size = 782, normalized size = 10.57

$$\frac{\frac{3a^4+12a^3b+18a^2b^2+12ab^3+3b^4+\frac{12a^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{40a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{24a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12b^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18a^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{56a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{8ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{18b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12a^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{40a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{24a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{16ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{12b^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^4(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{12a^3b(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{18a^2b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{12ab^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{3b^4(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^5+2a^4b+a^3b^2)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}-\frac{2\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3}+\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*((3*a^4 + 12*a^3*b + 18*a^2*b^2 + 12*a*b^3 + 3*b^4 + 12*a^4*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 40*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 56*a^3*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^4*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 + 40*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^
2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
3*a^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a^3*b*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 12*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^
3 + 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B]
time = 4.40, size = 142, normalized size = 1.92

$$\frac{\frac{3a+2b}{4a^2}+\frac{b\tan(e+fx)^2}{2a^2}}{f\left(2ab+a^2+b^2+\tan\left(e+fx\right)^2\left(2b^2+2ab\right)+b^2\tan\left(e+fx\right)^4\right)}+\frac{\operatorname{atanh}\left(\frac{4b^2\tan(e+fx)^2}{8b^2+\frac{8b^3}{a}+4b^2\tan(e+fx)^2+\frac{8b^3\tan(e+fx)^2}{a}}\right)}{a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((3*a + 2*b)/(4*a^2) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b + a^2 + b^2 +
tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + atanh((4*b^2*tan(e
+ f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x)^2
)/a))/(a^3*f)
```

3.366 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3} dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2}+\frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)}+\frac{b(3a^2+3ab+b^2)\log\left(b+a\cos^2(e\right.}{2a^3(a+b)^3f}$$

```
[Out] -1/4*b^3/a^3/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/f/(
b+a*cos(f*x+e)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^3/
f+ln(sin(f*x+e))/(a+b)^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 90}

$$-\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b(3a^2+3ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^3}+\frac{\log(\sin(e+fx))}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*b^3/(a^3*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^
3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*
Cos[e + f*x]^2])/(2*a^3*(a + b)^3*f) + Log[Sin[e + f*x]]/((a + b)^3*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b(3a^2+3ab+b^2)}{a^2(a+b)^3(b+ax)}\right) dx$$
$$2f$$
$$= -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{b(3$$

Mathematica [A]
time = 1.13, size = 158, normalized size = 1.22

$$\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(4\log(\sin(e+fx)) + \frac{2b(3a^2+3ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3} - \frac{b^3(a+b)^2}{a^3(a+b-a\sin^2(e+fx))^2} + \frac{2b^2(a+b)(3a+2b)}{a^3(a+b-a\sin^2(e+fx))}\right)}{32(a+b)^3f\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2*
b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)^2
)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*(a
+ b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.13, size = 144, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{b\left(\frac{b\left(3a^2+5ab+2b^2\right)}{a^3\left(b+a\left(\cos^2(fx+e)\right)\right)} - \frac{b^2\left(a^2+2ab+b^2\right)}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2} + \frac{\left(3a^2+3ab+b^2\right)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^3}\right)}{2(a+b)^3} + \frac{\ln(\cos(fx+e)-1)}{2(a+b)^3} + \frac{\ln(1+\cos(fx+e}{2(a+b)^3}}{f}$
default	$\frac{\frac{b\left(\frac{b\left(3a^2+5ab+2b^2\right)}{a^3\left(b+a\left(\cos^2(fx+e)\right)\right)} - \frac{b^2\left(a^2+2ab+b^2\right)}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2} + \frac{\left(3a^2+3ab+b^2\right)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^3}\right)}{2(a+b)^3} + \frac{\ln(\cos(fx+e)-1)}{2(a+b)^3} + \frac{\ln(1+\cos(fx+e}{2(a+b)^3}}{f}$

risch	$\frac{ix}{a^3} - \frac{2ix}{a^3+3a^2b+3a\,b^2+b^3} - \frac{2ie}{f(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6ibx}{a(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6ibe}{af(a^3+3a^2b+3a\,b^2+b^3)} - \frac{}{a^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b/(a+b)^3*(1/a^3*b*(3*a^2+5*a*b+2*b^2)/(b+a*cos(f*x+e)^2)-1/2*b^2*
(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+(3*a^2+3*a*b+b^2)/a^3*ln(b+a*cos(f
*x+e)^2))+1/2/(a+b)^3*ln(cos(f*x+e)-1)+1/2/(a+b)^3*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.26, size = 248, normalized size = 1.91

$$\frac{\frac{2\,(3\,a^2b+3\,ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{6\,a^2b^2+9\,ab^3+3\,b^4-2\,(3\,a^2b^2+2\,ab^3)\sin(fx+e)^2}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sin(fx+e)^2}+\frac{2\,\log\left(\sin(fx+e)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 + 2
*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(128) = 256$.
time = 6.50, size = 316, normalized size = 2.43

$$\frac{5\,a^2b^3+8\,ab^4+3\,b^5+2\,(3\,a^3b^2+5\,a^2b^3+2\,ab^4)\cos(fx+e)^2+2\left(3\,a^2b^3+3\,ab^4+b^5+(3\,a^4b+3\,a^3b^2+a^2b^3)\cos(fx+e)^4+2\,(3\,a^3b^2+3\,a^2b^3+ab^4)\cos(fx+e)^2\right)\log\left(a\cos(fx+e)^2+b\right)+4\left(a^5\cos(fx+e)^4+2\,a^4b\cos(fx+e)^2+a^3b^2\right)\log\left(\frac{1}{2}\,\sin(fx+e)\right)}{4\left((a^8+3\,a^7b+3\,a^6b^2+a^5b^3)f\cos(fx+e)^4+2\,(a^7b+3\,a^6b^2+3\,a^5b^3+a^4b^4)f\cos(fx+e)^2+(a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos(
f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(a*c
os(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x
 + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(124) = 248.

time = 0.58, size = 767, normalized size = 5.90

$$\frac{\frac{2\,(3\,a^2b+3\,ab^2+b^3)\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{2\,\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{9\,a^3b+18\,a^2b^2+12\,ab^3+3\,b^4+\frac{36\,a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{24\,a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{54\,a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12\,a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{8\,ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{18\,b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{36\,a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{24\,a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{16\,ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{12\,b^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{9\,a^3b(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{18\,a^2b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{12\,ab^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{3\,b^4(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^5+2\,a^4b+a^3b^2)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}-\frac{4\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 2*log(abs(-cos(f*x + e) + 1)/abs(cos
(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^3*b + 18*a^2*b^2 + 1
2*a*b^3 + 3*b^4 + 36*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b
^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 54*a^3*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1
8*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 36*a^3*b*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 9*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x
+ e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a*b
^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x + e) - 1)^4/(
cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2)^2) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)
/f
```

Mupad [B]

time = 4.86, size = 190, normalized size = 1.46

$$\frac{\ln(\tan(e+f\,x))}{f\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}-\frac{\frac{2\,b^2+5\,a\,b}{4\,a^2\,(a+b)}+\frac{b\tan(e+f\,x)^2\,(b^2+2\,a\,b)}{2\,a^2\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan(e+f\,x)^2\,(2\,b^2+2\,a\,b)+b^2\tan(e+f\,x)^4\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f}+\frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a^2+3\,a\,b+b^2)}{2\,a^3\,f\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - ((5*a*b + 2*b^2)/(4
*a^2*(a + b)) + (b*tan(e + f*x)^2*(2*a*b + b^2))/(2*a^2*(a + b)^2))/(f*(2*a
```

```
*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - lo
g(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b*log(a + b + b*tan(e + f*x)^2)*(3*a*b +
 3*a^2 + b^2))/(2*a^3*f*(a + b)^3)
```

3.367 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=154

$$\frac{b^4}{4a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2}-\frac{b^3(2a+b)}{a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)}-\frac{\csc^2(e+fx)}{2(a+b)^3f}-\frac{b^2(6a^2+4ab+b^2)\log(\ldots}{2a^3(a+\ldots}$$

```
[Out] 1/4*b^4/a^3/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2-b^3*(2*a+b)/a^3/(a+b)^3/f/(b+a*c
os(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^3/f-1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+a*co
s(f*x+e)^2)/a^3/(a+b)^4/f-(a+4*b)*ln(sin(f*x+e))/(a+b)^4/f
```

Rubi [A]
time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(6a^2+4ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^4}-\frac{\csc^2(e+fx)}{2f(a+b)^3}-\frac{(a+4b)\log(\sin(e+fx))}{f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^4/(4*a^3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) - (b^3*(2*a + b))/(a^3*(a
+ b)^3*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)^3*f) - (b^2*(6
*a^2 + 4*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^4*f) - ((a +
4*b)*Log[Sin[e + f*x]])/((a + b)^4*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2}{a^2}\right.\right.}{2f}$$

$$= \frac{b^4}{4a^3(a+b)^2f(b+a\cos^2(e+fx))^2} - \frac{b^3(2a+b)}{a^3(a+b)^3f(b+a\cos^2(e+fx))} - \frac{\csc^2}{2(a}$$

Mathematica [A]
time = 1.91, size = 176, normalized size = 1.14

$$-\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)\csc^2(e+fx)+4(a+4b)\log(\sin(e+fx))+\frac{2b^2(6a^2+4ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3}-\frac{b^4(a+b)^2}{a^3(a+b-a\sin^2(e+fx))^2}+\frac{4b^3(a+b)(2a+b)}{a^3(a+b-a\sin^2(e+fx))}\right)}{32(a+b)^4f(a+b\sec^2(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/32*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e + f
*x]^2 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a
+ b - a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]
^2)^2) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/((a +
 b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.16, size = 193, normalized size = 1.25

method	result
derivativedivides	$\frac{-\frac{b^2\left(-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a(\cos^2(fx+e)))^2}+\frac{(6a^2+4ab+b^2)\ln(b+a(\cos^2(fx+e)))}{a^3}+\frac{2b(2a^2+3ab+b^2)}{a^3(b+a(\cos^2(fx+e)))}\right)}{2(a+b)^4}+\frac{1}{4(a+b)^3(\cos(fx+e)-1)}+}{f}$
default	$\frac{-\frac{b^2\left(-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a(\cos^2(fx+e)))^2}+\frac{(6a^2+4ab+b^2)\ln(b+a(\cos^2(fx+e)))}{a^3}+\frac{2b(2a^2+3ab+b^2)}{a^3(b+a(\cos^2(fx+e)))}\right)}{2(a+b)^4}+\frac{1}{4(a+b)^3(\cos(fx+e)-1)}+}{f}$

risch	$-\frac{ix}{a^3}+\frac{2ib^4e}{a^3f(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{8ib^3x}{a^2(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{8ib^3e}{a^2f(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*b^2/(a+b)^4*(-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+(6
*a^2+4*a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)+2/a^3*b*(2*a^2+3*a*b+b^2)/(b+a*cos
(f*x+e)^2))+1/4/(a+b)^3/(cos(f*x+e)-1)+1/2*(-a-4*b)/(a+b)^4*ln(cos(f*x+e)-1
)-1/4/(a+b)^3/(1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(1+cos(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(153) = 306$.
time = 0.27, size = 351, normalized size = 2.28

$$-\frac{\frac{2\,(6\,a^2b^2+4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4}+\frac{2\,(a+4\,b)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,a^5+4\,a^4b+2\,a^3b^2+2\,(a^5-4\,a^2b^3-2\,ab^4)\sin(fx+e)^4-(4\,a^5+4\,a^4b-8\,a^2b^3-11\,ab^4-3\,b^5)\sin(fx+e)^2}{(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)\sin(fx+e)^6-2\,(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\sin(fx+e)^4+(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sin(fx+e)^2}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2)/
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b^2
+ 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a^2*b
^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sin
(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^
5)*sin(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(153) = 306$.
time = 6.92, size = 597, normalized size = 3.88

$$\frac{2\,a^4b^2+2\,a^3b^3+7\,a^2b^4+10\,ab^5+3\,b^6+2\,(a^6+a^5b-4\,a^3b^3-6\,a^2b^4-2\,ab^5)\cos(fx+e)^4+(4\,a^5b+4\,a^4b^2+8\,a^3b^3+5\,a^2b^4-6\,ab^5-3\,b^6)\cos(fx+e)^2-2\,\left((6\,a^4b^2+4\,a^3b^3+a^2b^4)\cos(fx+e)^6-6\,a^2b^4-4\,ab^5-b^6-(6\,a^4b^2-8\,a^3b^3-7\,a^2b^4-2\,ab^5)\cos(fx+e)^4-(12\,a^3b^3+2\,a^2b^4-2\,ab^5-b^6)\cos(fx+e)^2\right)\log\left(a\cos(fx+e)^2+b\right)-4\,\left((a^6+4\,a^5b)\cos(fx+e)^6-a^4b^2-4\,a^3b^3-(a^6+2\,a^5b-8\,a^4b^2)\cos(fx+e)^4-(2\,a^5b+7\,a^4b^2-4\,a^3b^3)\cos(fx+e)^2\right)\log\left(\frac{1}{2}\sin(fx+e)\right)}{4\left((a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)f\cos(fx+e)^6-(a^9+2\,a^8b-2\,a^7b^2-8\,a^6b^3-7\,a^5b^4-2\,a^4b^5)f\cos(fx+e)^4-(2\,a^8b+7\,a^7b^2+8\,a^6b^3+2\,a^5b^4-2\,a^4b^5-a^3b^6)f\cos(fx+e)^2-(a^7b^2+4\,a^6b^3+6\,a^5b^4+4\,a^4b^5+a^3b^6)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
- 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2 +
8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4*b^2
 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^6 + 4*
```

```
a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4*b^2)*c
os(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*log(1/2*s
in(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x
+ e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*
cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
 a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(148) = 296$.
time = 0.70, size = 1069, normalized size = 6.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 4*(a + 4*b)*log(abs(-
cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1)) - 2*(18*a^4*b^2 + 48*a^3*b^3 + 45
*a^2*b^4 + 18*a*b^5 + 3*b^6 + 72*a^4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 80*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^2*b^4*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^5*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 12*b^6*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 108*a^4*b^2*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 64*a^3*b^3*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 46*a^2*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 44*a*
b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^6*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + 72*a^4*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^
3 + 80*a^3*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 20*a^2*b^4*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 40*a*b^5*(cos(f*x + e) - 1)^3/(cos(f
```

```
*x + e) + 1)^3 - 12*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 18*a^4*
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 48*a^3*b^3*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 45*a^2*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 18*a*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^6*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
+ a^3*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 8*log(abs(-(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

Mupad [B]
time = 6.03, size = 272, normalized size = 1.77

$$\frac{\frac{\tan(e+f\,x)^2\,\left(-4\,a^2\,b+7\,a\,b^2+2\,b^3\right)}{4\,a^2\,\left(a^2+2\,a\,b+b^2\right)}-\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^4\,\left(-a^2\,b^2+3\,a\,b^3+b^4\right)}{2\,a^2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^2\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^4\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^6\right)}+\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+4\,b\right)}{f\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}-\frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(6\,a^2+4\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^2*(7*a*b^2 - 4*a^2*b + 2*b^3))/(4*a^2*(2*a*b + a^2 + b^2)) -
 1/(2*(a + b)) + (tan(e + f*x)^4*(3*a*b^3 + b^4 - a^2*b^2))/(2*a^2*(a + b)*
(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^2*(2*a*b + a^2 + b^2) + tan(e + f*x)
^4*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^6)) + log(tan(e + f*x)^2 + 1)/(2*a^3*
f) - (log(tan(e + f*x))*(a + 4*b))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^
2*b^2)) - (b^2*log(a + b + b*tan(e + f*x)^2)*(4*a*b + 6*a^2 + b^2))/(2*a^3*
f*(a + b)^4)
```

3.368 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)}+\frac{(2a+5b)\csc^2(e+fx)}{2(a+b)^4f}-\frac{\csc^4(e+}{4(a+b}$$

```
[Out] -1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2*b^4*(5*a+2*b)/a^3/(a+b)^4/f
/(b+a*cos(f*x+e)^2)+1/2*(2*a+5*b)*csc(f*x+e)^2/(a+b)^4/f-1/4*csc(f*x+e)^4/(
a+b)^3/f+1/2*b^3*(10*a^2+5*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^5/f+(a^2
+5*a*b+10*b^2)*ln(sin(f*x+e))/(a+b)^5/f
```

Rubi [A]
time = 0.19, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$-\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{f(a+b)^5}+\frac{b^3(10a^2+5ab+b^2)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^5}-\frac{\csc^4(e+fx)}{4f(a+b)^3}+\frac{(2a+5b)\csc^2(e+fx)}{2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/4*b^5/(a^3*(a + b)^3*f*(b + a*Cos[e + f*x]^2)^2) + (b^4*(5*a + 2*b))/(2*
a^3*(a + b)^4*f*(b + a*Cos[e + f*x]^2)) + ((2*a + 5*b)*Csc[e + f*x]^2)/(2*(
a + b)^4*f) - Csc[e + f*x]^4/(4*(a + b)^3*f) + (b^3*(10*a^2 + 5*a*b + b^2)*
Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^5*f) + ((a^2 + 5*a*b + 10*b^2)*Lo
g[Sin[e + f*x]])/((a + b)^5*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{11}}{(1-x^2)^3(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x)^3(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)^3}+\frac{-2a-5b}{(a+b)^4(-1+x)^2}+\frac{-a^2-5ab-10b^2}{(a+b)^5(-1+x)}-\frac{b^5}{a^2(a+b)^3(b+ax)^3}+\frac{}{a^2(}\right.\right.}{2f}\\ &= -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)}+\frac{(2}{}\end{aligned}$$

Mathematica [A]

time = 5.45, size = 208, normalized size = 1.08

$$\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)(2a+5b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2+5ab+10b^2)\log(\sin(e+fx))+\frac{2b^3(10a^2+5ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3}-\frac{b^5(a+b)^2}{a^3(a+b-a\sin^2(e+fx))^2}+\frac{2b^4(a+b)(5a+2b)}{a^3(a+b-a\sin^2(e+fx))}\right)}{32(a+b)^5f\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Csc
[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin[e
 + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3
 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*(5*a
 + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec[e +
f*x]^2)^3)
```

Maple [A]

time = 0.18, size = 254, normalized size = 1.32

method	result
derivativedivides	$\dfrac{\dfrac{b^3\left(\frac{b\left(5a^2+7ab+2b^2\right)}{a^3\left(b+a\left(\cos^2(fx+e)\right)\right)}-\frac{b^2\left(a^2+2ab+b^2\right)}{2a^3\left(b+a\left(\cos^2(fx+e)\right)\right)^2}+\frac{\left(10a^2+5ab+b^2\right)\ln\left(b+a\left(\cos^2(fx+e)\right)\right)}{a^3}\right)}{2(a+b)^5}-\frac{1}{16(a+b)^3(\cos(fx+e)-1)^2}-\frac{}{16(}}{}$

default	$\frac{b^3\left(\frac{b(5a^2+7ab+2b^2)}{a^3(b+a(\cos^2(fx+e)))}-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a(\cos^2(fx+e)))^2}+\frac{(10a^2+5ab+b^2)\ln(b+a(\cos^2(fx+e)))}{a^3}\right)}{2(a+b)^5}-\frac{1}{16(a+b)^3(\cos(fx+e)-1)^2}-$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b^3/(a+b)^5*(1/a^3*b*(5*a^2+7*a*b+2*b^2)/(b+a*cos(f*x+e)^2)-1/2*b^
2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+(10*a^2+5*a*b+b^2)/a^3*ln(b+a*co
s(f*x+e)^2))-1/16/(a+b)^3/(cos(f*x+e)-1)^2-1/16*(7*a+19*b)/(a+b)^4/(cos(f*x
+e)-1)+1/2*(a^2+5*a*b+10*b^2)/(a+b)^5*ln(cos(f*x+e)-1)-1/16/(a+b)^3/(1+cos(
f*x+e))^2-1/16*(-7*a-19*b)/(a+b)^4/(1+cos(f*x+e))+1/2*(a^2+5*a*b+10*b^2)/(a
+b)^5*ln(1+cos(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(188) = 376$.
time = 0.27, size = 462, normalized size = 2.41

$$\frac{\frac{2\,(10\,a^2b^3+5\,ab^4+b^5)\log\left(a\sin(fx+e)^2-a-b\right)}{a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5}+\frac{2\,(a^2+5\,ab+10\,b^2)\log\left(\sin(fx+e)^2\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\sin(fx+e)^6-a^6-3\,a^5b-3\,a^4b^2-a^3b^3-(9\,a^6+29\,a^5b+20\,a^4b^2-10\,a^2b^4-13\,ab^5-3\,b^6)\sin(fx+e)^4+2\,(3\,a^6+11\,a^5b+13\,a^4b^2+5\,a^3b^3)\sin(fx+e)^2}{(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)\sin(fx+e)^8-2\,(a^9+5\,a^8b+10\,a^7b^2+10\,a^6b^3+5\,a^5b^4+a^4b^5)\sin(fx+e)^6+(a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6)\sin(fx+e)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5*
a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b + 1
0*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 - a^6
 - 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10*a^2*
b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4*b^2 +
 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4
 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*
a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(188) = 376$.
time = 9.36, size = 876, normalized size = 4.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2*
(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x + e
)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b^5
+ 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3 - 2
*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b^4 +
a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3 - 5*a^
3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4 - 9*a^
2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4*a*b^6 -
 b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*b + 10*a^
5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7 + 4*a^6*b
 + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 35*a
^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 10
*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^9*b + 10*a^8*
b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*
b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 +
 a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^
7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*
b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^
5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. 2(182) = 364.
time = 0.67, size = 1912, normalized size = 9.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(32*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a + b + 2*a*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)/(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 32*(a
^2 + 5*a*b + 10*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - (12*a^3*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 60*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
```

+ 84*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 36*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a^2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + 16*a^7*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 144*a^5*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 112*a^4*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a^3*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 70*a^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 312*a^6*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 436*a^5*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 536*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 822*a^3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 672*a^2*b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 224*a*b^6*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 32*b^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*a^7*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 568*a^6*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 672*a^5*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 1096*a^4*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 852*a^3*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 384*a^2*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 512*a*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 128*b^7*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 145*a^7*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 612*a^6*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 838*a^5*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1572*a^4*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1777*a^3*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 704*a^2*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 576*a*b^6*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 192*b^7*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 76*a^7*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 392*a^6*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 720*a^5*b^2*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 1080*a^4*b^3*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 676*a^3*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 384*a^2*b^5*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 512*a*b^6*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 128*b^7*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 16*a^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 112*a^6*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 336*a^5*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 720*a^4*b^3*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 960*a^3*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 672*a^2*b^5*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 224*a*b^6*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 32*b^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2) - 64*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f

Mupad [B]
time = 6.42, size = 327, normalized size = 1.70

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+5\,a\,b+10\,b^2\right)}{f\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}-\frac{\frac{1}{4\,(a+b)}-\frac{\tan(e+f\,x)^2\,(a+3\,b)}{2\,(a+b)^2}+\frac{\tan(e+f\,x)^4\,\left(-4\,a^3\,b-15\,a^2\,b^2+9\,a\,b^3+2\,b^4\right)}{4\,a^2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}+\frac{\tan(e+f\,x)^6\,\left(-a^3\,b^2-4\,a^2\,b^3+4\,a\,b^4+b^5\right)}{2\,a^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^6\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^8\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f}+\frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(10\,a^2+5\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] (log(tan(e + f*x))*(5*a*b + a^2 + 10*b^2))/(f*(5*a*b^4 + 5*a^4*b + a^5 + b^
5 + 10*a^2*b^3 + 10*a^3*b^2)) - (1/(4*(a + b)) - (tan(e + f*x)^2*(a + 3*b))
/(2*(a + b)^2) + (tan(e + f*x)^4*(9*a*b^3 - 4*a^3*b + 2*b^4 - 15*a^2*b^2))/
(4*a^2*(a + b)*(2*a*b + a^2 + b^2)) + (tan(e + f*x)^6*(4*a*b^4 + b^5 - 4*a^
2*b^3 - a^3*b^2))/(2*a^2*(a + b)^2*(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^4
*(2*a*b + a^2 + b^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^8)
) - log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b^3*log(a + b + b*tan(e + f*x)^2)*
(5*a*b + 10*a^2 + b^2))/(2*a^3*f*(a + b)^5)
```

3.369 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=147

$$-\frac{x}{a^3}+\frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2}-\frac{(3a-4b)(a+}{8a^2b^2f\,(a+b+}$$

```
[Out] -x/a^3+1/8*(3*a^2-4*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)
^(1/2)/a^3/b^(5/2)/f-1/4*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^2-1/
8*(3*a-4*b)*(a+b)*tan(f*x+e)/a^2/b^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 481, 592, 536, 209, 211}

$$-\frac{x}{a^3}-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\,(a+b\tan^2(e+fx)+b)}+\frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(8*a^3*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(4*a*b*f*(a +
 b + b*Tan[e + f*x]^2)^2) - ((3*a - 4*b)*(a + b)*Tan[e + f*x])/(8*a^2*b^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a-b)x^2)}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+\right.}{4abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}(}{}$$

$$= -\frac{x}{a^3} + \frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(}{4abf\,(a+b+b\,\text{ta}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.57, size = 760, normalized size = 5.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*a^3 + a^2*b - 4*a*b^2 - 8*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*S
in[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))
*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*b^2*S
qrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*b^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]])))/(a + b*Sec[e + f*x]^2)^3 + ((a + 2*b + a*Cos[2*e + 2*f*x])*S
ec[2*e]*Sec[e + f*x]^6*(-24*a^2*b^2*f*x*Cos[2*e] - 64*a*b^3*f*x*Cos[2*e] -
64*b^4*f*x*Cos[2*e] - 16*a^2*b^2*f*x*Cos[2*f*x] - 32*a*b^3*f*x*Cos[2*f*x] -
 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] - 32*a*b^3*f*x*Cos[4*e + 2*f*x] - 4*a^2*b^
2*f*x*Cos[2*e + 4*f*x] - 4*a^2*b^2*f*x*Cos[6*e + 4*f*x] + 9*a^4*Sin[2*e] +
15*a^3*b*Sin[2*e] - 18*a^2*b^2*Sin[2*e] - 72*a*b^3*Sin[2*e] - 48*b^4*Sin[2*
e] - 9*a^4*Sin[2*f*x] - 13*a^3*b*Sin[2*f*x] + 28*a^2*b^2*Sin[2*f*x] + 32*a*
```

```
b^3*Sin[2*f*x] + 3*a^4*Sin[4*e + 2*f*x] - a^3*b*Sin[4*e + 2*f*x] - 20*a^2*b
^2*Sin[4*e + 2*f*x] - 16*a*b^3*Sin[4*e + 2*f*x] - 3*a^4*Sin[2*e + 4*f*x] +
3*a^3*b*Sin[2*e + 4*f*x] + 6*a^2*b^2*Sin[2*e + 4*f*x]))/(128*a^3*b^2*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.15, size = 134, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\left(\tan^3(fx+e)\right)}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\left(\tan^3(fx+e)\right)}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-3a^4 e^{6i(fx+e)}+a^3 b\, e^{6i(fx+e)}+20a^2b^2 e^{6i(fx+e)}+16a\, b^3 e^{6i(fx+e)}-9a^4 e^{4i(fx+e)}-15a^3 b\, e^{4i(fx+e)}+18a^2b^2 e^{4i\cdots}\right.}{4a^3b^2 f\left(a\, e^{4i(fx+\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))+(a+b)/a^3*((-1/8*a*(5*a-4*b)/b*tan(f*x+e)^3-
1/8*a*(3*a^2-a*b-4*b^2)/b^2*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2-4
*a*b+8*b^2)/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.46, size = 199, normalized size = 1.35

$$-\frac{\frac{(5a^2b+ab^2-4b^3)\tan(fx+e)^3+(3a^3+2a^2b-5ab^2-4b^3)\tan(fx+e)}{a^2b^4\tan(fx+e)^4+a^4b^2+2a^3b^3+a^2b^4+2(a^3b^3+a^2b^4)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}-\frac{(3a^3-a^2b+4ab^2+8b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3b^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((5*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e)^3 + (3*a^3 + 2*a^2*b - 5*a*b^
2 - 4*b^3)*tan(f*x + e))/(a^2*b^4*tan(f*x + e)^4 + a^4*b^2 + 2*a^3*b^3 + a^
2*b^4 + 2*(a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 - (3*a^3 -
```

```
a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a +
b)*b)*a^3*b^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(138) = 276$.
time = 4.28, size = 692, normalized size = 4.71

$$\left[-\frac{32\,a^2b^2fx\cos\left(fx+e\right)^4+64\,ab^3fx\cos\left(fx+e\right)^2+32\,b^4fx-\left(\left(3\,a^4-4\,a^3b+8\,a^2b^2\right)\cos\left(fx+e\right)^4+3\,a^2b^2-4\,ab^3+8\,b^4+2\left(3\,a^3b-4\,a^2b^2+8\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-\frac{a+b}{b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(ab+2\,b^2\right)\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{-\frac{a+b}{b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)+4\left(3\left(a^4-a^3b-2\,a^2b^2\right)\cos\left(fx+e\right)^3+\left(5\,a^3b+a^2b^2-4\,ab^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{32\left(a^5b^2f\cos\left(fx+e\right)^4+2\,a^4b^3f\cos\left(fx+e\right)^2+a^3b^4f\right)},\ -\frac{16\,a^2b^2fx\cos\left(fx+e\right)^4+32\,ab^3fx\cos\left(fx+e\right)^2+16\,b^4fx+\left(\left(3\,a^4-4\,a^3b+8\,a^2b^2\right)\cos\left(fx+e\right)^4+3\,a^2b^2-4\,ab^3+8\,b^4+2\left(3\,a^3b-4\,a^2b^2+8\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left(\left(a+2\,b\right)\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\left(a+b\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)+2\left(3\left(a^4-a^3b-2\,a^2b^2\right)\cos\left(fx+e\right)^3+\left(5\,a^3b+a^2b^2-4\,ab^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{16\left(a^5b^2f\cos\left(fx+e\right)^4+2\,a^4b^3f\cos\left(fx+e\right)^2+a^3b^4f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b^
4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3
 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a + b)/
b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4
*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3
)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f
*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*a*b^3*f*x
*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^
4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x
 + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a
+ b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3*b - 2*a^2*b^
2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e))*sin(f*x + e
))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3*b^4*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 2.62, size = 200, normalized size = 1.36

$$-\frac{\frac{8\,(fx+e)}{a^3}-\frac{\left(3\,a^3-a^2b+4\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^3b^2}+\frac{5\,a^2b\tan(fx+e)^3+ab^2\tan(fx+e)^3-4\,b^3\tan(fx+e)^3+3\,a^3\tan(fx+e)+2\,a^2b\tan(fx+e)-5\,ab^2\tan(fx+e)-4\,b^3\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2a^2b^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(f*x + e)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(pi*floor((f*x +
e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b
^2)*a^3*b^2) + (5*a^2*b*tan(f*x + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f
*x + e)^3 + 3*a^3*tan(f*x + e) + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e
) - 4*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b^2))/f
```

Mupad [B]
time = 4.90, size = 615, normalized size = 4.18

$$-\frac{\mathrm{atan}\left(\frac{25\tan(e+fx)}{32\left(\frac{5b}{4a}-\frac{3a}{16b}+\frac{9a^2}{32b^2}+\frac{25}{32}\right)}-\frac{3\tan(e+fx)}{16\left(\frac{9a}{32b}+\frac{25b}{32a}+\frac{5b^2}{4a^2}-\frac{3}{16}\right)}+\frac{9\tan(e+fx)}{32\left(\frac{25b^2}{32a^2}-\frac{3b}{16a}+\frac{5b^3}{4a^3}+\frac{9}{32}\right)}+\frac{5\tan(e+fx)}{4\left(\frac{25a}{32b}-\frac{3a^2}{16b^2}+\frac{9a^3}{32b^3}+\frac{5}{4}\right)}\right)}{a^3 f}-\frac{\frac{\tan(e+fx)^3\,(5a^2+ab-4b^2)}{8a^2b}-\frac{\tan(e+fx)\,(a+b)\,(-3a^2+ab+4b^2)}{8a^2b^2}}{f\,\left(2ab+a^2+b^2+\tan(e+fx)^2\,(2b^2+2ab)+b^2\tan(e+fx)^4\right)}-\frac{\mathrm{atanh}\left(\frac{27\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27ab^2}{256}-\frac{27b^3}{128}+\frac{171b^4}{256a}-\frac{7b^5}{64a^2}+\frac{5b^6}{32a^3}+\frac{5b^7}{4a^4}\right)}-\frac{81\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27a^2b}{256}-\frac{27ab^2}{128}+\frac{171b^3}{256}-\frac{7b^4}{64a}+\frac{5b^5}{32a^2}+\frac{5b^6}{4a^3}\right)}-\frac{35\tan(e+fx)\sqrt{-b^6-ab^5}}{32\left(\frac{171a^2b}{256}-\frac{7ab^2}{64}-\frac{27a^3}{128}+\frac{5b^3}{32}+\frac{5b^4}{4a}+\frac{27a^4}{256b}\right)}+\frac{5\tan(e+fx)\sqrt{-b^6-ab^5}}{4\left(\frac{5ab^2}{32}-\frac{7a^2b}{64}+\frac{171a^3}{256}+\frac{5b^3}{4}-\frac{27a^4}{128b}+\frac{27a^5}{256b^2}\right)}+\frac{63\tan(e+fx)\sqrt{-b^6-ab^5}}{64\left(\frac{171ab^2}{256}-\frac{27a^2b}{128}+\frac{27a^3}{256}-\frac{7b^3}{64}+\frac{5b^4}{32a}+\frac{5b^5}{4a^2}\right)}\right)\sqrt{-b^5\,(a+b)}\,(3a^2-4ab+8b^2)}{8a^3b^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] - atan((25*tan(e + f*x))/(32*((5*b)/(4*a) - (3*a)/(16*b) + (9*a^2)/(32*b^2)
 + 25/32)) - (3*tan(e + f*x))/(16*((9*a)/(32*b) + (25*b)/(32*a) + (5*b^2)/(
4*a^2) - 3/16)) + (9*tan(e + f*x))/(32*((25*b^2)/(32*a^2) - (3*b)/(16*a) +
(5*b^3)/(4*a^3) + 9/32)) + (5*tan(e + f*x))/(4*((25*a)/(32*b) - (3*a^2)/(16
*b^2) + (9*a^3)/(32*b^3) + 5/4)))/(a^3*f) - ((tan(e + f*x)^3*(a*b + 5*a^2 -
 4*b^2))/(8*a^2*b) - (tan(e + f*x)*(a + b)*(a*b - 3*a^2 + 4*b^2))/(8*a^2*b^
2))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*
x)^4)) - (atanh((27*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a*b^2)/25
6 - (27*b^3)/128 + (171*b^4)/(256*a) - (7*b^5)/(64*a^2) + (5*b^6)/(32*a^3)
+ (5*b^7)/(4*a^4))) - (81*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a^2
*b)/256 - (27*a*b^2)/128 + (171*b^3)/256 - (7*b^4)/(64*a) + (5*b^5)/(32*a^2
) + (5*b^6)/(4*a^3))) - (35*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(32*((171*a
^2*b)/256 - (7*a*b^2)/64 - (27*a^3)/128 + (5*b^3)/32 + (5*b^4)/(4*a) + (27*
a^4)/(256*b))) + (5*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(4*((5*a*b^2)/32 -
(7*a^2*b)/64 + (171*a^3)/256 + (5*b^3)/4 - (27*a^4)/(128*b) + (27*a^5)/(256
*b^2))) + (63*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(64*((171*a*b^2)/256 - (2
7*a^2*b)/128 + (27*a^3)/256 - (7*b^3)/64 + (5*b^4)/(32*a) + (5*b^5)/(4*a^2)
)))*(-b^5*(a + b))^(1/2)*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3*b^5*f)
```

3.370 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{x}{a^3}+\frac{(a^2-4ab-8b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,f}-\frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b+b\tan^2(e+f$$

```
[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/b^(3
/2)/f/(a+b)^(1/2)-1/4*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a-
4*b)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 481, 541, 536, 209, 211}

$$\frac{x}{a^3}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b\tan^2(e+fx)+b)}+\frac{(a^2-4ab-8b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}}-\frac{(a+b)\tan(e+fx)}{4abf\,(a+b\tan^2(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(8*a^3*b^(3/2)*Sqrt[a + b]*f) - ((a + b)*Tan[e + f*x])/(4*a*b*f*(a + b + b*
Tan[e + f*x]^2)^2) + ((a - 4*b)*Tan[e + f*x])/(8*a^2*b*f*(a + b + b*Tan[e +
 f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{a+b+(a-3b)x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx\right.}{4abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,f} - \frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.86, size = 1473, normalized size = 10.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
```

```
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

Maple [A]

time = 0.16, size = 124, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\left(\tan^3(fx+e)\right)-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\left(\tan^3(fx+e)\right)-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{i\left(a^3e^{6i(fx+e)}+12a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+3a^3e^{4i(fx+e)}+26a^2b\,e^{4i(fx+e)}+56a\,b^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}\right.}{4a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))+1/a^3*(((1/8*a^2-1/2*a*b)*tan(f*x+e)^3-1/8*a*
(a^2+5*a*b+4*b^2)/b*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a^2-4*a*b-8*b^2
)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.47, size = 169, normalized size = 1.23

$$\frac{\frac{(ab-4b^2)\tan(fx+e)^3-(a^2+5ab+4b^2)\tan(fx+e)}{a^2b^3\tan(fx+e)^4+a^4b+2a^3b^2+a^2b^3+2(a^3b^2+a^2b^3)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}+\frac{(a^2-4ab-8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3b}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(((a*b - 4*b^2)*tan(f*x + e)^3 - (a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a
^2*b^3*tan(f*x + e)^4 + a^4*b + 2*a^3*b^2 + a^2*b^3 + 2*(a^3*b^2 + a^2*b^3)
*tan(f*x + e)^2) + 8*(f*x + e)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(b*tan(f*x
 + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(128) = 256.
time = 3.79, size = 791, normalized size = 5.77

$$\left[\frac{32(a^3b^2+a^2b^3)fx\cos(fx+e)^4+64(a^2b^3+ab^4)fx\cos(fx+e)^2+32(ab^4+b^5)fx+((a^4-4a^3b-8a^2b^2)\cos(fx+e)^4+a^2b^2-4ab^3-8b^4+2(a^3b-4a^2b^2-8ab^3)\cos(fx+e)^2)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2-4((a+2b)\cos(fx+e)^3-b\cos(fx+e))\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)-4((a^4b+7a^3b^2+6a^2b^3)\cos(fx+e)^3-(a^3b^2-3a^2b^3-4ab^4)\cos(fx+e))\sin(fx+e)}{32((a^6b^2+a^5b^3)f\cos(fx+e)^4+2(a^5b^3+a^4b^4)f\cos(fx+e)^2+(a^4b^4+a^3b^5)f)},\ \frac{16(a^3b^2+a^2b^3)fx\cos(fx+e)^4+32(a^2b^3+ab^4)fx\cos(fx+e)^2+16(ab^4+b^5)fx-((a^4-4a^3b-8a^2b^2)\cos(fx+e)^4+a^2b^2-4ab^3-8b^4+2(a^3b-4a^2b^2-8ab^3)\cos(fx+e)^2)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2((a^4b+7a^3b^2+6a^2b^3)\cos(fx+e)^3-(a^3b^2-3a^2b^3-4ab^4)\cos(fx+e))\sin(fx+e)}{16((a^6b^2+a^5b^3)f\cos(fx+e)^4+2(a^5b^3+a^4b^4)f\cos(fx+e)^2+(a^4b^4+a^3b^5)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*x
*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos(f
*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*cos
(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^
3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^
3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a
^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*(a^2*b^3 +
 a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^3*b - 8*a^2
*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8
*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4*b + 7*a^3*b^2
 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e)
)*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^
4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 1.33, size = 159, normalized size = 1.16

$$\frac{\frac{8\,(fx+e)}{a^3}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-4\,ab-8\,b^2)}{\sqrt{ab+b^2}\,a^3b}+\frac{ab\tan(fx+e)^3-4\,b^2\tan(fx+e)^3-a^2\tan(fx+e)-5\,ab\tan(fx+e)-4\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2a^2b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(f*x + e)/a^3 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - 4*a*b - 8*b^2)/(sqrt(a*b + b^2)*a^3*b) +
(a*b*tan(f*x + e)^3 - 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*tan(f
*x + e) - 4*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b))/f
```

Mupad [B]
time = 5.40, size = 1117, normalized size = 8.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(a - 4*b))/(8*a^2) - (tan(e + f*x)*(a + b)*(a + 4*b))/(8*a
^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e +
 f*x)^4)) - atan(tan(e + f*x)/(32*(b/(4*a) - 1/32)) + tan(e + f*x)/(4*(a/(3
2*b) - 1/4)))/(a^3*f) + (atan(-(((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*
b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^
(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2
+ 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^
4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(1
6*(a^3*b^4 + a^4*b^3)) - ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)
/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(
```

```
4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2
))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128
*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*1i)/(16*(a^3*
b^4 + a^4*b^3)))/(((a*b^2)/4 - (a^2*b)/4 + a^3/32 + b^3)/(a^6*b) + ((((-b^3
*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6
*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*
(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (ta
n(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))
^(1/2)*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + ((((-b^3*(a + b))^
(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256
*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 +
 a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)
*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*
a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3))))*(-b^3*(a + b))^(1/2)*(4*a*b
- a^2 + 8*b^2)*1i)/(8*f*(a^3*b^4 + a^4*b^3))
```

3.371 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{x}{a^3}+\frac{(3a^2+12ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}f}+\frac{\tan(e+fx)}{4af\,\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(3a+4b)\tan(e+}{8a^2(a+b)f\,(a+b+b\tan}$$

```
[Out] -x/a^3+1/8*(3*a^2+12*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/
(a+b)^(3/2)/f/b^(1/2)+1/4*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a+4*
b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 482, 541, 536, 209, 211}

$$-\frac{x}{a^3}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\,(a+b\tan^2(e+fx)+b)}+\frac{(3a^2+12ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,f(a+b)^{3/2}}+\frac{\tan(e+fx)}{4af\,\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*f) + Tan[e + f*x]/(4*a*f*(a + b + b*Tan[
e + f*x]^2)^2) + ((3*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)*f*(a + b + b*Tan
[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{1-3x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af} \\
&= \frac{\tan(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}}{} \\
&= \frac{\tan(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\,(a+b+b\tan^2(e+fx))} - \frac{\text{Subst}}{} \\
&= -\frac{x}{a^3} + \frac{(3a^2+12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}f} + \frac{\tan(e+fx)}{4af\,(a+b+b\tan^2(e+f
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.67, size = 1473, normalized size = 10.67

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
```

```
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

Maple [A]
time = 0.18, size = 125, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\left(\tan^3(fx+e)\right)}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\left(\tan^3(fx+e)\right)}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(5a^3e^{6i(fx+e)}+20a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+58a^2b\,e^{4i(fx+e)}+88a\,b^2e^{4i(fx+e)}+48b^3e^{4i}\right.}{4a^3(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))+1/a^3*((1/8*a*b*(3*a+4*b)/(a+b)*tan(f*x+e)^3
+1/8*(5*a+4*b)*a*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b^2
)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.47, size = 197, normalized size = 1.43

$$\frac{\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}}+\frac{(3ab+4b^2)\tan(fx+e)^3+(5a^2+9ab+4b^2)\tan(fx+e)}{a^5+3a^4b+3a^3b^2+a^2b^3+(a^3b^2+a^2b^3)\tan(fx+e)^4+2(a^4b+2a^3b^2+a^2b^3)\tan(fx+e)^2}-\frac{8(fx+e)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
+ a^3*b)*sqrt((a + b)*b)) + ((3*a*b + 4*b^2)*tan(f*x + e)^3 + (5*a^2 + 9*a*
b + 4*b^2)*tan(f*x + e))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^3*b^2 +
a^2*b^3)*tan(f*x + e)^4 + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) -
 8*(f*x + e)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(129) = 258.
time = 5.95, size = 888, normalized size = 6.43

$$\left[-\frac{32(a^4b+2a^3b^2+a^2b^3)fx\cos(fx+e)^4+64(a^3b^2+2a^2b^3+ab^4)fx\cos(fx+e)^2+32(a^2b^3+2ab^4+b^5)fx+\left((3a^4+12a^3b+8a^2b^2)\cos(fx+e)^4+3a^2b^2+12ab^3+8b^4+2(3a^3b+12a^2b^2+8ab^3)\cos(fx+e)^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)-4\left((5a^4b+11a^3b^2+6a^2b^3)\cos(fx+e)^3+(3a^3b^2+7a^2b^3+4ab^4)\cos(fx+e)\right)\sin(fx+e)}{32\left((a^7b+2a^6b^2+a^5b^3)f\cos(fx+e)^4+2(a^6b^2+2a^5b^3+a^4b^4)f\cos(fx+e)^2+(a^5b^3+2a^4b^4+a^3b^5)f\right)},-\frac{16(a^4b+2a^3b^2+a^2b^3)fx\cos(fx+e)^4+32(a^3b^2+2a^2b^3+ab^4)fx\cos(fx+e)^2+16(a^2b^3+2ab^4+b^5)fx+\left((3a^4+12a^3b+8a^2b^2)\cos(fx+e)^4+3a^2b^2+12ab^3+8b^4+2(3a^3b+12a^2b^2+8ab^3)\cos(fx+e)^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2\left((5a^4b+11a^3b^2+6a^2b^3)\cos(fx+e)^3+(3a^3b^2+7a^2b^3+4ab^4)\cos(fx+e)\right)\sin(fx+e)}{16\left((a^7b+2a^6b^2+a^5b^3)f\cos(fx+e)^4+2(a^6b^2+2a^5b^3+a^4b^4)f\cos(fx+e)^2+(a^5b^3+2a^4b^4+a^3b^5)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2 +
 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x +
 ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*
b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
 e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^4*b
 + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e
)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4
*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e
)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^2*b^3 + 2
*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*
b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)
*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)
*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*
```

```
x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((
a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^
4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Giac [A]
time = 0.86, size = 172, normalized size = 1.25

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+12ab+8b^2)}{(a^4+a^3b)\sqrt{ab+b^2}}+\frac{3ab\tan(fx+e)^3+4b^2\tan(fx+e)^3+5a^2\tan(fx+e)+9ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^3+a^2b)\left(b\tan(fx+e)^2+a+b\right)^2}-\frac{8(fx+e)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 12*a*b + 8*b^2)/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (3*a*b*
tan(f*x + e)^3 + 4*b^2*tan(f*x + e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f*x
+ e) + 4*b^2*tan(f*x + e))/((a^3 + a^2*b)*(b*tan(f*x + e)^2 + a + b)^2) - 8
*(f*x + e)/a^3)/f
```

Mupad [B]
time = 7.44, size = 2405, normalized size = 17.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(5*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a*b + 4*b^2))/(8*a
^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*t
an(e + f*x)^4)) - atan((((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/
(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 +
 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*a^3)
+ (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))
/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 - ((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a
```

^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3)/(((9*a*b^3)/4 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3))/(a^3*f) - (atan((((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2)*1i)/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))/(((9*a*b^3)/4 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) - ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)) + ((-b*(a + b)^3)^(1/2)*((tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2))/(32*(2*a^5*b + a^6 + a^4*b^2)) + ((-b*(a + b)^3)^(1/2)*((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(512*(2*a^5*b + a^6 + a^4*b^2)*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2)))*(12*a*b + 3*a^2 + 8*b^2))/(16*(a^6*b + a^3*b^4 + 3*a^4*b^3 + 3*a^5*b^2))

```
))*(-b*(a + b)^3)^(1/2)*(12*a*b + 3*a^2 + 8*b^2)*1i)/(8*f*(a^6*b + a^3*b^4
+ 3*a^4*b^3 + 3*a^5*b^2))
```

3.372 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^3}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f}-\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{b(7a+}{8a^2(a+b)^2f(a}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1
/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8
*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 209, 211}

$$\frac{x}{a^3}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+}{4a(a+b)f\left(a+b+b\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.21, size = 332, normalized size = 2.31

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\text{ArcTan}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e+fx)))^2(\cos(2e)-i\sin(2e))}{(a+b)^{5/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}-\frac{4b^2((a+2b)\sin(2e)-a\sin(2fx))}{(a+b)f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}+\frac{b(a+2b+a\cos(2(e+fx)))((9a^2+28ab+16b^2)\sin(2e)-3a(3a+2b)\sin(2fx))}{(a+b)^2f(\cos(e)-\sin(e))(\cos(e)+\sin(e))}\right)}{64a^3(a+b\sec^2(e+fx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]
time = 0.00, size = 147, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\left(\tan^3(fx+e)\right)}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2b\,e^{4i(fx+e)}+120a\,b^2e^{4i(fx+e)}+48b^3e^{4i(f\cdots}{4a^3(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan
(f*x+e)^3+1/8*(9*a+4*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*
a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2))))
```

Maxima [A]
time = 0.46, size = 237, normalized size = 1.65

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}}+\frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)\tan(fx+e)^2}-\frac{8\,(fx+e)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x +
e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 +
2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x + e)/a
^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(135) = 270.
time = 4.48, size = 847, normalized size = 5.88

$$\left[\frac{32\,(a^4+2\,a^3b+a^2b^2)fx\cos(fx+e)^4+64\,(a^3b+2\,a^2b^2+ab^3)fx\cos(fx+e)^2+32\,(a^2b^2+2\,ab^3+b^4)fx+\left((15\,a^4+20\,a^3b+8\,a^2b^2)\cos(fx+e)^4+15\,a^2b^2+20\,ab^3+8\,b^4+2\,(15\,a^3b+20\,a^2b^2+8\,ab^3)\cos(fx+e)^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right)-4\left(3\,(3\,a^3b+2\,a^2b^2)\cos(fx+e)^3+(7\,a^2b^2+4\,ab^3)\cos(fx+e)\right)\sin(fx+e)}{32\left((a^7+2\,a^6b+a^5b^2)f\cos(fx+e)^4+2\,(a^6b+2\,a^5b^2+a^4b^3)f\cos(fx+e)^2+(a^5b^2+2\,a^4b^3+a^3b^4)f\right)},\ \frac{16\,(a^4+2\,a^3b+a^2b^2)fx\cos(fx+e)^4+32\,(a^3b+2\,a^2b^2+ab^3)fx\cos(fx+e)^2+16\,(a^2b^2+2\,ab^3+b^4)fx+\left((15\,a^4+20\,a^3b+8\,a^2b^2)\cos(fx+e)^4+15\,a^2b^2+20\,ab^3+8\,b^4+2\,(15\,a^3b+20\,a^2b^2+8\,ab^3)\cos(fx+e)^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-2\left(3\,(3\,a^3b+2\,a^2b^2)\cos(fx+e)^3+(7\,a^2b^2+4\,ab^3)\cos(fx+e)\right)\sin(fx+e)}{16\left((a^7+2\,a^6b+a^5b^2)f\cos(fx+e)^4+2\,(a^6b+2\,a^5b^2+a^4b^3)f\cos(fx+e)^2+(a^5b^2+2\,a^4b^3+a^3b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Giac [A]
time = 0.43, size = 196, normalized size = 1.36

$$-\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan\left(fx+e\right)^3+4\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)+13\,ab^2\tan\left(fx+e\right)+4\,b^3\tan\left(fx+e\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}-\frac{8\,\left(fx+e\right)}{a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B]
time = 8.43, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e +
 f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10
*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6
*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a
^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*
a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan
(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*
a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 8
```

56*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3)/((((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3 + (((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/a^3))/(a^3*f) - ((tan(e + f*x)^3*(7*a*b^2 + 4*b^3))/(8*a^2*(a + b)^2) + (tan(e + f*x)*(9*a*b + 4*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + (atan(((((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*1i)/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))/((((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) - (tan(e + f*x)*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^

```
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(512*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^
3 + 6*a^6*b^2)*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b
^2)))*(-b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 +
a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3 + 10*a^6*b^2)))*(-b*(a + b)^5)^(1/2)*(20*a
*b + 15*a^2 + 8*b^2))/(16*(5*a^7*b + a^8 + a^3*b^5 + 5*a^4*b^4 + 10*a^5*b^3
 + 10*a^6*b^2)) + (((tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856
*a^3*b^4 + 289*a^4*b^3))/(32*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b
^2)) + (((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10
*b^2)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (tan(e + f*x)*(-
b*(a + b)^5)^(1/2)*(20*a*b + 15*a^2 + 8*b^2)*(512*a^6*b^7 + 2304*a^7*b^6 +
4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 2...
```

3.373 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{x}{a^3}+\frac{b^{3/2}(35a^2+28ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f}-\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f}-\frac{b\cot(\cdots}{4a(a+b)f\,(a+b\cdots}$$

```
[Out] -x/a^3+1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(7/2)/f-1/8*(8*a^2-11*a*b-4*b^2)*cot(f*x+e)/a^2/(a+b)^3/f-1/
4*b*cot(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(9*a+4*b)*cot(f*x+e)/
a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 209, 211}

$$-\frac{x}{a^3}-\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2f(a+b)^3}-\frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b^{3/2}(35a^2+28ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{7/2}}-\frac{b\cot(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*f) - ((8*a^2 - 11*a*b - 4*b^2)*Cot[e +
f*x])/(8*a^2*(a + b)^3*f) - (b*Cot[e + f*x])/(4*a*(a + b)*f*(a + b + b*Tan[
e + f*x]^2)^2) - (b*(9*a + 4*b)*Cot[e + f*x])/(8*a^2*(a + b)^2*f*(a + b + b
*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-5bx^2}{x^2(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan\right.}{4a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx\right.}$$

$$= -\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8}$$

$$= -\frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{}{8}$$

$$= -\frac{x}{a^3} + \frac{b^{3/2}(35a^2+28ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} - \frac{(8a^2-11ab-4b^2)\,c}{8a^2(a+b)^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.18, size = 2089, normalized size = 11.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
(-1/64*(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])
)*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*(Cos[
2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sq
rt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] +
 a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[
```

```
4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f
*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f*x] + 56*a
^4*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*Cos[f*x] + 2
24*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*f*x] - 68*a^
4*b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*f*x*Cos[3*f*x
] - 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a^4*b*f*x*Cos[2
*e - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x*Cos[2*e - f*x]
 - 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x] - 8*a^5*f*x*Cos
[2*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*x*Cos[2*e + f*x]
- 296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e + f*x] - 64*b^5*f*
x*Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f*x*Cos[4*e + f*x] +
 184*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Cos[4*e + f*x] + 224*a*b^
4*f*x*Cos[4*e + f*x] + 64*b^5*f*x*Cos[4*e + f*x] + 12*a^5*f*x*Cos[2*e + 3*f
*x] + 68*a^4*b*f*x*Cos[2*e + 3*f*x] + 132*a^3*b^2*f*x*Cos[2*e + 3*f*x] + 10
8*a^2*b^3*f*x*Cos[2*e + 3*f*x] + 32*a*b^4*f*x*Cos[2*e + 3*f*x] - 12*a^5*f*x
*Cos[4*e + 3*f*x] - 68*a^4*b*f*x*Cos[4*e + 3*f*x] - 132*a^3*b^2*f*x*Cos[4*e
 + 3*f*x] - 108*a^2*b^3*f*x*Cos[4*e + 3*f*x] - 32*a*b^4*f*x*Cos[4*e + 3*f*x
] + 12*a^5*f*x*Cos[6*e + 3*f*x] + 68*a^4*b*f*x*Cos[6*e + 3*f*x] + 132*a^3*b
^2*f*x*Cos[6*e + 3*f*x] + 108*a^2*b^3*f*x*Cos[6*e + 3*f*x] + 32*a*b^4*f*x*C
os[6*e + 3*f*x] - 4*a^5*f*x*Cos[2*e + 5*f*x] - 12*a^4*b*f*x*Cos[2*e + 5*f*x
] - 12*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 4*a^2*b^3*f*x*Cos[2*e + 5*f*x] + 4*a^
5*f*x*Cos[4*e + 5*f*x] + 12*a^4*b*f*x*Cos[4*e + 5*f*x] + 12*a^3*b^2*f*x*Cos
[4*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[4*e + 5*f*x] - 4*a^5*f*x*Cos[6*e + 5*f*x]
 - 12*a^4*b*f*x*Cos[6*e + 5*f*x] - 12*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 4*a^2*
b^3*f*x*Cos[6*e + 5*f*x] + 4*a^5*f*x*Cos[8*e + 5*f*x] + 12*a^4*b*f*x*Cos[8*
e + 5*f*x] + 12*a^3*b^2*f*x*Cos[8*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[8*e + 5*f*
x] - 32*a^5*Sin[f*x] - 64*a^4*b*Sin[f*x] - 30*a^2*b^3*Sin[f*x] - 120*a*b^4*
Sin[f*x] - 48*b^5*Sin[f*x] + 32*a^5*Sin[3*f*x] + 64*a^4*b*Sin[3*f*x] + 26*a
^3*b^2*Sin[3*f*x] + 86*a^2*b^3*Sin[3*f*x] + 32*a*b^4*Sin[3*f*x] - 48*a^5*Si
n[2*e - f*x] - 128*a^4*b*Sin[2*e - f*x] - 128*a^3*b^2*Sin[2*e - f*x] - 30*a
^2*b^3*Sin[2*e - f*x] - 120*a*b^4*Sin[2*e - f*x] - 48*b^5*Sin[2*e - f*x] +
48*a^5*Sin[2*e + f*x] + 128*a^4*b*Sin[2*e + f*x] + 102*a^3*b^2*Sin[2*e + f*
x] - 86*a^2*b^3*Sin[2*e + f*x] - 136*a*b^4*Sin[2*e + f*x] - 48*b^5*Sin[2*e
+ f*x] - 32*a^5*Sin[4*e + f*x] - 64*a^4*b*Sin[4*e + f*x] + 26*a^3*b^2*Sin[4
*e + f*x] + 86*a^2*b^3*Sin[4*e + f*x] + 136*a*b^4*Sin[4*e + f*x] + 48*b^5*S
in[4*e + f*x] - 8*a^5*Sin[2*e + 3*f*x] - 26*a^3*b^2*Sin[2*e + 3*f*x] - 86*a
^2*b^3*Sin[2*e + 3*f*x] - 32*a*b^4*Sin[2*e + 3*f*x] + 32*a^5*Sin[4*e + 3*f*
x] + 64*a^4*b*Sin[4*e + 3*f*x] - 13*a^3*b^2*Sin[4*e + 3*f*x] - 36*a^2*b^3*S
in[4*e + 3*f*x] - 16*a*b^4*Sin[4*e + 3*f*x] - 8*a^5*Sin[6*e + 3*f*x] + 13*a
^3*b^2*Sin[6*e + 3*f*x] + 36*a^2*b^3*Sin[6*e + 3*f*x] + 16*a*b^4*Sin[6*e +
3*f*x] + 8*a^5*Sin[2*e + 5*f*x] + 13*a^3*b^2*Sin[2*e + 5*f*x] + 6*a^2*b^3*S
in[2*e + 5*f*x] - 13*a^3*b^2*Sin[4*e + 5*f*x] - 6*a^2*b^3*Sin[4*e + 5*f*x]
+ 8*a^5*Sin[6*e + 5*f*x]))/(512*a^3*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A]

time = 0.23, size = 149, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(35a^2+28ab+8b^2\right.}{8}\right.}{(a+b)^3a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(35a^2+28ab+8b^2\right.}{8}\right.}{(a+b)^3a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-8a^5e^{8i(fx+e)}+13a^3b^2e^{8i(fx+e)}+36a^2b^3e^{8i(fx+e)}+16ab^4e^{8i(fx+e)}-32a^5e^{6i(fx+e)}-64a^4be^{6i(fx+e)}+26a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))-1/(a+b)^3/tan(f*x+e)+b^2/(a+b)^3/a^3*(((11/8
*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(13*a^2+17*a*b+4*b^2)*tan(f*x+e))/(a+b
+b*tan(f*x+e)^2)^2+1/8*(35*a^2+28*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f
*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]

time = 0.48, size = 318, normalized size = 1.76

$$\frac{\frac{\left(35a^2b^2+28ab^3+8b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{(a+b)b}}-\frac{\left(8a^2b^2-11ab^3-4b^4\right)\tan(fx+e)^4+8a^4+16a^3b+8a^2b^2+\left(16a^3b+3a^2b^2-17ab^3-4b^4\right)\tan(fx+e)^2}{\left(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5\right)\tan(fx+e)^5+2\left(a^6b+4a^5b^2+6a^4b^3+4a^3b^4+a^2b^5\right)\tan(fx+e)^3+\left(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5\right)\tan(fx+e)}-\frac{8(fx+e)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)) - ((8*a^2*b^2 - 11
*a*b^3 - 4*b^4)*tan(f*x + e)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^2 + (16*a^3*b +
 3*a^2*b^2 - 17*a*b^3 - 4*b^4)*tan(f*x + e)^2)/((a^5*b^2 + 3*a^4*b^3 + 3*a^
3*b^4 + a^2*b^5)*tan(f*x + e)^5 + 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*
b^4 + a^2*b^5)*tan(f*x + e)^3 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
5*a^3*b^4 + a^2*b^5)*tan(f*x + e)) - 8*(f*x + e)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(171) = 342$.

time = 5.29, size = 1094, normalized size = 6.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b - 1
3*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4 +
8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2
+ 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(
f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5 + 3*a^4
*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*
x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 +
 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8*a^5 + 13*a^3
*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 + 5*a^2*b^3 + 4
*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (35*a^4*b + 28*a^
3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*b^3 + 8*a*b^4)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(8*a^3*b^2 -
11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*x*cos(
f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 247, normalized size = 1.36

$$\frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan(fx+e)^3+4\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+17\,ab^3\tan(fx+e)+4\,b^4\tan(fx+e)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\left(b\tan(fx+e)^2+a+b\right)^2}-\frac{8\,(fx+e)}{a^3}-\frac{8}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3 + 1
3*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e
)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Mupad [B]
time = 10.71, size = 2500, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^4*(11*a*b^3 + 4*b^4 - 8*a^2*b^2))/(8*a^2*(a + b)^3) - 1/(a +
 b) + (tan(e + f*x)^2*(13*a*b^2 - 16*a^2*b + 4*b^3))/(8*a^2*(a + b)^2))/(f*
(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b + a^2 + b^2) + b^2*ta
n(e + f*x)^5)) - atan((286720*a^6*b^15*tan(e + f*x))/(286720*a^6*b^15 + 361
9840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11
 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22166323
2*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 10
48576*a^18*b^3 + 65536*a^19*b^2) + (3619840*a^7*b^14*tan(e + f*x))/(286720*
a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 17717
2480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*
b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320
*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (21052416*a^8*b^13*tan(e +
 f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a
^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 +
334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^1
6*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (74346496*a
^9*b^12*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^
13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346
560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6
 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2
) + (177172480*a^10*b^11*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14
+ 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a
^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 1
05978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3
 + 65536*a^19*b^2) + (299796480*a^11*b^10*tan(e + f*x))/(286720*a^6*b^15 +
3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b
^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22166
```

3232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (369346560*a^12*b^9*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (334344192*a^13*b^8*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (221663232*a^14*b^7*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (105978880*a^15*b^6*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (35445760*a^16*b^5*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (7864320*a^17*b^4*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (1048576*a^18*b^3*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (65536*a^19*b^2*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2))/(a^3*f) - (atan((((-b^3*(a + b)^7)^(1/2)*(tan(e + f*x)*(131072*a^6*b^18 + 2031616*a^7*b^17 + 14745600*a^8*b^16 + 66232320*a^9*b^15 + 205112320*a^10*b^14 + 462013440*a^11*b^13 + 778473472*a^12*b^12 + 994283520*a^13*b^11 + 965376000*a^14*b^10 + 708392960*a^15*b^9 + 387272704*a^16*b^8 + 154054656*a^17*b^7 + 43115520*a^18*b^6 + 8135680*a^19*b^...

3.374 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{x}{a^3}-\frac{b^{5/2}(63a^2+36ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f}+\frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f}-\frac{(8a^2-39\ldots}{}$$

[Out] x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/(a+b)^(9/2)/f+1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*cot(f*x+e)/a^2/(a+b)^4/f-1/24*(8*a^2-39*a*b-12*b^2)*cot(f*x+e)^3/a^2/(a+b)^3/f-1/4*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+4*b)*cot(f*x+e)^3/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)

Rubi [A]
time = 0.33, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 209, 211}

$$\frac{x}{a^3}-\frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2f(a+b)^3}-\frac{b(11a+4b)\cot^3(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{b^{5/2}(63a^2+36ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{9/2}}+\frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2f(a+b)^4}-\frac{b\cot^3(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]

[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(9/2)*f) + ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3)*Cot[e + f*x])/(8*a^2*(a + b)^4*f) - ((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*x]^3)/(24*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^3)/(4*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 4*b)*Cot[e + f*x]^3)/(8*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-7bx^2}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan\right.}{4a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+f\right.}$$

$$= -\frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} -$$

$$= \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f}$$

$$= \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f}$$

$$= \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{(8a^3+32a^2b-15ab^2}{8a^2(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.83, size = 3340, normalized size = 14.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*
e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a
*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a + b
]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Cos[2*e
```

]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[
a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*
Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]])))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[f*x] - 336
*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f*x*Cos[f*x]
 - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^6*f*x*Cos[f*
x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^4*b^2*f*x*Cos
[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f*x] - 672*a*b^
5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2*e - f*x] + 336
*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x] + 3600*a^3*b^3*
f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 2496*a*b^5*f*x*Cos[2
*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2*e + f*x] + 336*a^
5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x] + 3600*a^3*b^3*f*x
*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + 2496*a*b^5*f*x*Cos[2*e
+ f*x] + 576*b^6*f*x*Cos[2*e + f*x] - 36*a^6*f*x*Cos[4*e + f*x] - 336*a^5*b
*f*x*Cos[4*e + f*x] - 1560*a^4*b^2*f*x*Cos[4*e + f*x] - 3600*a^3*b^3*f*x*Co
s[4*e + f*x] - 4260*a^2*b^4*f*x*Cos[4*e + f*x] - 2496*a*b^5*f*x*Cos[4*e + f
*x] - 576*b^6*f*x*Cos[4*e + f*x] - 36*a^6*f*x*Cos[2*e + 3*f*x] - 240*a^5*b*
f*x*Cos[2*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[2*e + 3*f*x] + 48*a^3*b^3*f*x*Co
s[2*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[2*e + 3*f*x] + 672*a*b^5*f*x*Cos[2*e +
 3*f*x] + 192*b^6*f*x*Cos[2*e + 3*f*x] + 36*a^6*f*x*Cos[4*e + 3*f*x] + 240*
a^5*b*f*x*Cos[4*e + 3*f*x] + 408*a^4*b^2*f*x*Cos[4*e + 3*f*x] - 48*a^3*b^3*
f*x*Cos[4*e + 3*f*x] - 732*a^2*b^4*f*x*Cos[4*e + 3*f*x] - 672*a*b^5*f*x*Cos
[4*e + 3*f*x] - 192*b^6*f*x*Cos[4*e + 3*f*x] - 36*a^6*f*x*Cos[6*e + 3*f*x]
- 240*a^5*b*f*x*Cos[6*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[6*e + 3*f*x] + 48*a^
3*b^3*f*x*Cos[6*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[6*e + 3*f*x] + 672*a*b^5*f
*x*Cos[6*e + 3*f*x] + 192*b^6*f*x*Cos[6*e + 3*f*x] - 12*a^6*f*x*Cos[2*e + 5
*f*x] - 144*a^5*b*f*x*Cos[2*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[2*e + 5*f*x] -
 624*a^3*b^3*f*x*Cos[2*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[2*e + 5*f*x] - 96*a
*b^5*f*x*Cos[2*e + 5*f*x] + 12*a^6*f*x*Cos[4*e + 5*f*x] + 144*a^5*b*f*x*Cos
[4*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[4*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[4*e
+ 5*f*x] + 396*a^2*b^4*f*x*Cos[4*e + 5*f*x] + 96*a*b^5*f*x*Cos[4*e + 5*f*x]
 - 12*a^6*f*x*Cos[6*e + 5*f*x] - 144*a^5*b*f*x*Cos[6*e + 5*f*x] - 456*a^4*b
^2*f*x*Cos[6*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[6*e + 5*f*x] - 396*a^2*b^4*f*
x*Cos[6*e + 5*f*x] - 96*a*b^5*f*x*Cos[6*e + 5*f*x] + 12*a^6*f*x*Cos[8*e + 5
*f*x] + 144*a^5*b*f*x*Cos[8*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[8*e + 5*f*x] +
 624*a^3*b^3*f*x*Cos[8*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[8*e + 5*f*x] + 96*a
*b^5*f*x*Cos[8*e + 5*f*x] - 12*a^6*f*x*Cos[4*e + 7*f*x] - 48*a^5*b*f*x*Cos[
4*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[4*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 7
*f*x] - 12*a^2*b^4*f*x*Cos[4*e + 7*f*x] + 12*a^6*f*x*Cos[6*e + 7*f*x] + 48*
a^5*b*f*x*Cos[6*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[6*e + 7*f*x] + 48*a^3*b^3*f
*x*Cos[6*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[6*e + 7*f*x] - 12*a^6*f*x*Cos[8*e
+ 7*f*x] - 48*a^5*b*f*x*Cos[8*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[8*e + 7*f*x]
- 48*a^3*b^3*f*x*Cos[8*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[8*e + 7*f*x] + 12*a^

```
6*f*x*Cos[10*e + 7*f*x] + 48*a^5*b*f*x*Cos[10*e + 7*f*x] + 72*a^4*b^2*f*x*C
os[10*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[10*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[10
*e + 7*f*x] - 128*a^6*Sin[f*x] - 440*a^5*b*Sin[f*x] - 1152*a^4*b^2*Sin[f*x]
 - 1920*a^3*b^3*Sin[f*x] + 228*a^2*b^4*Sin[f*x] + 1320*a*b^5*Sin[f*x] + 432
*b^6*Sin[f*x] + 48*a^6*Sin[3*f*x] + 104*a^5*b*Sin[3*f*x] + 640*a^4*b^2*Sin[
3*f*x] + 1511*a^3*b^3*Sin[3*f*x] - 528*a^2*b^4*Sin[3*f*x] + 264*a*b^5*Sin[3
*f*x] + 144*b^6*Sin[3*f*x] - 32*a^6*Sin[2*e - f*x] + 384*a^5*b*Sin[2*e - f*
x] + 2048*a^4*b^2*Sin[2*e - f*x] + 3072*a^3*b^3*Sin[2*e - f*x] + 228*a^2*b^
4*Sin[2*e - f*x] + 1320*a*b^5*Sin[2*e - f*x] + 432*b^6*Sin[2*e - f*x] + 32*
a^6*Sin[2*e + f*x] - 384*a^5*b*Sin[2*e + f*x] - 2048*a^4*b^2*Sin[2*e + f*x]
 - 2919*a^3*b^3*Sin[2*e + f*x] + 642*a^2*b^4*Sin[2*e + f*x] + 1416*a*b^5*Si
n[2*e + f*x] + 432*b^6*Sin[2*e + f*x] - 128*a^6*Sin[4*e + f*x] - 440*a^5*b*
Sin[4*e + f*x] - 1152*a^4*b^2*Sin[4*e + f*x] - 2073*a^3*b^3*Sin[4*e + f*x]
- 642*a^2*b^4*Sin[4*e + f*x] - 1416*a*b^5*Sin[4*e + f*x] - 432*b^6*Sin[4*e
+ f*x] - 144*a^6*Sin[2*e + 3*f*x] - 672*a^5*b*Sin[2*e + 3*f*x] - 960*a^4*b^
2*Sin[2*e + 3*f*x] + 153*a^3*b^3*Sin[2*e + 3*f*x] + 528*a^2*b^4*Sin[2*e + 3
*f*x] - 264*a*b^5*Sin[2*e + 3*f*x] - 144*b^6*Si...
```

Maple [A]
time = 0.27, size = 171, normalized size = 0.74

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3} - \frac{1}{3(a+b)^3\tan(fx+e)^3} - \frac{-a-4b}{(a+b)^4\tan(fx+e)} - \frac{b^3 \frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e \ldots}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2} \ldots}{a^3(a+b)^4 \ldots}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3} - \frac{1}{3(a+b)^3\tan(fx+e)^3} - \frac{-a-4b}{(a+b)^4\tan(fx+e)} - \frac{b^3 \frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e \ldots}{8}}{\left(a+b+b\left(\tan^2(fx+e)\right)\right)^2} \ldots}{a^3(a+b)^4 \ldots}}{f}$
risch	$\frac{x}{a^3} - \frac{i\left(-48a^6e^{4i(fx+e)}-144b^6e^{4i(fx+e)}-80a^6e^{2i(fx+e)}-48a^6e^{12i(fx+e)}-144a^6e^{10i(fx+e)}+144b^6e^{10i(fx+e)}-128a^6 \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*arctan(tan(f*x+e))-1/3/(a+b)^3/tan(f*x+e)^3-(-a-4*b)/(a+b)^4/tan
(f*x+e)-b^3/a^3/(a+b)^4*(((15/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(17*a^2
+21*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(63*a^2+36*a*b+8*b^2)
/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.48, size = 417, normalized size = 1.81

$$-\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{(a+b)b}}-\frac{3\,(8\,a^3b^2+32\,a^2b^3-15\,ab^4-4\,b^5)\tan(fx+e)^6-8\,a^5-24\,a^4b-24\,a^3b^2-8\,a^2b^3+(48\,a^4b+232\,a^3b^2+133\,a^2b^3-63\,ab^4-12\,b^5)\tan(fx+e)^4+8\,(3\,a^5+16\,a^4b+23\,a^3b^2+10\,a^2b^3)\tan(fx+e)^2}{(a^6b^2+4\,a^5b^3+6\,a^4b^4+4\,a^3b^5+a^2b^6)\tan(fx+e)^7+2\,(a^7b+5\,a^6b^2+10\,a^5b^3+10\,a^4b^4+5\,a^3b^5+a^2b^6)\tan(fx+e)^5+(a^8+6\,a^7b+15\,a^6b^2+20\,a^5b^3+15\,a^4b^4+6\,a^3b^5+a^2b^6)\tan(fx+e)^3}-\frac{24\,(fx+e)}{a^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)
*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)) -
(3*(8*a^3*b^2 + 32*a^2*b^3 - 15*a*b^4 - 4*b^5)*tan(f*x + e)^6 - 8*a^5 - 24*
a^4*b - 24*a^3*b^2 - 8*a^2*b^3 + (48*a^4*b + 232*a^3*b^2 + 133*a^2*b^3 - 63
*a*b^4 - 12*b^5)*tan(f*x + e)^4 + 8*(3*a^5 + 16*a^4*b + 23*a^3*b^2 + 10*a^2
*b^3)*tan(f*x + e)^2)/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b
^6)*tan(f*x + e)^7 + 2*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3
*b^5 + a^2*b^6)*tan(f*x + e)^5 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*tan(f*x + e)^3) - 24*(f*x + e)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(219) = 438$.
time = 4.07, size = 1691, normalized size = 7.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4*
(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(
f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^
5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6
 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 -
16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos
(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
 (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*
x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*b^2 + 32*a
^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 -
8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2
+ 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*
a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b +
6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*
b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^
```

```
7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), 1/48
*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 2*(24*a
^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(f*x +
 e)^5 - 2*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^5)*co
s(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 - 63
*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 - 16*a*
b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos(f*x
+ e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a
 + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(8*a^4*b^2 + 32*a^3*
b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 48*((a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a
^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8
*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3
*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b + 6*a
^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2
 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b
^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b
^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.60, size = 302, normalized size = 1.31

$$-\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{ab+b^2}}+\frac{3\left(15\,ab^4\tan(fx+e)^3+4\,b^5\tan(fx+e)^3+17\,a^2b^3\tan(fx+e)+21\,ab^4\tan(fx+e)+4\,b^5\tan(fx+e)\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\left(b\tan(fx+e)^2+a+b\right)^2}-\frac{24\,(fx+e)}{a^3}-\frac{8\left(3\,a\tan(fx+e)^2+12\,b\tan(fx+e)^2-a-b\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(fx+e)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b^5*
tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^5*ta
n(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(f*x +
 e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*tan(f*x
```

```
 + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
3))/f
```

Mupad [B]
time = 11.98, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan((860160*a^6*b^20*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200
*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^1
4*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^
9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 871628
80*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (145
15200*a^7*b^19*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 1153474
56*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b
^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12
+ 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350
2829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21
*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (115347456*
a^8*b^18*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8
*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 +
9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606
6462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 350282956
8*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 +
 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (570587136*a^9*b^
17*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18
+ 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 957730
8160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 1606646272
0*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18
*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762
560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (1961717760*a^10*b^16*t
an(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 57
0587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160
*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^
15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8
 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*
a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (4965811200*a^11*b^15*tan(e
 + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587
136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^1
2*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b
^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1
```

329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (9577308160*a^12*b^14*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (14379552768*a^13*b^13*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (17038737408*a^14*b^12*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (16066462720*a^15*b^11*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (12106321920*a^16*b^10*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 4965811200*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) + (7294187520*a^17*b^9*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 + 115347456*a^8*b^18 +...

3.375 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=285

$$-\frac{x}{a^3}+\frac{b^{7/2}(99a^2+44ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f}-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f}+$$

```
[Out] -x/a^3+1/8*b^(7/2)*(99*a^2+44*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(11/2)/f-1/8*(8*a^4+40*a^3*b+80*a^2*b^2-19*a*b^3-4*b^4)*cot(
f*x+e)/a^2/(a+b)^5/f+1/24*(8*a^3+32*a^2*b-51*a*b^2-12*b^3)*cot(f*x+e)^3/a^2
/(a+b)^4/f-1/40*(8*a^2-75*a*b-20*b^2)*cot(f*x+e)^5/a^2/(a+b)^3/f-1/4*b*cot(
f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(13*a+4*b)*cot(f*x+e)^5/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.42, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 209, 211}

$$-\frac{x}{a^3}-\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2f(a+b)^3}-\frac{b(13a+4b)\cot^5(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b^{7/2}(99a^2+44ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{11/2}}+\frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2f(a+b)^4}-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2f(a+b)^5}-\frac{b\cot^5(e+fx)}{4af(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(11/2)*f) - ((8*a^4 + 40*a^3*b + 80*a^2*b^2 -
 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(8*a^2*(a + b)^5*f) + ((8*a^3 + 32*a^2*b -
 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(24*a^2*(a + b)^4*f) - ((8*a^2 - 75*a*b
 - 20*b^2)*Cot[e + f*x]^5)/(40*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^5)/(4*a*(
a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(8
*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-9bx^2}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(\right.}{4a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx)}$$

$$= -\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} -$$

$$= \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx}{40a^2(a+b)^3f}$$

$$= -\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-}{24a^2(a+}$$

$$= -\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-}{24a^2(a+}$$

$$= -\frac{x}{a^3} + \frac{b^{7/2}(99a^2+44ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{(8a^4+40a^3b+80a^2b}{8a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.53, size = 976, normalized size = 3.42

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(x*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(a^3*(a + b*Sec[e
+ f*x]^2)^3) + ((11*a*Cos[e] + 26*b*Cos[e])*(a + 2*b + a*Cos[2*e + 2*f*x])^
3*Csc[e]*Csc[e + f*x]^2*Sec[e + f*x]^6)/(120*(a + b)^4*f*(a + b*Sec[e + f*x
]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Cot[e]*Csc[e + f*x]^4*Sec[e + f
*x]^6)/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((99*a^2 + 44*a*b + 8*b^
2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1/64*(b^4*ArcTan[Sec[f
*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[
2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
in[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] -
 I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt
[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e
] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[
2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^5*(a +
 b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*
x]^5*Sec[e + f*x]^6*Sin[f*x])/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^6*(-11*
a*Sin[f*x] - 26*b*Sin[f*x]))/(120*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3) + (
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]*Sec[e + f*x]^6*(23*a^2
*Sin[f*x] + 106*a*b*Sin[f*x] + 173*b^2*Sin[f*x]))/(120*(a + b)^5*f*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6
*(a*b^5*Sin[2*e] + 2*b^6*Sin[2*e] - a*b^5*Sin[2*f*x]))/(16*a^3*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[2*e]*Sec[
e + f*x]^6*(-21*a^2*b^4*Sin[2*e] - 52*a*b^5*Sin[2*e] - 16*b^6*Sin[2*e] + 21
*a^2*b^4*Sin[2*f*x] + 6*a*b^5*Sin[2*f*x]))/(64*a^3*(a + b)^5*f*(a + b*Sec[e
 + f*x]^2)^3)
```

Maple [A]

time = 0.30, size = 199, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{-a-4b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2+5ab+10b^2}{(a+b)^5\tan(fx+e)}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a(2\cdots}{\cdots}}{\left(a+b+b\left(\tan^2(fx\cdots\right.\right.}\cdots\right.}{\cdots}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{-a-4b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2+5ab+10b^2}{(a+b)^5\tan(fx+e)}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\left(\tan^3(fx+e)\right)+\frac{a(2\cdots}{\cdots}}{\left(a+b+b\left(\tan^2(fx\cdots\right.\right.}\cdots\right.}{\cdots}}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*arctan(tan(f*x+e))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(-a-4*b)/(a+b)^
4/tan(f*x+e)^3-(a^2+5*a*b+10*b^2)/(a+b)^5/tan(f*x+e)+b^4/a^3/(a+b)^5*(((19/
8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(21*a^2+25*a*b+4*b^2)*tan(f*x+e))/(a+
b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+44*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(
f*x+e)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 529, normalized size = 1.86

$$\frac{\frac{15\,(99\,a^2b^4+44\,ab^5+8\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sqrt{(a+b)b}}-\frac{15\,(8\,a^4b^2+40\,a^3b^3+80\,a^2b^4-19\,ab^5-4\,b^6)\tan(fx+e)^8+5\,(48\,a^5b+280\,a^4b^2+680\,a^3b^3+385\,a^2b^4-75\,ab^5-12\,b^6)\tan(fx+e)^6+24\,a^6+96\,a^5b+144\,a^4b^2+96\,a^3b^3+24\,a^2b^4+8\,(15\,a^6+95\,a^5b+258\,a^4b^2+291\,a^3b^3+113\,a^2b^4)\tan(fx+e)^4-8\,(5\,a^6+29\,a^5b+57\,a^4b^2+47\,a^3b^3+14\,a^2b^4)\tan(fx+e)^2}{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7)\tan(fx+e)^9+2\,(a^8b+6\,a^7b^2+15\,a^6b^3+20\,a^5b^4+15\,a^4b^5+6\,a^3b^6+a^2b^7)\tan(fx+e)^7+(a^9+7\,a^8b+21\,a^7b^2+35\,a^6b^3+35\,a^5b^4+21\,a^4b^5+7\,a^3b^6+a^2b^7)\tan(fx+e)^5}-\frac{120\,(fx+e)}{a^3}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt
((a + b)*b)) - (15*(8*a^4*b^2 + 40*a^3*b^3 + 80*a^2*b^4 - 19*a*b^5 - 4*b^6)
*tan(f*x + e)^8 + 5*(48*a^5*b + 280*a^4*b^2 + 680*a^3*b^3 + 385*a^2*b^4 - 7
5*a*b^5 - 12*b^6)*tan(f*x + e)^6 + 24*a^6 + 96*a^5*b + 144*a^4*b^2 + 96*a^3
*b^3 + 24*a^2*b^4 + 8*(15*a^6 + 95*a^5*b + 258*a^4*b^2 + 291*a^3*b^3 + 113*
a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 + 29*a^5*b + 57*a^4*b^2 + 47*a^3*b^3 + 1
4*a^2*b^4)*tan(f*x + e)^2)/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5
+ 5*a^3*b^6 + a^2*b^7)*tan(f*x + e)^9 + 2*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)*tan(f*x + e)^7 + (a^9 + 7*a
^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*
b^7)*tan(f*x + e)^5) - 120*(f*x + e)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(273) = 546$.
time = 4.71, size = 2279, normalized size = 8.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*
cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945
*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b -
1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f
*x + e)^5 + 20*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^
2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5
)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b
^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*
```

```
a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 3
6*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e
) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e
) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*
x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a
*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b
^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5
*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10
+ 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f
*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e)), -1/240*(2
*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*cos(f*x +
e)^9 - 2*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945*a^3*b^4 -
 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 2*(120*a^7 + 40*a^6*b - 1416*a^5*b
^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f*x + e)^5
+ 10*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^2*b^5 + 36
*a*b^6)*cos(f*x + e)^3 + 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5)*cos(f*x
+ e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b^4 - 36*a^
2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*a^2*b^5 +
12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 36*a*b^6 -
8*b^7)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(8*a
^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e) + 240
*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*x*cos(f
*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f
*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*
a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^
3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 +
10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10 + 5*a^9
*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a
^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)
^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*
b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*
b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6
*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 388, normalized size = 1.36

$$\frac{\frac{15\left(99\,a^{2}b^{4}+44\,ab^{5}+8\,b^{6}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{8}+5\,a^{7}b+10\,a^{6}b^{2}+10\,a^{5}b^{3}+5\,a^{4}b^{4}+a^{3}b^{5}\right)\sqrt{ab+b^{2}}}+\frac{15\left(19\,ab^{5}\tan\left(fx+e\right)^{3}+4\,b^{6}\tan\left(fx+e\right)^{3}+21\,a^{2}b^{4}\tan\left(fx+e\right)+25\,ab^{5}\tan\left(fx+e\right)+4\,b^{6}\tan\left(fx+e\right)\right)}{\left(a^{7}+5\,a^{6}b+10\,a^{5}b^{2}+10\,a^{4}b^{3}+5\,a^{3}b^{4}+a^{2}b^{5}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{2}}-\frac{120\left(fx+e\right)}{a^{3}}-\frac{8\left(15\,a^{2}\tan\left(fx+e\right)^{4}+75\,ab\tan\left(fx+e\right)^{4}+150\,b^{2}\tan\left(fx+e\right)^{4}-5\,a^{2}\tan\left(fx+e\right)^{2}-25\,ab\tan\left(fx+e\right)^{2}-20\,b^{2}\tan\left(fx+e\right)^{2}+3\,a^{2}+6\,ab+3\,b^{2}\right)}{\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)\tan\left(fx+e\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2
+ 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(f*x
 + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan(f*x
 + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 - 8*(15
*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4 - 5*a^
2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 + 3*a^2 +
6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*t
an(f*x + e)^5))/f
```

Mupad [B]
time = 15.21, size = 2500, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

```
[Out] atan(((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 1944780
80*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^
16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^1
9*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^2
2*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^
25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^2
8*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 +
340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2
 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^
```

26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))/(2*a^3) - (((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 760217600*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(2*a^3))*1i)/(2*a^3) - tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 40960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 134472684544*a^14*b^20 + 236839037952*a^15*b^19 + 348859675648*a^16*b^18 + 434405198848*a^17*b^17 + 461878103040*a^18*b^16 + 423083193344*a^19*b^15 + 336152947712*a^20*b^14 + 232369345536*a^21*b^13 + 139446393856*a^22*b^12 + 72073323520*a^23*b^11 + 31662619648*a^24*b^10 + 11616461824*a^25*b^9 + 3481927680*a^26*b^8 + 829030400*a^27*b^7 + 150732800*a^28*b^6 + 19660800*a^29*b^5 + 1638400*a^30*b^4 + 65536*a^31*b^3))/(2*a^3))/(27354112*a^10*b^21 - ((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*a^19*b^18 + 585044721664*a^20*b^17

+ 828584034304*a^21*b^16 + 989821665280*a^22*b^15 + 1000564490240*a^23*b^1
4 + 856970493952*a^24*b^13 + 621538574336*a^25*b^12 + 380751118336*a^26*b^1
1 + 196065116160*a^27*b^10 + 84230471680*a^28*b^9 + 29853974528*a^29*b^8 +
8588754944*a^30*b^7 + 1957904384*a^31*b^6 + 340787200*a^32*b^5 + 42598400*a
^33*b^4 + 3407872*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(524288*a^12*b
^28 + 13369344*a^13*b^27 + 163840000*a^14*b^26 + 1284505600*a^15*b^25 + 723
5174400*a^16*b^24 + 31171543040*a^17*b^23 + 106779115520*a^18*b^22 + 298450
944000*a^19*b^21 + 693069414400*a^20*b^20 + 1354635673600*a^21*b^19 + 22493
25281280*a^22*b^18 + 3193847152640*a^23*b^17 + 3894935552000*a^24*b^16 + 40
89682329600*a^25*b^15 + 3700188774400*a^26*b^14 + 2882252308480*a^27*b^13 +
 1927993098240*a^28*b^12 + 1102610432000*a^29*b^11 + 535553638400*a^30*b^10
 + 218864025600*a^31*b^9 + 74281123840*a^32*b^8 + 20559953920*a^33*b^7 + 45
21984000*a^34*b^6 + 760217600*a^35*b^5 + 917504...

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^5(e+fx)\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{(a+b\sec$$

```
[Out] -1/3*(a+2*b)*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b^
2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^
(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 90, 52, 65, 214}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f - ((a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2*f) + (a +
b*Sec[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{x} + \frac{(a+bx)^{3/2}}{b}\right) dx, x, \sec$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2 f} + $$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{(a+}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{(a+}{}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [F]
time = 2.40, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^5(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(95) = 190$.
time = 0.75, size = 924, normalized size = 8.32

method	result	size
default	Expression too large to display	924

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(cos(f*x+e)-1)*(15*l
n(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*
x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^5*
a^(1/2)*(a+b)^(1/2)*b^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)^5*(a+b)^(1/2)*a^2+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)^5*(a+b)^(1/2)*a*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)^5*(a+b)^(1/2)*b^2+15*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e
)^5*a*b^2-15*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^5*a*b^2+2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4*(a+b)^(1/2)*a^2+10*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4*(a+b)^(1/2)*a*b-15*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4*(a+b)^(1/2)*b^2-((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*(a+b)^(1/2)*a*b+10*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*(a+b)^(1/2)*b^2-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(1/2)*a*b+10*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(1/2)*b^2-3*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)*b^2-3*(a+b)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^2/cos(f*x+e)^4/(a+b)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(99) = 198$.
time = 11.85, size = 484, normalized size = 4.36

$$\left[\frac{15\,\sqrt{a}\,b^{2}\cos\left(fx+e\right)^{4}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+b^{4}-8\left(16\,a^{3}\cos\left(fx+e\right)^{8}+24\,a^{2}b\cos\left(fx+e\right)^{6}+10\,ab^{2}\cos\left(fx+e\right)^{4}+b^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\right)-8\left(\left(2\,a^{2}+10\,ab-15\,b^{2}\right)\cos\left(fx+e\right)^{4}-\left(ab-10\,b^{2}\right)\cos\left(fx+e\right)^{2}-3\,b^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{120\,b^{2}f\cos\left(fx+e\right)^{4}},\ \frac{15\,\sqrt{-a}\,b^{2}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{4}+8\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}+3\,a^{2}b\cos\left(fx+e\right)^{2}+ab^{2}\right)}\right)\cos\left(fx+e\right)^{4}-4\left(\left(2\,a^{2}+10\,ab-15\,b^{2}\right)\cos\left(fx+e\right)^{4}-\left(ab-10\,b^{2}\right)\cos\left(fx+e\right)^{2}-3\,b^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{60\,b^{2}f\cos\left(fx+e\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f
*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(
f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b
*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 -
4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f*x + e)^2
 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^
4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2{(e+fx)}}\,\tan^5{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(95) = 190.
time = 1.99, size = 1280, normalized size = 11.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(15*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(1
5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*
e)^2 + a + b))^9*a - 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*a + 20*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*(
27*a^2 - 5*a*b - 16*b^2) - 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
```

```
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(33*a^2 - 83*a*b + 32*b^2)*sqr
t(a + b) - 2*(15*a^3 + 1230*a^2*b - 625*a*b^2 - 416*b^3)*(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5 + 1
0*(81*a^3 + 90*a^2*b - 391*a*b^2 + 256*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) - 20
*(33*a^4 - 45*a^3*b - 157*a^2*b^2 + 161*a*b^3 - 16*b^4)*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3 + 20
*(3*a^4 - 75*a^3*b + 49*a^2*b^2 + 159*a*b^3 - 160*b^4)*(sqrt(a + b)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(
a + b) + 5*(27*a^5 + 140*a^4*b - 286*a^3*b^2 + 76*a^2*b^3 + 667*a*b^4 - 576
*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b)) - (45*a^5 + 100*a^4*b - 450*a^3*b^2 + 1028*a^2*b^3 - 153
9*a*b^4 + 768*b^5)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b
) + a - 3*b)^5)*sgn(cos(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+fx)^5\sqrt{a+\frac{b}{\cos{(e+fx)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}$$

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*
a^(1/2)/f-(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 81, 52, 65, 214}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - Sqrt[a + b*Sec[e
+ f*x]^2]/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx\right.}{2f}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf} - \frac{a\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf} - \frac{a\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^3(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(68) = 136$.
time = 0.14, size = 648, normalized size = 8.10

method	result
default	$-\frac{\sqrt{4}\,\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\,(\cos(fx+e)-1)\left(3\ln\left(4\cos(fx+e)\sqrt{a}\,\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a(\cos}{(1+\cos}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(cos(f*x+e)-1)*(3*ln
(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x
+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(1/2)*cos(f*
x+e)^3*(a+b)^(1/2)*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
^3*(a+b)^(1/2)*a-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3
*(a+b)^(1/2)*b+3*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a*b-3*ln
(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a*b+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(1/2)*a-3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(1/2)*b+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*b)/sin(f*x+e)^2/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)/b/(a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.
time = 5.01, size = 412, normalized size = 5.15

$$\left[\frac{3\sqrt{a}\,b\cos(fx+e)^2\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4+8\left(16\,a^3\cos(fx+e)^8+24\,a^2b\cos(fx+e)^6+10\,ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)+8\left((a-3\,b)\cos(fx+e)^2+b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\,bf\cos(fx+e)^2},-\frac{3\sqrt{-a}\,b\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2-4\left((a-3\,b)\cos(fx+e)^2+b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\,bf\cos(fx+e)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*co
s(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 +
 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)
```

^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*co
s(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2(e + fx)}\,\tan^3(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)

[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(68) = 136.
time = 0.92, size = 800, normalized size = 10.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")

[Out] -2/3*(3*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(3*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^5*a - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(3*a - 4*b)*sqrt(a + b) + 2*(sqrt(a + b
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^3*(3*a^2 - 9*a*b + 8*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + a*b - 4*b^2)*sqrt(a +
b) - 3*(3*a^3 - 2*a^2*b - 5*a*b^2 + 16*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (3*a^3 - 6*a^2*
b + 19*a*b^2 - 20*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x

```
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
 + b) + a - 3*b)^3)*sgn(cos(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1/
2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 272, 52, 65, 214}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(54) = 108.

time = 0.50, size = 119, normalized size = 2.20

$$\frac{\left(-2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{b}}\right)\cos(e+fx)+\sqrt{2}\,\sqrt{b}\,\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}\,\sqrt{b}\,f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] ((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2]
*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^2]
)/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

Maple [A]
time = 0.03, size = 58, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f}$	58
default	$\frac{\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{f}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a+b*sec(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^
(1/2))/sec(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(48) = 96.
time = 3.94, size = 332, normalized size = 6.15

$$\left[\frac{\sqrt{a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+b^{4}-8\left(16\,a^{3}\cos\left(fx+e\right)^{8}+24\,a^{2}b\cos\left(fx+e\right)^{6}+10\,ab^{2}\cos\left(fx+e\right)^{4}+b^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\right)+8\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{8\,f},\ \frac{\sqrt{-a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{4}+8\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}+3\,a^{2}b\cos\left(fx+e\right)^{2}+ab^{2}\right)}\right)+4\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2(e + fx)}\,\tan(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(46) = 92.
time = 0.58, size = 377, normalized size = 6.98

$$\frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)b+\sqrt{a+b}\,b\right)}{\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)^2-2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)\sqrt{a+b}+a-3b}\right)\operatorname{sgn}(\cos(fx+e))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 2*((sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))*b + sqrt(a + b)*b)/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a -
 3*b))*sgn(cos(f*x + e))/f
```

Mupad [B]

time = 5.49, size = 46, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{\cos\left(e+f x\right)^{2}}}}{f}-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f x\right)^{2}}}}{\sqrt{a}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] (a + b/cos(e + f*x)^2)^(1/2)/f - (a^(1/2)*atanh((a + b/cos(e + f*x)^2)^(1/2
)/a^(1/2)))/f
```

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 457, 85, 65, 214}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a+b)\text{Subst}\cdots}{\cdots}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right)}{f}$$

Mathematica [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

[Out] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(58) = 116.
time = 0.17, size = 570, normalized size = 8.14

method	result
default	$-\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\,\sqrt{4}\,\cos(fx+e)\left(2\sqrt{a}\,\ln\left(4\cos(fx+e)\sqrt{a}\,\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a(\cos}{(1+\cos(}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f*x+e)*(2*a^(1/2
)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos
(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(1/2
)+ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos
(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a+ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2
))*b-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/(cos(f*x+e)-1))*a-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
x+e)(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b)*(cos(f*x+e)-1)/sin(f*x+e)^2/((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b)^(1/2)

Maxima [C] Result contains complex when optimal does not.
time = 0.73, size = 3495, normalized size = 49.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

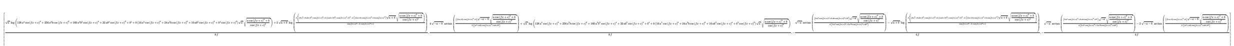
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/4*(a^(3/2)*log(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*s

qrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*
sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*
f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2) -
2*sqrt(a + b)*a*log(4*(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)
^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2
 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2
*a*b)*cos(2*f*x + 2*e))*(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 +
4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*
(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*
x + 2*e))*(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^2
+ 2*a*b + b^2)*abs(2*e^(2*I*f*x + 2*I*e) - 2)^2 + 4*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a + b)*abs(2*e^(2*I*
f*x + 2*I*e) - 2) + 4*a + 4*b)*sqrt(a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2 + 8*(a^2 + 2*a*b + b^2)*abs(2*e^
(2*I*f*x + 2*I*e) - 2))/abs(2*e^(2*I*f*x + 2*I*e) - 2)^2) - (a^(3/2) + 2*sq
rt(a)*b)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 +
16*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e
)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos
(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a
+ 2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a
*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^
2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos
(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^

```
2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*
x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*c
os(2*f*x + 2*e) + a))^2)) + 2*(a^(3/2) + sqrt(a)*b)*log(a^2*cos(2*f*x + 2*e
)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 +
 a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*a^(3/2)*sin(2*f*x + 2*e)*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) + sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f
*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x ...
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(60) = 120$.
time = 5.17, size = 1019, normalized size = 14.56



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a + b)*l
og(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x
 + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, -1/4*(
```

```
sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3
*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\cot(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(58) = 116.
time = 0.82, size = 402, normalized size = 5.74

$$-\frac{\left(\frac{4a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right|\right)-\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\sqrt{a+b}\right|\right)+\sqrt{a+b}\log\left(\left|\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(4*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + sqrt(
a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) - sqrt(a + b))) + sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*
(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(e+f\,x)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}\,f}-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/2*(2*a+b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*sec
(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 101, 162, 65, 214}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a + b)*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]*f) - (Cot[e +
f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{bf}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{2\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.36, size = 527, normalized size = 4.83

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2}-\frac{-2i\sqrt{a}\sqrt{a+b}fx+(2a+b)\log\left(1-e^{2i(e+fx)}\right)+\sqrt{a}\sqrt{a+b}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+\sqrt{a}\sqrt{a+b}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-2a\log\left(a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-b\log\left(a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)}{\sqrt{a+b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^2
- ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x))]
+ Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt[a +
b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a + b +
a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*E^((2*I)*
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2
]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(91) = 182$.

time = 0.13, size = 2528, normalized size = 23.19

method	result	size
default	Expression too large to display	2528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(cos(f*x+e)-1)*(4*(a+b)^(3/2)*a^(1/2)*cos(f*x+e)^2*ln(4*cos(f*x+e)*a
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*4^(1/2)*b+4*(a+b)^(3/2)*a^(3/
2)*cos(f*x+e)^2*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2))*4^(1/2)-4*(a+b)^(3/2)*a^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*4^(1/2)*b+5*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b
)^(1/2))*4^(1/2)*a^2*b+4*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*
4^(1/2)*a*b^2-5*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a^2*b-4*cos(f*x+e)^2*ln(-4
*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*
x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+
e)-1))*4^(1/2)*a*b^2+2*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*4^(1/2)*a-2*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*4^(1/2)*b-2*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*a-2*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*b-8*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-16*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(3/2)-2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1
/2)*a^3-ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1/2)*b^3+2*ln(-4*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*4^(1
/2)*a^3+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
```

```
+b)/(cos(f*x+e)-1))*4^(1/2)*b^3-8*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(3/2)+4*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^
(1/2)*b-4*(a+b)^(3/2)*a^(3/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2))*4^(1/2)+2*cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2
))*4^(1/2)*a^3+cos(f*x+e)^2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1/2)*b^
3-2*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a^3-cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*b
^3-5*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*4^(1/2)*a^2*b-4*ln(-2*(cos(f*x+e)
-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*
x+e)^2/(a+b)^(1/2))*4^(1/2)*a*b^2+5*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*4^(1/2)*a^2*b+4*ln(-4*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))
*4^(1/2)*a*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/(a+b)^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(95) = 190$.
time = 5.29, size = 1422, normalized size = 13.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^
2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 +
4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a
 + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^2 - 2*a - b
)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a + b)*cos(
f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f
*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^
3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + ((2*a + b)*cos(f*x +
e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a
 + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x
+ e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2
+ a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\,\cot^3{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(91) = 182.
time = 0.96, size = 574, normalized size = 5.27

$$\frac{\left(\frac{16\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\,(2a+b)\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{2\,(2a+b)\log\left(\left|-\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)(a+b)+\sqrt{a+b}\,(a-b)\right|\right)}{\sqrt{a+b}}+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}-\frac{2\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)(a-b)-(a+b)^{\frac{3}{2}}\right)}{\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}\right)^{2}-a-b}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*(2*
a + b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) + 2*(2*a + b)*log(ab
s(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) + sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b) - (a + b)^(3/
2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f}+\frac{(4a+3b)\cot}{}$$

```
[Out] -1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+
b)^(3/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/8*(4*a+3*b
)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f-1/4*cot(f*x+e)^4*(a+b*sec(f
*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4224, 457, 101, 156, 162, 65, 214}

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 12*a*b
+ 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f)
- (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+b}}\right.}{4j}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a}}{4}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a}}{4}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a}}{4}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+12ab+3b^2)\tanh}{8(a}$$

Mathematica [F]
time = 5.53, size = 0, normalized size = 0.00

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7345 vs. 2(139) = 278.
time = 0.15, size = 7346, normalized size = 45.63

method	result	size
default	Expression too large to display	7346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(145) = 290$.
time = 12.38, size = 2049, normalized size = 12.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^
2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f
*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)
, 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^
2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sq
rt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
```

```
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^2 + 11*a*b + 5*
b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/32*(8*((a^
2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^
2 + 2*a*b + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2
)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a
*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^
4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(4*((a^2 + 2*a*b + b^2)*c
os(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*
a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 -
 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-
a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) +
2*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b +
 b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,\cot^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(139) = 278.
time = 1.38, size = 866, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2*e
)^2 - (11*a + 9*b)/(a + b)) - 128*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/
sqrt(-a))/sqrt(-a) + 8*(8*a^2 + 12*a*b + 3*b^2)*arctan(-(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-
a - b))/((a + b)*sqrt(-a - b)) - 4*(8*a^2 + 12*a*b + 3*b^2)*log(abs(-(sqrt(
a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*
(3*a^2 - 2*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*ta
n(1/2*f*x + 1/2*e)^2 + a + b))^2*(7*a^2 + 10*a*b + 3*b^2)*sqrt(a + b) - 2*(
2*a^3 + 2*a^2*b - 3*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(a + b)))*sgn(cos(f*x + e
))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^6(e+fx)\,dx$

Optimal. Leaf size=219

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(a^3+5a^2b+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{16b^{5/2}f}$$

```
[Out] 1/16*(a^3+5*a^2*b+15*a*b^2-5*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))*a^(1/2)/f-1/16*(a-b)*(a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f
+1/24*(a-5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]

time = 0.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 489, 596, 537, 223, 212, 385, 209}

$$\frac{(a^3+5a^2b+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}+\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - b)*(a + 5*b)*Tan[e + f*
x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 5*b)*Tan[e + f*x]^3*S
qrt[a + b + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b\left(1+x^2\right)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5(a+b)+(-\cdots}{(1+x^2)\sqrt{a+\cdots}}\right.}{\cdots}$$

$$= \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\cdots}{\cdots}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-\cdots}{\cdots}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-\cdots}{\cdots}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-\cdots}{\cdots}$$

$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15a\cdots}{\cdots}$$

Mathematica [A]

time = 3.93, size = 263, normalized size = 1.20

$$-\frac{\left(16\sqrt{a}\,b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a^3+5a^2b+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8\sqrt{2}\,b^2f\sqrt{a+2b+a\cos(2e+2fx)}}-\frac{(9a^2+34ab-59b^2+4(3a^2+12ab-7b^2)\cos(2(e+fx))+(3a^2+14ab-33b^2)\cos(4(e+fx)))\sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)}{384b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -1/8*((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e +
f*x]^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2 + 3
4*a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2 + 14*
a*b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(384*b^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 2756, normalized size = 12.58

method	result	size
default	Expression too large to display	2756

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-cos(f*x+e)^5*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^4*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b+10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2-10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^
2-96*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^6*a*b^2+14*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*cos(f*x+e)^6*a*b^2+40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x
+e)^4*a*b^2-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^3+26*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3+33*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^3-26*cos(f*x+e)^3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b^3+30*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+
90*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-15*cos(f*x+e)^6*sin(f*x+e)*
```

```
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*cos(f*x+e)^6*sin(f*x
+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+33*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^2-30*cos(f*x+e)^6*sin(f*x+e)*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*b^3+15*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-8*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-14*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^2*b-40*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b^2-3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(
f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+e)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/cos(f
*x+e)^5/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Fricas [**A**]
time = 18.45, size = 1873, normalized size = 8.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(b)
*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
- 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*(a^3 + 5*a^2*b +
15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b
^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/19
2*(48*sqrt(a)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 - 3*(a^3 + 5*a^2*b + 15*a*b
^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^
3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(24*sqrt(a)*b^3*arctan(1
```

```
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^5 + 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(
a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,\tan^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e + f\,x)^6\,\sqrt{a + \frac{b}{\cos\left(e + f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{(a-3b}{}$$

```
[Out] -1/8*(a^2+6*a*b-3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/b^(3/2)/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/
f+1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]

time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 489, 596, 537, 223, 212, 385, 209}

$$-\frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8bf}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b\left(1+x^2\right)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(-a\right.}{\left(1+x^2\right)\sqrt{a+b}\right.}}{}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{}}{}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{}}{}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{}}{}$$

$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(a^2+6ab-3b^2\right)\tan}{}$$

Mathematica [A]
time = 2.93, size = 208, normalized size = 1.26

$$\frac{\left(8\sqrt{a}\,b\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \frac{\left(a^2+6ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4\sqrt{2}\,bf\sqrt{a+2b+a\cos(2e+2fx)}} + \frac{(a-b+(a-5b)\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)}{16bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
 - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin
```

```
[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]
*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.21, size = 2005, normalized size = 12.15

method	result	size
default	Expression too large to display	2005

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-2*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*sin(f*x+e)*cos(f*x+e)^4*a^2-12*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+
6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*a^2-2*cos(f*x+e)^4*sin(f*x+e)*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*cos(f*x+e)^4*sin(f*x+e)*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
```

```
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+16*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x
+e)*cos(f*x+e)^4*a*b+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2-5*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^4*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*cos(f*x+e)^4*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b-5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*co
s(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*
x+e)-1)/(b+a*cos(f*x+e)^2)/cos(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(151) = 302$.
time = 11.65, size = 1715, normalized size = 10.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a*b - 5*b^2
```

```
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x + e)^3*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b -
 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b
^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x
+ e)^3), -1/32*(8*sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b
- 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4) - 4*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), -1/16*(4*
sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*
arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))
)*cos(f*x + e)^3 - 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\,\tan^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}+\tan(e+fx)\,\sqrt{\ }$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(a-b)*
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*(a+b+b
*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 489, 537, 223, 212, 385, 209}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\,\tan^2(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+b+(-a+}{(1+x^2)\sqrt{a+b}}\right.}{} \\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+}}\right.}{} \\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x\right.}{} \\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\tanh^{-1}\left(\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.62, size = 526, normalized size = 4.46

$$
\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{-2\sqrt{a}\sqrt{b}fx+i\sqrt{a}\sqrt{b}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-i\sqrt{a}\sqrt{b}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-a\log\left(\frac{2\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{(a-b)\left(1+e^{2i(e+fx)}\right)}\right)+b\log\left(\frac{2\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{(a-b)\left(1+e^{2i(e+fx)}\right)}\right)}{\sqrt{b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}f\sqrt{a+2b+a\cos(2e+2fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*
x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((2*I)
*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*
```

```
E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((
2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 1331, normalized size = 11.28

method	result	size
default	Expression too large to display	1331

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*
x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+sin(f*x+e)*cos(f*x+
e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b-4*sin(f*x+e)*cos(f*x+
e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*
(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*sin(f*x
+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*
x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(
```

```
a+b))^(1/2)*b)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)/cos(f*x+e)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(106) = 212$.
time = 5.23, size = 1561, normalized size = 13.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
```

```
e) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 +
 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2
)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(b*f*cos(f*x + e)), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a
- b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2{(e+fx)}}\,\tan^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(e+f\,x\right)}^2\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4213, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\right.}{f}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.}{}$$
$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e\right.}}\right.}{f}$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 588, normalized size = 7.44

method	result
default	$-\frac{\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{(\cos(fx+e)-1)\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}}{\sin(fx+e)}, \sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^{2}+6ab-b^{2}}{(a+b)^{2}}}\right)a+\mathrm{EllipticF}\left(\frac{(\cos(fx+e)}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f*2^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a+EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*Ellipti
cPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)*sin(f*x+e)^2*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)/(cos(f*x
+e)-1)/(b+a*cos(f*x+e)^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(71) = 142$.

time = 6.88, size = 1293, normalized size = 16.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2{(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-cot(f*x+e)
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4226, 2000, 486, 12, 385, 209}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+b}}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+}}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x,\right.}{}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b}}{f}$$

Mathematica [A]
time = 0.66, size = 130, normalized size = 1.88

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}+\sqrt{2}\sqrt{a}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)\right)}{\sqrt{a+b}\,f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/S
qrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1004, normalized size = 14.55

method	result
default	$-\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{\cos(fx+e)^2}}\cos(fx+e)\left(\text{EllipticF}\left(\frac{(\cos(fx+e)-1)\sqrt{\frac{2i\sqrt{a}\sqrt{b}+a-b}{a+b}}}{\sin(fx+e)},\sqrt{-\frac{4ia^{\frac{3}{2}}\sqrt{b}-4i\sqrt{a}\,b^{\frac{3}{2}}-a^2+6ab-b^2}{(a+b)^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(EllipticF((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)*sin(f*x
+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a-2*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*a+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*
x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(
1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x+e)^
2)/sin(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(65) = 130$.
time = 5.53, size = 528, normalized size = 7.65

$$\left[\frac{\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\,\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\,\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)\sin\left(fx+e\right)-8\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{8\,f\sin\left(fx+e\right)},\ \frac{\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\,\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-4\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{4\,f\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^{2}{\left(e + fx \right)}}\cot^{2}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}-\frac{\cot^3(e+fx)\sqrt{\cdots}}{\cdots}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/3*(3*a+2*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 486, 597, 12, 385, 209}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\,(1+x^2)}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{b-3(a+b)}{x^2(1+x^2)\sqrt{a-}}\right.}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx}{3(}$$

Mathematica [A]
time = 0.78, size = 176, normalized size = 1.54

$$-\frac{\sqrt{2}\,\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{3/2}}{a+b}-3\csc(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\left(1+\frac{\sqrt{a}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)}{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)\right)}{3f\sqrt{a+2b+a\cos(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a +
 b - a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e +
 f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))))/(f*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 3855, normalized size = 33.82

method	result	size
default	Expression too large to display	3855

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-6*2^(1/2)*((I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*co
s(f*x+e)^3*sin(f*x+e)*a*b+3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^2+3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2
-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*E
llipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+3*cos(f*x+e)^2*sin(f*x+e)*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
```

```
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+3*2^(1/2)*((I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2
)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b+6*cos(f*x+e)*sin(f*x+e)*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a^2+6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-3*sin(f*x+e
)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-3*sin(f*x+e
)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+4*cos(f*x+e
)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^4*a*b+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.

time = 7.45, size = 664, normalized size = 5.82

$$\left[\frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5a^4-14a^3b+5a^2b^2\right)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32\left(a^4-7a^3b+7a^2b^2-ab^3\right)\cos(fx+e)^2-8\left(16a^3\cos(fx+e)^7-24\left(a^3-a^2b\right)\cos(fx+e)^5+2\left(5a^3-14a^2b+5ab^2\right)\cos(fx+e)^3-\left(a^3-7a^2b+7ab^2-b^3\right)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)\sin(fx+e)+8\left((4a+3b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left((a+b)f\cos(fx+e)^2-(a+b)f\right)\sin(fx+e)},\ -\frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)\sin(fx+e)-4\left((4a+3b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left((a+b)f\cos(fx+e)^2-(a+b)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e)
^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*
a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*b)*cos(
f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1/12*(3*((
a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*
a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sec^2(e+fx)}\cot^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f}-$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-1/15*(15*a
^2+25*a*b+8*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/15*(-4*b
-5*a)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b
+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 486, 597, 12, 385, 209}

$$-\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)^2*f) - ((b - 5*(a + b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(15*(a + b)*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(
5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{b-5(a+}{x^4(1+x^2)\sqrt{a}}\right.}{}$$

$$= -\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e}{}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(}{}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(}{}$$

$$= -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(}{}$$

$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+25ab+8b}{}$$

Mathematica [A]
time = 1.76, size = 178, normalized size = 1.07

$$-\frac{\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}} - \frac{\cot(e+fx)\left(23a^2+40ab+15b^2-(11a^2+21ab+10b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}{15(a+b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e
+ 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b +
```

```
10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e + f*x
]^2])/(15*(a + b)^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 8605, normalized size = 51.53

method	result	size
default	Expression too large to display	8605

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(157) = 314.
time = 15.23, size = 890, normalized size = 5.33

$$\left[\frac{15\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b^2)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32(a^4-7a^3b+7a^2b^2-ab^3)\cos(fx+e)^2+8\left(16a^3\cos(fx+e)^7-24(a^3-a^2b)\cos(fx+e)^5+2(5a^3-14a^2b+5ab^2)\cos(fx+e)^3-(a^3-7a^2b+7ab^2-b^3)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)\sin(fx+e)-8\left((23a^2+40ab+15b^2)\cos(fx+e)^5-(35a^2+59ab+20b^2)\cos(fx+e)^3+(15a^2+25ab+8b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{120\left((a^2+2ab+b^2)f\cos(fx+e)^4-2(a^2+2ab+b^2)f\cos(fx+e)^2+(a^2+2ab+b^2)f\right)\sin(fx+e)}, \frac{15\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^3\cos(fx+e)^5-8(a^3-ab)\cos(fx+e)^3+(a^3-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^6-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^4\right)\sin(fx+e)}\right)\sin(fx+e)-4\left((23a^2+40ab+15b^2)\cos(fx+e)^5-(35a^2+59ab+20b^2)\cos(fx+e)^3+(15a^2+25ab+8b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{60\left((a^2+2ab+b^2)f\cos(fx+e)^4-2(a^2+2ab+b^2)f\cos(fx+e)^2+(a^2+2ab+b^2)f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b + 15*b^2)
*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*
a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*b + b^2)*cos
```

```
(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 15*b^2)*cos(f*x +
e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*a*b + 8*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 +
2*a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec^2(e + fx)}\,\cot^6(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e + f\,x)^6\,\sqrt{a + \frac{b}{\cos\left(e + f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.389 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=135

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec \ldots\right.}{5b^2 \ldots}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f-1/5*(a+2*b)*(a+b*sec(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*sec(f*x+e)^2)^
(7/2)/b^2/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 90, 52, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) - ((a + 2*b)*(a + b
*Sec[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Sec[e + f*x]^2)^(7/2)/(7*b^2*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^{3/2}}{b}+\frac{(a+bx)^{3/2}}{x}+\frac{(a+bx)^{5/2}}{b}\right)dx, x, \sec^2(e+\right.}{2f}$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}+$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{(a}{}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)(a}{}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)(a}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx}}{f}$$

Mathematica [F]
time = 3.09, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2605 vs. 2(115) = 230.
time = 0.23, size = 2606, normalized size = 19.30

method	result	size

default	Expression too large to display	2606

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)

[Out] 1/420/f*4^(1/2)*(cos(f*x+e)-1)^3*(-210*cos(f*x+e)^7*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b^2-945*cos(f*x+e)^7*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b^3-1575*cos(f*x+e)^7*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^4-1155*cos(f*x+e)^7*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^5-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^4-315*cos(f*x+e)^7*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^6+210*cos(f*x+e)^7*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b^2+945*cos(f*x+e)^7*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b^3+1575*cos(f*x+e)^7*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^4+1155*cos(f*x+e)^7*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^5+315*cos(f*x+e)^7*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^6-70*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^5*(a+b)^(7/2)*b^4-70*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4*(a+b)^(7/2)*b^4+84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*(a+b)^(7/2)*b^4+84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(7/2)*b^4-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(7/2)*b^4-30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^3+12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^7*(a+b)^(7/2)*a^4+12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^6*(a+b)^(7/2)*a^4+210*cos(f*x+e)^7*a^(3/2)*ln(4*cos

```
(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*b^3+96*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^7*(a+b)^(7/2)*a^3*b-1
96*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^7*(a+b)^(7/2)*a^2
*b^2-280*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^7*(a+b)^(7/
2)*a*b^3+96*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^6*(a+b)^
(7/2)*a^3*b-196*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^6*(a
+b)^(7/2)*a^2*b^2-280*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)^6*(a+b)^(7/2)*a*b^3-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)^5*(a+b)^(7/2)*a^3*b+162*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)^5*(a+b)^(7/2)*a^2*b^2+98*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)^5*(a+b)^(7/2)*a*b^3-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)^4*(a+b)^(7/2)*a^3*b+162*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)^4*(a+b)^(7/2)*a^2*b^2+98*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)^4*(a+b)^(7/2)*a*b^3-48*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)^3*(a+b)^(7/2)*a^2*b^2+36*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*(a+b)^(7/2)*a*b^3-48*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(7/2)*a^2*b^2+36*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(7/2)*a*b^3-30*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(7/2)*a*b^3+210*cos(f*x+e)
^7*a^(5/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(
a+b)^(7/2)*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/cos(f*x+e)^4/b^2/(a+b)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(120) = 240$.
time = 28.51, size = 557, normalized size = 4.13

$$\left[\frac{105\,a^{\frac{3}{2}}b^{2}\cos\left(fx+e\right)^{6}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+b^{4}-8\left(16\,a^{3}\cos\left(fx+e\right)^{8}+24\,a^{2}b\cos\left(fx+e\right)^{6}+10\,ab^{2}\cos\left(fx+e\right)^{4}+b^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\right)-8\left(2\left(3\,a^{3}+21\,a^{2}b-70\,ab^{2}\right)\cos\left(fx+e\right)^{6}-\left(3\,a^{2}b-84\,ab^{2}+35\,b^{3}\right)\cos\left(fx+e\right)^{4}-15\,b^{3}-6\left(4\,ab^{2}-7\,b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{840\,b^{2}f\cos\left(fx+e\right)^{6}},\ \frac{105\sqrt{-a}\,ab^{2}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{4}+8\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}+3\,a^{2}b\cos\left(fx+e\right)^{2}+ab^{2}\right)}\right)\cos\left(fx+e\right)^{6}-4\left(2\left(3\,a^{3}+21\,a^{2}b-70\,ab^{2}\right)\cos\left(fx+e\right)^{6}-\left(3\,a^{2}b-84\,ab^{2}+35\,b^{3}\right)\cos\left(fx+e\right)^{4}-15\,b^{3}-6\left(4\,ab^{2}-7\,b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{420\,b^{2}f\cos\left(fx+e\right)^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
```

```
b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84
*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)
^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6), 1/
420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4*(2*(3*a^3
 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 35*b^3)*cos(
f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^5\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. $2(115) = 230$.
time = 3.32, size = 2026, normalized size = 15.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/105*(105*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
 + e))/sqrt(-a) - 2*(105*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^13*a^2*sgn(cos(f*x + e)) - 1575*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^12*sqrt(a + b)*a^2*sgn(cos(f*x + e)) + 70*(129*a^3 + 21*a^2*b - 96*
a*b^2 - 32*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b))^11*sgn(cos(f*x + e)) - 70*(387*a^3 - 417*a^2*b
 - 96*a*b^2 + 128*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
```

*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^10*sqrt(a + b)*sgn(cos(f*x + e)) + 7*(6525*a^4 - 11790*a^3*b - 2235*a^2*b^2 + 9088*a*b^3 - 1344*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*sgn(cos(f*x + e)) - 7*(5355*a^4 - 28290*a^3*b + 28995*a^2*b^2 - 3008*a*b^3 - 896*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*sgn(cos(f*x + e)) - 4*(1575*a^5 + 50505*a^4*b - 80115*a^3*b^2 - 4949*a^2*b^3 + 45920*a*b^4 - 5024*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) + 28*(1755*a^5 + 2325*a^4*b - 17655*a^3*b^2 + 20207*a^2*b^3 - 3232*a*b^4 + 768*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(f*x + e)) - 7*(7335*a^6 - 9300*a^5*b - 47430*a^4*b^2 + 98572*a^3*b^3 - 13353*a^2*b^4 - 55296*a*b^5 - 2944*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) + 7*(3225*a^6 - 20460*a^5*b + 9990*a^4*b^2 + 79028*a^3*b^3 - 136023*a^2*b^4 + 48000*a*b^5 + 256*b^6)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) + 14*(45*a^7 + 7125*a^6*b - 13230*a^5*b^2 - 8006*a^4*b^3 + 52305*a^3*b^4 - 35199*a^2*b^5 - 21472*a*b^6 - 2592*b^7)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) - 14*(375*a^7 + 2895*a^6*b - 11370*a^5*b^2 + 19246*a^4*b^3 + 547*a^3*b^4 - 43053*a^2*b^5 + 39072*a*b^6 + 3456*b^7)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) + 7*(315*a^8 + 1530*a^7*b - 5955*a^6*b^2 + 18508*a^5*b^3 - 21499*a^4*b^4 - 10598*a^3*b^5 + 58275*a^2*b^6 - 43392*a*b^7 - 3648*b^8)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (315*a^8 + 1050*a^7*b - 4515*a^6*b^2 + 28364*a^5*b^3 - 81403*a^4*b^4 + 133050*a^3*b^5 - 133053*a^2*b^6 + 53952*a*b^7 + 4992*b^8)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b)^7)/f

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^5 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.390 $\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=104

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}+\frac{(a+b\sec^2(e+fx)}{5bf}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/3*(a+b*sec(f*x+e)^2)^
(3/2)/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b/f-a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 81, 52, 65, 214}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(a+b\sec^2(e+fx))^{5/2}}{5bf}-\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (a*Sqrt[a + b*Sec
[e + f*x]^2])/f - (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) + (a + b*Sec[e + f*x]^
2)^(5/2)/(5*b*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x}\,dx,x,\sec(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{a\text{Subst}\Big(}{}$$
$$=-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b\sec}{}$$
$$=-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b\sec}{}$$
$$=\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [F]
time = 2.09, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3, x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2149 vs. 2(88) = 176.
time = 0.17, size = 2150, normalized size = 20.67

method	result	size
default	Expression too large to display	2150

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)

[Out] -1/60/f*4^(1/2)*(cos(f*x+e)-1)^3*(-34*(a+b)^(7/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+30*(a+b)^(7/2)*cos(f*x+e)^5*a^(5/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-40*(a+b)^(7/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2-34*(a+b)^(7/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b-40*(a+b)^(7/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+12*(a+b)^(7/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+2*(a+b)^(7/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+12*(a+b)^(7/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+2*(a+b)^(7/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+6*(a+b)^(7/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+30*(a+b)^(7/2)*cos(f*x+e)^5*a^(3/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^2+6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+6*(a+b)^(7/2)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-10*(a+b)^(7/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-10*(a+b)^(7/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+6*(a+b)^(7/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3+6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+6*(a+b)^(7/2)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-30*cos(f*x+e)^5*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b-135*cos(f*x+e)^5*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b^2-225*cos(f*x+e)^5*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^3-165*cos(f*x+e)^5*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^4-45*cos(f*x+e)^5*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^5+30*cos(f*x+e)^5*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6*b+135*cos(f*x+e)^5*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b^2+225*cos(f*x+e)^5*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)

```
-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^3+165*cos(f*x+e)^5*ln(-2*(
cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a
+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^4+45*cos(f*x+e)^5*ln(-2*(cos(f*x+e)-1)*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)
^2/(a+b)^(1/2))*a^2*b^5)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)
^6/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/b/(a+b)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(92) = 184.
time = 11.44, size = 471, normalized size = 4.53

$$\left[\frac{15\,a^{\frac{3}{2}}b\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+b^4+8\left(16\,a^3\cos\left(fx+e\right)^8+24\,a^2b\cos\left(fx+e\right)^6+10\,ab^2\cos\left(fx+e\right)^4+b^3\cos\left(fx+e\right)^2\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\right)+8\left(\left(3\,a^2-20\,ab\right)\cos\left(fx+e\right)^4+\left(6\,ab-5\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{120\,bf\cos\left(fx+e\right)^4},\ -\frac{15\sqrt{-a}\,ab\arctan\left(\frac{\left(8\,a^2\cos\left(fx+e\right)^4+8\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\left(2\,a^3\cos\left(fx+e\right)^4+3\,a^2b\cos\left(fx+e\right)^2+ab^2\right)}\right)\cos\left(fx+e\right)^4-4\left(\left(3\,a^2-20\,ab\right)\cos\left(fx+e\right)^4+\left(6\,ab-5\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{60\,bf\cos\left(fx+e\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2
+ 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4),
 -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*((3*a^2 -
 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(88) = 176$.
time = 1.76, size = 1431, normalized size = 13.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
+ e))/sqrt(-a) - 2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*a^2*sgn(cos(f*x + e)) - 15*(sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
))^8*(7*a^2 - 8*a*b - 4*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(15*a^3 - 2
1*a^2*b - 12*a*b^2 + 8*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) - 20*(21*a^3 -
 63*a^2*b + 60*a*b^2 - 4*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn(cos(f*x + e))
+ 2*(105*a^4 - 630*a^3*b + 1065*a^2*b^2 + 360*a*b^3 - 16*b^4)*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^
5*sgn(cos(f*x + e)) + 10*(21*a^4 + 42*a^3*b - 303*a^2*b^2 + 336*a*b^3 + 4*b
^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))^4*sqrt(a + b)*sgn(cos(f*x + e)) - 20*(21*a^5 - 21*a^4*b -
69*a^3*b^2 + 225*a^2*b^3 + 4*a*b^4 + 8*b^5)*(sqrt(a + b)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)
) + 20*(15*a^5 - 39*a^4*b + 57*a^3*b^2 + 87*a^2*b^3 - 244*a*b^4 - 28*b^5)*(
sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 5*(21*a^6 - 60*a^5*b + 222*a^
4*b^2 - 172*a^3*b^3 - 491*a^2*b^4 + 848*a*b^5 + 96*b^6)*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
```

```
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(co
s(f*x + e)) + (15*a^6 - 60*a^5*b + 390*a^4*b^2 - 980*a^3*b^3 + 1543*a^2*b^4
 - 984*a*b^5 - 132*b^6)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))*sqrt(a + b) + a - 3*b)^5)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.391 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 272, 52, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right.}{2f}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subst}\left(\int\right.}{}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.30, size = 84, normalized size = 1.08

$$\frac{2b\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{a\cos^2(e+fx)}{b}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f\sqrt{1+\frac{a\cos^2(e+fx)}{b}}\,(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*Se
c[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos[2
*(e + f*x)]))
```

Maple [A]
time = 0.02, size = 77, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{a+b\left(\sec^2(fx+e)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2(fx+e)\right)}}{\sec(fx+e)}\right)\right)}{f}$
default	$\frac{\frac{\left(a+b\left(\sec^2(fx+e)\right)\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{a+b\left(\sec^2(fx+e)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2(fx+e)\right)}}{\sec(fx+e)}\right)\right)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*(a+b*sec(f*x+e)^2)^(3/2)+a*((a+b*sec(f*x+e)^2)^(1/2)-a^(1/2)*ln((2
*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(69) = 138$.
time = 6.26, size = 399, normalized size = 5.12

$$\left[\frac{3\,a^{\frac{3}{2}}\cos(fx+e)^2\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4-8\left(16\,a^3\cos(fx+e)^8+24\,a^2b\cos(fx+e)^6+10\,ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)+8\left(4\,a\cos(fx+e)^2+b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\,f\cos(fx+e)^2},\ \frac{3\sqrt{-a}\,a\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2+4\left(4\,a\cos(fx+e)^2+b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\,f\cos(fx+e)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
 + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^
2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(66) = 132.
time = 1.12, size = 867, normalized size = 11.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 2/3*(3*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x + e
))/sqrt(-a) + 2*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))^5*(2*a*b + b^2)*sgn(cos(f*x + e)) - 3*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^4*(6*a*b - b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(6*a^2*b - 15*a*b
^2 - b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*
```

```
x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^2*b + 7*a*b^2 + b^3)*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(6*a^3*b - a^2*b^2 - 28*a*b^
3 - 5*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (6*a^3*b - 21*a^2*b^2 + 28*a*b^
3 + 7*b^4)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) + a - 3*b)^3)/f
```

Mupad [B]
time = 7.16, size = 66, normalized size = 0.85

$$\frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{3\,f}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sqrt{a}}\right)}{f}+\frac{a\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] (a + b/cos(e + f*x)^2)^(3/2)/(3*f) - (a^(3/2)*atanh((a + b/cos(e + f*x)^2)^
(1/2)/a^(1/2)))/f + (a*(a + b/cos(e + f*x)^2)^(1/2))/f
```

3.392 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(3/2)*arctanh((a+
b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f+b*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4224, 457, 86, 162, 65, 214}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((a + b)^(3/2)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f + (b*Sqrt[a + b*Sec[e + f*
x]^2])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2\right.}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+\right.}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2}\right.}{bf}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\sqrt{\right.}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.73, size = 506, normalized size = 5.56

$$\frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{2b}{1+e^{2i(e+fx)}}+\frac{-2ia^{3/2}fx+2(a+b)^{3/2}\log\left(1-e^{2i(e+fx)}\right)+a^{3/2}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+a^{3/2}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-2a\sqrt{a+b}\log\left(a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-2b\sqrt{a+b}\log\left(a+b+ae^{2i(e+fx)}+be^{2i(e+fx)}+\sqrt{a+b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)(a+b\sec^2(e+fx))^{3/2}}{f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a^(
3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((
2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^
((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2423 vs. 2(77) = 154.
time = 0.12, size = 2424, normalized size = 26.64

method	result	size
default	Expression too large to display	2424

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(cos(f*x+e)-1)^
3*(2*cos(f*x+e)*(a+b)^(7/2)*a^(5/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2))+2*cos(f*x+e)*(a+b)^(7/2)*a^(3/2)*ln(4*cos(f*x+e)*a^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(7/2)*a*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(7/2)*b^2+2*a*b*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(7/2)+2*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(7/2)+cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^6+3*cos(
```

f*x+e)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^5*b+6*cos(f*x+e)*ln(-2*(cos(f
x+e)-1)(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/s
in(f*x+e)^2/(a+b)^(1/2))*a^4*b^2+11*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b
)^(1/2))*a^3*b^3+12*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b^4
+6*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a*b^5+cos(f*x+e)*ln(-2*(
cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)
^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a
+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^6-cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^6-6*cos(f*x+e)*l
n(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+co
s(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(
f*x+e)-1))*a^5*b-15*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^4*b^2-20*cos(f*x+e)*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*
a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1)
)*a^3*b^3-15*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^2*b^4-6*cos(f*x+e)*ln(-4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a*b^5-
cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/(cos(f*x+e)-1))*b^6+3*cos(f*x+e)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2)
)*a^5*b+9*cos(f*x+e)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^4*b^2+9*cos(f*x
+e)*ln(-4*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
x+e)(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-c
os(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^3*b^3+3*cos(f*x+e)*ln(-4*(cos(f*
x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/si
n(f*x+e)^2/(a+b)^(1/2))*a^2*b^4)*4^(1/2)/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(

```
1+cos(f*x+e))^2)^(3/2)/(a+b)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(80) = 160$.
time = 6.30, size = 1139, normalized size = 12.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(3/2)
*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x +
e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*(4*(a +
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*b*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*
cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b
^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b
 + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x +
e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e
)^4 - 2*cos(f*x + e)^2 + 1)) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*(a + b)*sqrt(-a - b)*ar
ctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
```

```
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 4*b*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 718 vs. 2(77) = 154.
time = 4.09, size = 718, normalized size = 7.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x +
e))/sqrt(-a) - (a + b)^(3/2)*log(abs(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos(f*x
 + e)) + (a + b)^(3/2)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos(f*x + e)
) + (a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)
))*sgn(cos(f*x + e))/sqrt(a + b) - 8*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*b^2*sgn(cos(f*x + e)) +
sqrt(a + b)*b^2*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) + a - 3*b))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.393 $\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=114

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}-\frac{(a+b)\cot^2(e$$

```
[Out] -a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a-b)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-1/2*(a+b)*cot(f*x+e)^2*(a
+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 100, 162, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2/Sqrt[a]])/f) + ((2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*f) - ((a + b)*
Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^{3/2}}{x(-1+x^2)^2}\,dx,x,\sec(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{(-1+x)^2x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$
$$=-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\text{Subst}\left(\int\frac{a^2\cdots}{(-1+x\cdots}\right.}{\cdots}$$
$$=-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{a^2\text{Subst}\left(\int\frac{\cdots}{x\sqrt{\cdots}}\right.}{\cdots}$$
$$=-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{a^2\text{Subst}\left(\int\frac{\cdots}{-\frac{a}{b}\cdots}\right.}{\cdots}$$
$$=-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a-b)\sqrt{a+b}\,\tan\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.10, size = 622, normalized size = 5.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E^(
(2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b^2)
*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x
)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt
[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^
2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]
```

```
*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec
[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(96) = 192$.
time = 0.17, size = 1609, normalized size = 14.11

method	result	size
default	Expression too large to display	1609

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(cos(f*x+e)-1)^2*cos
(f*x+e)^3*(-4*(a+b)^(3/2)*a^(3/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*cos(f*x+e)+4*a^(3/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)+2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a+2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*b-2*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*c
os(f*x+e)*a^3-3*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^2*b+ln(-2
*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+
b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)
*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*b^3+2*ln(-4*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a^3
+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(
cos(f*x+e)-1))*cos(f*x+e)*a^2*b-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*b^3+2*ln(-2*(cos(f*x
+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin
(f*x+e)^2/(a+b)^(1/2))*a^3+3*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a^2*b-ln(
-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^3-2*a^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)
```

```
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))-3*a^2*ln(-4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b
+b^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1
/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/(cos(f*x+e)-1)))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/
(a+b)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(100) = 200.
time = 6.36, size = 1380, normalized size = 12.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^
4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b
+ b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a +
b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x +
e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*
((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*cos(f*x + e)^2
- a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^
2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)
^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(f*cos(f*x + e)^2
 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
```

```
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a -
b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1
/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 +
(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a - b)*cos(f*x
 + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*
x + e)^2 + a*b + b^2)))/(f*cos(f*x + e)^2 - f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.394 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}\,f}+\frac{(4a-b)\cot^2(e\cdot$$

[Out] a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+4*a*b-b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/8*(4*a-b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*(a+b)*cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4224, 457, 100, 156, 162, 65, 214}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*Sqrt[a + b]*f) +
((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - ((a + b)*Cot[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^3x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{2a^2}{(-1+x}\right.}{}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+}{}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+}{}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+}{}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+4ab-b^2)\tanh}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.14, size = 684, normalized size = 4.30

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e +
f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)*(e
 + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]*f*x
 + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt[a +
b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((2*I)*(e +
 f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) +
```

```
 a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*(e + f*x))
+ b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4954 vs. $2(137) = 274$.

time = 0.13, size = 4955, normalized size = 31.16

method	result	size
default	Expression too large to display	4955

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/f*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(cos(f*x+e)-1)^2*cos
(f*x+e)^3*(ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a
+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*b^4-8*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/
2))*cos(f*x+e)^2*a^4+ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^2*b^4+
8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(c
os(f*x+e)-1))*cos(f*x+e)^2*a^4-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*b^4-8*ln(-2*(cos(f*
x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/si
n(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)*a^4+ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))
*cos(f*x+e)*b^4+8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a^4-ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)*b^4+20*
a^3*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-c
os(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b+15*a^2*ln(-2*(cos(f*x+e)-1)*(((b
```

```
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(
a+b)^(1/2))*b^2+2*b^3*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*a-20*a^3*ln(-4*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+
e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)
-1))*b-15*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(cos(f*x+e)-1))*b^2-2*b^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a-8*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*cos(f*x+e)^
3*a^4+8*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a^4-ln(-2*(cos(f*
x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/si
n(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*b^4+16*a^(3/2)*ln(4*cos(f*x+e)*a^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)-b^4*ln(-2*(cos(f*x+e)-1
)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+
e)^2/(a+b)^(1/2))-8*a^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))+b^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))+8*a^4*ln(-2*(cos(f*x+e)
-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*
x+e)^2/(a+b)^(1/2))-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/
2)*cos(f*x+e)^3*b+8*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)
*cos(f*x+e)*a-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*cos
(f*x+e)*b+20*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*a^3*b+15*ln(
-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(
a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+
e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*cos(f*x+e)^3*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(143) = 286$.
time = 9.27, size = 1897, normalized size = 11.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 +
 a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 + 4*a*b
 - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4
*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 -
 (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f
), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*co
s(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*cos(f*x + e)^4 - 2
*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)) - 2*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*
b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/32*(8*((
a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) + ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a
^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
```

```
 + 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e
)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/16*(4*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(-a)*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)
*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*c
os(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^2 + 7*a*b + b^2)*cos(f
*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
+ (a + b)*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.395 $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$

Optimal. Leaf size=290

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\tanh^{-1}\left(\frac{\sqrt{b}\ \tan(e}{\sqrt{a+b+b\tan}}\right.}{128b^{5/2}f}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/128*(3*a
^4+20*a^3*b+90*a^2*b^2-60*a*b^3-5*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*ta
n(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+17*a^2*b-55*a*b^2-5*b^3)*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-50*a*b-5*b^2)*(a+b+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan
(f*x+e)^5/f+1/8*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.37, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4226, 2000, 488, 596, 537, 223, 212, 385, 209}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\ \tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf}-\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{128b^2f}+\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\tanh^{-1}\left(\frac{\sqrt{b}\ \tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{128b^{5/2}f}+\frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3 + 17
*a^2*b - 55*a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 50*a*b - 5*b^2)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(192*b*f) + ((9*a + b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^
2])/(48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b(1+x^2)\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6((a+b)(8a-}{(1+x^2)\sqrt{a}}\right.}{}$$

$$= \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+f}{}$$

$$= \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9}{}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{128b^2f}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{128b^2f}$$

$$= -\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{128b^2f}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+20a^3b+9}{}$$

Mathematica [A]

time = 6.90, size = 353, normalized size = 1.22

$$-\frac{\left(128a^{3/2}b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{32\sqrt{2}\,b^2f(a+2b+a\cos(2e+2fx))^{3/2}}-\frac{(90a^3+498a^2b-1594ab^2-626b^3+(135a^3+759a^2b-2303ab^2+513b^3)\cos(2(e+fx))+2(27a^3+159a^2b-523ab^2-191b^3)\cos(4(e+fx))+9a^3\cos(6(e+fx))+57a^2b\cos(6(e+fx))-337ab^2\cos(6(e+fx))+15b^3\cos(6(e+fx)))\sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\tan(e+fx)}{12288b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -1/32*((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
+ f*x]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(S
qrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]
^3*(a + b*Sec[e + f*x]^2)^(3/2))/(Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e + 2*f*
x])^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 + 759*a
^2*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b - 523
*a*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2*b*Cos[
6*(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)])*Sec[e
+ f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.53, size = 3583, normalized size = 12.36

method	result	size
default	Expression too large to display	3583

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/f*sin(f*x+e)*(-301*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)
^7*a^2*b^2+455*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^3+3
01*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a^2*b^2-455*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^6*a*b^3-380*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a*b^3+380*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*cos(f*x+e)^4*a*b^3-768*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2
*b^2-15*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+15*cos(f*x
+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-136*cos(f*x+e)^3*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+540*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
```

2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^2*b^2-3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+3*
cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+78*cos(f*x+e)^5*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-78*cos(f*x+e)^4*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+120*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^3-120*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b^3+118*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*b^4-1
18*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^4+136*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^4-57*cos(f*x+e)^9*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+337*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^2*b^2-15*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b^3+57*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-337
*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+15*cos(f*x+e)
^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-9*sin(f*x+e)*cos(f*x+e)^8*
2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4+15*sin(f*x+e)*cos(f*x+e)
^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)
/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4+18*sin(f*x+e)*cos(f*x
+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a^4-30*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^4-48*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-360*sin(f*x+e)*cos(f*x+e)^8*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*a*b^3-60*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^

```
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*a^3*b+114*sin(f*x+e)*cos(f*x+e)^8*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Fricas [A]
time = 57.19, size = 2075, normalized size = 7.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
[Out] [1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*
b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*
x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(
9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a*b^3*cos(f*x +
 e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*
(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
```

```
7 - 2*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^
2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^3*f*cos(f*x + e)^7), 1/1536*(384*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x +
e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3
*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a
*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x
 + e)^7), 1/768*(192*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
- a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
 (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^7 + 3*(3*a^4 +
 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((
9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 -
122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.396 $\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=214

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(a-b)*
(a^2+10*a*b+b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(
3/2)/f+1/16*(a^2-8*a*b-b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/24*
(7*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b+b*tan(f*x+e)^2
)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.31, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 488, 596, 537, 223, 212, 385, 209}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6f}+\frac{(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a + b)*Tan[e + f*x]^3*Sqrt[a + b
 + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x
]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}+\frac{\text{Subst}\left(\int \frac{x^4((a+b)(6a-}{(1+x^2)\sqrt{a}}\right.}{}$$

$$=\frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f}+\frac{b\tan^5(e+f}{}$$

$$=\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}+\frac{(7a+}{}$$

$$=\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}+\frac{(7a+}{}$$

$$=\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}+\frac{(7a+}{}$$

$$=\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)\left(a^2+10ab\right.}{}$$

Mathematica [A]

time = 4.53, size = 258, normalized size = 1.21

$$\frac{\left(16a^{3/2}b\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4\sqrt{2}\,bf(a+2b+a\cos(2e+2fx))^{3/2}}+\frac{\left(9a^2-58ab+17b^2+4\left(3a^2-24ab-11b^2\right)\cos(2(e+fx))+\left(3a^2-38ab+3b^2\right)\cos(4(e+fx))\right)\sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{384bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] ((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b
 - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)
)/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a*b +
 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a*b +
3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e +
f*x])/(384*b*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 2757, normalized size = 12.88

method	result	size
default	Expression too large to display	2757

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*sin(f*x+e)*(-17*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2*b+17*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-22*co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+22*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-38*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^6*a^2*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^6*a*b^2-52*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*
b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*b^3-14*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^3-3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^5*b^3+14*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^3+54*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-54*cos(f*x
+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+c
os(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+21*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
```

```
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+27*cos(f*x+e)^6*sin(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^7*a*b^2-6*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
b^3+3*cos(f*x+e)^6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-96*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin
(f*x+e)*cos(f*x+e)^6*a^2*b+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+38
*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+52*cos(f*x+e)^5
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)^7*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^3-8*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*((
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(cos(f*x+e)-1)/(b+a*cos(f*x+e)^2)^2/c
os(f*x+e)^3/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [A]
time = 18.02, size = 1875, normalized size = 8.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(b)*
cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) +
 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*
a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 + 2*((3*a^2*b - 38*a*b^2 + 3*b^3)*co
s(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^5), -1/192*(48*a
^(3/2)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
+ (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f
*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3
)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) - 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b
```

```
^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^5), -1/96*(24*a^(3/2)*b^2*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f
*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2
*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*co
s(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
2*f*cos(f*x + e)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e + f\,x)^4\,\left(a + \frac{b}{\cos\left(e + f\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.397 $\int (a + b\sec^2(e + fx))^{3/2} \tan^2(e + fx)\, dx$

Optimal. Leaf size=166

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^2-6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{(5a$$

[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/8*(3*a^2
-6*a*b-b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2
)+1/8*(5*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f

Rubi [A]
time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 488, 596, 537, 223, 212, 385, 209}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{(3a^2-6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}\,f}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^2 - 6*a*b - b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2((a+b)(4}{(1+x^2)\sqrt{}}\right.}{}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+f}{}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+f}{}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+f}{}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.94, size = 703, normalized size = 4.23

$$\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(5a\left(1+e^{2i(e+fx)}\right)^2-b\left(1-6e^{2i(e+fx)}+e^{4i(e+fx)}\right)\right)}{\left(1+e^{2i(e+fx)}\right)^4}+\frac{-8a^{3/2}\sqrt{b}fx+4ia^{3/2}\sqrt{b}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-4ia^{3/2}\sqrt{b}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-3a^2\log\left(\frac{4\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{\left(3a^2-6ab-b^2\right)\left(1+e^{2i(e+fx)}\right)}\right)+6ab\log\left(\frac{4\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{\left(3a^2-6ab-b^2\right)\left(1+e^{2i(e+fx)}\right)}\right)+b^2\log\left(\frac{4\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{\left(3a^2-6ab-b^2\right)\left(1+e^{2i(e+fx)}\right)}\right)}{\sqrt{b}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2\sqrt{2}f(a+2b+a\cos(2e+2fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)*(
e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1 + E
^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]*Log
[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((2*I)*(
e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*Log[(4*(Sqr
t[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x)))
)] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 +
E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b +
 a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 2002, normalized size = 12.06

method	result	size
default	Expression too large to display	2002

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*(5*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+
e)*cos(f*x+e)^4*a^2+6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a*b+cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^2+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
```

```
pticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^4*a^2-12*cos(f
*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-2*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*b^2-16*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+5*cos(f*x+e)^5*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b+7*cos(f*x+e)^
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^3*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^2-7*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^2+2*cos(
f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(cos(f*x+e)-
1)/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(152) = 304.
time = 9.54, size = 1721, normalized size = 10.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b - b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*
a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*
b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^
4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f
*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + (3*a^2 - 6*a*b -
 b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)
^3), 1/32*(8*a^(3/2)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos
(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*
a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 - (3*a^2 - 6*a*b - b^2
)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(4*a^(3/2)*b*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e)))*cos(f*x + e)^3 + (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x +
e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.398 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+f\ldots}{}$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(a+
b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4213, 427, 537, 223, 212, 385, 209}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,d\right.}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\text{t}}{\sqrt{a+b+}}\right.}{f}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+b+}}\right.}{2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.16, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}\,e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)+ia^{3/2}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-3a\sqrt{b}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)-b^{3/2}\log\left(\frac{-2\sqrt{b}\left(-1+e^{2i(e+fx)}\right)f+2i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}{b(3a+b)\left(1+e^{2i(e+fx)}\right)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f\left(a+2b+a\cos(2e+2fx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.12, size = 1557, normalized size = 13.19

method	result	size
default	Expression too large to display	1557

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*
x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f
*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f
*x+e)^2*a*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*b^2+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+6*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
sin(f*x+e)*cos(f*x+e)^2*a*b+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((
```

```
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(b+a*cos(f*
x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(106) = 212$.
time = 4.54, size = 1547, normalized size = 13.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
```

```
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a+b)\cot(e+fx\dots}{}$$

```
[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(3/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-(a+b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 485, 537, 223, 212, 385, 209}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/f - ((a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1+x^2)\right)^{3/2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{\cdots}{(1+\cdots}\right.}{\cdots}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a^2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\cdots}{\sqrt{a\cdots}}\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.63, size = 410, normalized size = 3.69

$$\frac{\sqrt{2}\,e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{2i(a+b)}{-1+e^{2i(e+fx)}}+\frac{ia^{3/2}\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-ia^{3/2}\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-2\left(a^{3/2}fx+b^{3/2}\log\left(\frac{\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{b^2\left(1+e^{2i(e+fx)}\right)}\right)\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f\left(a+2b+a\cos(2e+2fx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x))) +
 (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^((2*I
)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[((Sqrt[
b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2
))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 1952, normalized size = 17.59

method	result	size
default	Expression too large to display	1952

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(sin(f*x+e)*cos(f
*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2^(1/2)*((I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)*b^2-2*cos(f*x+e)*sin(f*x
+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b
)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*a^2+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)*b^2+2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
```

```
(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+2
*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(
1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*b^2*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2)/sin(f*x+e)/(b+a*cos(f*x+e)^2)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(103) = 206.
time = 5.69, size = 1536, normalized size = 13.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e)
+ sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
```

```
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e))/(f*sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + b^(
3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*si
n(f*x + e) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e))/(f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(e+f\,x)^2\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}-\frac{(a+b)\cot^3(e+$$

```
[Out] a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-b)
*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*cot(f*x+e)^3*(a+b+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 485, 597, 12, 385, 209}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f) - ((a + b)*C
ot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1+x^2)\right)^{3/2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+f}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.36, size = 100, normalized size = 0.89

$$-\frac{2(a+b)\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{a\sin^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f(a+2b+a\cos(2(e+fx)))\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e + f
*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e +
f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.15, size = 2014, normalized size = 17.98

method	result	size
default	Expression too large to display	2014

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+3*cos(f*x+e)^2*sin(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-6*cos(f*x+e)^2*si
n(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a^2-3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1
/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+4*
cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
```

```
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(
-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f
*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x
+e)-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)^3
*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^2/sin(f*x+e)^3/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(104) = 208$.
time = 5.89, size = 632, normalized size = 5.64

$$\left[\frac{3\,\left(a\cos\left(fx+e\right)^{2}-a\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\,\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}-8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\,\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)\sin\left(fx+e\right)+8\,\left(4\,a\cos\left(fx+e\right)^{3}-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{24\,\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)},\ -\frac{3\,\left(a\cos\left(fx+e\right)^{2}-a\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\,\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-4\,\left(4\,a\cos\left(fx+e\right)^{3}-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{12\,\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3 - (3*a
- b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x
 + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
```

```
 + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 - f)*sin(f*x +
e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e + f\,x)^4\,\left(a + \frac{b}{\cos\left(e + f\,x\right)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.401 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f}+$$

[Out] -a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*a^2+10*a*b-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f+1/15*(5*a-b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/5*(a+b)*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 485, 597, 12, 385, 209}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f) - ((15*a^2 + 10*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*(a + b)*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*f) - ((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1+x^2)\right)^{3/2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} - \frac{(a+b)\cot^5}{}$$

$$= -\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} +$$

$$= -\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} +$$

$$= -\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} +$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+10ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.45, size = 139, normalized size = 0.84

$$\frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{3}{4}(a+2b+a\cos(2(e+fx)))^2\csc^2(e+fx)+\frac{5(a+b)^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{a\sin^2(e+fx)}{a+b}\right)}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)}{15(a+b)f(a+2b+a\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2, -1
/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))
/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 5850, normalized size = 35.45

method	result	size
default	Expression too large to display	5850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(155) = 310.
time = 15.88, size = 808, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*
b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b
```

```
)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*arctan(1/4*(8*
a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(
f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*b - 5*
b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*co
s(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^6\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}$$

[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-(a+2*b)*(a+b*sec(f*x+e)^2)^(1/2)/b^2/f

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4224, 457, 90, 65, 214}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) - ((a + 2*b)*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f) + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b^2*f)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}} + \frac{1}{x\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a-\cdots}}\right.}{\cdots}$$

$$= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{(a+\cdots}{\cdots}$$

Mathematica [F]
time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(77) = 154$.
time = 0.20, size = 358, normalized size = 4.02

method	result
default	$\frac{(\sin^2(fx+e))\left(2(\cos^4(fx+e))a^{\frac{5}{2}}+3\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\ \ln\left(4\cos(fx+e)\sqrt{a}\ \sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a+4\sqrt{a}\ \sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)^2*(2*cos(f*x+e)^4*a^(5/2)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*cos(f*x+e)^4*b^2+6*cos(f*x+e)^4*a^(3/2)*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
))*cos(f*x+e)^3*b^2+cos(f*x+e)^2*a^(3/2)*b+6*cos(f*x+e)^2*a^(1/2)*b^2-a^(1/
2)*b^2)/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-
1)/b^2/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(80) = 160$.
time = 6.40, size = 436, normalized size = 4.90

$$\left[\frac{3\sqrt{a}b^2\cos(fx+e)^2\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\cos(fx+e)^2+b^4-8\left(16a^3\cos(fx+e)^8+24a^2b\cos(fx+e)^6+10ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)-8\left(2\left(a^2+3ab\right)\cos(fx+e)^2-ab\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24ab^2f\cos(fx+e)^2}, \frac{3\sqrt{-a}b^2\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2-4\left(2\left(a^2+3ab\right)\cos(fx+e)^2-ab\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12ab^2f\cos(fx+e)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)
^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(77) = 154.
time = 2.02, size = 765, normalized size = 8.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(3*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b))^5 - 21*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) + 2*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(3*a - 17*b)
 + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^2*(5*a + 9*b)*sqrt(a + b) - 3*(sqrt(a + b)*tan(1/2*f*x +
```

```
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(3*a^2 - 18*a*
b - 37*b^2) - (9*a^2 + 10*a*b - 47*b^2)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))*sqrt(a + b) + a - 3*b)^3)/(f*sgn(cos(f*x + e)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+(a+b*sec(f*x+e)^2)^(1/2
)/b/f
```

Rubi [A]

time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4224, 457, 81, 65, 214}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}
\end{aligned}
$$

Mathematica [F]
time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(48) = 96$.
time = 0.16, size = 303, normalized size = 5.41

method	result
default	$-\frac{\left(\sin^2(fx+e)\right)\left(a^{\frac{3}{2}}\left(\cos^2(fx+e)\right)+\left(\cos^2(fx+e)\right)\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\ \ln\left(4\cos(fx+e)\sqrt{a}\ \sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}+4\cos(fx+\right.\right.}{f\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*sin(f*x+e)^2*(a^(3/2)*cos(f*x+e)^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2))*b+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+a^(1/2)*b)/cos(f*x
+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-1)/b/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(50) = 100$.
time = 4.83, size = 348, normalized size = 6.21

$$\left[\frac{\sqrt{a}\,b\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4+8\left(16\,a^3\cos(fx+e)^8+24\,a^2b\cos(fx+e)^6+10\,ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)+8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{8\,abf},\ -\frac{\sqrt{-a}\,b\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)-4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,abf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(48) = 96.

time = 1.03, size = 373, normalized size = 6.66

$$-\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right)}{\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)^2-2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)\sqrt{a+b}+a-3b}\right)}{f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a +
b) + sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b))/
(f*sgn(cos(f*x + e)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4224, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}\end{aligned}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [A]
time = 0.04, size = 42, normalized size = 1.27

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42

default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2(fx+e)\right)}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(28) = 56.
time = 4.50, size = 277, normalized size = 8.39

$$\left[\frac{\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4-8\left(16\,a^3\cos(fx+e)^8+24\,a^2b\cos(fx+e)^6+10\,ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)}{8\sqrt{a}\,f},\ \frac{\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)}{4\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 2
4*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(-a)*
arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*co
s(f*x + e)^2 + a*b^2))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(27) = 54$.
time = 0.68, size = 111, normalized size = 3.36

$$\frac{2 \arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\, f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*f*sgn(cos(f*x
+ e)))
```

Mupad [B]
time = 5.09, size = 27, normalized size = 0.82

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] -atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f)
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4224, 457, 88, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,\right.}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \surd\right.}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

Mathematica [F]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(58) = 116$.
time = 0.16, size = 376, normalized size = 5.37

method	result
default	$-\frac{\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\cos(fx+e)\sqrt{a+b}+\sqrt{\frac{b+a(\cos^2(fx+e))}{(1+\cos(fx+e))^2}}\sqrt{a+b}-\cos\right.}{\sin(fx+e)^2\sqrt{a+b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(cos(f*x+e)-1)*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2
/(a+b)^(1/2))*a^(1/2)+2*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2))*(a+b)^(1/2)-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*a^(1/2))*sin(f*x+e)^2/((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/(a+b)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(60) = 120$.
time = 4.01, size = 1071, normalized size = 15.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(
a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*co
s(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a
 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 +
 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*
x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sqrt(-a)*(a + b)
*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*c
os(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*
x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), -1/4*(sqr
t(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^
4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*a*sqrt(-a - b)*arctan(1/2*((2*a +
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^2 + a*b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(58) = 116.
time = 1.02, size = 403, normalized size = 5.76

$$-\frac{\frac{4\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right|\right)}{\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\sqrt{a+b}\right|\right)}{\sqrt{a+b}}+\frac{\log\left(\left|\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{\sqrt{a+b}}}{2f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + log(abs
(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*ta
n(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
)^2 + a + b) + sqrt(a + b)))/sqrt(a + b) - log(abs(-sqrt(a + b)*tan(1/2*f*x
 + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*
a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b
)))/sqrt(a + b) + log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt
(a + b))/(f*sgn(cos(f*x + e)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f}-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(}}{2(a+b)f}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-a
rctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*cot(f*x+e)^2*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 105, 162, 65, 214}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ((2*a + 3*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)*f) - (Cot[
e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2\right.}{2(a+b)f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+f\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e\right.}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+f}}{\sqrt{a+b}}\right.}{2(a+b)^{3/2}f}$$

Mathematica [F]
time = 5.20, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4244 vs. $2(98) = 196$.
time = 0.14, size = 4245, normalized size = 36.59

method	result	size
default	Expression too large to display	4245

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*
x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/si
n(f*x+e)^2/(a+b)^(1/2))-4*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2))*a-4*(a+b)^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4
*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-5*a^
(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f
*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/s
in(f*x+e)^2/(a+b)^(1/2))*b+5*a^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b-3*a^(1/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^2+3*a^(1/2)
*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^2-3
*a^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(co
s(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(
1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b
)/sin(f*x+e)^2/(a+b)^(1/2))*b^2+3*a^(1/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b^2+4*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*a+4*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2))*b+5*a^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b-5*a^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b+3*a^(1/2)*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/2))*b^2-3*a
^(1/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*
```

```
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)*a+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1))*b
^2-4*(a+b)^(3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*
x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a-4*(a+b)^(3/
2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)
*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-5*a^(3/2)*cos(f*x+e)^3*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/sin(f*x+e)^2/(a+b)^(1/
2))*b+5*a^(3/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos
(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f
*x+e)-1))*b-2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a+b)^(1/2)+cos(f*x+e)
*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(cos(f*x+e)-1
))+2*(a+b)^(3/2)*a^(3/2)*cos(f*x+e)^3+2*(a+b)^(3/2)*a^(1/2)*cos(f*x+e)*b+4*
(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a+4*(a+b)^(3/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(102) = 204$.
time = 5.54, size = 1630, normalized size = 14.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
```

```
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e
)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(
-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*
b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)
^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^3 + 2*a^2*b + a
*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a^2 + 2*a*b +
 b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*
x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
 ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2
 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4
*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^
3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/4*(2*
(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + ((
a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*
(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^
2 + a*b^2)) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)
*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^3 +
 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(98) = 196.

time = 1.08, size = 595, normalized size = 5.13

$$\frac{\frac{16 \arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}+\sqrt{a+b}}{2 \sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4 (2 a+3 b) \arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}}{\sqrt{-a-b}}\right)}{(a+b) \sqrt{-a-b}}+\frac{2 (2 a+3 b) \log\left(\left|-\left(\sqrt{a+b}\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}\right)(a+b)+\sqrt{a+b}(a-b)\right|\right)}{(a+b)^{\frac{3}{2}}}+\frac{\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}}{a+b}-\frac{2 \left(\left(\sqrt{a+b}\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}\right)(a-b)-(a+b)^{\frac{3}{2}}\right)}{\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+2 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a+b}\right)^{2}-a-b\right)(a+b)}}{8 f \mathrm{sgn}\left(\cos\left(f x+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*(2*a
+ 3*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) + 2*(2*a +
 3*b)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) +
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)/(a + b) - 2*((sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))
*(a - b) - (a + b)^(3/2))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)*(a + b)))/(f*sgn(cos(f*
x + e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{(8a^2+20ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f}+\frac{(4a+7b)\cot^2(e+ \ldots}{8 \ldots}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a
+b)^(5/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a+7*
b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f-1/4*cot(f*x+e)^4*(a+b*se
c(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]

time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4224, 457, 105, 156, 162, 65, 214}

$$-\frac{(8a^2+20ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}+\frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ((8*a^2 + 20*a*b
+ 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(5/2)
*f) + ((4*a + 7*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)^2*
f) - (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*(a + b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(\right.}{4(a+b)f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+}}{4(a+b)f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+}}{4(a+b)f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+}}{4(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{(8a^2+20ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2}}{\sqrt{a+}}\right.}{8(a+b)^{5/2}f}$$

Mathematica [F]
time = 7.92, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10440 vs. $2(144) = 288$.
time = 0.16, size = 10441, normalized size = 62.90

method	result	size
default	Expression too large to display	10441

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(150) = 300$.
time = 7.97, size = 2353, normalized size = 14.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*a*b^2)
*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*
b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^
4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 +
b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)
*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos
(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b + 15*a*b^2)*co
s(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2
)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 +
```

```
a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
+ e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(3*(2*a^3 +
 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/32*(8*((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*co
s(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2
)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*
a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/1
6*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3
+ 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqr
t(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2))
 + 2*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(144) = 288.

time = 1.38, size = 932, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*((a + b)*tan(1/2*f*x
 + 1/2*e)^2/(a^2 + 2*a*b + b^2) - (11*a + 17*b)/(a^2 + 2*a*b + b^2)) - 4*(8
*a^2 + 20*a*b + 15*b^2)*sqrt(a + b)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a
+ b)*(a - b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 128*arctan(-1/2*(sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 8*(8*a^2 + 20*a*b + 15*b^2)*arctan(-(
sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b)) + 4*(2*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))^3*(3*a^2 + 2*a*b - 4*b^2) - 7*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + 2*a*b + b^2)*sqrt
(a + b) - 2*(2*a^3 + 4*a^2*b - 3*a*b^2 - 5*b^3)*(sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + (5*a^3 + 19*
a^2*b + 23*a*b^2 + 9*b^3)*sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)^2*(a^2 + 2
*a*b + b^2)))/(f*sgn(cos(f*x + e)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=173

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+ \cdots}{}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(
1/2)-1/8*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.22, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 490, 596, 537, 223, 212, 385, 209}

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(4*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+7b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x,\right.}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b}}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b}}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b}}{4bf}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{}{\sqrt{a+}}\right.}{8b^{5/2}f}$$

Mathematica [A]
time = 4.67, size = 230, normalized size = 1.33

$$-\frac{\left(\frac{8b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\,\sec(e+fx)}{8\sqrt{2}\,b^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(3a+5b+3(a+3b)\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{32b^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -1/8*(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[
a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec
[e + f*x])/(Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*Cos[2
*(e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4*Tan[
e + f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.23, size = 1993, normalized size = 11.52

method	result	size
default	Expression too large to display	1993

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*(6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
Pi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^4*a^2+20*cos(f*x+e
)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos
(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((c
os(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+30*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*b^2-3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)
^4*a^2-10*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a*b-7*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*
```

```
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*b^2-16*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a
+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*cos(f*x+e)^5*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-9*cos(f*x+e)^5*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^2+9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*a*b-cos(f
*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-9*cos(f*x+e)^3*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b+9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*b^
2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/cos(f*x+e)^5/((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(159) = 318$.
time = 7.94, size = 1767, normalized size = 10.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x
 + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e
```

```
)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(2*a
*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sqrt(-a)*b^3*c
os(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a
^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^
3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
 e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(2*a*b^2
- 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/32*(8*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x + e)^
3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 +
4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*a*b^2 -
 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/16*(4*sqrt(a)*b^3*arctan(1/
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2
)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x +
 e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e
)^3 + 2*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+}}{2}$$

```
[Out] -1/2*(a+3*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)
/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/2*(a+b
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A]
time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 490, 537, 223, 212, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) -
 ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+3b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x\right.}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x\right.}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e}{\sqrt{a+b+b\,\mathrm{t}}}\right.}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx}{\sqrt{a+b+b\tan^2(e}}\right.}{2b^{3/2}f}$$

Mathematica [A]
time = 2.94, size = 196, normalized size = 1.63

$$\frac{\left(\frac{2b\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{2\sqrt{2}\,bf\sqrt{a+b\sec^2(e+fx)}} + \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{4bf\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^
2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2]*b
*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.17, size = 1322, normalized size = 11.02

method	result	size
default	Expression too large to display	1322

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+6*sin(f*x+e)*
cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*b-4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-cos(f
*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(cos(f*x+e)-1)/cos(f*x+e
)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(108) = 216$.
time = 4.92, size = 1597, normalized size = 13.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(sqrt(-a)*b
^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
+ e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*
a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*a*b*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(2*sqrt
(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e)))*cos(f*x + e) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/
4*(sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a^2 + 3*a*b)*sqrt(-b)*arctan(
```

```
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) - 2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
a*b^2*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 494, 223, 212, 385, 209}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}\,f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\right.}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{\sqrt{b}\,f}$$

Mathematica [F]
time = 2.93, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 404, normalized size = 5.05

method	result
default	$-\frac{2\sqrt{2}\sqrt{\frac{i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}-\cos(fx+e)a-b\right.}{(1+\cos(fx+e))(a+b)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticPi((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))-EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e
)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(72) = 144.
time = 3.89, size = 1325, normalized size = 16.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3
)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
```

```
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*c
os(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8
 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^
3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
 (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
))) + a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*c
os(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 2*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4213, 385, 209}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 0.14, size = 87, normalized size = 2.23

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 380, normalized size = 9.74

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}-\cos(fx+e)a-b\right)}{(1+\cos(fx+e))(a+b)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-2*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(35) = 70$.
time = 0.58, size = 1046, normalized size = 26.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e

```
)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4
*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a)) + 4*a + 4*b))/(sqrt(a)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(35) = 70$.
time = 3.21, size = 427, normalized size = 10.95

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\left(fx+e\right)^4+a^4-28\,a^3b+70\,a^2b^2-28\,ab^3+b^4-32\,(a^4-7\,a^3b+7\,a^2b^2-ab^3)\cos\left(fx+e\right)^2+8\,(16\,a^3\cos\left(fx+e\right)^7-24\,(a^3-a^2b)\cos\left(fx+e\right)^5+2\,(5\,a^3-14\,a^2b+5\,ab^2)\cos\left(fx+e\right)^3-(a^3-7\,a^2b+7\,ab^2-b^3)\cos\left(fx+e\right))\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)\right)}{8\,af}, -\frac{\arctan\left(\frac{\left(8\,a^2\cos\left(fx+e\right)^5-8\,(a^2-ab)\cos\left(fx+e\right)^3+(a^2-6\,ab+b^2)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\left(2\,a^3\cos\left(fx+e\right)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}\right)}{4\,\sqrt{a}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f

Rubi [A]
time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4226, 2000, 491, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-b}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e}{\sqrt{a+b+b\,t}}\right.}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx}}{(a+b)f}$$

Mathematica [A]
time = 0.22, size = 127, normalized size = 1.72

$$-\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+\sqrt{a}\csc(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\right)}{\sqrt{2}\sqrt{a}(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.19, size = 1865, normalized size = 25.20

method	result	size
default	Expression too large to display	1865

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
a+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)*cos(f*x+e)-2*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)*cos(
f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)
-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*
((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(
1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1
+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin
(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
```

```
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/cos(f*x+e)/si
n(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.
time = 3.87, size = 554, normalized size = 7.49

$$\left[-\frac{\sqrt{-a}(a+b)\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5a^2b^2)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32(a^4-7a^3b+7a^2b^2-ab^3)\cos(fx+e)^2-8\left(16a^3\cos(fx+e)^7-24(a^3-a^2b)\cos(fx+e)^5+2(5a^3-14a^2b+5ab^2)\cos(fx+e)^3-(a^3-7a^2b+7ab^2-b^3)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)\sin(fx+e)+8a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{8(a^2+ab)f\sin(fx+e)}, \frac{(a+b)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)\sin(fx+e)-4a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{4(a^2+ab)f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=119

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f}-\frac{\cot^3(e+fx)\sqrt{a+}}{3(a}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/3*(3*a+5*
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]

time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 491, 597, 12, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) +
 ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3a-5b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx\right.}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan}}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan}}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan}}{3(a+b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2}}{3(a+b)^2f}$$

Mathematica [A]
time = 1.95, size = 168, normalized size = 1.41

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(-a-2b+(2a+3b)\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.17, size = 5619, normalized size = 47.22

method	result	size
default	Expression too large to display	5619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(111) = 222.
time = 4.40, size = 758, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
- 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
+ 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x +
 e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f*cos
(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*((a^2 + 2*
a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f
*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e
```

```
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f
*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=172

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f}+\frac{(5a$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-1/15*(15*a
^2+40*a*b+33*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f+1/15*(5*a
+9*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a
+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 491, 597, 12, 385, 209}

$$-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}\,f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}+\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(15*(a + b)^3*f) + ((5*a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5a-9b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,d\right.}{5(a+b)f} \\
&= \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2 f} - \frac{\cot^5(e+fx)\sqrt{a+b+b}}{5(a+b)f} \\
&= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3 f} + \frac{(5a+9b)\,\mathrm{co}}{} \\
&= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3 f} + \frac{(5a+9b)\,\mathrm{co}}{} \\
&= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3 f} + \frac{(5a+9b)\,\mathrm{co}}{} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{}}{15(a+b)^3}
\end{aligned}
$$

Mathematica [A]

time = 4.49, size = 199, normalized size = 1.16

$$
-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}\,f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\left(23a^2+60ab+45b^2-(11a^2+26ab+15b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sec(e+fx)}{30(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*a*b
```

```
+ 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e +
f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 11267, normalized size = 65.51

method	result	size
default	Expression too large to display	11267

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(162) = 324.
time = 9.92, size = 1028, normalized size = 5.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b
 + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-
a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x
 + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3 + 94*a^2
*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
```

```
x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), 1/60*(15*(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a
^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.415 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(a+b)^2/a/b^2/f/(a+b*s
ec(f*x+e)^2)^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$,
Rules used = {4224, 457, 89, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + (a + b)^2/(a*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Sqrt[a + b*Sec[e + f*x]^2]/(b^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{2a}$$
$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b}}{}$$

Mathematica [F]
time = 5.18, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6592 vs. $2(78) = 156$.
time = 0.26, size = 6593, normalized size = 74.92

method	result	size
default	Expression too large to display	6593

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(81) = 162$.
time = 4.34, size = 484, normalized size = 5.50

$$\left[\frac{(ab^2\cos(fx+e)^2+b^3)\sqrt{a}\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\cos(fx+e)^2+b^4-8\left(16a^3\cos(fx+e)^8+24a^2b\cos(fx+e)^6+10ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)+8\left(a^2b+(2a^3+2a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{8\left(a^3b^2f\cos(fx+e)^2+a^2b^3f\right)}, \frac{(ab^2\cos(fx+e)^2+b^3)\sqrt{-a}\arctan\left(\frac{(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)+4\left(a^2b+(2a^3+2a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(a^3b^2f\cos(fx+e)^2+a^2b^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
, 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
```

```
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(a^
2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(78) = 156.
time = 2.44, size = 560, normalized size = 6.36

$$-\frac{\frac{\frac{\left(a^4 b\,\mathrm{sgn}(\cos(fx+e))+2a^3b^2\,\mathrm{sgn}(\cos(fx+e))+a^2b^3\,\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^3b^3}-\frac{a^4 b\,\mathrm{sgn}(\cos(fx+e))+2a^3b^2\,\mathrm{sgn}(\cos(fx+e))+a^2b^3\,\mathrm{sgn}(\cos(fx+e))}{a^3b^3}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}}-\frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(fx+e))}-\frac{4\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)^2-2\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}\right)\sqrt{a+b}+a-3b\right)b\,\mathrm{sgn}(\cos(fx+e))}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^4*b*sgn(cos(f*x + e)) + 2*a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(cos
(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^3*b^3) - (a^4*b*sgn(cos(f*x + e)) + 2
*a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(cos(f*x + e)))/(a^3*b^3))/sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt
(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))) - 4*(sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a
+ b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
+ 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*b)*b*sgn(cos(f*
x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.416 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(-a-b)/a/b/f/(a+b*sec(f
*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 79, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (a + b)/(a*b*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}\\
&= -\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.17, size = 187, normalized size = 2.97

$$
\frac{3(a+b)F_1\left(2;\frac{1}{2},\frac{3}{2};3;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\tan^4(e+fx)}{2f\left(a+b\sec^2(e+fx)\right)^{3/2}\left(6(a+b)F_1\left(2;\frac{1}{2},\frac{3}{2};3;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+\left(3aF_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(3;\frac{3}{2},\frac{3}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sin^2(e+fx)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^4)/(2*f*(a + b*Sec[e + f*x]^2)^(3/2)*(6*(a + b)*AppellF1[
2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1
[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[3, 3/2, 3/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(58) = 116$.
time = 0.16, size = 2371, normalized size = 37.63

method	result	size
default	Expression too large to display	2371

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(cos(f*x+e)-1)^2*(1+cos(f*x+e))^2*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3
*b^3-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4-((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2))*a*b^5-cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^5*b-2*cos(f*x+e)^5
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^4*b^2-cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2))*a^3*b^3-cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(
f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^5*b-2*cos
(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^4*b^2-cos(f*x+e)^4*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2))*a^3*b^3-2*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
```

```
))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*
a^4*b^2-4*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3-2*cos(f*x+e)
^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))*a^4*b^2-4*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+
4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b
^3-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+
e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4-cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3-2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2))*a^2*b^4-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos
(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a+
4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^5+2*cos(f*x+e)^3
*a^(5/2)*b^4+cos(f*x+e)*a^(9/2)*b^2+3*cos(f*x+e)*a^(7/2)*b^3+3*cos(f*x+e)*a
^(5/2)*b^4+cos(f*x+e)*a^(3/2)*b^5+3*cos(f*x+e)^5*a^(11/2)*b+3*cos(f*x+e)^5*
a^(9/2)*b^2+cos(f*x+e)^5*a^(7/2)*b^3+2*cos(f*x+e)^3*a^(11/2)*b+6*cos(f*x+e)
^3*a^(9/2)*b^2+6*cos(f*x+e)^3*a^(7/2)*b^3+cos(f*x+e)^5*a^(13/2))*cos(f*x+e)
^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(a+b)/(b+a*cos(f*x+e)^2)^4/sin(f
*x+e)^4/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. $2(57) = 114$.
time = 0.74, size = 2674, normalized size = 42.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e
), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 4*a*b*cos(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(1/2*arctan2(a*sin(4*f*x + 4*e
) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*
```

x + 2*e) + a))^2 - 4*(a^2 + a*b)*sin(2*f*x + 2*e)*sin(1/2*arctan2(a*sin(4*f
*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*
cos(2*f*x + 2*e) + a)) - 4*(a^2 + 2*a*b + (a^2 + a*b)*cos(2*f*x + 2*e))*cos
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a^2*cos(4*f*x + 4*e)^2 + a^
2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
+ 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*
f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x +
 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((b*cos(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b
)*cos(2*f*x + 2*e) + a))^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a))^2)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2 + 16
*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4
*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*
sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4
*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a*sin(2*f*x + 2*e)*s
in(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f
*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a*cos(2*f*x + 2*e) + a +
2*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)))*sqrt(a) + 4*sqrt(a^2*
cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2
*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2
+ 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*(a*cos(1/2
*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
e) + 2(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a))^2)) + (b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + b*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*log(4*sqrt(a^2*c
os(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*
f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 +
a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^
2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(
2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
 + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x +
 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(
2*f*x + 2*e) + a))^2 + 16*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2

```
+ 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 +
2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*
b)*cos(2*f*x + 2*e))^(1/4)*(a + b)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*
e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f
*x + 2*e) + a)) + 16*a^2 + 32*a*b + 16*b^2))*sqrt(a))/((a^2*b*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*b*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(
a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^
2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*co
s(2*f*x + 2*e))^(1/4)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(57) = 114$.
time = 3.43, size = 443, normalized size = 7.03

$$\left[-\frac{8\left(a^{2}+ab\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)^{2}-\left(ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+b^{4}+8\left(16\,a^{3}\cos\left(fx+e\right)^{8}+24\,a^{2}b\cos\left(fx+e\right)^{6}+10\,ab^{2}\cos\left(fx+e\right)^{4}+b^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\right)}{8\left(a^{3}bf\cos\left(fx+e\right)^{2}+a^{2}b^{2}f\right)},\ -\frac{4\left(a^{2}+ab\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)^{2}+\left(ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{4}+8\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}+3\,a^{2}b\cos\left(fx+e\right)^{2}+ab^{2}\right)}\right)}{4\left(a^{3}bf\cos\left(fx+e\right)^{2}+a^{2}b^{2}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 +
b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*
f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(55) = 110.
time = 1.33, size = 251, normalized size = 3.98

$$\frac{\frac{\frac{\left(a^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+a^2b\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^3b}-\frac{a^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+a^2b\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{a^3b}}{\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}}-\frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] (((a^3*sgn(cos(f*x + e)) + a^2*b*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^3*b) - (a^3*sgn(cos(f*x + e)) + a^2*b*sgn(cos(f*x + e)))/(a^3*b))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))))/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan\left(e+f\,x\right)^3}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)

[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)

3.417 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sec(f*x+e)^
2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 272, 53, 65, 214}

$$\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}\\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.96, size = 382, normalized size = 6.70

$$\frac{(a+2b+a\cos(2(e+fx)))^{3/2}\sec^2(e+fx)\left(-\frac{2}{b\sqrt{a+2b+a\cos(2(e+fx))}}+\frac{\sqrt{2}\,e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\sqrt{a}\,(a+4b)\left(1+e^{2i(e+fx)}\right)}{b\left(4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}+\frac{4ifx-2\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-2\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sec(e+fx)}{a^{3/2}}\right)}{16f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)
*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(e +
 f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)) + ((
4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)*(e +
f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*x]^2)^(3/
2))
```

Maple [A]
time = 0.02, size = 62, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{a\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{a^{\frac{3}{2}}}}{f}$	62
default	$\frac{\frac{1}{a\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^2\left(fx+e\right)\right)}}{\sec\left(fx+e\right)}\right)}{a^{\frac{3}{2}}}}{f}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a/(a+b*sec(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+
e)^2)^(1/2))/sec(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(51) = 102$.

time = 3.59, size = 418, normalized size = 7.33

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+b^4-8\left(16\,a^3\cos\left(fx+e\right)^8+24\,a^2b\cos\left(fx+e\right)^6+10\,ab^2\cos\left(fx+e\right)^4+b^3\cos\left(fx+e\right)^2\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\right)}{8\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}, \frac{4\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos\left(fx+e\right)^4+8\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\left(2\,a^3\cos\left(fx+e\right)^4+3\,a^2b\cos\left(fx+e\right)^2+ab^2\right)}\right)}{4\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*c
os(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*
a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^
3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(a^
3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4*(8*a^2
*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [A]
time = 7.33, size = 53, normalized size = 0.93

$$\frac{1}{af\sqrt{a+b\sec^2\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2\left(e+fx\right)}}{\sqrt{-a}}\right)}{af\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(a*f*sqrt(a + b*sec(e + f*x)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt
(-a))/(a*f*sqrt(-a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(49) = 98.
time = 0.98, size = 216, normalized size = 3.79

$$-\frac{\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}-\frac{1}{a\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}}-\frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((tan(1/2*f*x + 1/2*e)^2/(a*sgn(cos(f*x + e))) - 1/(a*sgn(cos(f*x + e))))/
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
```

```
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a +
b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))))/f
```

Mupad [B]
time = 5.56, size = 49, normalized size = 0.86

$$\frac{1}{a\,f\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] 1/(a*f*(a + b/cos(e + f*x)^2)^(1/2)) - atanh((a + b/cos(e + f*x)^2)^(1/2)/a
^(1/2))/(a^(3/2)*f)
```

3.418 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f}-\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4224, 457, 87, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - b/(a*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e\right.}{2a(a+b)f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx\right.}{2af}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+\right.}}{abf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{}{a(}$$

Mathematica [F]
time = 6.46, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9692 vs. $2(86) = 172$.
time = 0.20, size = 9693, normalized size = 96.93

method	result	size
default	Expression too large to display	9693

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(89) = 178.
time = 3.94, size = 1649, normalized size = 16.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)
*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^
2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8
 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x + e)^
2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2
 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x + e)^2 + a^2*
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*s
qrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*
cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^5 + 2*a^4*b + a^3*b
^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4*(4*(a^2*b + a
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a^2*b +
 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/
4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e
)^2 + a*b^2)) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*
```

```
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4
*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2
*b*cos(f*x + e)^2 + a*b^2)) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-a - b)*a
rctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)}{{\left(a+\frac{b}{{\cos(e+f\,x)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.419 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f}-\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/2*(2*a+5*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/2*(a-2*b)*b/a/(a+b)^2
/f/(a+b*sec(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/
2)
```

Rubi [A]
time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4224, 457, 105, 157, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((2*a + 5*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/2)*f) - ((a -
 2*b)*b)/(2*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - Cot[e + f*x]^2/(2*(
a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{3bx}{2}}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+\right.}{2(a+b)f}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx}}{\sqrt{a+b}}\right.}{2(a+b)^{5/2}f}$$

<u>**Mathematica**</u> [**F**]
time = 10.64, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19967 vs. $2(131) = 262$.
time = 0.67, size = 19968, normalized size = 130.51

method	result	size
default	Expression too large to display	19968

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(136) = 272$.
time = 6.88, size = 2443, normalized size = 15.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b
^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a
)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a
^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*cos(f*x
+ e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2
)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cos(f*x + e)
^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
- (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b
^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2
*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a
- b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - ((
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a
```

```
*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a)*log(12
8*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos
(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f
*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 -
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/8*(2*((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 +
2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b
- 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*
b^2 + 2*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 +
 a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*
x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/4*(((a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4
- (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^
2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a
*b^2)) - ((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 +
 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*co
s(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2
*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^
2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.420 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{(8a^2+28ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f}+\frac{b(4a^2+11a}{8a(a+b)^3f\sqrt{a+}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/8*(8*a^2+28*a*b+35*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+1/8*b*(4*a^2
+11*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)+1/8*(4*a+9*b)*cot(f*x+e
)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec(f*
x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4224, 457, 105, 156, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{b(4a^2+11ab-8b^2)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{(8a^2+28ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^4(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(4a+9b)\cot^2(e+fx)}{8f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ((8*a^2 + 28*a*b
+ 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(7/2)
*f) + (b*(4*a^2 + 11*a*b - 8*b^2))/(8*a*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]
^2]) + ((4*a + 9*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cot[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{5bx}{2}}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+\right.}{4(a+b)f}$$

$$= \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}(}{}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a}$$

$$= \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{}{4(a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.}{8(a+b)^{7/2}f}$$

Mathematica [F]
time = 14.19, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32564 vs. $2(187) = 374$.
time = 1.01, size = 32565, normalized size = 152.89

method	result	size
default	Expression too large to display	32565

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(194) = 388$.
time = 16.22, size = 3613, normalized size = 16.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 +
a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^
2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2
- 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x
+ e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*c
os(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 +
10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)) + ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 +
 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a
^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f
*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
```

*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*
b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 +
16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b +
6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a
^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b
- 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), 1/16*(((8*a^5 + 28*a^4*
b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^
5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b
- 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a +
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4
*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*co
s(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
+ 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a
^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 +
9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15
*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(
f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5
)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2
*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a
^2*b^5)*f), -1/32*(8*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f
*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*
b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b
- 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*arctan(
1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x +
e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b
+ 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*
cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^
2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b
^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*
sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 -
2*cos(f*x + e)^2 + 1)) + 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*
a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4
)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b

```
^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 +
 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5
*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5
*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/16*(4*((a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a
^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.421 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{(a+b)\tan^3}{abf\sqrt{a+b+b\,t}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1/2*(3*a+5
*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/2*(3
*a+2*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/b^2/f-(a+b)*tan(f*x+e)^3/a/
b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 481, 596, 537, 223, 212, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}+\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - ((3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]
^2]])/(2*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(a*b*f*Sqrt[a + b + b*Tan[e
+ f*x]^2]) + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*b^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a+2b)x^2\right)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{2ab^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{2ab^2f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{2ab^2f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e\cdot}{\sqrt{a+b+b\tan}}\right.}{2b^{5/2}f}$$

Mathematica [A]

time = 10.10, size = 247, normalized size = 1.44

$$-\frac{\left(\frac{2b^2\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}+\frac{a(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{4\sqrt{2}\,ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{(a+2b+a\cos(2(e+fx)))\left(3a^2+6ab+2b^2+(3a^2+4ab+2b^2)\cos(2(e+fx))\right)\sec^4(e+fx)\tan(e+fx)}{8ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
)/Sqrt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Si
n[e + f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^
3)/(Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e
+ f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f*x)])
*Sec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 3860, normalized size = 22.44

method	result	size
default	Expression too large to display	3860

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(8*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)
^2*2^(1/2)*b^(7/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)+7*arctanh(1/4*(co
s(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*cos(f*x+e)^2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2-7*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^
(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4
^(1/2))*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-4*EllipticF((cos(f*x+e)-1)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*2^
(1/2)*b^(7/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)+7*arctanh(1/4*(cos(f*x
+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*sin(f*x+e)*cos(f*x+e)^4*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2*b-7*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)
-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2
))*sin(f*x+e)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-8*cos(f*x+e)^3*b^(5/2)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+4*cos(f*x+e)^4*b^(5/2)*((2*I*a^(1/2)*b^(1/2
```

```
)+a-b)/(a+b))^(1/2)*a-8*cos(f*x+e)^3*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2+8*cos(f*x+e)^4*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2-8*cos(f*x+e)^5*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-4*
cos(f*x+e)^5*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+8*cos(f*x+e)
^2*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-2*cos(f*x+e)*b^(5/2)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+8*cos(f*x+e)^2*b^(3/2)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+6*cos(f*x+e)^4*a^3*b^(1/2)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)-6*cos(f*x+e)^5*a^3*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)-10*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*b^(3/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*a^2+12*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*s
in(f*x+e)*cos(f*x+e)^4*2^(1/2)*a^3*b^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)-4*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*b^(5/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*a+4*cos(f*x+e)^2*b^(7/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)-4*cos(f*x+e)^3*b^(7/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)*a-6*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*2^(1/2
)*a^3*b^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)+20*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*b^(5/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*a-10*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*b^(5/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(160) = 320$.
time = 8.49, size = 2001, normalized size = 11.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f
*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)*cos(f*
x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2
+ 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), -1/8*(2*((3*
a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(-b
)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e
))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^
4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(a^2*b^2 + (3*a^3*b + 4*
a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), 1/8*
(2*(a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + ((3*a
^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*
log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*
```

```
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(a^2*b^2 + (
3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x
 + e)), 1/4*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(
8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*c
os(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*co
s(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)
)) - ((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e)
)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*si
n(f*x + e))) + 2*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f
*x + e)^3 + a^2*b^4*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.422 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+f}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+arctanh(b^(
1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-(a+b)*tan(f*x+e)/a/b/
f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 481, 537, 223, 212, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) +
 ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
 - ((a + b)*Tan[e + f*x])/(a*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+ax^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(\right.}{abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(\right.}{af}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e}}\right.}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{b^{3/2}f}$$

Mathematica [A]
time = 4.54, size = 201, normalized size = 1.73

$$\frac{\left(\frac{b\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}\,abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(a+b)(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2abf\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a]
 + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*f*(
a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[
e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 2662, normalized size = 22.95

method	result	size
default	Expression too large to display	2662

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(4*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)*sin(f*x+e)*cos
(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a-2*EllipticF((cos
(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*sin(f*
x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a+4*Ellipt
icPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b)
)^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)
/(1+cos(f*x+e))/(a+b))^(1/2)-2*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(5/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)
*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e))/(a+b))^(1/2)-2*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))
^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/
(1+cos(f*x+e))/(a+b))^(1/2)*a^2+4*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*b^(1/2)*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^
(1/2)*a^2-2*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*b^(3/2)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/
```

```
2)*a+4*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)*sin(f*x+e)*2^(1/2
)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*a-2*b^(3/2)*cos(f*x+e)^3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+sin(f*x+e)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1
/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^2-sin(f*x+e)
*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1/2)-2*cos
(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2))*a^2-2*b^(5/2)*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+
2*b^(3/2)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-2*b^(1/2)*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*b^(5/2)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*b^(3/2)*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a+2*b^(1/2)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2+sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(cos(f*x+e)*4^(1
/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*4^(1/2))*a*b-sin(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*(cos(f*x+e)-
1)*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2))*a*b+2*a*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^(3/2))*sin(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(3/2)/b^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(108) = 216$.
time = 4.54, size = 1749, normalized size = 15.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x + e)^2 +
 a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 4*(a^3*cos(f*
x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*b^2*f*cos(
f*x + e)^2 + a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*
sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e))) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 +
a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*arcta
n(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
 b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2
*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f
*x + e))) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a^3*b^2*f*cos(f*x +
 e)^2 + a^2*b^3*f)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.423 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}+\frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+tan(f*x+e)
/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4226, 2000, 482, 385, 209}

$$\frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) + Tan[e + f*x]/(a*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+
\\
&= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+
\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(71) = 142.
time = 2.68, size = 169, normalized size = 2.38

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\operatorname{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{4a^{3/2}\sqrt{a+b}\,f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e
+ f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqrt[
a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(a^(3/2)
*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 569, normalized size = 8.01

method	result
default	$\frac{(b+a(\cos^2(fx+e)))\left(\sqrt{2}\sqrt{\frac{i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}\right.}{(1+\cos(fx+e))(a+b)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(
f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)/(cos(f*x+e)
-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2119 vs. $2(67) = 134$.
time = 0.67, size = 2119, normalized size = 29.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 2*a*cos(1/2*ar
ctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
 + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) + 2*(a*cos(1/2*arcta
n2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) +
2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a*cos(2*f*x + 2*e))*sin(1/2*arctan2(
a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(
a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x +
 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(
4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2
+ a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^
2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2
*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2
*e) + a))^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x +
2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*
a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4
*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(
a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*
cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a +
 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
 a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*
e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a
^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e)
+ 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x
+ 2*e) + a)) + 2*a + 4*b) - (cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*
b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*
cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4
*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)
*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4
```

```
*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2
 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*
(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*s
in(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*
f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x +
4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^
2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*
b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*
cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a^2*cos(1/2*arc
tan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e)
+ 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + a^2*sin(1/2*arctan2(a*sin(4*f*x +
4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2
*f*x + 2*e) + a))^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(67) = 134$.
time = 3.02, size = 579, normalized size = 8.15

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\,(a^{4}-a^{3}b)\cos\left(fx+e\right)^{6}+32\,(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2})\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\,(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3})\cos\left(fx+e\right)^{2}-8\,\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\,(a^{3}-a^{2}b)\cos\left(fx+e\right)^{5}+2\,(5\,a^{3}-14\,a^{2}b+5\,ab^{2})\cos\left(fx+e\right)^{3}-(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3})\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{8\,\left(a^{3}f\cos\left(fx+e\right)^{2}+a^{2}bf\right)},\ \frac{4\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos(fx+e)^{5}-8\,(a^{2}-ab)\cos(fx+e)^{3}+(a^{2}-6\,ab+b^{2})\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,\left(2\,a^{3}\cos(fx+e)^{4}-a^{2}b+ab^{2}-(a^{3}-3\,a^{2}b)\cos(fx+e)^{2}\right)\sin(fx+e)}\right)}{4\,\left(a^{3}f\cos\left(fx+e\right)^{2}+a^{2}bf\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
+ e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*co
s(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(e+f\,x\right)}^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.424 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+e
)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4213, 390, 385, 209}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e}{\sqrt{a+b+b\,t}}\right.}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.
time = 1.48, size = 168, normalized size = 2.18

$$\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\,\sqrt{a}\,b\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\sin(e+fx)\right)}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.00, size = 1007, normalized size = 13.08

method	result
default	$-\frac{(b+a(\cos^2(fx+e)))\left(\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(1+\cos(fx+e))(a+b)}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{a}\,\sqrt{b}-i\sqrt{a}\,\sqrt{\right.}{(1+\cos(fx+e))(a+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(b+a*cos(f*x+e)^2)*(2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*Ell
ipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*s
in(f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x
+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(
1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b)*sin(f*x+e)/(cos(f*x+e)-1)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(3/2)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. $2(73) = 146$.
time = 0.70, size = 2169, normalized size = 28.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e
) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*
x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2*(a*
b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a
+ 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e)
+ a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin
(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a
^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*co
s(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a
^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(
((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b)*sin(1/
2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*sin(2*f*x + 2*e) +
 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^
2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b
)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1
/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x
+ 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*
b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2
 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x +
2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*
f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a +
 4*b) - ((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x
 + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a + b
)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(
4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*(a^2*cos(4*f
*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x +
2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b
 + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin
```

```
(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2
 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2
*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*arctan2(a*
sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) + 4*a + 4*b))*sqrt(a))/((a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)
*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((a^3 + a^2*b)*cos(1
/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x +
 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + (a^3 + a^2*b)*sin(1/2*arctan
2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2
*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(73) = 146$.
time = 3.48, size = 632, normalized size = 8.21

$$\left[-\frac{8\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)}{8\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)},\ -\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{4\left(\left(a^{4}+a^{3}b\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b+a^{2}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec^{2}\left(e + f x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.425 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(1/2)/a/(a+b)^2/f
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 483, 597, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2}-\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((a - b
)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx,\right.}{a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2}}{a(a+b)^2 f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2}}{a(a+b)^2 f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2}}{a(a+b)^2 f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A]
time = 4.62, size = 182, normalized size = 1.53

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}\,a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(a+2b+a\cos(2(e+fx)))\,(a^2+2ab-b^2+(a^2+b^2)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4a(a+b)^2 f\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2 +
```

```
(a^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)^2*f
*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.16, size = 2836, normalized size = 23.83

method	result	size
default	Expression too large to display	2836

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(b+a*cos(f*x+e)^2)^2*(-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-4*2
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f
*x+e)*b^2+sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a^2+2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a*b+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e
)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e
)*b^2-2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
```

```
*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-2*2
^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*b^2*sin(f*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^2*sin(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF
((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f
*x+e)+2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+cos(f*x
+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*cos(f*x+e)^2*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)^3*((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/(a+b)^2/((2*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(115) = 230$.

time = 5.25, size = 776, normalized size = 6.52

$$\left[-\frac{\left(a^{2} b+2 a b^{2}+b^{3}+\left(a^{3}+2 a^{2} b+a b^{2}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{-a} \log \left(128 a^{4} \cos \left(f x+e\right)^{8}-256 \left(a^{4}-a^{3} b\right) \cos \left(f x+e\right)^{6}+32 \left(5 a^{4}-14 a^{3} b+5 a^{2} b^{2}\right) \cos \left(f x+e\right)^{4}+a^{4}-28 a^{3} b+70 a^{2} b^{2}-28 a b^{3}+b^{4}-32 \left(a^{4}-7 a^{3} b+7 a^{2} b^{2}-a b^{3}\right) \cos \left(f x+e\right)^{2}-8 \left(16 a^{3} \cos \left(f x+e\right)^{7}-24 \left(a^{3}-a^{2} b\right) \cos \left(f x+e\right)^{5}+2 \left(5 a^{3}-14 a^{2} b+5 a b^{2}\right) \cos \left(f x+e\right)^{3}-\left(a^{3}-7 a^{2} b+7 a b^{2}-b^{3}\right) \cos \left(f x+e\right)\right) \sqrt{-a} \sqrt{\frac{a \cos \left(f x+e\right)^{2}+b}{\cos \left(f x+e\right)^{2}}} \sin \left(f x+e\right)\right) \sin \left(f x+e\right)+8 \left(\left(a^{3}+a b^{2}\right) \cos \left(f x+e\right)^{3}+\left(a^{2} b-a b^{2}\right) \cos \left(f x+e\right)\right) \sqrt{\frac{a \cos \left(f x+e\right)^{2}+b}{\cos \left(f x+e\right)^{2}}}}{8 \left(\left(a^{5}+2 a^{4} b+a^{3} b^{2}\right) f \cos \left(f x+e\right)^{2}+\left(a^{4} b+2 a^{3} b^{2}+a^{2} b^{3}\right) f\right) \sin \left(f x+e\right)}, \frac{\left(a^{2} b+2 a b^{2}+b^{3}+\left(a^{3}+2 a^{2} b+a b^{2}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{a} \arctan \left(\frac{\left(8 a^{2} \cos \left(f x+e\right)^{5}-8 \left(a^{2}-a b\right) \cos \left(f x+e\right)^{3}+\left(a^{2}-6 a b+b^{2}\right) \cos \left(f x+e\right)\right) \sqrt{a} \sqrt{\frac{a \cos \left(f x+e\right)^{2}+b}{\cos \left(f x+e\right)^{2}}}}{4 \left(2 a^{3} \cos \left(f x+e\right)^{4}-a^{2} b+a b^{2}-\left(a^{3}-3 a^{2} b\right) \cos \left(f x+e\right)^{2}\right) \sin \left(f x+e\right)}\right) \sin \left(f x+e\right)-4 \left(\left(a^{3}+a b^{2}\right) \cos \left(f x+e\right)^{3}+\left(a^{2} b-a b^{2}\right) \cos \left(f x+e\right)\right) \sqrt{\frac{a \cos \left(f x+e\right)^{2}+b}{\cos \left(f x+e\right)^{2}}}}{4 \left(\left(a^{5}+2 a^{4} b+a^{3} b^{2}\right) f \cos \left(f x+e\right)^{2}+\left(a^{4} b+2 a^{3} b^{2}+a^{2} b^{3}\right) f\right) \sin \left(f x+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(
f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^3*b^2)*f*
cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), 1/4*((a^2*
b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b +
 a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+fx)^2}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.426 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(3a-b)(a+3b)\cot(e+fx)}{3a(a+}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+e
)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(a+3*b)*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/3*(a-3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x
+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A]
time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 483, 597, 12, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2}-\frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Cot[e + f*x]^3)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a -
b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^3*f)
 - ((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^2
*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-3b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx\right.}{a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\,\mathrm{t}}}{3a(a+b)^2f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+}}{3a(a+b)^3f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A]

time = 5.83, size = 224, normalized size = 1.29

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}\,a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2e+2fx))^2\sec^3(e+fx)\left(\frac{(4a+9b)\csc(e+fx)}{12(a+b)^3f} - \frac{\csc^3(e+fx)}{12(a+b)^2f} - \frac{b^3\sin(e+fx)}{2a(a+b)^3f(a+2b+a\cos(2e+2fx))}\right)}{(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
+ f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4*a
+ 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f) - (
b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(a + b
*Sec[e + f*x]^2)^(3/2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 7541, normalized size = 43.34

method	result	size
default	Expression too large to display	7541

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(166) = 332$.
time = 9.49, size = 1102, normalized size = 6.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a
^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin
```

```
(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*
b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*
b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*
x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3
*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x +
e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*co
s(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 + 9*a^3*b + 3*a*b^3)*
cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (
3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2
+ 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.427 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}-\frac{(15a^3+55a^2b+73ab^2-15b}{1}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+55*a^2*b+73*a*b^2-15
*b^3)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^4/f+1/15*(5*a^2+14*a*b-
15*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/5*(a-5*b)*cot
(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

Rubi [A]
time = 0.32, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 483, 597, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3}-\frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^4}-\frac{(a-5b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5af(a+b)^2}-\frac{b\cot^5(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x]^5)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*
a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*a*(a + b)^4*f) + ((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(15*a*(a + b)^3*f) - ((a - 5*b)*Cot[e + f*x]^5*Sq
rt[a + b + b*Tan[e + f*x]^2])/(5*a*(a + b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-5b-6bx^2}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, \right.}{a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan}}{5a(a+b)^2 f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{}}{15a(a+b)^3 f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e}{15a(a+}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e}{15a(a+}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e}{15a(a+}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

<u>Mathematica</u> [A]

time = 11.31, size = 237, normalized size = 0.98

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}\,a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+2b+a\cos(2(e+fx)))^2\left(\frac{30b^4}{a(a+2b+a\cos(2(e+fx)))} - (23a^2+80ab+90b^2)\csc^2(e+fx) + (a+b)(11a+20b)\csc^4(e+fx) - 3(a+b)^2\csc^6(e+fx)\right)\sec^2(e+fx)\tan(e+fx)}{60(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*Sec
[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a + 2
*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 + (a
+ b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x
]^2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.24, size = 14137, normalized size = 58.66

method	result	size
default	Expression too large to display	14137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(231) = 462.
time = 24.31, size = 1564, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6
 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3
*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b
^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
```

```
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^5
+ 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b +
80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 5
6*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2
 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e
)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos
(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^
5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)
*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 +
 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2
*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*a
rctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*s
in(f*x + e)))*sin(f*x + e) - 4*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)
*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
+ 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^
5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^
5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.428 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(a+b)^2/a/b^2/f/(a
+b*sec(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4224, 457, 89, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + (a + b)^2/(3*a
*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (a^(-2) - b^(-2))/(f*Sqrt[a + b*Sec[
e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}} + \frac{a^2-b^2}{a^2b(a+bx)^{3/2}} + \frac{1}{a^2x\sqrt{a+bx}}\right) dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{}$$
$$= \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{}{f\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

time = 8.87, size = 187, normalized size = 1.93

$$\frac{4(a+b)F_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\tan^6(e+fx)}{3f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(8(a+b)F_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+\left(5aF_1\left(4;\frac{1}{2},\frac{7}{2};5;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(4;\frac{3}{2},\frac{5}{2};5;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\right)\sin^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF1[
3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1
[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10946 vs. 2(85) = 170.
time = 1.27, size = 10947, normalized size = 112.86

method	result	size
default	Expression too large to display	10947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(88) = 176.
time = 5.80, size = 596, normalized size = 6.14

$$\left[\frac{3\left(a^2b^2\cos(fx+e)^4+2ab^3\cos(fx+e)^2+b^4\right)\sqrt{a}\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\cos(fx+e)^2+b^4-8\left(16a^3\cos(fx+e)^8+24a^2b\cos(fx+e)^6+10ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)-8\left(2\left(a^4-a^3b-2a^2b^2\right)\cos(fx+e)^4+3\left(a^3b-ab^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left(a^5b^2f\cos(fx+e)^4+2a^4b^3f\cos(fx+e)^2+a^3b^4f\right)}, \frac{3\left(a^2b^2\cos(fx+e)^4+2ab^3\cos(fx+e)^2+b^4\right)\sqrt{-a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)-4\left(2\left(a^4-a^3b-2a^2b^2\right)\cos(fx+e)^4+3\left(a^3b-ab^3\right)\cos(fx+e)^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left(a^5b^2f\cos(fx+e)^4+2a^4b^3f\cos(fx+e)^2+a^3b^4f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b
*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*b^2)*
cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2
 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b
^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4
 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*
x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3
*b^4*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(85) = 170.
time = 3.05, size = 466, normalized size = 4.80

$$\frac{\frac{\left(\left(\frac{\left(2a^{11}\operatorname{sgn}(\cos(fx+e))+a^{10}b\operatorname{sgn}(\cos(fx+e))-4a^{9}b^{2}\operatorname{sgn}(\cos(fx+e))-3a^{8}b^{3}\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{10}b^{2}}-\frac{3\left(2a^{11}\operatorname{sgn}(\cos(fx+e))-3a^{10}b\operatorname{sgn}(\cos(fx+e))-4a^{9}b^{2}\operatorname{sgn}(\cos(fx+e))+a^{8}b^{3}\operatorname{sgn}(\cos(fx+e))\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\frac{3\left(2a^{11}\operatorname{sgn}(\cos(fx+e))-3a^{10}b\operatorname{sgn}(\cos(fx+e))-4a^{9}b^{2}\operatorname{sgn}(\cos(fx+e))+a^{8}b^{3}\operatorname{sgn}(\cos(fx+e))\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\frac{2a^{11}\operatorname{sgn}(\cos(fx+e))+a^{10}b\operatorname{sgn}(\cos(fx+e))-4a^{9}b^{2}\operatorname{sgn}(\cos(fx+e))-3a^{8}b^{3}\operatorname{sgn}(\cos(fx+e))}{a^{10}b^{2}}}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b\right)^{\frac{3}{2}}}+\frac{6\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\operatorname{sgn}(\cos(fx+e))}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((((2*a^11*sgn(cos(f*x + e)) + a^10*b*sgn(cos(f*x + e)) - 4*a^9*b^2*sg
n(cos(f*x + e)) - 3*a^8*b^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^10
*b^2) - 3*(2*a^11*sgn(cos(f*x + e)) - 3*a^10*b*sgn(cos(f*x + e)) - 4*a^9*b^
2*sgn(cos(f*x + e)) + a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x +
1/2*e)^2 + 3*(2*a^11*sgn(cos(f*x + e)) - 3*a^10*b*sgn(cos(f*x + e)) - 4*a^9
*b^2*sgn(cos(f*x + e)) + a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x
 + 1/2*e)^2 - (2*a^11*sgn(cos(f*x + e)) + a^10*b*sgn(cos(f*x + e)) - 4*a^9*
b^2*sgn(cos(f*x + e)) - 3*a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))/(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) + 6*arctan(-1/2*(sqrt(a + b)*tan(1
```

```
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt
(a + b))/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.429 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}}-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(-a-b)/a/b/f/(a+b*s
ec(f*x+e)^2)^(3/2)-1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4224, 457, 79, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - (a + b)/(3*a*b*f*
(a + b*Sec[e + f*x]^2)^(3/2)) - 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= -\frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\,(a+b\sec^2(e+fx))^{3/2}} - \frac{}{a^2f\sqrt{a}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 10.68, size = 613, normalized size = 6.89

$$-\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}} + \frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{96b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}} - \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\,(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}\left(1+e^{2i(e+fx)}\right)\left(-96b^3e^{2i(e+fx)}+a^3\left(1+e^{2i(e+fx)}\right)^2-32ab^2\left(1+e^{2i(e+fx)}\right)^2-6a^2b\left(1+e^{2i(e+fx)}+e^{4i(e+fx)}\right)\right)}{b^2\left(4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)^2} + \frac{24ifx-12\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-12\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sec^5(e+fx)}{96\sqrt{2}\,a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(96*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
```

```
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10838 vs. $2(81) = 162$.
time = 1.32, size = 10839, normalized size = 121.79

method	result	size
default	Expression too large to display	10839

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(80) = 160$.
time = 5.31, size = 554, normalized size = 6.22

$$\left[\frac{3\left(a^2b\cos(fx+e)^4+2ab^2\cos(fx+e)^2+b^3\right)\sqrt{a}\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\cos(fx+e)^2+b^4+8\left(16a^3\cos(fx+e)^8+24a^2b\cos(fx+e)^6+10ab^2\cos(fx+e)^4+b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right)-8\left(3ab^2\cos(fx+e)^2+(a^3+4a^2b)\cos(fx+e)^4\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left(a^5bf\cos(fx+e)^4+2a^4b^2f\cos(fx+e)^2+a^3b^3f\right)}, -\frac{3\left(a^2b\cos(fx+e)^4+2ab^2\cos(fx+e)^2+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)+4\left(3ab^2\cos(fx+e)^2+(a^3+4a^2b)\cos(fx+e)^4\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left(a^5bf\cos(fx+e)^4+2a^4b^2f\cos(fx+e)^2+a^3b^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 + (a^3
+ 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^
5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/12*(3*(a
^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 +
```

```
 a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^
2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(77) = 154.
time = 1.87, size = 418, normalized size = 4.70

$$\frac{\frac{\left(\left(\frac{\left(a^{10}b\,\mathrm{sgn}(\cos(fx+e))+4a^{9}b^{2}\mathrm{sgn}(\cos(fx+e))+3a^{8}b^{3}\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{10}b^{2}}-\frac{3\left(a^{10}b\,\mathrm{sgn}(\cos(fx+e))+4a^{9}b^{2}\mathrm{sgn}(\cos(fx+e))-a^{8}b^{3}\mathrm{sgn}(\cos(fx+e))\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\frac{3\left(a^{10}b\,\mathrm{sgn}(\cos(fx+e))+4a^{9}b^{2}\mathrm{sgn}(\cos(fx+e))-a^{8}b^{3}\mathrm{sgn}(\cos(fx+e))\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\frac{a^{10}b\,\mathrm{sgn}(\cos(fx+e))+4a^{9}b^{2}\mathrm{sgn}(\cos(fx+e))+3a^{8}b^{3}\mathrm{sgn}(\cos(fx+e))}{a^{10}b^{2}}}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b\right)^{\frac{3}{2}}}-\frac{6\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}(\cos(fx+e))}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((((a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3
*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^10*b^2) - 3*(a^10*b*sgn(cos(f
*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^10*b
^2))*tan(1/2*f*x + 1/2*e)^2 + 3*(a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*sgn(c
os(f*x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/2*e)^
2 - (a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3*sgn
(cos(f*x + e)))/(a^10*b^2))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3
/2) - 6*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2*sgn(
cos(f*x + e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan(e+f\,x)}^3}{{\left(a+\frac{b}{{\cos(e+f\,x)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.430 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sec(f*x+e
)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 272, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sec[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af} \\
&= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}\right.}{} \\
&= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{}{a^2f\sqrt{a}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.20, size = 613, normalized size = 7.39

$$-\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}}+\frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{32b^2f(a+2b+a\cos(2(e+fx)))^{3/2}\left(a+b\sec^2(e+fx)\right)^{5/2}}+\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\,(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}\left(1+e^{2i(e+fx)}\right)\left(-96b^3e^{2i(e+fx)}+a^3\left(1+e^{2i(e+fx)}\right)^2-32ab^2\left(1+e^{2i(e+fx)}\right)^2-6a^2b\left(1+e^{2i(e+fx)}+e^{4i(e+fx)}\right)\right)}{b^2\left(4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)^2}+\frac{24ifx-12\log\left(a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-12\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sec^5(e+fx)}{96\sqrt{2}\,a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*
Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e +
f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e
 + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(32*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(
5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a
^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a
^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Se
c[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.02, size = 86, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\frac{1}{3a\left(a+b\left(\sec^{2}(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{a+b\left(\sec^{2}(fx+e)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}(fx+e)\right)}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{a}}{f}$	86
default	$\frac{\frac{1}{3a\left(a+b\left(\sec^{2}(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{a+b\left(\sec^{2}(fx+e)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sec^{2}(fx+e)\right)}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{a}}{f}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3/a/(a+b*sec(f*x+e)^2)^(3/2)+1/a*(1/a/(a+b*sec(f*x+e)^2)^(1/2)-1/a^(
3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(74) = 148$.
time = 4.42, size = 526, normalized size = 6.34

$$\left[\frac{3\left(a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}+256a^{3}b\cos\left(fx+e\right)^{6}+160a^{2}b^{2}\cos\left(fx+e\right)^{4}+32ab^{3}\cos\left(fx+e\right)^{2}+b^{4}-8\left(16a^{3}\cos\left(fx+e\right)^{8}+24a^{2}b\cos\left(fx+e\right)^{6}+10ab^{2}\cos\left(fx+e\right)^{4}+b^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\right)+8\left(4a^{2}\cos\left(fx+e\right)^{4}+3ab\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{24\left(a^{5}f\cos\left(fx+e\right)^{4}+2a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}, \frac{3\left(a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{4}+8ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}+3a^{2}b\cos\left(fx+e\right)^{2}+ab^{2}\right)}\right)+4\left(4a^{2}\cos\left(fx+e\right)^{4}+3ab\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{12\left(a^{5}f\cos\left(fx+e\right)^{4}+2a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*cos(f
*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*
b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4*a^2*cos(f
*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [A]
time = 13.51, size = 78, normalized size = 0.94

$$\frac{1}{3af\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{a^{2}f\sqrt{a+b\sec^{2}\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\sec^{2}\left(e+fx\right)}}{\sqrt{-a}}\right)}{a^{2}f\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + 1/(a**2*f*sqrt(a + b*sec(e + f*x
)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(a**2*f*sqrt(-a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(71) = 142$.
time = 1.45, size = 370, normalized size = 4.46

$$-\frac{\frac{\left(\left(\frac{\left(4a^{9}b^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+3a^{8}b^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{10}b^{2}}-\frac{3\left(4a^{9}b^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-a^{8}b^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\frac{3\left(4a^{9}b^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-a^{8}b^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{a^{10}b^{2}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\frac{4a^{9}b^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+3a^{8}b^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{a^{10}b^{2}}}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b\right)^{\frac{3}{2}}}-\frac{6\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3*sgn(cos(f*x + e)))*tan(1/
2*f*x + 1/2*e)^2/(a^10*b^2) - 3*(4*a^9*b^2*sgn(cos(f*x + e)) - a^8*b^3*sgn(
cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/2*e)^2 + 3*(4*a^9*b^2*sgn(cos(f*
x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/2*e)^2 - (
4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))/(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) - 6*arctan(-1/2*(sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
+ sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(f*x + e))))/f
```

Mupad [B]
time = 8.17, size = 68, normalized size = 0.82

$$\frac{\frac{a+\frac{b}{\cos(e+f\,x)^2}}{a^2}+\frac{1}{3\,a}}{f\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}{\sqrt{a}}\right)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b/cos(e + f*x)^2)/a^2 + 1/(3*a))/(f*(a + b/cos(e + f*x)^2)^(3/2)) - a
tanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(5/2)*f)
```

3.431 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f}-\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}-$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((a+b*sec(f*x+e)
^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3*b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/
2)-b*(2*a+b)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4224, 457, 87, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - b/(3*a*(a + b)*f*(a + b*
Sec[e + f*x]^2)^(3/2)) - (b*(2*a + b))/(a^2*(a + b)^2*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e\right.}{2a(a+b)f}$$

$$= -\frac{b}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} -$$

$$= -\frac{b}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} -$$

$$= -\frac{b}{3a(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{}{3a}$$

Mathematica [F]
time = 9.55, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 68988 vs. 2(119) = 238.
time = 1.89, size = 68989, normalized size = 503.57

method	result	size
default	Expression too large to display	68989

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(123) = 246$.
time = 7.06, size = 2375, normalized size = 17.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f
*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 +
160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(
f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a^5*cos
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 +
 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 8*((7
*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3
+ a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8
 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 +
 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 +
 a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
```

```
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*((7*a
^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 +
 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3
*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a
^3*b^5)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*(a^5*cos(f*x + e
)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b +
 b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b
)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((7*a^4*b +
 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7
*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b
^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 6*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*
x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^
2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 4*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^
3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*
x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e + f\,x)}{\left(a + \frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.432 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f}-\frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e}$$

```
[Out] -arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*a+7*b)*arctanh(
(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f-1/6*(3*a-2*b)*b/a/(a+b)
^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^(
3/2)-1/2*b*(a^2-6*a*b-2*b^2)/a^2/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4224, 457, 105, 157, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(a^2-6ab-2b^2)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{b(3a-2b)}{6af(a+b)^2(a+b\sec^2(e+fx))^{3/2}}-\frac{\cot^2(e+fx)}{2f(a+b)(a+b\sec^2(e+fx))^{3/2}}+\frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ((2*a + 7*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(7/2)*f) - ((3*a
 - 2*b)*b)/(6*a*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/
(2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (b*(a^2 - 6*a*b - 2*b^2))/(2*a
^2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{5bx}{2}}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e\right.}{2(a+b)f}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} +$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+}}{\sqrt{a+b}}\right.}{2(a+b)^{7/2}f}$$

Mathematica [F]
time = 21.04, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105236 vs. $2(174) = 348$.
time = 2.40, size = 105237, normalized size = 526.18

method	result	size
default	Expression too large to display	105237

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(180) = 360.
time = 16.29, size = 3619, normalized size = 18.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
- a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*
b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^
2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 -
 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4
*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2
+ 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*
((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((
3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 + 2*(
3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x + e)^4 + 3*(
a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*
```

```
b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4
- 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^
4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 + 4*a^4*b^5 + a^3*b^6)*f), -1/24*(6*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2
*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4*a
^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*
a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 3*((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2
*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 -
 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2
*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
 + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
 + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/24*(6*((a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4
- 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*
b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^
5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*
cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 3*((2*a^6 + 7*a^
5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2
)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)
*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)
*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b
^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8
*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 +
 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 +
 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/12*(3*((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*
a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^
4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4
```

```
- 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*((2*a^
6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14
*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.433 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(8a^2+36ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f}+\frac{b(12a^2+}{24a(a+b)^3f\,(a}$$

```
[Out] arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/8*(8*a^2+36*a*b+63*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(9/2)/f+1/24*b*(12*a
^2+39*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)+1/8*(4*a+11*b)*cot(f*
x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*sec
(f*x+e)^2)^(3/2)+1/8*b*(4*a^3+15*a^2*b-32*a*b^2-8*b^3)/a^2/(a+b)^4/f/(a+b*s
ec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4224, 457, 105, 156, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{b(12a^2+39ab-8b^2)}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(8a^2+36ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}+\frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^4(e+fx)}{4f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{(4a+11b)\cot^2(e+fx)}{8f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ((8*a^2 + 36*a*b
+ 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(9/2)
*f) + (b*(12*a^2 + 39*a*b - 8*b^2))/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^(3/2)) + ((4*a + 11*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sec[e + f*x]
^2)^(3/2)) - Cot[e + f*x]^4/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (b
*(4*a^3 + 15*a^2*b - 32*a*b^2 - 8*b^3))/(8*a^2*(a + b)^4*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{7bx}{2}}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e\right.}{4(a+b)f}$$

$$= \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\text{Su}}{}$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(8a^2+36ab+63b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\text{s}}}{\sqrt{}}\right.}{8(a+b)^{9/2}f}$$

Mathematica [F]

time = 31.99, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 145924 vs. $2(238) = 476$.
time = 23.22, size = 145925, normalized size = 544.50

method	result	size
default	Expression too large to display	145925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(246) = 492$.
time = 54.07, size = 4879, normalized size = 18.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(12*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*
cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x
 + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 +
```

a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), 1/48*(3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 6*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b

```
^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4
- 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^
7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a
^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x +
 e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^
7)*f), -1/96*(24*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a
*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*
cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2
*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3
*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 +
 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6
 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4
 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.434 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+f\right.}$$

[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+(1/a^2-1/b^2)*tan(f*x+e)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^(3/2)

Rubi [A]
time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4226, 2000, 481, 592, 537, 223, 212, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a^(-2) - b^(-2))*Tan[e + f*x])/(f*Sqrt[a + b + b*Tan[e + f*x]^2])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3ax^2)}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right.}{b^{5/2}f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(157) = 314.

time = 11.84, size = 316, normalized size = 2.01

$$-\frac{\left(\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}\,a^2b^2f\left(a+b\sec^2(e+fx)\right)^{5/2}} + \frac{(a+2b+a\cos(2e+2fx))^3\sec^5(e+fx)\left(\frac{-a^2\sin(e+fx)-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a+2b+a\cos(2e+2fx))^2} + \frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2\sin(e+fx)}{12a^2b^2f(a+2b+a\cos(2e+2fx))}\right)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/
Sqrt[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2
]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*
a^2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e + f*
x])/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*x] + a
*b*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos[2*e +
2*f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.07, size = 2256, normalized size = 14.37

method	result	size
default	Expression too large to display	2256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*
x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/
2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*a*b^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+3*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
```

```
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-
6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b
+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^2*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(cos(f*x+e)-1)/cos(f*x+e)^5/((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(147) = 294$.
time = 9.72, size = 2145, normalized size = 13.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a
^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6*(a^5*c
os(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^2 - 4*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4
 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*cos(f*x + e)^4 +
2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x +
 e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*
f*cos(f*x + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*co
s(f*x + e)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 3*(a^5*cos(f*x + e)^4 + 2*a^4*
b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x + e)^3 + (
4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x
 + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e
)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e))) + 6*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
```

```
*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e))) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x
+ e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4
*f*cos(f*x + e)^2 + a^3*b^5*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.435 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(a-3*b)
*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)/a/b/f/(
a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4226, 2000, 481, 541, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 ((a + b)*Tan[e + f*x])/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a -
3*b)*Tan[e + f*x])/(3*a^2*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(a-2b)x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(120) = 240$.
time = 6.61, size = 409, normalized size = 3.41

$$\frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e+fx)}{a+b}+\frac{16\left(a+b-a\sin^2(e+fx)\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(-\frac{6a(a+b)\sin^2(e+fx)}{a+2b+a\cos(2(e+fx))}+\frac{a^2(a+b)\sin^4(e+fx)}{\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{3\sqrt{a}\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)}{a^3}\right)}{\left(a+b-a\sin^2(e+fx)\right)^{3/2}}+\frac{8(2a+3b+a\cos(2(e+fx)))\tan(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^{3/2}}-\frac{12(b+(3a+2b)\cos(2(e+fx)))\tan(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^{3/2}}\right)}{384f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x]
*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin
[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e
 + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/
(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Si
n[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(a
+ b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[e +
 f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b + (3*a + 2
*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)
])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.04, size = 1142, normalized size = 9.52

method	result	size
default	Expression too large to display	1142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^2*sin(f*
x+e)*a+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-6*2^(1
/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos
(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-c
os(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*b*sin(f*x+e)+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a-4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2
```

```
)+a-b)/(a+b))^(1/2)*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(cos(f*x+e)-1)/cos(f*x+e)^5/((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(112) = 224.
time = 6.35, size = 698, normalized size = 5.82

$$\left[-\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5a^4-14a^3b+5a^2b^2\right)\cos(fx+e)^4+a^4-28a^3b+70a^2b^2-28ab^3+b^4-32\left(a^4-7a^3b+7a^2b^2-ab^3\right)\cos(fx+e)^2+8\left(16a^3\cos(fx+e)^7-24\left(a^3-a^2b\right)\cos(fx+e)^5+2\left(5a^3-14a^2b+5ab^2\right)\cos(fx+e)^3-\left(a^3-7a^2b+7ab^2-b^3\right)\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)+8\left(4a^2\cos(fx+e)^3-\left(a^2-3ab\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{24\left(a^5f\cos(fx+e)^4+2a^4bf\cos(fx+e)^2+a^3b^2f\right)},\ -\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\left(4a^2\cos(fx+e)^3-\left(a^2-3ab\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)}{12\left(a^5f\cos(fx+e)^4+2a^4bf\cos(fx+e)^2+a^3b^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^2*cos(f
*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a
^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e
)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.436 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}+\frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(2*a+3
*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4226, 2000, 482, 541, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + Tan[e + f*x]/(3*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((2*a + 3*b)*Ta
n[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3af} \\
&= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \\
&= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \\
&= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} +
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 410 vs. $2(119) = 238$.

time = 4.82, size = 410, normalized size = 3.45

$$
\frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e+fx)}{a+b}+\frac{16\left(a+b-a\sin^2(e+fx)\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(-\frac{6a(a+b)\sin^2(e+fx)}{a+2b+a\cos(2(e+fx))}+\frac{a^2(a+b)\sin^4(e+fx)}{\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{3\sqrt{a}\sqrt{a+b}\,\text{ArcSin}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)}{a^3}\right)}{\left(a+b-a\sin^2(e+fx)\right)^{3/2}}+\frac{8(2a+3b+a\cos(2(e+fx)))\tan(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^{3/2}}-\frac{4(b+(3a+2b)\cos(2(e+fx)))\tan(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^{3/2}}\right)}{384f\left(a+b\sec^2(e+fx)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f*
x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[
e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b + (3*a +
 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.08, size = 2112, normalized size = 17.75

method	result	size
default	Expression too large to display	2112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*a^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+3*cos(f*x+e)^2*sin(f
*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+3*2^(1/2)*((I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+
b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b-6*2^(1/2)*((I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(
a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
```

```
a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a*b*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticP
i((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/
2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/
(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*b^2*sin(f*x+e)+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^2+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*co
s(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-4*cos(f*x+e)^2*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*b^2)/(cos(f*x+e)-1)/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*
x+e)^2)^(5/2)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(111) = 222.
time = 5.23, size = 810, normalized size = 6.81

$$\left[-\frac{3\left(\left(a^{3}+a^{2}b\right)\cos\left(fx+e\right)^{4}+ab^{2}+b^{3}+2\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28\,a^{3}b+70\,a^{2}b^{2}-28\,ab^{3}+b^{4}-32\left(a^{4}-7\,a^{3}b+7\,a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}-8\left(16\,a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5\,a^{3}-14\,a^{2}b+5\,ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)-8\left(\left(3\,a^{3}+4\,a^{2}b\right)\cos\left(fx+e\right)^{3}+\left(2\,a^{2}b+3\,ab^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{24\left(\left(a^{6}+a^{5}b\right)f\cos\left(fx+e\right)^{4}+2\left(a^{5}b+a^{4}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b^{2}+a^{3}b^{3}\right)f\right)},\ \frac{3\left(\left(a^{3}+a^{2}b\right)\cos\left(fx+e\right)^{4}+ab^{2}+b^{3}+2\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^{3}\cos\left(fx+e\right)^{5}-8\left(a^{3}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2\,a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)+4\left(\left(3\,a^{3}+4\,a^{2}b\right)\cos\left(fx+e\right)^{3}+\left(2\,a^{2}b+3\,ab^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{12\left(\left(a^{6}+a^{5}b\right)f\cos\left(fx+e\right)^{4}+2\left(a^{5}b+a^{4}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b^{2}+a^{3}b^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*c
os(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b
^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(
a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((3*a^3 + 4*a^2*b
)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^
5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.437 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a+
3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a
/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 425, 541, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \text{t}\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 6.50, size = 1927, normalized size = 15.42

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.16, size = 3024, normalized size = 24.19

method	result	size
default	Expression too large to display	3024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e
))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x
+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a^3-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b
))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(
a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*cos(f*x+e)^2*s
in(f*x+e)*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b
))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a
+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
```

```
))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e
)-12*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+
b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((cos(f*x+e)-
1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^(1/2)*(-2*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(
a+b))^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+3*2^
(1/2)*((I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e))/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*((I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
)/(a+b))^(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+
e)*a-b)/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))/(a+b))^
(1/2)*(-2*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+6*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-
4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(117) = 234$.

time = 5.48, size = 918, normalized size = 7.34

$$\left[-\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\log\left(128a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5a^{4}-14a^{3}b+5a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-28a^{3}b+70a^{2}b^{2}-28ab^{3}+b^{4}-32\left(a^{4}-7a^{3}b+7a^{2}b^{2}-ab^{3}\right)\cos\left(fx+e\right)^{2}+8\left(16a^{3}\cos\left(fx+e\right)^{7}-24\left(a^{3}-a^{2}b\right)\cos\left(fx+e\right)^{5}+2\left(5a^{3}-14a^{2}b+5ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}-7a^{2}b+7ab^{2}-b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)\right)+8\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{24\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)},\ -\frac{3\left(\left(a^{4}+2a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{2}b^{2}+2ab^{3}+b^{4}+2\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-8\left(a^{2}-ab\right)\cos\left(fx+e\right)^{3}+\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\left(2a^{3}\cos\left(fx+e\right)^{4}-a^{2}b+ab^{2}-\left(a^{3}-3a^{2}b\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)+4\left(2\left(3a^{3}b+2a^{2}b^{2}\right)\cos\left(fx+e\right)^{3}+\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{12\left(\left(a^{7}+2a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+2a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.438 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(7*a
+3*b)*cot(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(3*a+
b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)/a/(
a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}-\frac{b(7a+3b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\cot(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*
(7*a + 3*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a^
2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-4bx^2}{x^2(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x,\right.}{3a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right.}$$

Mathematica [A]
time = 7.91, size = 247, normalized size = 1.42

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}\,a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{(a+2b+a\cos(2(e+fx)))\left(3(3a^4+8a^3b+5a^2b^2-12ab^3-4b^4)+4(3a^4+6a^3b+8ab^3+3b^4)\cos(2(e+fx))+a(3a^3+9ab^2+4b^3)\cos(4(e+fx))\right)\csc(e+fx)\sec^5(e+fx)}{48a^2(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*b +
 5*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*Cos[
2*(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f*x]*S
ec[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.27, size = 7586, normalized size = 43.60

method	result	size
default	Expression too large to display	7586

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(164) = 328.
time = 10.53, size = 1138, normalized size = 6.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(
f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
 + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
```

```
b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5
 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
+ 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^
2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*s
in(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5 + (6*a^4*b
- 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2 - 8*a^2*b^3
- 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e + fx)}{(a + b\sec^2(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e + f\,x)^2}{\left(a + \frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.439 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(3a+b)\cot^3(e+f}{a^2(a+b)^2f\sqrt{a+b+b\tan^2}}$$

```
[Out] arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-b*(3*a+b)*c
ot(f*x+e)^3/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(a-b)*(3*a^2+14*a*
b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/3*(a^2-10*a*
b-3*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*
x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.32, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}+\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^4}-\frac{b(3a+b)\cot^3(e+fx)}{a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\cot^3(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Cot[e + f*x]^3)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(
3*a + b)*Cot[e + f*x]^3)/(a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
 ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]
^2])/(3*a^2*(a + b)^4*f) - ((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(3*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-b)-6bx^2}{x^4(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \right.}{3a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2 f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2 f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2 f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2 f\sqrt{a+b+b\tan^2(e+}}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2 f\sqrt{a+b+b\tan^2(e+}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/}}$$

Mathematica [A]

time = 14.86, size = 234, normalized size = 0.99

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}\,a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}-\frac{(a+2b+a\cos(2(e+fx)))^3\sec^5(e+fx)\left(-4(a+3b)\csc(e+fx)+(a+b)\csc^3(e+fx)+\frac{4b^3(6a^2+13ab+3b^2+2a(3a+b)\cos(2(e+fx)))\sin(e+fx)}{a^2(a+2b+a\cos(2(e+fx)))^2}\right)}{24(a+b)^4f\,(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
+ f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*(a
+ 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b + 3*b
^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*Cos[2*
(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.41, size = 15128, normalized size = 64.10

method	result	size
default	Expression too large to display	15128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. 2(226) = 452.
time = 24.50, size = 1626, normalized size = 6.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b
^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8
*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b -
 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4
*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*
b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x +
 e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*c
os(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^
6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x +
 e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*(a^6 + 3*a^5*b + 3*a^
3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 -
a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^
5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^
8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b -
2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b
+ 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2
 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e + fx)}{(a + b\sec^2(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.440 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=315

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(11a+3b)\cot^5(e}{3a^2(a+b)^2f\sqrt{a+b+bt}}$$

```
[Out] -arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(11*
a+3*b)*cot(f*x+e)^5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4+7
0*a^3*b+128*a^2*b^2-70*a*b^3-15*b^4)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/
a^2/(a+b)^5/f+1/15*(5*a^3+19*a^2*b-65*a*b^2-15*b^3)*cot(f*x+e)^3*(a+b+b*tan
(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/5*(a^2-20*a*b-5*b^2)*cot(f*x+e)^5*(a+b+b*t
an(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*
x+e)^2)^(3/2)
```

Rubi [A]
time = 0.40, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3}-\frac{b(11a+3b)\cot^5(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15a^2f(a+b)^4}-\frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15a^2f(a+b)^5}-\frac{b\cot^5(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x]^5)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (
b*(11*a + 3*b)*Cot[e + f*x]^5)/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - ((15*a^4 + 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^2*(a + b)^5*f) + ((5*a^3 + 19*a^2*b
 - 65*a*b^2 - 15*b^3)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^
2*(a + b)^4*f) - ((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[
e + f*x]^2])/(5*a^2*(a + b)^3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b(1+x^2))^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-5b-8bx^2}{x^6(1+x^2)(a+b+bx^2)^{3/2}}\,dx, \text{:}\right.}{3a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx\right.}$$

Mathematica [A]

time = 24.86, size = 272, normalized size = 0.86

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f(a+b\sec^2(e+fx))^{5/2}}+\frac{(a+2b+a\cos(2(e+fx)))^3\left(-\frac{20b^5(a+b)}{a^2(a+2b+a\cos(2(e+fx)))^2}+\frac{10b^4(15a+4b)}{a^2(a+2b+a\cos(2(e+fx)))}-(23a^2+100ab+150b^2)\csc^2(e+fx)+(a+b)(11a+25b)\csc^4(e+fx)-3(a+b)^2\csc^6(e+fx)\right)\sec^4(e+fx)\tan(e+fx)}{120(a+b)^5f(a+b\sec^2(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*Sec
[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b))/
(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a + 2*
b + a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2 + (a
 + b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*
x]^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.49, size = 22712, normalized size = 72.10

method	result	size
default	Expression too large to display	22712

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(301) = 602.
time = 83.14, size = 2112, normalized size = 6.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5
)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f
*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5
+ a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^
5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 100*a^6*b + 1
50*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7 + 118*a^6*b
+ 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x +
 e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15
*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4
 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*
a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 +
a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a
^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3
 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b
+ 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2
 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
*sin(f*x + e)), 1/60*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b
^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 -
 a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b
^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
+ e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(
f*x + e) - 4*((23*a^7 + 100*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*
cos(f*x + e)^9 - (35*a^7 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b
^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^
4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 1
05*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e
)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 1
0*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 +
4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (
a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 +
a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 -
4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 +
```

```
10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.441 $\int (a+b\sec^2(e+fx))^p\,(d\tan(e+fx))^m\,dx$

Optimal. Leaf size=105

$$\frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}\,(a+b+b\tan^2(e+fx))^p\,\Bigl(1+\frac{b\tan^2(}{a+}}{df(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*(d*t
an(f*x+e))^(1+m)*(a+b+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/(a+b))
^p)
```

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4226, 2000, 525, 524}

$$\frac{(d\tan(e+fx))^{m+1}\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(1
+ m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^m(a+b(1+x^2))^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(dx)^m(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx \ldots}{\ldots}\right.}{f}$$

$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+\ldots}{df(1+\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(105) = 210.
time = 3.86, size = 259, normalized size = 2.47

$$\frac{F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\sin(e+fx)(d\tan(e+fx))^m}{f(1+m)\left(F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)+\frac{2\left(bpF_1\left(\frac{3+m}{2};1-p,1;\frac{5+m}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)-(a+b)F_1\left(\frac{3+m}{2};-p,2;\frac{5+m}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\right)\tan^2(e+fx)}{(a+b)(3+m)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e + f
*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1, (5 +
 m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[(
3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
)*Tan[e + f*x]^2)/((a + b)*(3 + m))))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(d\tan\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
[Out] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^m\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.442 $\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=122

$$-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}+\frac{(a+b\sec^2}{2b^2f}$$

[Out] -1/2*(a+2*b)*(a+b*sec(f*x+e)^2)^(1+p)/b^2/f/(1+p)-1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/a/f/(1+p)+1/2*(a+b*sec(f*x+e)^2)^(2+p)/b^2/f/(2+p)

Rubi [A]
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4224, 457, 90, 67}

$$-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]

[Out] -1/2*((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(b^2*f*(1 + p)) - (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(2*b^2*f*(2 + p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^p}{b} + \frac{(a+bx)^p}{x} + \frac{(a+bx)^{1+p}}{b}\right)\,dx, x, \sec^2(e+fx$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} + \frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)} + \text{S}$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} - \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2}{}\right.}{2af}$$

Mathematica [A]
time = 4.30, size = 106, normalized size = 0.87

$$-\frac{(a+b\sec^2(e+fx))^p\,(a+b+b\tan^2(e+fx))\left(b^2(2+p)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b+b\tan^2(e+fx)}{a}\right)+a(a+b(3+p)-b(1+p)\tan^2(e+fx))\right)}{2ab^2f(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -1/2*((a + b*Sec[e + f*x]^2)^p*(a + b + b*Tan[e + f*x]^2)*(b^2*(2 + p)*Hype
rgeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] + a*(a + b*(3
+ p) - b*(1 + p)*Tan[e + f*x]^2)))/(a*b^2*f*(1 + p)*(2 + p))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2(fx+e)\right)\right)^p\left(\tan^5(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.443 $\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=86

$$\frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)}+\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

```
[Out] 1/2*(a+b*sec(f*x+e)^2)^(1+p)/b/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],1+b*sec
(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4224, 457, 81, 67}

$$\frac{(a+b\sec^2(e+fx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}+\frac{(a+b\sec^2(e+fx))^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] (a + b*Sec[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec^2(e+fx))^p\tan^3(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} + \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a}{2af(1+p)}
\end{aligned}
$$

Mathematica [A]
time = 2.76, size = 84, normalized size = 0.98

$$
\frac{(a+2b+a\cos(2(e+fx)))\left(a+b\,{}_2F_1\left(1,1+p;2+p;\frac{a+b+b\tan^2(e+fx)}{a}\right)\right)\sec^2(e+fx)\,(a+b\sec^2(e+fx))^p}{4abf(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(a + b*Hypergeometric2F1[1, 1 + p, 2 + p, (
a + b + b*Tan[e + f*x]^2)/a])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*a
*b*f*(1 + p))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$
\int \left(a+b\left(\sec^2(fx+e)\right)\right)^p \left(\tan^3(fx+e)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec ^2(e+fx)\right)^p \tan ^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.444 $\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx$

Optimal. Leaf size=54

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(1+p)/
a/f/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4224, 272, 67}

$$-\frac{(a+b\sec^2(e+fx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*S
ec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

Mathematica [**A**]
time = 0.10, size = 54, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*S
ec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2(fx+e)\right)\right)^p \tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^p \tan\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e + f\,x)\,\left(a + \frac{b}{\cos\left(e + f\,x\right)^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=114

$$-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)}+\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(\right.}{2af(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^
(1+p)/(a+b)/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*s
ec(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A]

time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4224, 457, 88, 70, 67}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a
 + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*f*(1 + p)) + (Hypergeometric2F1[1, 1
 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a
*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] &&  !IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{x(-1+x^2)}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec\right.}{2f}\\
&= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)} + \frac{{}_2}{}
\end{aligned}
$$

Mathematica [A]
time = 2.32, size = 115, normalized size = 1.01

$$\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b+b\tan^2(e+fx)}{a}\right)-a\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a+b}\right)\right)\sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p}{4a(a+b)f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p,
 (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
```

```
b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*a*(
a + b)*f*(1 + p))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \cot (fx+e) \left(a+b\left(\sec ^2(fx+e)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec ^2(e+fx)\right)^p \cot (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.446 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=157

$$-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}+\frac{(a+b-bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+}}{2(a+b)^2f(1+p)}$$

```
[Out] -1/2*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1+p)/(a+b)/f+1/2*(-b*p+a+b)*hypergeom
([1, 1+p],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^(1+p)/(a+b)^2/
f/(1+p)-1/2*hypergeom([1, 1+p],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)
^(1+p)/a/f/(1+p)
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4224, 457, 105, 162, 70, 67}

$$\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)^2}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*f) + ((a + b
- b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(
a + b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)^2*f*(1 + p)) - (Hypergeometric2F1
[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))
/(2*a*f*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a+b-bpx}{(-1+x)x}\right.}{2(a+}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \text{s}\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{(a+b-bp)\,{}_2F_1\left(1,1+\right.}{}$$

Mathematica [A]
time = 3.85, size = 139, normalized size = 0.89

$$-\frac{\left(b+(a+b)\cot^2(e+fx)\right)\left(a(a+b)(1+p)\cot^2(e+fx)+(a+b)^2\,{}_2F_1\left(1,1+p;2+p;\frac{a+b+b\tan^2(e+fx)}{a}\right)-a(a+b-bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a+b}\right)\right)\left(a+b\sec^2(e+fx)\right)^p\tan^2(e+fx)}{2a(a+b)^2f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/2*((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a +
 b)^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] - a*
(a + b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/(a
+ b)])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(a*(a + b)^2*f*(1 + p))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cot^3\left(fx+e\right)\right)\left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.447 $\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]

time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4226, 2000, 525, 524}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b(1+x^2))^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\right.}{1+}\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+}{5f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2777 vs. $2(88) = 176$.
time = 18.62, size = 2777, normalized size = 31.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/
2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)
/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]
```

*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p,
 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b
*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*
Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan
[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^
2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + (2*p*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]^2*((9*(a +
 b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometri
c2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*T
an[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^p*Tan[e + f*x]*((-18*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Cos[e + f*x]^2*((2*b*p*Appe
llF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (2*b*p*Sec[e + f*x]^2*Tan
[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p)*(-3*Hypergeometric2F1[1
/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2))/(a + b) - (9*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[
e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b

```
)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f
*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*
b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*App
ellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,...
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p \left(\tan^4\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.448 $\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4226, 2000, 525, 524}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b(1+x^2))^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{b}{a}\right.}{1+x}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2465 vs. $2(88) = 176$.

time = 16.88, size = 2465, normalized size = 28.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a
 + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x
]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*(e + f*x
```

)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]^2*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((-2*b*p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) + (6*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Cos[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -

```
(2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/
2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*Ap
pellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2)^2)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p} \left(\tan^{2}\left(fx+e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.449 $\int (a+b\sec^2(e+fx))^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a+b\sec^2(e+fx))^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2137 vs. $2(83) = 166$.
time = 6.26, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
```

```
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sec^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x)^2)^p,x)
[Out] int((a + b/cos(e + f*x)^2)^p, x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=84

$$-\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e)*(a+
b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4226, 2000, 525, 524}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
 + b))^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+b(1+x^2))^p}{x^2(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\frac{(a+b+bx^2)^p}{x^2(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left((a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a+}\right.}{x^2(1+}\right.}{f}$$

$$=-\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\,(a}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2469 vs. $2(84)=168$.
time = 17.41, size = 2469, normalized size = 29.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/
(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b + a*Cos[2*
```

(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Si
n[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b
*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - (a + 2*b +
a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F
1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) - 2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*Cot[e + f*x]*(Sec
[e + f*x]^2)^p*Sin[2*(e + f*x)]*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Ta
n[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin
[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a
Cos[2(e + f*x)])^p*Cot[e + f*x]*(Sec[e + f*x]^2)^p*((2*b*p*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (6*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a +
b)*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
 -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) - (Csc[e + f*x]*Sec[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e
+ f*x]^2)/(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e +
 f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*

```
(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*A
ppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)
) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)^2))
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)\left(a+b\left(\sec^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e
)^3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4226, 2000, 525, 524}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]
^2)/(a + b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+b(1+x^2))^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right.}{x^4(1+x}\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\,(}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3033 vs. $2(88) = 176$.
time = 18.88, size = 3033, normalized size = 34.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*Appe
llF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*T
an[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
```

f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f
*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f*x]^2)^p*(
(9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a +
 b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*T
an[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (a + 2*b + a*Cos[2*
(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*((9*(a + b)*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
in[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2
) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]
^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (2*a*p*(a + 2*b + a*Cos[2*(e + f*
x)])^(-1 + p)*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((9*(a + b
)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x
)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*((18*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e +
f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (18*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Tan[e + f
x]^3)/(3(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Sin[e +
f*x]^2*Tan[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
(2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p,
1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[

```
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) + (2*b*p*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b)
)^(-1 - p)*(Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))
] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e
 + f*x]^2))/(a + b) - (9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2*(4*(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (
a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]
^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 -...
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(fx+e\right)\right) \left(a+b\left(\sec^{2}\left(fx+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.452 $\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$

Optimal. Leaf size=92

$$-\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}$$

```
[Out] -a*ln(cos(f*x+e))/f-a*sec(f*x+e)^2/f+1/3*b*sec(f*x+e)^3/f+1/4*a*sec(f*x+e)^
4/f-2/5*b*sec(f*x+e)^5/f+1/7*b*sec(f*x+e)^7/f
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4223, 1816}

$$\frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*f
) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]
^7)/(7*f)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^3)}{x^8}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^8}-\frac{2b}{x^6}+\frac{a}{x^5}+\frac{b}{x^4}-\frac{2a}{x^3}+\frac{a}{x}\right)\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(}{}$$

Mathematica [A]
time = 0.30, size = 87, normalized size = 0.95

$$\frac{b\sec^3(e+fx)}{3f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}-\frac{a(4\log(\cos(e+fx))+2\tan^2(e+fx)-\tan^4(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x]^3)/(3*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]^7)/
(7*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

Maple [A]
time = 0.12, size = 141, normalized size = 1.53

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+b\left(\frac{\sin^6(fx+e)}{7\cos(fx+e)^7}+\frac{\sin^6(fx+e)}{35\cos(fx+e)^5}-\frac{\sin^6(fx+e)}{105\cos(fx+e)^3}+\frac{\sin^6(fx+e)}{35\cos(fx+e)}+\frac{\left(\frac{8}{3}\right.}{}\right.}{f}$
default	$\frac{a\left(\frac{\left(\tan^4(fx+e)\right)}{4}-\frac{\left(\tan^2(fx+e)\right)}{2}-\ln(\cos(fx+e))\right)+b\left(\frac{\sin^6(fx+e)}{7\cos(fx+e)^7}+\frac{\sin^6(fx+e)}{35\cos(fx+e)^5}-\frac{\sin^6(fx+e)}{105\cos(fx+e)^3}+\frac{\sin^6(fx+e)}{35\cos(fx+e)}+\frac{\left(\frac{8}{3}\right.}{}\right.}{f}$
risch	$iax+\frac{2iae}{f}-\frac{4\left(105a\,e^{12i(fx+e)}-70b\,e^{11i(fx+e)}+420a\,e^{10i(fx+e)}+56b\,e^{9i(fx+e)}+735a\,e^{8i(fx+e)}-228b\,e^{7i(fx+e)}+\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2-ln(cos(f*x+e)))+b*(1/7*sin(f*x+e)
^6/cos(f*x+e)^7+1/35*sin(f*x+e)^6/cos(f*x+e)^5-1/105*sin(f*x+e)^6/cos(f*x+e
)^3+1/35*sin(f*x+e)^6/cos(f*x+e)+1/35*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)))
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.86

$$-\frac{420\,a\log\left(\cos\left(fx+e\right)\right)+\frac{420\,a\cos(fx+e)^{5}-140\,b\cos(fx+e)^{4}-105\,a\cos(fx+e)^{3}+168\,b\cos(fx+e)^{2}-60\,b}{\cos(fx+e)^{7}}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x + e
)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)/f
```

Fricas [A]
time = 2.56, size = 88, normalized size = 0.96

$$-\frac{420\,a\cos\left(fx+e\right)^{7}\log\left(-\cos\left(fx+e\right)\right)+420\,a\cos\left(fx+e\right)^{5}-140\,b\cos\left(fx+e\right)^{4}-105\,a\cos\left(fx+e\right)^{3}+168\,b\cos\left(fx+e\right)^{2}-60\,b}{420\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 14
0*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/(f
*cos(f*x + e)^7)
```

Sympy [A]
time = 1.14, size = 119, normalized size = 1.29

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{4}\left(e+fx\right)}{4f}-\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)\sec^{3}\left(e+fx\right)}{7f}-\frac{4b\tan^{2}\left(e+fx\right)\sec^{3}\left(e+fx\right)}{35f}+\frac{8b\sec^{3}\left(e+fx\right)}{105f} & \text{for}\: f\neq 0\\ x\left(a+b\sec^{3}\left(e\right)\right)\tan^{5}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(e
+ f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)),
(x*(a + b*sec(e)**3)*tan(e)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(84) = 168.
time = 1.83, size = 340, normalized size = 3.70

$$\frac{420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right)+\frac{1089\,a+64\,b+\frac{8463\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{448\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28749\,a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{1344\,b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{51555\,a(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{2240\,b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{51555\,a(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{4480\,b(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{28749\,a(\cos(fx+e)-1)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{8463\,a(\cos(fx+e)-1)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{1089\,a(\cos(fx+e)-1)^{7}}{(\cos(fx+e)+1)^{7}}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^{7}}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 420*a*l
og(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (1089*a + 64*b + 8463
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 448*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 28749*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1344*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 51555*a*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 2240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 51555
*a*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 4480*b*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4 + 28749*a*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 +
8463*a*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 1089*a*(cos(f*x + e) - 1
)^7/(cos(f*x + e) + 1)^7)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^7)/f
```

Mupad [B]
time = 8.79, size = 227, normalized size = 2.47

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}-\frac{2\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-14\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+\left(32\,a+\frac{32\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\left(\frac{16\,b}{3}-32\,a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(14\,a+\frac{16\,b}{5}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(-2\,a-\frac{16\,b}{15}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{16\,b}{105}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((16*b)/105 - tan(e/2 + (f*x)/2)^2*(2
*a + (16*b)/15) + tan(e/2 + (f*x)/2)^4*(14*a + (16*b)/5) - tan(e/2 + (f*x)/
2)^6*(32*a - (16*b)/3) + tan(e/2 + (f*x)/2)^8*(32*a + (32*b)/3) - 14*a*tan(
e/2 + (f*x)/2)^10 + 2*a*tan(e/2 + (f*x)/2)^12)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1))
```

3.453 $\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$

Optimal. Leaf size=61

$$\frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}$$

```
[Out] a*ln(cos(f*x+e))/f+1/2*a*sec(f*x+e)^2/f-1/3*b*sec(f*x+e)^3/f+1/5*b*sec(f*x+
e)^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4223, 1816}

$$\frac{a\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + (a*Sec[e + f*x]^2)/(2*f) - (b*Sec[e + f*x]^3)/(3*
f) + (b*Sec[e + f*x]^5)/(5*f)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^3)}{x^6}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^6}-\frac{b}{x^4}+\frac{a}{x^3}-\frac{a}{x}\right)\,dx,x,\cos(e+fx)\right)}{f}\\ &= \frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+}{5f}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 59, normalized size = 0.97

$$-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}+\frac{a(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] -1/3*(b*Sec[e + f*x]^3)/f + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f*
x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A]
time = 0.09, size = 101, normalized size = 1.66

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+b\left(\frac{\sin^4(fx+e)}{5\cos(fx+e)^5}+\frac{\sin^4(fx+e)}{15\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{15\cos(fx+e)}-\frac{\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{15}\right)}{f}$
default	$\frac{a\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)+b\left(\frac{\sin^4(fx+e)}{5\cos(fx+e)^5}+\frac{\sin^4(fx+e)}{15\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{15\cos(fx+e)}-\frac{\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{15}\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2a\,e^{8i(fx+e)}-\frac{8b\,e^{7i(fx+e)}}{3}+6a\,e^{6i(fx+e)}+\frac{16b\,e^{5i(fx+e)}}{15}+6a\,e^{4i(fx+e)}-\frac{8b\,e^{3i(fx+e)}}{3}+2a\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1/2*tan(f*x+e)^2+ln(cos(f*x+e)))+b*(1/5*sin(f*x+e)^4/cos(f*x+e)^5+1
/15*sin(f*x+e)^4/cos(f*x+e)^3-1/15*sin(f*x+e)^4/cos(f*x+e)-1/15*(2+sin(f*x+
e)^2)*cos(f*x+e)))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.90

$$\frac{30\,a\log\left(\cos\left(fx+e\right)\right)+\frac{15\,a\cos(fx+e)^3-10\,b\cos(fx+e)^2+6\,b}{\cos(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2 +
 6*b)/cos(f*x + e)^5)/f
```

Fricas [A]
time = 3.04, size = 64, normalized size = 1.05

$$\frac{30\,a\cos\left(fx+e\right)^{5}\log\left(-\cos\left(fx+e\right)\right)+15\,a\cos\left(fx+e\right)^{3}-10\,b\cos\left(fx+e\right)^{2}+6\,b}{30\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*c
os(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

Sympy [A]
time = 0.49, size = 82, normalized size = 1.34

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\tan^{2}\left(e+fx\right)\sec^{3}\left(e+fx\right)}{5f}-\frac{2b\sec^{3}\left(e+fx\right)}{15f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^{3}\left(e\right)\right)\tan^{3}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f, 0
)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(55) = 110.
time = 0.87, size = 271, normalized size = 4.44

$$-\frac{60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right)+\frac{137\,a+16\,b+\frac{805\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{1730\,a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{80\,b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{1730\,a(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{240\,b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{805\,a(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{137\,a(\cos(fx+e)-1)^{5}}{(\cos(fx+e)+1)^{5}}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^{5}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 60*a*log
(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (137*a + 16*b + 805*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1730*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 1730*a*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 + 240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 805*a*(cos(f*x
+ e) - 1)^4/(cos(f*x + e) + 1)^4 + 137*a*(cos(f*x + e) - 1)^5/(cos(f*x + e)
 + 1)^5)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^5)/f
```

Mupad [B]

time = 8.39, size = 167, normalized size = 2.74

$$\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\left(-6\,a-4\,b\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(6\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(-2\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{4\,b}{15}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] ((4*b)/15 - tan(e/2 + (f*x)/2)^2*(2*a + (4*b)/3) - tan(e/2 + (f*x)/2)^6*(6*
a + 4*b) + tan(e/2 + (f*x)/2)^4*(6*a - (4*b)/3) + 2*a*tan(e/2 + (f*x)/2)^8)
/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/
2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (2*a*atanh(ta
n(e/2 + (f*x)/2)^2))/f
```

3.454 $\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx$

Optimal. Leaf size=30

$$-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[Out] -a*ln(cos(f*x+e))/f+1/3*b*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 14}

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx&=-\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^4}\,dx,x,\cos(e+fx)\right)}{f}\\&=-\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a}{x}\right)\,dx,x,\cos(e+fx)\right)}{f}\\&=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.00

$$-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Maple [A]
time = 0.05, size = 26, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\left(\sec^3(fx+e)\right)}{3}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\left(\sec^3(fx+e)\right)}{3}+a\ln(\sec(fx+e))}{f}$	26
risch	$iax+\frac{2iae}{f}+\frac{8b\,\mathrm{e}^{3i(fx+e)}}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b*sec(f*x+e)^3+a*ln(sec(f*x+e)))
```

Maxima [A]
time = 0.28, size = 30, normalized size = 1.00

$$-\frac{a\log\left(\cos\left(fx+e\right)^3\right)-\frac{b}{\cos(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

Fricas [A]
time = 2.82, size = 40, normalized size = 1.33

$$-\frac{3\,a\cos\left(fx+e\right)^3\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

Sympy [A]
time = 0.20, size = 42, normalized size = 1.40

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^3\left(e+fx\right)}{3f} & \text{for } f\neq 0\\ x\left(a+b\sec^3\left(e\right)\right)\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(f
, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(28) = 56.
time = 0.53, size = 181, normalized size = 6.03

$$\frac{6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right)+\frac{11\,a+4\,b+\frac{33\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{33\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{11\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 6*a*log(abs
(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (11*a + 4*b + 33*a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 33*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 12*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 11*a*(cos(f*x + e) -
 1)^3/(cos(f*x + e) + 1)^3)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/
f
```

Mupad [B]
time = 5.26, size = 83, normalized size = 2.77

$$\frac{2\,a\,\mathrm{atanh}\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)}{f}-\frac{2\,b\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+\frac{2\,b}{3}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((2*b)/3 + 2*b*tan(e/2 + (f*x)/2)^4)/
(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6
- 1))
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] 1/2*(a+b)*ln(1-cos(f*x+e))/f+1/2*(a-b)*ln(1+cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 1816}

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((a + b)*Log[1 - Cos[e + f*x]])/(2*f) + ((a - b)*Log[1 + Cos[e + f*x]])/(2*
f) + (b*Sec[e + f*x])/f
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^2(1-x^2)}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{2(-1+x)}+\frac{b}{x^2}+\frac{-a+b}{2(1+x)}\right)dx,x,\cos(e+fx)\right)}{f}\\ &= \frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\text{s}}{}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 1.20

$$-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{a(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b*Log[Cos[(e + f*x)/2]])/f) + (b*Log[Sin[(e + f*x)/2]])/f + (a*(Log[Cos[
e + f*x]] + Log[Tan[e + f*x]]))/f + (b*Sec[e + f*x])/f
```

Maple [A]
time = 0.09, size = 42, normalized size = 0.78

method	result
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
default	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}+\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*ln(sin(f*x+e))+b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.89

$$\frac{(a-b)\log\left(\cos\left(fx+e\right)+1\right)+(a+b)\log\left(\cos\left(fx+e\right)-1\right)+\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/co
s(f*x + e))/f
```

Fricas [A]
time = 2.78, size = 66, normalized size = 1.22

$$\frac{(a-b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+2\,b}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x + e
)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec^3(e + fx))\cot(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.61

$$\frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 2\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) + \frac{4\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 2*a*log(ab
s(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) + 4*b/((cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 1))/f
```

Mupad [B]
time = 4.64, size = 72, normalized size = 1.33

$$\frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f} - \frac{a\,\ln\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}{f} - \frac{2\,b}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)} + \frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)))/f - (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (2*b)
/(f*(tan(e/2 + (f*x)/2)^2 - 1)) + (b*log(tan(e/2 + (f*x)/2)))/f
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal. Leaf size=72

$$-\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(1+\cos(e+fx))}{4f}$$

```
[Out] -1/2*(a+b*cos(f*x+e))*csc(f*x+e)^2/f-1/4*(2*a-b)*ln(1-cos(f*x+e))/f-1/4*(2*
a+b)*ln(1+cos(f*x+e))/f
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4223, 1828, 647, 31}

$$-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/2*((a + b*Cos[e + f*x])*Csc[e + f*x]^2)/f - ((2*a - b)*Log[1 - Cos[e + f
*x]])/(4*f) - ((2*a + b)*Log[1 + Cos[e + f*x]])/(4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^3}{(1-x^2)^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{\text{Subst}\left(\int \frac{-b+2ax}{1-x^2}\,dx,x,\cos(e\right.}{2f}\\ &= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{(2a-b)\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\right.}{4f}\\ &= -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}-\frac{(2a-b)\log(1-\cos(e+fx}{4f}\end{aligned}$$

Mathematica [A]
time = 1.11, size = 114, normalized size = 1.58

$$-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
[Out] -1/8*(b*Csc[(e + f*x)/2]^2)/f - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Si
n[(e + f*x)/2]])/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[T
an[e + f*x]]))/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

Maple [A]
time = 0.10, size = 63, normalized size = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$iax+\frac{2iae}{f}+\frac{b\,e^{3i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+b*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2
*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.27, size = 66, normalized size = 0.92

$$-\frac{(2\,a+b)\log\left(\cos\left(fx+e\right)+1\right)+(2\,a-b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,(b\cos(fx+e)+a)}{\cos(fx+e)^2-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) - 2
*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 3.40, size = 105, normalized size = 1.46

$$\frac{2\,b\cos\left(fx+e\right)-\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-\left((2\,a-b)\cos\left(fx+e\right)^2-2\,a+b\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+2\,a}{4\,\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f*
x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e)
+ 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec^3\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(66) = 132.
time = 0.49, size = 172, normalized size = 2.39

$$-\frac{2\,(2\,a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a - b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*l
og(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

Mupad [B]
time = 4.65, size = 86, normalized size = 1.19

$$\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a}{8}-\frac{b}{8}\right)}{f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a}{8}+\frac{b}{8}\right)}{f}-\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(a-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

```
[Out] (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (tan(e/2 + (f*x)/2)^2*(a/8 - b/8))/f
- (cot(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (log(tan(e/2 + (f*x)/2))*(a - b/2)
)/f
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=219

$$-\frac{(a^{2/3}+2b^{2/3})\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,\cos(e+fx)}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{4/3}f}-\frac{(a^{2/3}-2b^{2/3})\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f}+\frac{(a^{2/3}-2b^{2/3})}{}$$

```
[Out] -1/3*(a^(2/3)-2*b^(2/3))*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(4/3)/f+1
/6*(a^(2/3)-2*b^(2/3))*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*
x+e)^2)/a^(1/3)/b^(4/3)/f-1/3*ln(b+a*cos(f*x+e)^3)/a/f+sec(f*x+e)/b/f-1/3*(
a^(2/3)+2*b^(2/3))*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(4/3)/f*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4223, 1848, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{(a^{2/3}+2b^{2/3})\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,\cos(e+fx)}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{4/3}f}+\frac{(a^{2/3}-2b^{2/3})\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\,\sqrt[3]{b}\,\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{4/3}f}-\frac{(a^{2/3}-2b^{2/3})\log\left(\sqrt[3]{a}\,\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{4/3}f}-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}+\frac{\sec(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -(((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)*f)) - ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)*f) + ((a^(2/3) - 2*b^(2/3))*
Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)*f) - Log[b + a*Cos[e + f*x]^3]/(3*a*f) + Sec[e + f*x]/(b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-2b-ax+bx^2}{b(b+ax^3)}\right)\,dx, x, \cos(e+fx)\right)}{f} \\
&= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{-2b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf} \\
&= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{-2b-ax}{b+ax^3}\,dx, x, \cos(e\right.}{bf} \\
&= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\left(-a\sqrt[3]{b}-4\sqrt[3]{a}\,b\right)+\sqrt[3]{a}\left(-\right.}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^2}\right.}{3\sqrt[3]{a}\,b} \\
&= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e}{b} \\
&= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\right.}}{6\sqrt[3]{}} \\
&= -\frac{\left(a^{2/3}+2b^{2/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{4/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\cos(e\right.}{3\sqrt[3]{a}\,b^{4/3}f}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.39, size = 251, normalized size = 1.15

$$\frac{3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)-\text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&,\frac{-4a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+4ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1-8ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1+ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2-b^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2}{4a-4a\#1+a\#1^2-b\#1^2}\&\right]+3a\sec(e+fx)}{3abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

[Out] (3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 - b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) &] + 3*a*Sec[e + f*x])/(3*a*b*f)

Maple [A]
time = 0.35, size = 267, normalized size = 1.22

method	result
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,\mathrm{e}^{i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)}-i\left(\sum_{\_R=\mathrm{RootOf}\left(27a^3b^4f^3\_Z^3+27ia^2b^4f^2\_Z^2+\left(-18a^3b^2f-9a\,b^4f\right)\_Z-ia^4+2ia\right.}\right.$
derivativedivides	$\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)+a\left(-\frac{\ln}{}\right.}{}$
default	$\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)+a\left(-\frac{\ln}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)

[Out] 1/f*((2*b*(1/3/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))-1/6/a/(1/a*b)^(2/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3/a/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1)))+a*(-1/3/a/(1/a*b)^(1/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/a/(1/a*b)^(1/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3*3^(1/2)/a/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1)))-1/3*b/a*ln(b+a*cos(f*x+e)^3))/b+1/b/cos(f*x+e))

Maxima [A]
time = 0.50, size = 223, normalized size = 1.02

$$\frac{\frac{2\sqrt{3}\left(2ab\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{b}{a}\right)+3a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+2b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{3\left(2b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)-a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{6\left(b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}-2\right)+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+\cos(fx+e)\right)}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{18}{b\cos(fx+e)}}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(2*a*b*(3*(b/a)^(1/3) - b/a) + 3*a^2*(b/a)^(2/3) + 2*b^2)*a
rctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b^2) - 3*
(2*b*((b/a)^(2/3) + 1) - a*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*co
s(f*x + e) + (b/a)^(2/3))/(a*b*(b/a)^(2/3)) - 6*(b*((b/a)^(2/3) - 2) + a*(b
/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a*b*(b/a)^(2/3)) + 18/(b*cos(f*
x + e)))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 7.24, size = 4438, normalized size = 20.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/
27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2
*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) +
1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))
*a*b*f*cos(f*x + e)*log(1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)
/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2
*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 - ((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^
2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f
^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4
*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^
3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f + 4*a^2*b + 5*b^3 + (a^3 + 8*a*b^2)*co
s(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2)
)/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a
^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/
```

(a*f))*a*b*f*cos(f*x + e) + 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(
a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^
2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
+ 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^
2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3
) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*
(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 -
2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3)
+ 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*
(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 +
36*b^2)/(a^2*b^2*f^2))*cos(f*x + e) - 18*b*cos(f*x + e))*log(1/36*((-I*sqr
t(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/
54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4
- 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^
3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/
54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 -
((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f
^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1
/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27
/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f
^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f
+ 4*a^2*b + 5*b^3 - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2
+ b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*
f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3
*b^4*f^3))^(1/3) + 6/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) +
1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^
2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a
^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1
/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^
4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-
I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3)
+ 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54
*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a
^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3)
- 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 2
88*a^2 + 36*b^2)/(a^2*b^2*f^2)) - 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - (((-I*s
qrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) +
1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a
^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*
f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) -
1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b*f*cos(f*x
+ e) - 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b

```
^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^3 + a), x)
```

Mupad [B]
time = 7.26, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(-(262144*(148*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z +
 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17 - 1920*a*b^15 - 156*b^16*
cos(e + f*x) + 300*b^16 + 16*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4
*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*b^18 + 5232*a^2*b^14 - 7
872*a^3*b^13 + 7080*a^4*b^12 - 3840*a^5*b^11 + 1200*a^6*b^10 - 192*a^7*b^9
+ 12*a^8*b^8 - 5916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a
^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15 + 4820*root(27*a^3*b^4*z^3
 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)
*a^3*b^14 + 5933*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*
b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13 - 12882*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a
^5*b^12 + 8891*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11 - 2872*root(27*a^3*b^4*z^3 + 27
*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^7*
b^10 + 447*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
```

- 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9 - 26*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^9*b^8 + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7 + 1396*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a*b^17 + 192*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a*b^18 - 3936*a^2*b^14*cos(e + f*x) + 7152*a^3*b^13*cos(e + f*x) - 7800*a^4*b^12*cos(e + f*x) + 5136*a^5*b^11*cos(e + f*x) - 1920*a^6*b^10*cos(e + f*x) + 336*a^7*b^9*cos(e + f*x) - 12*a^8*b^8*cos(e + f*x) - 768*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^2*b^16 + 4772*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^3*b^15 - 13924*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^4*b^14 + 6927*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^5*b^13 + 5747*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^6*b^12 - 5944*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^7*b^11 + 2004*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^8*b^10 - 239*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^9*b^9 + 13*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*a^10*b^8 + 4296*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^2*b^17 - 11856*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^3*b^16 + 16956*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^4*b^15 - 17916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^5*b^14 + 11175*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^6*b^13 - 4608*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^7*b^12 + 2118*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^8*b^11 - 372*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^9*b^10 + 15*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^3*a^10*b^9 + 864*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^2*b^18 + 3240*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^3*b^17 - 12996*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^4*b^16 + 4140*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^5*b^15 + 16668*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^6*b^14 - 16011*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a

```
^7*b^13 + 4959*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^
2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^8*b^12 - 873*root(27*a^3*b^4*z^3 + 2
7*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a
^9*b^11 + 9*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z
 - 2*a^2*b^2 + b^4 + a^4, z, k)^4*a^10*b^10 + 1728*root(27*a^3*b^4*z^3 + 27
*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^5*a^
3*b^18 - 5724*root(27*a^3*b^4*z^3 + 27*a^2*b^4*...
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=166

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}f}-\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f}+\frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2\right.}{6\sqrt[3]{a}\,b^{2/3}f}$$

```
[Out] -1/3*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(2/3)/f+1/6*ln(b^(2/3)-a^(1/3
)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(2/3)/f+1/3*ln(b+a*cos
(f*x+e)^3)/a/f+1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2
))/a^(1/3)/b^(2/3)/f*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4223, 1885, 206, 31, 648, 631, 210, 642, 266}

$$\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}f}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}f}-\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}f}+\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/
3)*b^(2/3)*f) - Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)*f) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(6*a^
(1/3)*b^(2/3)*f) + Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, \cos(e+fx)\right)}{3b^{2/3}f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}}{b^{2/3}-\sqrt[3]{a}}\right.}{}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e\right.}{6\sqrt[3]{a}\,b^{2/3}f}$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.28, size = 242, normalized size = 1.46

$$\frac{-3\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+\text{RootSum}\left[-a-b+3a\#1-3b\#1-3a\#1^2-3b\#1^2+a\#1^3-b\#1^3\&,\frac{-a\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-b\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4a\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1-2b\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1+a\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2-b\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2}{a-b-2a\#1-2b\#1+a\#1^2-b\#1^2}\&\right]}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2 -
 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*Log
[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*Log
[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 - b*Lo
g[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 - b*#1^
2) & ])/(3*a*f)
```

Maple [A]
time = 0.21, size = 133, normalized size = 0.80

method	result

risch	$-\frac{ix}{a}-\frac{2ie}{af}+i\left(\sum_{\_R=\mathrm{RootOf}\left(27b^2a^3f^3\_Z^3+27ib^2a^2f^2\_Z^2-9\_Zab^2f+ia^2-ib^2\right)}\_R\ln\left(\mathrm{e}^{2i(fx+e)}+\left(-6ib\right.\right.\right.$
derivativedivides	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\left(\cos^3\right.\right.}{3a}}{f}$
default	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\left(\cos^3\right.\right.}{3a}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/a/(1/a*b)^(2/3)*
ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))-1/3/a/(1/a*b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1))+1/3/a*ln(b+a*co
s(f*x+e)^3))
```

Maxima [A]
time = 0.50, size = 163, normalized size = 0.98

$$-\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{2b}{a}\right)+2b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{6\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+\cos(fx+e)\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*(3*(b/a)^(1/3) - 2*b/a) + 2*b)*arctan(-1/3*sqrt(3)*((b/
a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b) - 3*(2*(b/a)^(2/3) + 1)*log(c
os(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a*(b/a)^(2/3)) - 6
*((b/a)^(2/3) - 1)*log((b/a)^(1/3) + cos(f*x + e))/(a*(b/a)^(2/3)))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 54.38, size = 2294, normalized size = 13.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")

[Out] -1/12*(6*sqrt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 - 4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^3 - 8*a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2*f^2)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) - 6*sqrt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 - 4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) - sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^3 - 8*a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2*f^2)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2

```
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) + (3*(I*sqrt(3) +
 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(
1/3) - 2/(a*f))*a*f*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b
^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 +
a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + (a^2*b*f*cos(f*x + e) + a*b^2*f)*
(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(
a^3*b^2*f^3))^(1/3) - 2/(a*f)) + b^2) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3
) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f
 + 6)*log((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^
2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)
^2 - 4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3
) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3)
)^(1/3) - 2/(a*f)) + 4*b^2))/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sec(f*x + e)^3 + a), x)
```

Mupad [B]
time = 8.54, size = 1620, normalized size = 9.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] symsum(log(262144*(a - b)^2*(8*a - 8*b + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2 + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^2 - 3*root(27*a^3*b^2*z^3 - 27*a^2*b
```

^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3 - 24*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^2 - 36*root(27*a^3*b^2*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^3*b + 28*root(27*a^3*b^
2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b + 36*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^2*b^2)*(16*a^2*
tan(e/2 + (f*x)/2)^2 + 32*b^2*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^3 - 4*root(27*a^3*b^2*z^3
 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^3 - 8*a^2 + 8*b^2 + 3*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4 - 3
*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^4*
tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
+ a^2 - b^2, z, k)^2*a*b^3 + 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b
^2*z + a^2 - b^2, z, k)^2*a^3*b + 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
 9*a*b^2*z + a^2 - b^2, z, k)^3*a^4*b - 48*a*b*tan(e/2 + (f*x)/2)^2 + 24*ro
ot(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2
 - 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3
*a^2*b^3 + 14*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2,
 z, k)*a^3*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z + a^2 - b^2, z, k)*b^3*tan(e/2 + (f*x)/2)^2 - 32*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2 - 32*root(27*a^3*
b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2*b - 146*root(27
*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b^2*tan(e/2
+ (f*x)/2)^2 + 64*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 -
b^2, z, k)*a^2*b*tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^3*tan(e/2 + (f*x)/2)^2 + 57*root(
27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3*b*tan(
e/2 + (f*x)/2)^2 - 54*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^
2 - b^2, z, k)^3*a^4*b*tan(e/2 + (f*x)/2)^2 + 84*root(27*a^3*b^2*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^2*b^2*tan(e/2 + (f*x)/2)^2 -
36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^
2*b^3*tan(e/2 + (f*x)/2)^2 + 198*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a
*b^2*z + a^2 - b^2, z, k)^3*a^3*b^2*tan(e/2 + (f*x)/2)^2))*root(27*a^3*b^2*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k), k, 1, 3)/f - log(tan(e
/2 + (f*x)/2)^2 + 1)/(a*f)

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

```
[Out] -1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4223, 266}

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$-\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
[Out] -1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Maple [A]
time = 0.06, size = 35, normalized size = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\left(\sec^3(fx+e)\right)\right)}{3a}}{f}$	35
default	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\left(\sec^3(fx+e)\right)\right)}{3a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{6i(fx+e)}+3\,e^{4i(fx+e)}+\frac{8b\,e^{3i(fx+e)}}{a}+3\,e^{2i(fx+e)}+1\right)}{3af}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/a*ln(sec(f*x+e))-1/3/a*ln(a+b*sec(f*x+e)^3))
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.96

$$-\frac{\log\left(a\cos\left(fx+e\right)^{3}+b\right)}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Fricas [A]
time = 3.54, size = 22, normalized size = 0.96

$$-\frac{\log\left(a\cos\left(fx+e\right)^{3}+b\right)}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(19) = 38$.

time = 21.47, size = 139, normalized size = 6.04

$$\begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^3(e)} & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x \tan(e)}{a + b \sec^3(e)} & \text{for } f = 0 \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b = 0 \\ -\frac{1}{3bf\sec^3(e+fx)} & \text{for } a = 0 \\ -\frac{\log\left(-\sqrt[3]{-\frac{a}{b}} + \sec(e+fx)\right)}{3af} + \frac{\log(\tan^2(e+fx)+1)}{2af} - \frac{\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-\frac{a}{b}}\sec(e+fx) + 4\sec^2(e+fx)\right)}{3af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x*tan(
e)/(a + b*sec(e)**3), Eq(f, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0
)), (-1/(3*b*f*sec(e + f*x)**3), Eq(a, 0)), (-log(-(-a/b)**(1/3) + sec(e +
f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(4*(-a/b)**(2/3) + 4*
(-a/b)**(1/3)*sec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(21) = 42$.

time = 0.56, size = 177, normalized size = 7.70

$$-\frac{\frac{\log\left(\left|a+b+\frac{3\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{3\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right|\right)}{a}-\frac{3\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] -1/3*(log(abs(a + b + 3*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1
)^3/(cos(f*x + e) + 1)^3 - b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3))/a
- 3*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

Mupad [B]

time = 5.19, size = 114, normalized size = 4.96

$$\frac{3\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)-\ln\left(a+b-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\right)}{3\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

```
[Out] (3*log(tan(e/2 + (f*x)/2)^2 + 1) - log(a + b - 3*a*tan(e/2 + (f*x)/2)^2 + 3
*a*tan(e/2 + (f*x)/2)^4 - a*tan(e/2 + (f*x)/2)^6 + 3*b*tan(e/2 + (f*x)/2)^2
 + 3*b*tan(e/2 + (f*x)/2)^4 + b*tan(e/2 + (f*x)/2)^6))/(3*a*f)
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=295

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f}+\frac{\log(1-\cos(e+fx))}{2(a+b)f}+\frac{\log(1+\cos(e+fx))}{2(a-b)f}-\frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{\ldots}\right.}{3\sqrt[3]{a}\left(a\ldots\right.}$$

```
[Out] 1/2*ln(1-cos(f*x+e))/(a+b)/f+1/2*ln(1+cos(f*x+e))/(a-b)/f-1/3*(a^(2/3)+b^(2
/3))*b^(2/3)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)/f+1/6*(a^(2/3
)+b^(2/3))*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)
^2)/a^(1/3)/(a^2-b^2)/f-1/3*b^2*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)/f-1/3*b^(2
/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^(
4/3)+a^(2/3)*b^(2/3)+b^(4/3))/f*3^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {4223, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}f\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)}-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)}-\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}f\left(a^2-b^2\right)}+\frac{\log(1-\cos(e+fx))}{2f(a+b)}+\frac{\log(\cos(e+fx)+1)}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))*f)) + Log[1 - Cos[e +
f*x]]/(2*(a + b)*f) + Log[1 + Cos[e + f*x]]/(2*(a - b)*f) - ((a^(2/3) + b^(
2/3))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)*f
) + ((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x]
 + a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)*f) - (b^2*Log[b + a*Cos[
e + f*x]^3])/(3*a*(a^2 - b^2)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} - \frac{1}{2(a-b)(1+x)} - \frac{b(b-ax+bx^2)}{(-a^2+b^2)(b+ax^3)}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\text{Subst}\left(\int \frac{b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e-\right.}{(a^2-b^2)f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\text{Subst}\left(\int \frac{b-ax}{b+ax^3}\,dx, x, \cos(e+f\right.}{(a^2-b^2)f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a(a^2-b^2)f} - \cdots$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{\cdots}\right.}{3\sqrt[3]{a}\,(a^2-b^2)f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{\cdots}\right.}{3\sqrt[3]{a}\,(a^2-b^2)f}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f} + \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+\cdots}{2(a-b)f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.44, size = 290, normalized size = 0.98

$$\frac{3\left(a(a+b)\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)\right)+b^2\log\left(\sec^2\left(\tfrac{1}{2}(e+fx)\right)\right)+a(a-b)\log\left(\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)-b\text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&,\frac{-4a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+4ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1-2ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1+ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2-b^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2}{4a-4a\#1+a\#1^2-b\#1^2}\&\right]}{3a(a-b)(a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*(a*(a + b)*Log[Cos[(e + f*x)/2]] + b^2*Log[Sec[(e + f*x)/2]^2] + a*(a -
b)*Log[Sin[(e + f*x)/2]]) - b*RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + Ta
n[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 2*a*b*Log[1
 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
- b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^
2) & ])/(3*a*(a - b)*(a + b)*f)
```

Maple [A]

time = 0.36, size = 303, normalized size = 1.03

method	result
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ie}{f(a+b)}-\frac{ix}{a-b}-\frac{ie}{f(a-b)}-\frac{2ia^2b^2f^3x}{-a^5f^3+a^3b^2f^3}-\frac{2ia^2b^2f^2e}{-a^5f^3+a^3b^2f^3}+\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}+\frac{\ln(e^{i(fx+e)}\cdots}{f(a-b)\cdots}$
derivativedivides	$\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\cdots\left(-b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\cdots\right.}{\cdots}\cdots\right.\right.$
default	$\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\cdots\left(-b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos^2(fx+e)-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\cdots\right.}{\cdots}\cdots\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(2*a-2*b)*ln(1+cos(f*x+e))+1/(2*a+2*b)*ln(cos(f*x+e)-1)+(-b*(1/3/a/(
1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))-1/6/a/(1/a*b)^(2/3)*ln(cos(f*x+e)
^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3/a/(1/a*b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1)))+a*(-1/3/a/(1/a*b)^(1/3)*ln(
cos(f*x+e)+(1/a*b)^(1/3))+1/6/a/(1/a*b)^(1/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)
```

```
*cos(f*x+e)+(1/a*b)^(2/3))+1/3*3^(1/2)/a/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/a*b)^(1/3)*cos(f*x+e)-1)))-1/3*b/a*ln(b+a*cos(f*x+e)^3))*b/(a+b)/(a-b)
)
```

Maxima [A]
time = 0.50, size = 312, normalized size = 1.06

$$-\frac{\frac{2\,\sqrt{3}\left(ab^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{2\,b}{a}\right)-3\,a^2b\left(\frac{b}{a}\right)^{\frac{2}{3}}-2\,b^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\,\cos\left(fx+e\right)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}-a^2b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{3\left(b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos\left(fx+e\right)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{6\left(b^2\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)+ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+\cos\left(fx+e\right)\right)}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{9\,\log\left(\cos\left(fx+e\right)+1\right)}{a-b}-\frac{9\,\log\left(\cos\left(fx+e\right)-1\right)}{a+b}}{18\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*b^2*(3*(b/a)^(1/3) + 2*b/a) - 3*a^2*b*(b/a)^(2/3) - 2*b
^3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^4*(
b/a)^(2/3) - a^2*b^2*(b/a)^(2/3))*(b/a)^(1/3)) + 3*(b^2*(2*(b/a)^(2/3) - 1)
 - a*b*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(
2/3))/(a^3*(b/a)^(2/3) - a*b^2*(b/a)^(2/3)) + 6*(b^2*((b/a)^(2/3) + 1) + a*
b*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^3*(b/a)^(2/3) - a*b^2*(b/
a)^(2/3)) - 9*log(cos(f*x + e) + 1)/(a - b) - 9*log(cos(f*x + e) - 1)/(a +
b))/f
```

Fricas [C] Result contains complex when optimal does not.
time = 6.30, size = 6487, normalized size = 21.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f
^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 -
 a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*
b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^
3*f - a*b^2*f))*f*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^
2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^
2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/
54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/
18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^
3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/
(a^3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2
 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*
f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5
```

*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/
(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f))
- 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*
(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - a*b*cos(f*x + e) + 2*b^2
) - ((a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))
*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/
((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f
^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f
- a*b^2*f))*f + 3*sqrt(1/3)*(a^3 - a*b^2)*f*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b
^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) +
1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*
f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*
a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f
^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b^4/(a^3*f - a*b^2*f)^2
+ b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f
)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*
f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(
a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) -
1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(
I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f
^2)) - 18*b^2)*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f
^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 -
a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*
b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^
3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 +
b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^
3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^
3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^
3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1
/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*
sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 + 1/12*sqrt(1/3)*(a^4 - a^2*b
^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) +
1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*
f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*
a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f^2
*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^
2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^

```
4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2
*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2
*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^
5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1
) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a
*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3)
+ 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^3 + a), x)
```

Mupad [B]
time = 7.90, size = 2500, normalized size = 8.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

```
[Out] log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(f*(a + b)) - log(1/cos(e/2 + (f
*x)/2)^2)/(f*(a + b)) + (a*symsum(log((262144*(832*root(27*a^3*b^2*z^3 - 27
*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*b^6
*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2
*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2*z^3 -
 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*root(27*
a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^3*b^4
- 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
```

z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9
*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a
^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^2*z^3 - 27*a
^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e + f*x) - a^2*b
^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 +
9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^3 - 27*a^5*z^3
- 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*root(27*a^3*b^2*
z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^4*b^4 - 9*ro
ot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*
a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^3*b^6 + 216667*root(27*a^3*b^2*z^3 -
27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^4*b^5 - 44745*roo
t(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a
^5*b^4 + 1584*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^3*a^6*b^3 - 12720*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^2*b^8 + 14028*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^3*b^7 + 156387*root(2
7*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^4*
b^6 - 457125*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z
- b^2, z, k)^4*a^5*b^5 + 228117*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^6*b^4 - 24723*root(27*a^3*b^2*z^3 - 27*
a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^7*b^3 + 486*root(27*a
^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a^8*b^2
+ 864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
z, k)^5*a^2*b^9 + 18792*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
+ 9*a*b^2*z - b^2, z, k)^5*a^3*b^8 - 151488*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^4*b^7 + 577008*root(27*a^3*
b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^5*b^6 -
414504*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2,
z, k)^5*a^6*b^5 - 144432*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2
+ 9*a*b^2*z - b^2, z, k)^5*a^7*b^4 + 63702*root(27*a^3*b^2*z^3 - 27*a^5*z^
3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^8*b^3 + 486*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^5*a^9*b^2 - 172
8*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k
)^6*a^3*b^9 + 3672*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*
b^2*z - b^2, z, k)^6*a^4*b^8 + 69444*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*
a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^5*b^7 - 637794*root(27*a^3*b^2*z^3
- 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^6*b^6 + 1468908
*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^6*a^7*b^5 - 1112400*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*
a*b^2*z - b^2, z, k)^6*a^8*b^4 + 210384*root(27*a^3*b^2*z^3 - 27*a^5*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^9*b^3 - 486*root(27*a^3*b^2*z^3
- 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^6*a^10*b^2 + 1296*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7

*a^4*b^9 - 23004*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^5*b^8 + 195534*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^6*b^7 - 778734*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^7*b^6 + 1175796*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^8*b^5 - 690768*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^9*b^4 + 120366*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^10*b^3 - 486*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^7*a^11*b^2 - 8702*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a*b^6 - 272*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7*cos(e + f*x) + 62*a*b^5*cos(e + f*x) + 13774*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z -...

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=393

$$\frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f}-\frac{1}{4(a+b)f(1-\cos(e+fx))}-\frac{1}{4(a-b)f(1+\cos(e+fx))}$$

```
[Out] -1/4/(a+b)/f/(1-cos(f*x+e))-1/4/(a-b)/f/(1+cos(f*x+e))-1/4*(2*a+5*b)*ln(1-c
os(f*x+e))/(a+b)^2/f-1/4*(2*a-5*b)*ln(1+cos(f*x+e))/(a-b)^2/f-1/3*b^(4/3)*(
a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^
2)^2/f+1/6*b^(4/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(2/3)-a^(1/3)*b^(1/3)
*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)^2/f-1/3*b^2*(2*a^2+b^2)
*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)^2/f+1/3*b^(4/3)*(a^2-3*a^(2/3)*b^(4/3)+2*
b^2)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)/(a^
2-b^2)^2/f*3^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4223, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{b^2\left(2a^2+b^2\right)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}f\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2}-\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}f\left(a^2-b^2\right)^2}-\frac{1}{4f(a+b)(1-\cos(e+fx))}-\frac{1}{4f(a-b)(\cos(e+fx)+1)}-\frac{(2a+5b)\log(1-\cos(e+fx))}{4f(a+b)^2}-\frac{(2a-5b)\log(\cos(e+fx)+1)}{4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[
e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*f) - 1/(4*(a +
 b)*f*(1 - Cos[e + f*x])) - 1/(4*(a - b)*f*(1 + Cos[e + f*x])) - ((2*a + 5*
b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2*f) - ((2*a - 5*b)*Log[1 + Cos[e + f*
x]])/(4*(a - b)^2*f) - (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2*f) + (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2
/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*f) - (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{2a+5b}{4(a+b)^2(-1+x)} - \frac{1}{4(a-b)(1+x)^2} + \frac{2a-5b}{4(a-b)^2(1+x)} + \frac{b^2(a^2+2b^2-3a}{(a^2-b^2}\right.\right.}{f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-}{4(a+b)}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-}{4(a+b)}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-}{4(a+b)}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-}{4(a+b)}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-}{4(a+b)}$$

$$= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{1}{4(a+b)f(1-\cos(e+fx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.14, size = 336, normalized size = 0.85

$$\frac{-\frac{3\csc^2\left(\frac{1}{2}(e+fx)\right)}{a+b}+\frac{12(-2a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)^2}-\frac{12(2a+5b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)^2}+\frac{8b^2\left(3(2a^2+b^2)\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+(-a+b)\text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&,\frac{8a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4ab\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-6a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1+2a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2+b^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\#1^2}{4a-4a\#1+a\#1^2-b\#1^2}\&\right]\right)}{a(a^2-b^2)^2}-\frac{3\sec^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])/
(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*(2
*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a
*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4*a*b
*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + Tan[(e
+ f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 - b^2)^2)
 - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

Maple [A]
time = 0.75, size = 374, normalized size = 0.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(4*a+4*b)/(cos(f*x+e)-1)+1/4/(a+b)^2*(-2*a-5*b)*ln(cos(f*x+e)-1)-((a
^2+2*b^2)*(1/3/a/(1/a*b)^(2/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))-1/6/a/(1/a*b)^(
2/3)*ln(cos(f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3/a/(1/a*b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1)))-3*a*b*(-1
/3/a/(1/a*b)^(1/3)*ln(cos(f*x+e)+(1/a*b)^(1/3))+1/6/a/(1/a*b)^(1/3)*ln(cos(
f*x+e)^2-(1/a*b)^(1/3)*cos(f*x+e)+(1/a*b)^(2/3))+1/3*3^(1/2)/a/(1/a*b)^(1/3
)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*cos(f*x+e)-1)))+1/3*(2*a^2+b^2)/a*ln(
b+a*cos(f*x+e)^3))*b^2/(a-b)^2/(a+b)^2-1/(4*a-4*b)/(1+cos(f*x+e))+1/4/(a-b)
^2*(-2*a+5*b)*ln(1+cos(f*x+e)))
```

Maxima [A]
time = 0.50, size = 496, normalized size = 1.26

$$\frac{\frac{4\sqrt{3}\left(a^2b^3\left(9\left(\frac{b}{a}\right)^{\frac{2}{3}}+4\right)-a^3b^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{4b}{a}\right)-2ab^4\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{b}{a}\right)+2b^5\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^6\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^4b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+a^2b^4\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{6\left(a^2b^2\left(4\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)+2b^4\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-3ab^3\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a^5\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+ab^4\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{12\left(a^2b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)+b^4\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+2\right)+3ab^3\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+\cos(fx+e)\right)}{a^5\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+ab^4\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{9(2a-5b)\log(\cos(fx+e)+1)}{a^2-2ab+b^2}-\frac{9(2a+5b)\log(\cos(fx+e)-1)}{a^2+2ab+b^2}-\frac{18(b\cos(fx+e)-a)}{(a^2-b^2)\cos(fx+e)^2-a^2+b^2}}{36f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b^3*(9*(b/a)^(2/3) + 4) - a^3*b^2*(3*(b/a)^(1/3) + 4*b
/a) - 2*a*b^4*(3*(b/a)^(1/3) + b/a) + 2*b^5)*arctan(-1/3*sqrt(3)*((b/a)^(1/
3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^6*(b/a)^(2/3) - 2*a^4*b^2*(b/a)^(2/3)
 + a^2*b^4*(b/a)^(2/3))*(b/a)^(1/3)) - 6*(a^2*b^2*(4*(b/a)^(2/3) - 1) + 2*b
^4*((b/a)^(2/3) - 1) - 3*a*b^3*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3
```

```
)*cos(f*x + e) + (b/a)^(2/3))/(a^5*(b/a)^(2/3) - 2*a^3*b^2*(b/a)^(2/3) + a*
b^4*(b/a)^(2/3)) - 12*(a^2*b^2*(2*(b/a)^(2/3) + 1) + b^4*((b/a)^(2/3) + 2)
+ 3*a*b^3*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^5*(b/a)^(2/3) - 2
*a^3*b^2*(b/a)^(2/3) + a*b^4*(b/a)^(2/3)) - 9*(2*a - 5*b)*log(cos(f*x + e)
+ 1)/(a^2 - 2*a*b + b^2) - 9*(2*a + 5*b)*log(cos(f*x + e) - 1)/(a^2 + 2*a*b
 + b^2) - 18*(b*cos(f*x + e) - a)/((a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2))
/f
```

Fricas [C] Result contains complex when optimal does not.
time = 10.57, size = 10762, normalized size = 27.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*a^4 - 18*a^2*b^2 + 2*((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 -
 (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f
 + a*b^4*f))*log(1/12*(a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^
2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/
18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f -
2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*
b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((
a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1
/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b
^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/
(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 + 2*a^2*b^2 + 7*b^4 + 1/6*(a^5 + 16*
a^3*b^2 + 10*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*
b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4
/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6
*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27
*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)
*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 +
 a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*
```

```
b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))
^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f
))*f - (a^3*b + 8*a*b^3)*cos(f*x + e)) - 18*(a^3*b - a*b^3)*cos(f*x + e) +
(36*a^2*b^2 + 18*b^4 - 18*(2*a^2*b^2 + b^4)*cos(f*x + e)^2 - ((a^5 - 2*a^3*
b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2
 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^
4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2
)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^
3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)
- 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 +
b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*
b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54
*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*
b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f)) + 3*sqrt(1/3)*((a^5 - 2*a^3*b^2
 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4
 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^
5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^
5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2
*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^
4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2
)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
- 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f +
 a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3
) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2
*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sec(f*x + e)^3 + a), x)
```

Mupad [B]
time = 19.52, size = 2500, normalized size = 6.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

```
[Out] -(a^3*cos(e/2 + (f*x)/2)^4 + a^3*sin(e/2 + (f*x)/2)^4 - a*b^2*cos(e/2 + (f*
x)/2)^4 + a*b^2*sin(e/2 + (f*x)/2)^4 + 2*a^2*b*sin(e/2 + (f*x)/2)^4 - 8*a^3
*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*co
s(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 + 8*b^3*log((cos(e/2 + (f*x)/2)^2 +
 sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(e/2 +
 (f*x)/2)^2 + 8*a^3*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (
f*x)/2))*sin(e/2 + (f*x)/2)^2 - 8*a^4*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/
2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*sin(e/2 +
 (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(e/2 + (f*
x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^
2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 + (f*x)/2)^2
 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 -
27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)/2)^2 - 1176*
a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)^2 + 952*root(
54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*
z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 2352*a*b^10*cos(e/
2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 5
4*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*sin(e/2 + (f*x
)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^5*b^2*z^3 - 27*a^3*
b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z
, k)^2*b^13*cos(e/2 + (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
- 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b
^14*cos(e/2 + (f*x)/2)^2 + 1032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a
^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a*b^11*si
```

n(e/2 + (f*x)/2)^2 + 39032*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*cos(e/2 + (f*x)/2)^2 + 30296*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*cos(e/2 + (f*x)/2)^2 + 7420*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*cos(e/2 + (f*x)/2)^2 - 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*cos(e/2 + (f*x)/2)^2 - 168*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*cos(e/2 + (f*x)/2)^2 + 14240*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*cos(e/2 + (f*x)/2)^2 + 4064*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*cos(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*cos(e/2 + (f*x)/2)^2 - 27888*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^2*b^10*sin(e/2 + (f*x)/2)^2 - 55576*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^3*b^9*sin(e/2 + (f*x)/2)^2 - 32174*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^4*b^8*sin(e/2 + (f*x)/2)^2 - 3318*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^5*b^7*sin(e/2 + (f*x)/2)^2 + 252*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*a^6*b^6*sin(e/2 + (f*x)/2)^2 + 24840*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a*b^12*sin(e/2 + (f*x)/2)^2 + 7856*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*a*b^13*sin(e/2 + (f*x)/2)^2 + 384*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^4*a*b^14*sin(e/2 + (f*x)/2)^2 + 107772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^2*b^11*cos(e/2 + (f*x)/2)^2 + 156216*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^3*b^10*cos(e/2 + (f*x)/2)^2 + 55448*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^4*b^9*cos(e/2 + (f*x)/2)^2 + 21772*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^5*b^8*cos(e/2 + (f*x)/2)^2 + 35364*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*a^6*b^7*cos(e/2 + (f*x)/2)^2 + 3588*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z -...

3.462 $\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left((a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m, x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$

Mathematica [A]
time = 3.44, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p (d\tan(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
[Out] Integral((d*tan(e + f*x))**m*(a + b*(c*sec(e + f*x))**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (d\tan(e+f\,x))^m \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.463 $\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=226

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}-\frac{{}_2F_1\left(\frac{2}{n},-p;\frac{2+n}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e}{}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1
+p)/a/f/n/(1+p)-hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*
x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)+1/4*hypergeo
m([-p, 4/n],[(4+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*x+e)^4*(a+b*(c*sec(f*x+e
))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A]
time = 0.37, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4224, 6874, 374, 12, 272, 67, 372, 371}

$$\frac{\sec^4(e+fx)(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{n},-p;\frac{n+4}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{4f}-\frac{\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{f}-\frac{(a+b(c\sec(e+fx))^n)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p))) - (Hypergeometric2F1[2/n, -p,
 (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f
*x])^n)^p)/(f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p) + (Hypergeometric2F1[4/n, -
p, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b(cx)^n)^p}{x} - 2x(a+b(cx)^n)^p + x^3(a+b(cx)^n)^p\right) d}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3(a+b(c\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x^3(a+bx^n)^p}{c^3}\right.}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3(a+bx^n\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left((a+b(c\sec(e+\right.}{}$$

$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))}{afn(1+p)}$$

Mathematica [A]
time = 10.40, size = 245, normalized size = 1.08

$$\frac{(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^{-p}\left(-4an(1+p)\,{}_2F_1\left(\frac{2}{n},-p;\frac{2+n}{n};-\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\sec^2(e+fx)+an(1+p)\,{}_2F_1\left(\frac{4}{n},-p;\frac{4+n}{n};-\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\sec^4(e+fx)-4\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)\left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^p\right)}{4afn(1+p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*(-4*a*n*(1 + p)*Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2 + a*n*(1 + p)*
Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*
Sec[e + f*x]^4 - 4*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sqrt[Sec[e
+ f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n)*(1 + (b*(c*Sqrt[Sec[e +
 f*x]^2])^n)/a)^p))/(4*a*f*n*(1 + p)*(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)
^p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p\left(\tan^5(fx+e)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^5\,\left(a+b\,\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.464 $\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=143

$$\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}+\frac{{}_2F_1\left(\frac{2}{n},-p;\frac{2+n}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+ \ldots}{}$$

```
[Out] hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1+
p)/a/f/n/(1+p)+1/2*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec
(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

Rubi [A]
time = 0.22, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4224, 6874, 374, 12, 272, 67, 372, 371}

$$\frac{\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f}+\frac{(a+b(c\sec(e+fx))^n)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*(c
*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*x]
)^n)^p)/(2*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b(cx)^n)^p}{x} + x(a+b(cx)^n)^p\right)\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}(\int x(a+b(c$$
$$= -\frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\right.}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}(\int x(a+bx^n)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left((a+b(c\sec(e+\right.}{}$$
$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1}}{afn(1+p)}$$

Mathematica [A]
time = 4.81, size = 162, normalized size = 1.13

$$\frac{(a+b(c\sec(e+fx))^n)^p\left(\frac{2\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)}{an(1+p)}+{}_2F_1\left(\frac{2}{n},-p;\frac{2+n}{n};-\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\sec^2(e+fx)\left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^{-p}\right)}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c*Sqrt[Sec[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n))/(a*n*(1
 + p)) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2
])^n)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2*f)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p \left(\tan^3(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c \sec (e + fx))^n)^p \tan^3 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e + f\,x)^3\,\left(a + b\,\left(\frac{c}{\cos\left(e + f\,x\right)}\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.465 $\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx$

Optimal. Leaf size=59

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(1
+p)/a/f/n/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4224, 374, 12, 272, 67}

$$-\frac{(a+b(c\sec(e+fx))^n)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
```

```
b, c, d, m, n, p}, x]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int (a+b(c\sec(e+fx))^n)^p\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)}{afn(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p\tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c \sec (e + fx))^n)^p \tan (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+fx) \left(a+b\left(\frac{c}{\cos\left(e+fx\right)}\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.466 $\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 4.17, size = 0, normalized size = 0.00

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(a+b\left(c\sec\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c\sec{(e + fx)})^{n})^{p} \cot{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(e + f\,x)\,\left(a + b\,\left(\frac{c}{\cos{(e + f\,x)}}\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.467 $\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

Mathematica [A]
time = 38.36, size = 0, normalized size = 0.00

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cot^3\left(fx+e\right)\right)\left(a+b(c\sec\left(fx+e\right))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
[Out] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(e+f\,x)^3\,\left(a+b\,\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.468 $\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b(c\sec(e+fx))^n)^p\tan^2(e+fx),x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi steps

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx$$

Mathematica [A]
time = 2.72, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(e+fx))^n)^p\tan^2(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p\left(\tan^2(fx+e)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c\sec{(e + fx)})^n)^p \tan^2{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(e + f\,x)^2 \left(a + b\left(\frac{c}{\cos{(e + f\,x)}}\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.469 $\int (a+b(c\sec(e+fx))^n)^p \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((a+b(c\sec(e+fx))^n)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\sec(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec(e+fx))^n)^p \, dx$$

Mathematica [A]
time = 1.39, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+b(c\sec(fx+e))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c \sec (e + fx))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a + b\left(\frac{c}{\cos (e + f\,x)}\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/cos(e + f*x))^n)^p,x)
[Out] int((a + b*(c/cos(e + f*x))^n)^p, x)
```

3.470 $\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Defer[Int][Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 2.07, size = 0, normalized size = 0.00

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
[Out] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)\left(a+b(c\sec(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c \sec{(e + fx)})^{n})^{p} \cot^{2}{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{cot}(e+f\,x)^2\,\left(a+b\,\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```